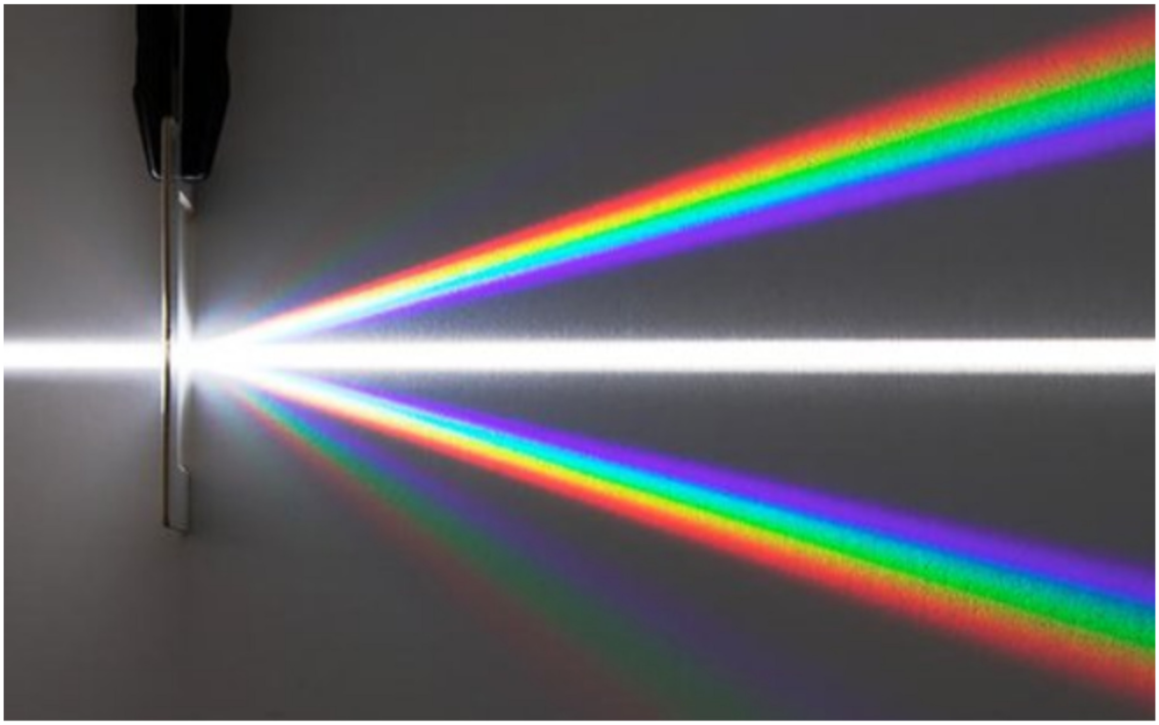
7 SUPERPOSITION OF WAVES IN 2D OR 3D

Here is the spectrum obtained on a screen when light passes through a grating with 100 slits/mm. In the first-order spectrum, a colour is missing, 4.12 cm from the central maximum, which means that this colour is absent from the light emitted by the source. What is the wavelength of the missing light knowing that the distance between the grating and the screen is 70 cm?



fineartamerica.com/featured/light-dispersed-by-diffraction-grating-giphotostock.html?product=greeting-card

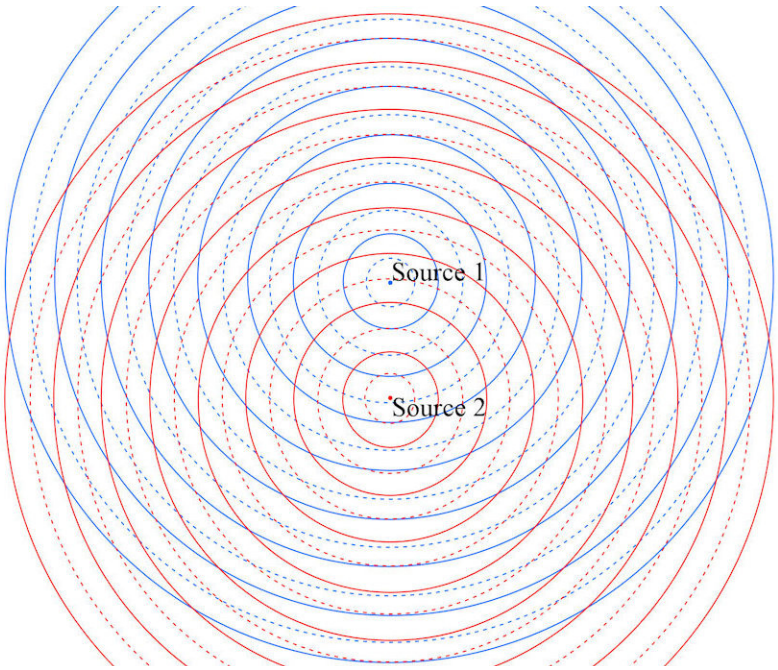
Discover how to solve this problem in this chapter.

7.1 SUPERPOSITION OF TWO WAVES

Now, the superimposition of two waves spreading in 2 or 3 dimensions will be considered.

The Result of the Superposition Varies According to the Position

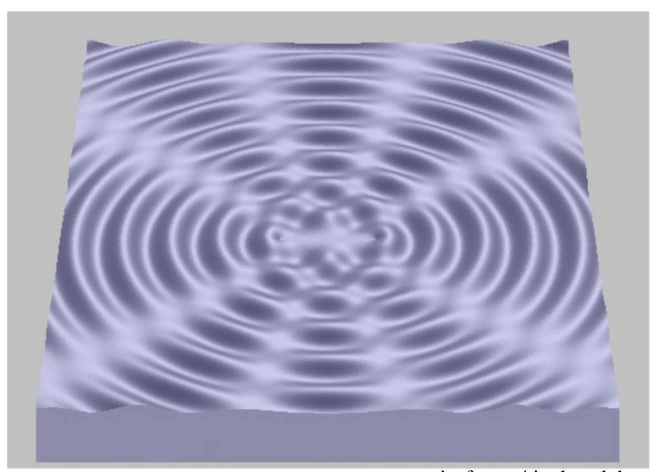
This is the waves emitted by two sources. Here, the solid lines represent the maximum of the wave (crests) and the dotted lines represent the minima of the wave (trough).



The result of this superposition looks like this (diagram on the right).

There are places where the resulting wave has a large amplitude. In these places, there is constructive interference.

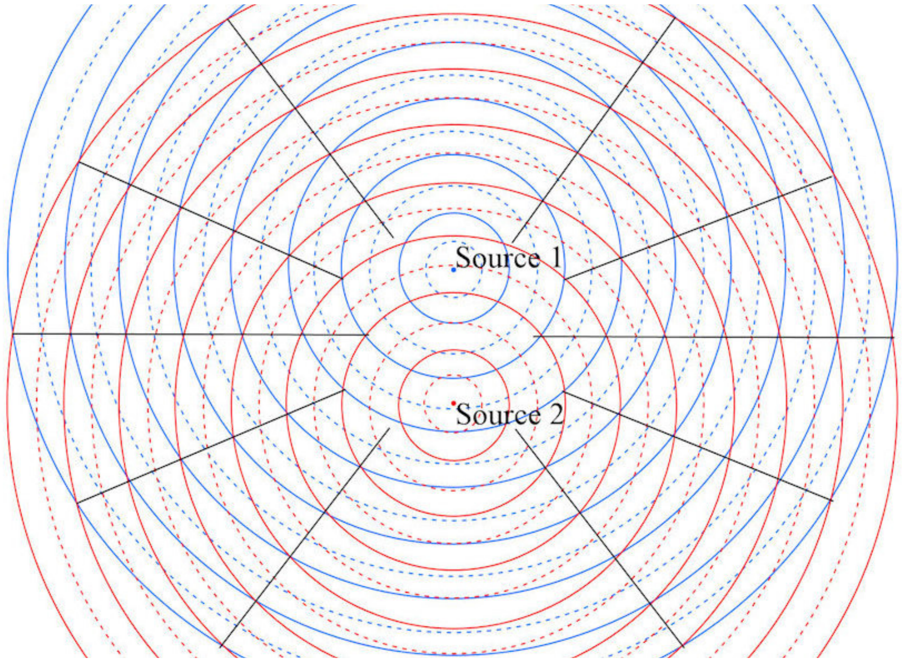
There are other places where the resulting wave has an amplitude of almost zero. In these places, there is destructive interference.



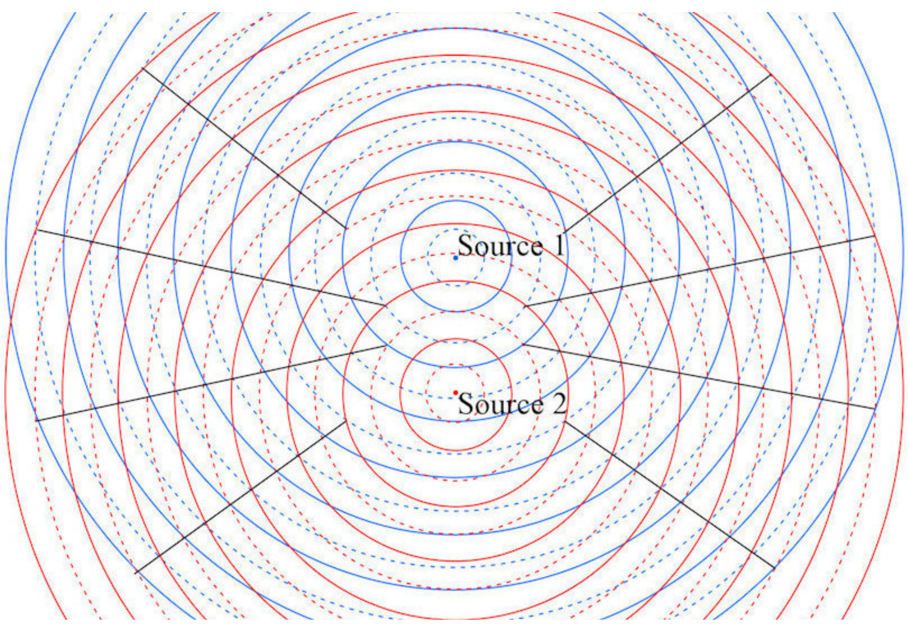
www.ottisoft.com/rippletank.htm

This can also be seen in this clip: http://www.youtube.com/watch?v=5PmnaPvAvQY

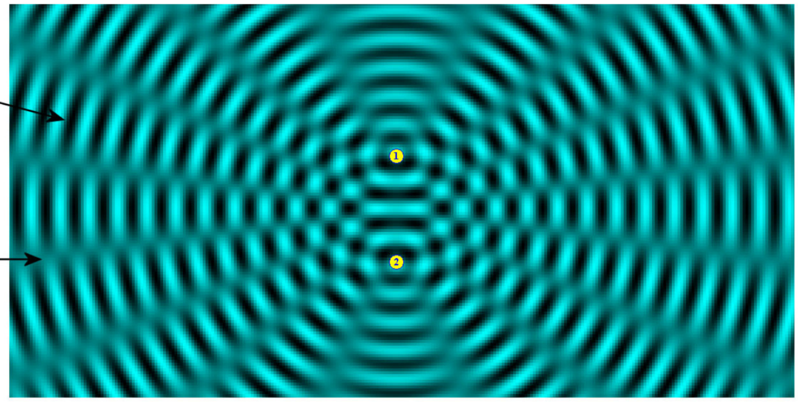
At the places where there is constructive interference, the maximum of the two waves (circles with full lines) are superimposed to give a wave with a large amplitude. This is what is happening along the lines shown in the diagram. Along these lines, the circles representing the maximum of the waves intersect.



At the locations where there is destructive interference, the maxima of one the waves (circles with full lines) are superimposed with the minima of the other wave (dotted circles) so that the resulting amplitude vanishes. Along these lines, there is destructive interference.



Thus, the result of the addition of 2 waves varies from one place to the other. In some places, there is constructive interference and in some other places, there is destructive interference.



Here is an animation of the superposition of waves with the wavefronts:
http://www.youtube.com/watch?v=PCYv0_qPk-4

Phase difference for Two Sources Next to Each Other

The total phase difference between the two waves is still

$$\Delta\phi = \Delta\phi_T + \Delta\phi_S + \Delta\phi_R$$

For two sources next to each other, there is no reflection. Using the formula for phase shift due to the time difference, the phase difference is

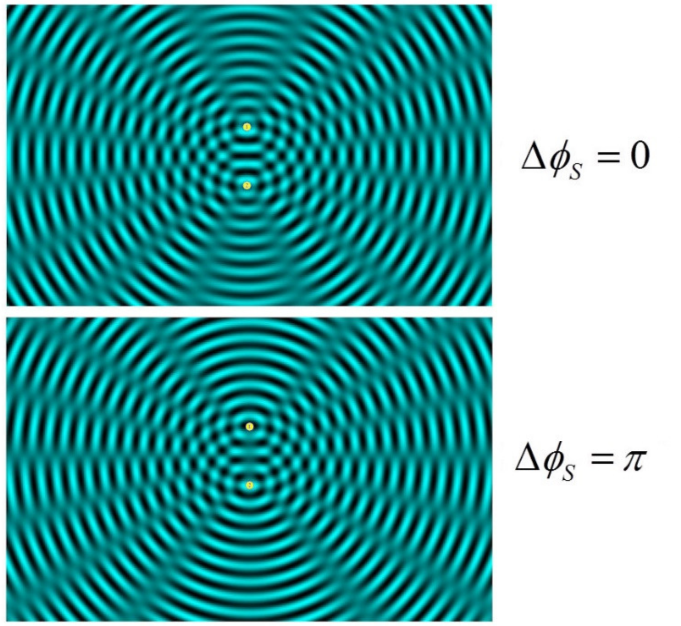
$$\Delta\phi = -\frac{\Delta t}{T}2\pi + \Delta\phi_S$$

<u>If the speed of the waves is the same</u>, this phase difference is

$$\Delta\phi = -\frac{\Delta r}{\lambda}2\pi + \Delta\phi_S$$

This formula means that if the two sources are not the same distance from the observer, there will be a phase difference between the received waves, and this difference will influence the amplitude of the resulting wave. As soon as the observer changes position, the path-length difference change and the amplitude of the resultant wave can change. Therefore, the amplitude of the resulting wave can change from one place to another. That's why there is constructive interference in some places and destructive interference in some other places.

Note that if $\Delta\phi_S$ is changed, the resulting interference would change everywhere. For example, phase difference between the source is changed from 0 to t of π, the places where there was constructive interference (where the phase shift was an even number of π) would become the places where there is destructive interference (where the phase shift is an odd number of π) since π would have been added to the total phase shift.



The video (a bit old) sums up the effects of changes of $\Delta\phi_T$ and $\Delta\phi_S$.
http://www.youtube.com/watch?v=J_xd9hUZ2AY

(Note that the formula

$$\Delta\phi = -\frac{\Delta r}{\lambda}2\pi + \Delta\phi_S$$

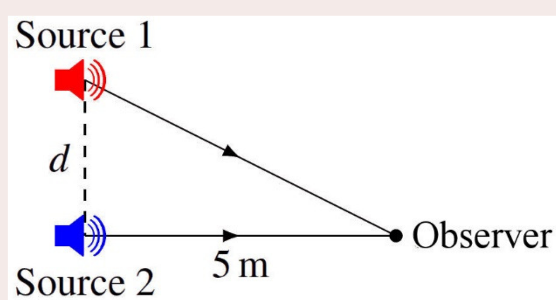
is valid only if the speed of the two waves is the same. The speed might be different here. For example, with sources of sound waves, the speed could be different if the air was warmer on the path of the wave coming from the source 1. In the case of light sources, there could be a transparent substance, a piece of glass for example, in the path of one light wave to slow it down. These situations will not be explored here, but if one day you have to analyze one of these situations, you need to calculate the phase shift with

$$\Delta\phi = -\frac{\Delta t}{T}2\pi + \Delta\phi_S$$

and calculate the time of arrival of each wave.)

Example 7.1.1

Two sound sources out of phase by 1/8 of a cycle (source 1 is emitting ahead of source 2) emit waves with a 1360 Hz frequency. What should the minimum value of d be so that there is constructive



interference at the place where the observer is? (The speed of sound is 340 m/s.)

To have constructive interference, the total phase difference must be

$$\Delta\phi = 2m\pi$$

This phase difference can come from three things: the difference in arrival time, the phase difference of the sources and phase difference due to reflections. Obviously, the phase difference due to reflections is zero since there is not any reflection here.

$$\Delta\phi = \Delta\phi_T + \Delta\phi_S + \cancel{\Delta\phi_R}$$

The phase difference $\Delta\phi_T$ is

$$\begin{aligned}
\Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\
&= -\frac{r_2 - r_1}{\lambda}2\pi \\
&= -\frac{5m - hypotenuse}{\lambda}\cdot 2\pi \\
&= \frac{hypotenuse - 5m}{\lambda}\cdot 2\pi \\
&= \frac{\sqrt{(5m)^2 + d^2} - 5m}{\lambda}\cdot 2\pi
\end{aligned}$$

The wavelength of the sound is

$$\begin{aligned}
\lambda &= \frac{v}{f} \\
&= \frac{340\frac{m}{s}}{1360Hz} \\
&= 0.25m
\end{aligned}$$

The phase difference $\Delta\phi_T$ is thus

$$\Delta\phi_T = \frac{\sqrt{(5m)^2 + d^2} - 5m}{0.25m}\cdot 2\pi$$

For $\Delta\phi_S$, it will be assumed that source 1 has a vanishing phase constant. Since source 2 lags behind source 1, the phase constant of source 2 must be negative. As source 2 lags by 1/8 of a cycle, the constant phase of source 2 is

$$\phi_{source\ 2} = -\frac{\pi}{4}$$

The phase difference $\Delta\phi_S$ is thus

$$\begin{aligned}\Delta\phi_S &= \phi_{source\ 2} - \phi_{source\ 1}\\ &= -\frac{\pi}{4} rad - 0 rad\\ &= -\frac{\pi}{4} rad\end{aligned}$$

Therefore, the total phase difference is

$$\begin{aligned}\Delta\phi &= \Delta\phi_T + \Delta\phi_S\\ &= \frac{\sqrt{(5m)^2 + d^2} - 5m}{0,25m} \cdot 2\pi + -\frac{\pi}{4}\end{aligned}$$

Using the condition for constructive interference, we obtain

$$2m\pi = \frac{\sqrt{(5\mathrm{m})^2 + d^2} - 5\mathrm{m}}{0.25\mathrm{m}} \cdot 2\pi + -\frac{\pi}{4}$$

$$m = \frac{\sqrt{(5\mathrm{m})^2 + d^2} - 5\mathrm{m}}{0.25\mathrm{m}} - \frac{1}{8}$$

(Be careful not to confuse the symbol for the metre *m* and the integer *m*. In these last three lines, the integer *m* is italicized, but not the metre symbol.)

This equation indicated that the integer *m* must be zero or positive. As the first term is necessarily positive (the hypotenuse is surely longer than the other side of the triangle), it is impossible to have m = -1 by subtracting 1/8 to a positive number. If this equation is solved for *d*, the result is

$$d = \sqrt{\left(\left(m + \tfrac{1}{8}\right) \cdot (0.25\mathrm{m}) + 5\mathrm{m}\right)^2 - (5\mathrm{m})^2}$$

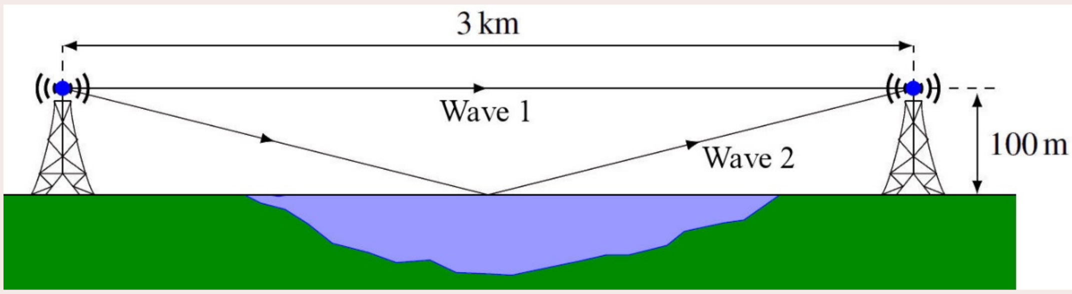
The minimum value of *d* is found by calculating the value of *d* for different values of *m*.

$m = 0$ gives $d = 0.56$ m
$m = 1$ gives $d = 1.70$ m
Greater values of *m* give values of *d* greater than 1.70 m.

Therefore, the minimum value is $d = 0.56$ m.

Example 7.1.2

A 100 m high antenna sends radio waves whose wavelength is 1 m to a receiving antenna 3 km away. The height of the receiving tower is also 100 m. However, there are waves reflected off a lake located halfway between the towers. The receiving antenna thus picks up the waves coming directly from the antenna and the reflected waves, and these two waves interfere. What is the amplitude of the wave received compared that what would be received if there was no reflection if it is assumed that the reflected wave amplitude is equal to half of the amplitude of the wave coming directly from the source?



The amplitude is found with this formula.

$$A_{tot} = \sqrt{A_1^2 + 2A_1A_2\cos(\Delta\phi) + A_2^2}$$

The phase difference between the two waves at the receiving antenna must be found first. The total phase difference is

$$\Delta\phi = \Delta\phi_T + \Delta\phi_S + \Delta\phi_R$$

The phase difference of the sources is zero here because the two waves are emitted by the same source. The phase difference due to the difference in time of arrival is

$$\begin{aligned}
\Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\
&= -\frac{r_2 - r_1}{\lambda}2\pi \\
&= -\frac{2\sqrt{(1500m)^2 + (100m)^2} - 3000m}{1m}\cdot 2\pi \\
&= -\frac{6.659m}{1m}\cdot 2\pi \\
&= -41.84rad
\end{aligned}$$

For the phase difference due to reflections, we notice that wave 2 is reflected on a medium whose index of refraction is greater than in the initial medium of propagation. The wave is, therefore, inverted and undergoes a phase change of π. Thus $\phi_{R2} = \pi$. The other wave (the wave that goes directly from the source to the receiver) is not reflected and does not undergo any change of phase. Thus $\phi_{R1} = 0$. Therefore, the phase shift is

$$\begin{aligned}\Delta\phi_R &= \phi_{R2} - \phi_{R1}\\ &= \pi - 0\\ &= \pi\end{aligned}$$

The total phase difference is thus

$$\begin{aligned}\Delta\phi &= \Delta\phi_T + \Delta\phi_S + \Delta\phi_R\\ &= -41.84rad + 0 + \pi rad\\ &= -38.70rad\end{aligned}$$

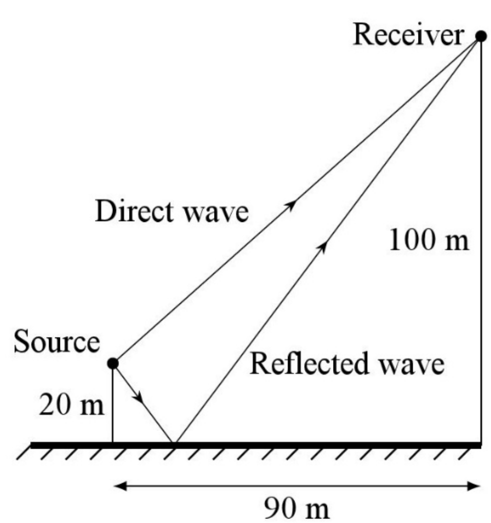
Since $A_2 = A_1/2$, the amplitude is

$$\begin{aligned}A_{tot} &= \sqrt{A_1^2 + 2A_1A_2\cos(\Delta\phi) + A_2^2}\\ &= \sqrt{A_1^2 + 2A_1\tfrac{A_1}{2}\cos(-38.70rad) + \tfrac{A_1^2}{4}}\\ &= \sqrt{A_1^2 + A_1^2\cos(-38.70rad) + \tfrac{A_1^2}{4}}\\ &= \sqrt{A_1^2\left(1 + \cos(-38.70rad) + \tfrac{1}{4}\right)}\\ &= A_1\sqrt{\left(1 + \cos(-38.70rad) + \tfrac{1}{4}\right)}\\ &= A_1 \cdot 1.338\end{aligned}$$

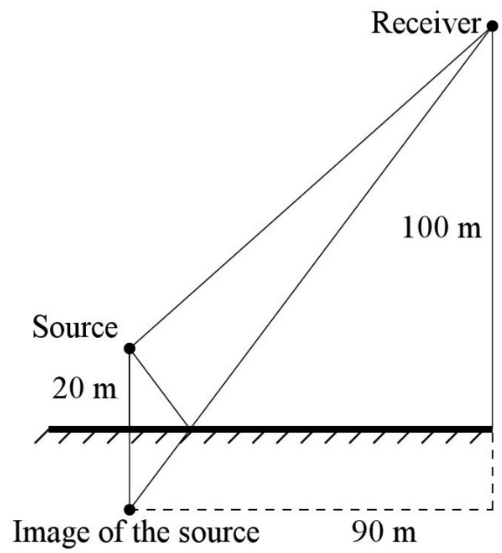
If there were no reflection, the amplitude of the wave coming directly from the source would be A_1. With the reflection, the amplitude is 1.338 times greater.

Here, it was pretty easy to know the length of the path of wave 2 because the two towers had the same height. Then, the reflection happens exactly halfway between the two towers. It would have been more difficult to find the point of reflection if the towers had not the same height.

The length of the path of the reflected wave can, however, be easily found in more complex situations. Consider the case of a source 20 metres away from a wall and a receiver 100 metres away from a wall, as shown in the following diagram.



To calculate the length of the reflected path, we must realize that the length of this path is the same as for a wave going directly from the image of the source to the receiver. With the image, the length of the path can easily be calculated with a single hypotenuse.



Here, the length of this path is

$$r = \sqrt{(90m)^2 + (120m)^2} = 150m$$

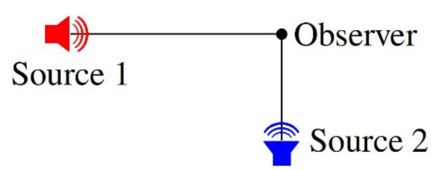
Direction of the Oscillation

Note that there is another subtlety with interference in 2 or 3 dimensions. For the oscillations to add up or to cancel out according to the formulas given, the oscillations made by each wave must be in the same direction.

Longitudinal Waves

With a longitudinal wave (such as sound), the oscillation of the medium is done in the same direction as the direction of propagation of the wave.

To illustrate what this implies, imagine the two sound wave sources in the following situation.



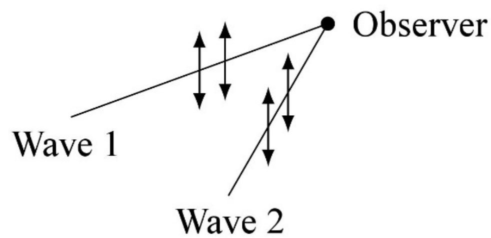
The following image shows the direction of the oscillation of the air molecules made by the sound wave emitted by each speaker.

Clearly, the oscillation motions are not in the same direction for the two waves. When the oscillations are not in the same direction, they must be added as vectors and the result is different from what was obtained previously. For example, for the situation shown in the diagram, a vanishing amplitude cannot be obtained even if the two waves have the same amplitude and were out of phase by π. Just to illustrate the complexity of this situation, the molecules would do a circular motion if the two waves had the same amplitude and were out of phase by π/2!

However, the results obtained for the conditions for constructive (greater amplitude) and destructive (smaller amplitude) interference remain valid as long as the angle between the two sources as measured from the position where the observer is stays small (say roughly 45°). As for the formula of the amplitude of the wave, it gives the correct value only if both sources are in the same direction.

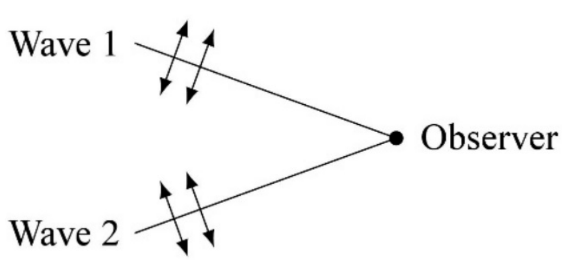
Transverse Waves

On the other hand, this problem does not exist with transverse waves. Regardless of their positions, two transverse-wave sources (electromagnetic waves for example) can generate oscillations in the same direction. This situation will be explored further in the chapter on polarization. For now, let's assume that electromagnetic waves sources generate oscillations in the same direction and that all the formulas used here give the correct values for these waves.



In this case, the formulas for the conditions of constructive and destructive interference and the formulas for the resulting amplitude of the wave remain valid. So, for the result of example 7.2.2 (the one where the radio waves are reflected on the lake) to be valid, it would have been necessary to specify that the waves were polarized horizontally to have two oscillations in the same direction when they arrive at the observer.

If the waves do not have this polarization, the oscillations will not be in the same direction when they arrive at the observer.

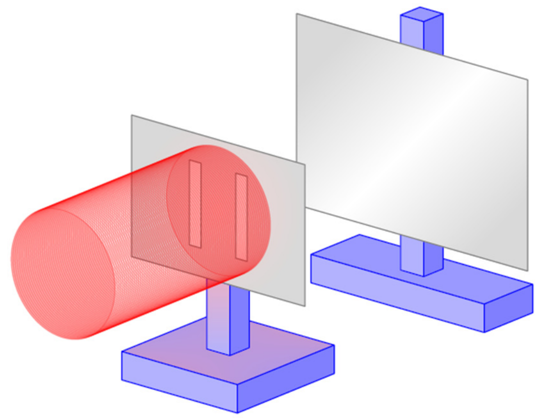


In this case, the same thing happens as with longitudinal waves: If the angle between the two sources is not too great, the results for the conditions of constructive and destructive interference remain valid and the formula of the amplitude of the resulting wave becomes an approximation.

7.2 YOUNG'S EXPERIMENT

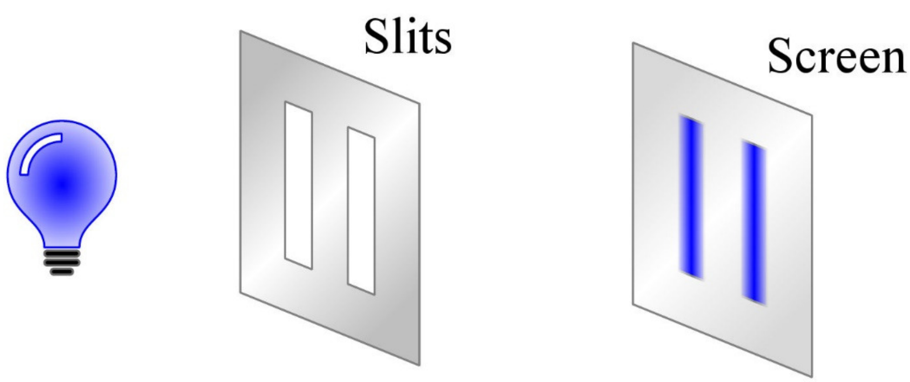
What Is Young's Experiment?

In 1803, Thomas Young sent light through two thin slits and observed the result on a screen. (The width of the slits and the distance between the slits are much smaller than what is shown in the diagram. Typically, the distance between the slits is around 0.1 mm.)

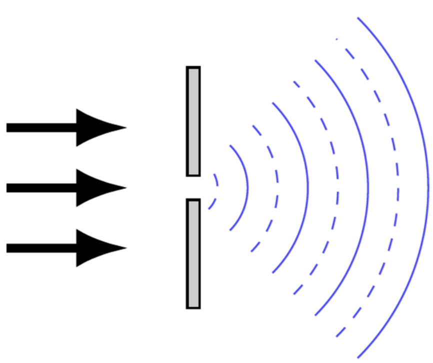


This experiment was crucial because it was the first experiment that allowed to determine the nature of the light. Remember that at this time, it was not known if light was a wave or a particle.

According to corpuscular theory, two bright lines should be observed on the screen. These lines are created by light particles passing through each slit.



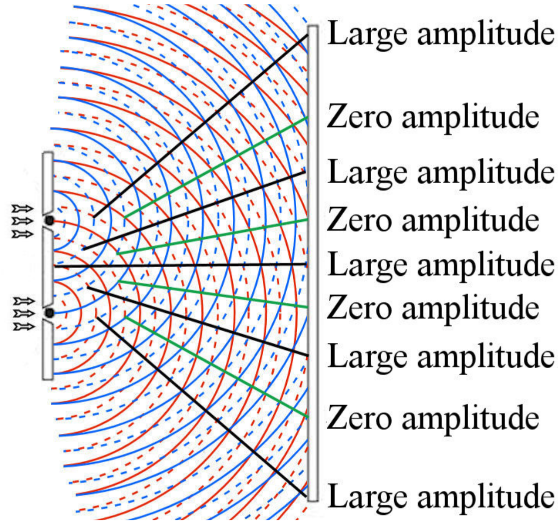
According to wave theory, the situation is more complex.



When the light from a source passes through a thin slit, the light wave spreads. (In the next chapter, this spreading of the wave when it passes through a small opening will be considered with more details.)

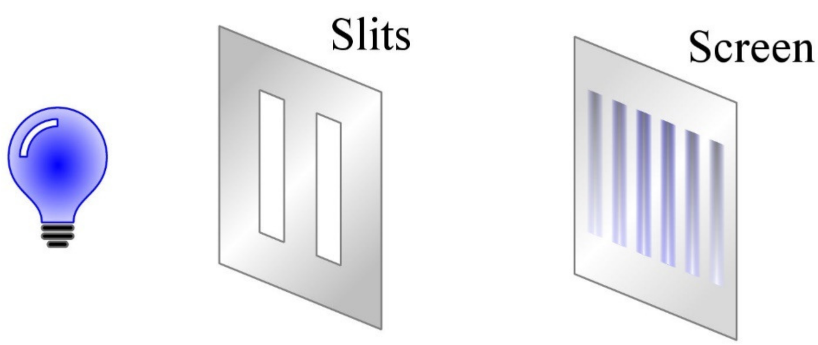
If the light passes through two thin slits, then the waves from each of the slits are superimposed. The situation shown on the diagram on the right is then obtained.

The resultant wave reaching the screen (which is to the right in the diagram) has, sometimes, a very large amplitude. This occurs when the maxima (full-line circles) intersect, resulting in constructive interference. Sometimes the resulting wave has a zero amplitude. This occurs when the maximum of a wave (full-line circles) intersects the minimum of the other wave (dashed line circles), resulting in destructive interference.



When the amplitude is great, the intensity of the light is great. Areas where the intensity is great are called *Bright fringes*. When the amplitude is zero, the light intensity is zero. Areas where the intensity is zero are called *Dark fringes*.

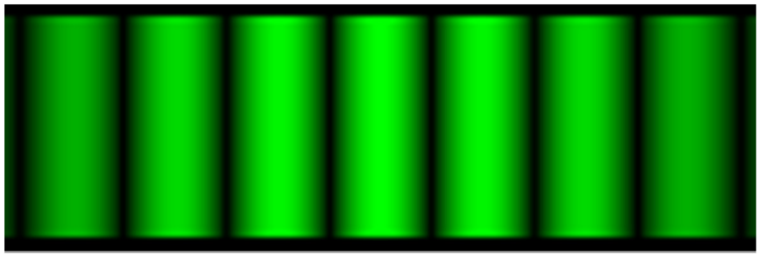
Therefore, if the wave theory is true, alternating bright and dark fringes should be seen on the screen.



This same explanation is given in this video.
http://www.youtube.com/watch?v=dNx70orCPnA
This video shows the difference between wave and corpuscular theories.
http://www.youtube.com/watch?v=JMrvp9vzPys

Therefore, only two bright areas should be seen on the screen according to corpuscular theory while multiple bright and dark areas should be seen according to wave theory. Which one is correct?

Here is what is seen when this experiment is performed with green light.



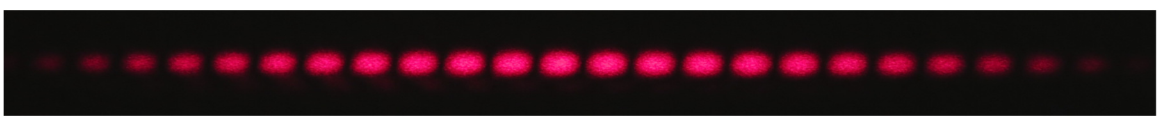
www.itp.uni-hannover.de/~zawischa/ITP/multibeam.html

Alternating dark and bright lines are seen, just as predicted by the wave theory. The bright lines are called *bright fringes*, and the dark areas are called *dark fringes*. In 1803, this experiment was the first solid proof that light is a wave.

(However, some supporters of the corpuscular theory saved the corpuscular theory with some complications. Giving an elongated shape and a rotation on themselves to the particles of light, they were able to provide an explanation of the appearance of bright and dark fringes in Young's experiment.)

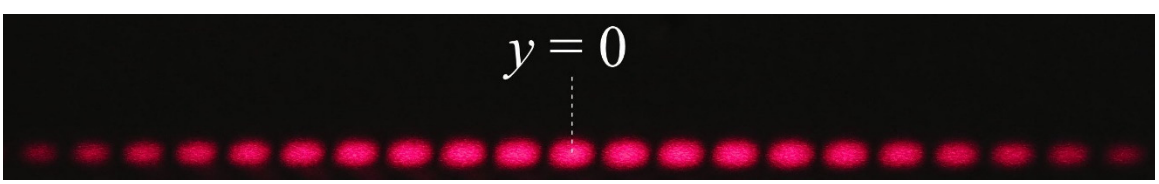
Calculation of the Position of the Bright and Dark Fringes

Often, Young's experiment is performed with a laser. Because the laser beam is highly concentrated, the laser does not illuminate the entire height of the slits. Thus, the interference figure obtained on a screen is not as vertically extended and looks more like this.

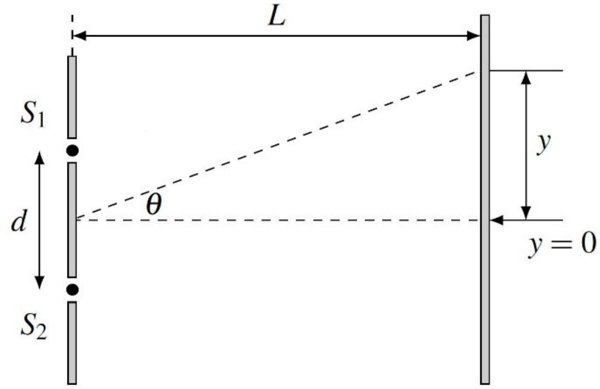


Now, the position of places where there is a lot of light (constructive interference) and places where there is little light (destructive interference) will be calculated.

The position of a point on the screen is denoted by y (even if this position is not necessarily vertical). The $y = 0$ is exactly opposite the center of the slits. Thus, the $y = 0$ is in the center of the interference pattern on the screen.



Link Between the Position and the Angle



The position can also be given by θ, the angle shown in the diagram.

According to the diagram, the link between y and θ is

Position From the Angle in Young's Experiment

$$\tan\theta = \frac{y}{L}$$

The Phase Difference $\Delta\phi$

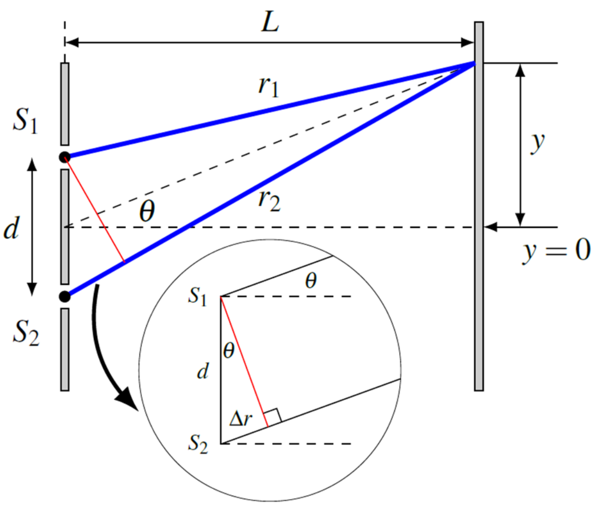
To find the result of wave interference on the screen, the phase difference between the waves when they arrive at a place on the screen must be known. The phase difference is

$$\Delta\phi = \Delta\phi_T + \Delta\phi_S + \Delta\phi_R$$

Here, $\Delta\phi_S$ is zero because the same wave produces the two waves coming from each slit. $\Delta\phi_R$ is also zero since there is not any reflection in this experiment. The phase difference between waves is, therefore,

$$\begin{aligned}\Delta\phi &= \Delta\phi_T \\ &= -\frac{\Delta r}{\lambda}2\pi\end{aligned}$$

To find the path-length difference, an approximation is made: it will be assumed that the distance of the screen (L) is much greater than the distance between the slits (d).



Then, it can be seen that, from the red line, the two waves have the same distance to travel to get to a point on the screen. The path-length difference is simply the small length of the small line indicated in the diagram by Δr. Moreover, the lines r_1 and r_2 are virtually parallel if the screen is far away, and the red line is almost perpendicular to r_1 and r_2. Using the right triangle (triangle with sides d and Δr, and the angle θ) in the diagram, the path-length difference is

Path-Length Difference in Young's Experiment

$$\Delta r = d \sin\theta$$

The phase difference is thus

$$\begin{aligned}\Delta\phi &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{d\sin\theta}{\lambda}2\pi\end{aligned}$$

Since the sign of the phase difference can always be changed, the negative sign will be dropped to arrive at

Phase Difference Between the Two Waves in Young's Experiment

$$\Delta\phi = \frac{d\sin\theta}{\lambda}2\pi$$

<u>*Maxima on the Screen*</u>

To have constructive interference, this phase difference must be equal to 2 $m\pi$. Therefore,

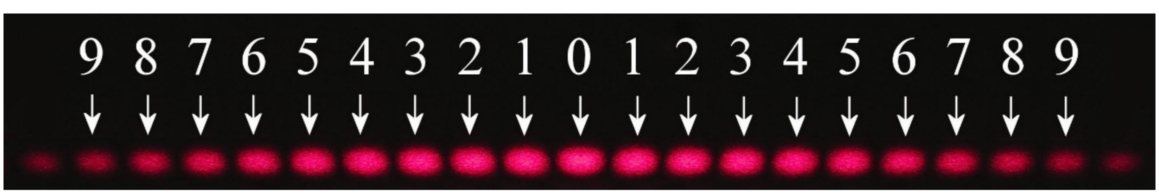
$$2m\pi = -\frac{d\sin\theta}{\lambda}2\pi$$

This can be written as

Angles of the Bright Fringes in Young's Experiment

$$d\sin\theta = m\lambda$$

where m is an integer called the *order of the maximum*. Here, the m are positive integers. Here's what these m values are on the interference pattern.

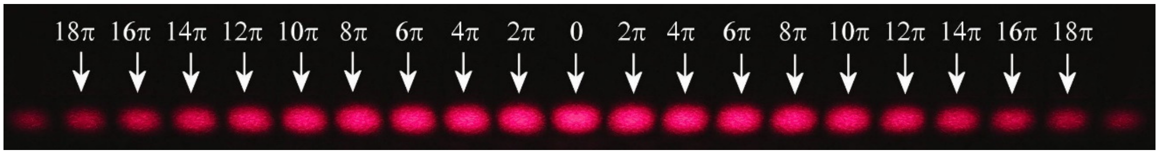


(Here, we stop at 9, but there can be many more...)

Since $d\sin\theta = m\lambda$, the phase difference between the two waves at these maxima is

$$\begin{aligned}\Delta\phi &= \frac{d\sin\theta}{\lambda}2\pi \\ &= \frac{m\lambda}{\lambda}2\pi \\ &= m2\pi\end{aligned}$$

The following diagram shows the phase difference between the waves for each maximum.

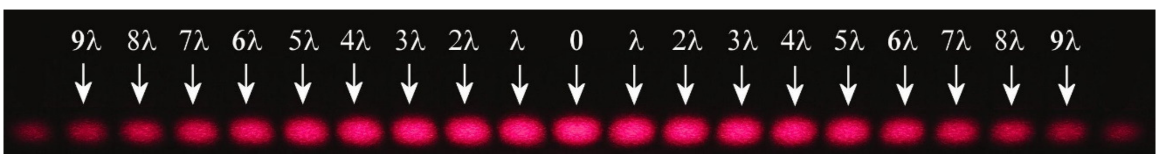


These results are obviously consistent with the fact that the maxima correspond to constructive interference and therefore to a phase difference equal to an even number of π.

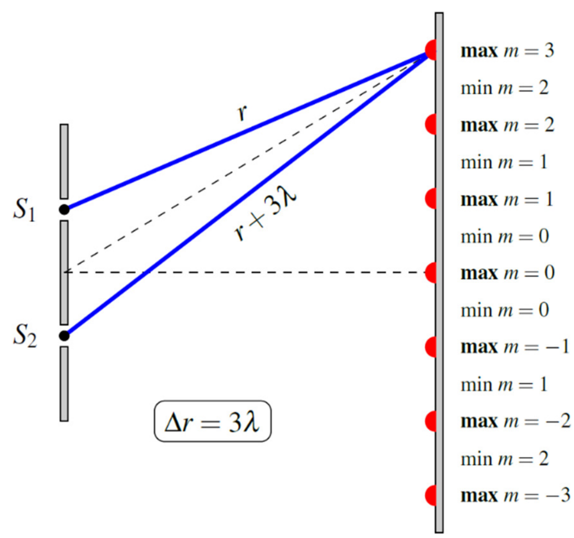
Since $\Delta r = d\sin\theta$ and $d\sin\theta = m\lambda$, the difference in walking between the two waves at these maxima is then

$$\Delta r = m\lambda$$

The following diagram shows the path-length difference between the waves for each maximum.



Let's clarify this concept a little by taking, for example, the order-3 maximum. For this maximum, the path-length difference is 3λ. This means that the length of the path travelled by one of the waves to get to the screen is 3λ longer compared to the length of the path travelled by the other wave.



Minima on the Screen

To have destructive interference, the phase difference must be equal to $(2m+1)\pi$. Therefore,

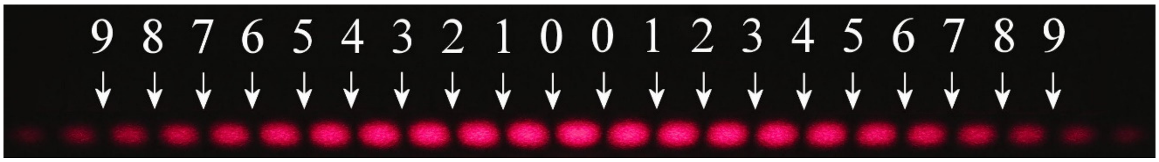
$$(2m+1)\pi = \frac{d\sin\theta}{\lambda}2\pi$$

Thus, the angles of the minima are given by

Angles of the Dark Fringes in Young's Experiment

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$

where m is the *order of the minimum* and are still positive integers. Here's what these m values are on the interference pattern.



(Again here, we stop at 9, but there can be many more ...)

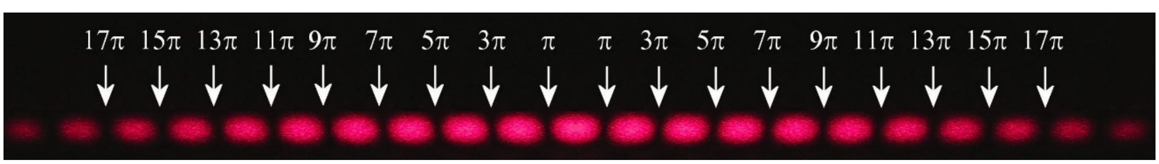
Common Mistake: using m = 1 for the first minimum

Any integer value of m is good in this formula. The value that gives the smallest angle is m = 0. You must, therefore, use m = 0 to find the position of the first minimum.

Since $d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$, the phase difference between the two waves at these minima is

$$\begin{aligned}\Delta\phi &= \frac{d\sin\theta}{\lambda}2\pi \\ &= \frac{\left(m+\tfrac{1}{2}\right)\lambda}{\lambda}2\pi \\ &= \left(2m+1\right)\pi\end{aligned}$$

The following diagram shows the phase difference between the waves for each minimum.

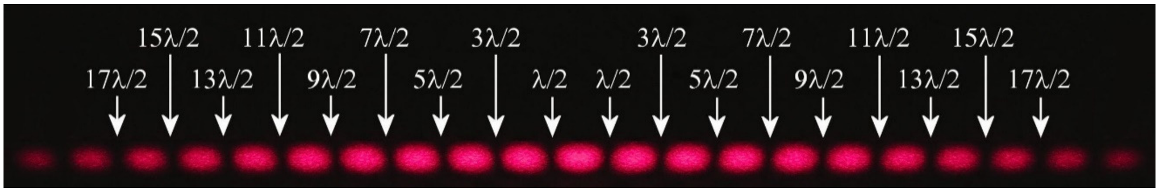


These results are obviously consistent with the fact that the minima correspond to destructive interference and therefore to a phase difference equal to an odd number of π.

Since $\Delta r = d\sin\theta$ and $d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$, the difference in walking between the two waves at these minima is then

$$\Delta r = \left(m + \tfrac{1}{2}\right)\lambda$$

The following diagram shows the path-length difference between the waves for each minimum.



Approximation for the Calculation of the Position of the Maxima and Minima If θ Is Small

If the angle is small (this is not always the case), then $\sin\theta \approx \theta$ and $\tan\theta \approx \theta$, and the position of the maximum can be found with the following approximations.

$$d\theta = m\lambda \qquad \text{and} \qquad \theta = \frac{y}{L}$$

Combining these equations, we obtain

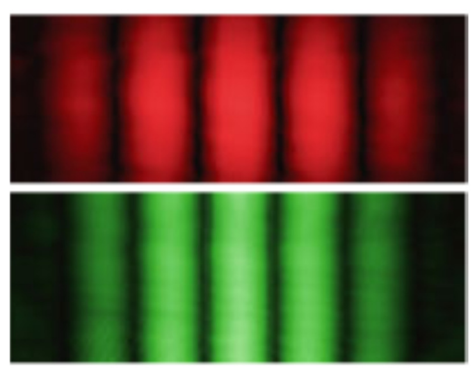
$$y = \frac{m\lambda L}{d} \quad \text{(Position of the bright fringes if } \theta \text{ is small.)}$$

The same can be done for the dark fringes to get

$$y = \frac{\left(m+\frac{1}{2}\right)\lambda L}{d} \quad \text{(Position of the dark fringes if } \theta \text{ is small.)}$$

These approximations can often be used with Young's experiment, but one should be careful because the distance between the slits may be small enough for the angle to be large. The problem is that if you take the approximation, the angle was never calculated and thus you do not know if it is small...

Changes to the Interference Pattern with the Wavelength



www.sciencephoto.com/media/157198/view

The formula to find the position of the maxima

$$d\sin\theta = m\lambda$$

indicates that the position of the maxima depends on the wavelength. With a smaller wavelength, the angles are smaller, and the maxima are closer to the central maximum. On the images to the left, the experiment was done with red light and green light, keeping the same distance between the slits and the same distance between the slits and the screen. The angles of the maxima for green light (which has a smaller wavelength) are clearly smaller. The maxima are, therefore, closer to the central maximum than those for red light.

If the experiment is done with white light, the angles of the maxima are different for each colour, except for the central maximum which is at the same position ($\theta = 0$) for every colour. The first-order maximum of violet light will be the one closest to the central maximum since it has the smallest visible wavelength, and the first-order maximum of red light will be the farthest from the central maximum since it has the largest wavelength in the visible spectrum. The following interference pattern is thus obtained.

www.itp.uni-hannover.de/~zawischa/ITP/multibeam.html

Notice how the first-order maximum (that are on each side of the central maximum) goes from violet to red moving away from the central maximum. It is a bit more complicated for the other maxima because there are several overlaps. For example, the third-order maximum of violet light is closer to the central maximum than the second-order maximum of red light.

Example 7.2.1

Light having a 632 nm wavelength passes through 2 slits 0.2 mm apart. We then look at the interference pattern on a screen 3 m away from the slits.

a) What is the distance between the central maximum to the first maximum?

The angle of the first maximum is

$$d \sin\theta = m\lambda$$
$$0.2\times10^{-3} m \sin\theta = 1\cdot 632\times10^{-9} m$$
$$\theta = 0.181°$$

Thus, the position of this maximum on the screen is

$$\tan\theta = \frac{y}{L}$$
$$\tan 0.181° = \frac{y}{3m}$$
$$y = 9.492mm$$

As the central maximum is necessarily at y = 0, the distance between the central maximum and the first-order maximum is 9.492 mm.

b) What is the distance between the central maximum to the first minimum?

The angle of the first minimum is

$$d\sin\theta = \left(m+\tfrac{1}{2}\right)\lambda$$
$$0.2\times10^{-3}m\cdot\sin\theta = \left(0+\tfrac{1}{2}\right)\cdot 632\times10^{-9}m$$
$$\theta = 0.0905°$$

Thus, the position of this minimum on the screen is

$$\tan\theta = \frac{y}{L}$$
$$\tan 0.0905° = \frac{y}{3m}$$
$$y = 4.74mm$$

As the central maximum is necessarily at y = 0, the distance between the central maximum and the first minimum is 4.74 mm.

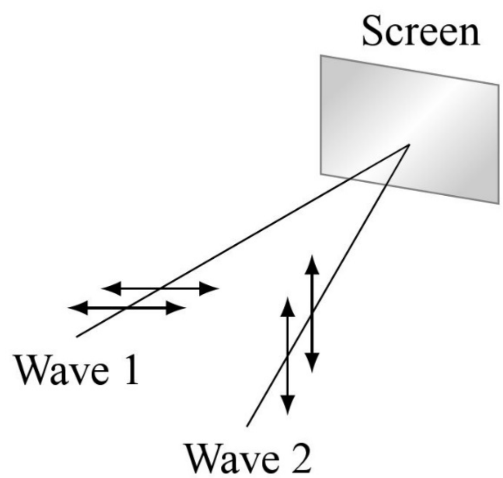
Necessary Conditions for Light Interference to Be Observed

Why is it necessary to use slits to see an interference pattern? Why doesn't it work when two bulbs are used to light up a wall to see the interference pattern? Actually, two elements are missing to observe the pattern with two bulbs.

First, the two sources must be very close to each other to see the interference pattern. The position of the first maximum is found with $d\sin\theta = \lambda$. If d increases θ decreases. This means that the first maximum will be very close to the central maximum if the distance between the sources becomes too great. With two bulbs, the distance is too great, and the bright fringes will be too close to each other and the sequence of bright and dark fringes will not be seen. As it is almost impossible to have light bulbs close enough, the interference pattern can never be seen. That's why the slits are always very close to each other in Young's experiment.

Second, the bulbs do not emit a nice continuous wave. They emit several waves lasting 10^{-8} seconds each. However, the phase constant of these waves is never the same and this means that the phase difference between the sources ($\Delta\phi_s$) changes every 10^{-8} seconds. This change of phase difference constantly shifts the position of the maxima and the minima. With the maxima changing positions every 10^{-8} second, it is impossible to see the pattern. When light coming from a single source passes through two slits, we ensure that the phase difference between the two sources is always zero because both waves actually come from the same source. Actually, it's almost impossible to obtain an interference pattern with light coming from two different light sources. In virtually all cases, the interference is obtained by combining two beams of light coming from the same source.

Finally, the interfering waves must have the same polarization or not be polarized. Two waves with perpendicular polarization do not interfere at all. With different directions of oscillation, the two waves cannot cancel each other at the positions where there should be destructive interference. In this case, the interference pattern disappears completely. (This phenomenon was discovered by François Arago and Augustin Fresnel in 1819.) However, the two waves have, very often, the same polarization and there is interference. If the waves are not polarized, then the 2 polarization components are present at the same time (say, horizontal and vertical). In this case, the 2 vertical polarizations interfere with each other, and the 2 horizontal polarizations interfere with each other, and an interference pattern can be seen.



Light Intensity on the Screen

When two interfering waves with the same amplitude (as in Young's experiment), the resulting amplitude is given by

$$A_{tot} = \left| 2A\cos\left(\frac{\Delta\phi}{2}\right) \right|$$

Here, the wave is an electromagnetic wave. This means that the amplitude is, in fact, the amplitude of the electric field.

$$E_{0tot} = \left| 2E_0\cos\left(\frac{\Delta\phi}{2}\right) \right|$$

As light intensity is given by

$$I_{tot} = \frac{cn\varepsilon_0 E_{0tot}^2}{2}$$

the intensity of the interfering light is

$$I_{tot} = \frac{cn\varepsilon_0}{2}\left(2E_0\cos\left(\frac{\Delta\phi}{2}\right)\right)^2$$

This intensity will be compared to the intensity obtained with a wave coming from a single source (that will be called I_1.) The intensity of this wave, which has a E_0 amplitude, is

$$I_1 = \frac{cn\varepsilon_0 E_0^2}{2}$$

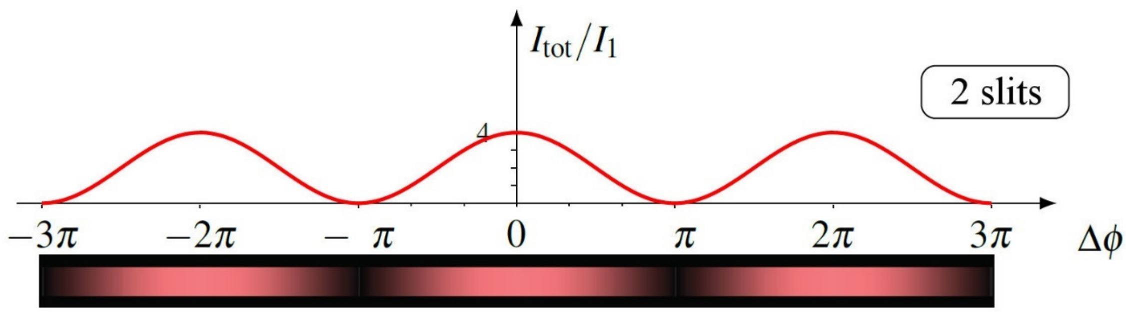
Dividing one by the other, the result is

$$\frac{I_{tot}}{I_1} = \left(2\cos\left(\frac{\Delta\phi}{2}\right)\right)^2$$

Therefore, the intensity is

$$I_{tot} = 4I_1\cos^2\frac{\Delta\phi}{2}$$

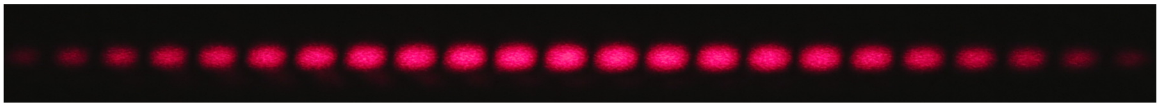
This graph shows the light intensity in Young's experiment.



Underneath the graph, the interference pattern on a screen corresponding to this graph can be seen.

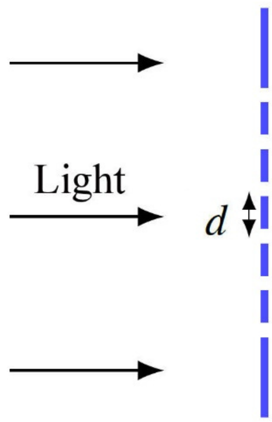
At the maxima, the intensity is four times greater than the intensity coming from a single source (a single slit in this case). The average intensity is $2I_1$, which is quite normal because there are two sources, and each has an intensity I_1.

However, the maxima should all have the same intensity according to this formula. But if you look at a real image of an interference pattern obtained in Young's experiment, it is obvious that this is not true.



Our intensity formula is quite all right for now. This problem will be resolved in the next chapter.

7.3 INTERFERENCE WITH MANY SLITS

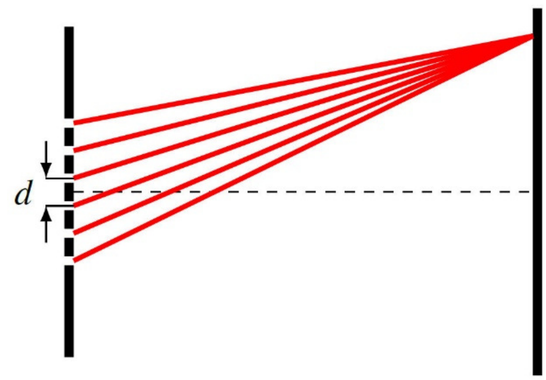


In this section, we will look for the result of superimposing waves from several regularly spaced slits. For example, on the diagram, there are six slits.

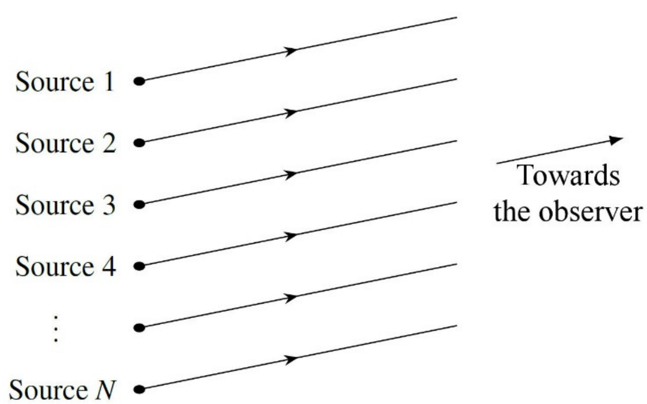
The distance between the slits is *d*.

It's like Young's experience, except that there are several slits rather than just 2 slits.

The result can then be examined on a screen. The intensity of the light received at one point on the screen is the result of the addition of waves coming from each slit.



The Sum of All Received Oscillations



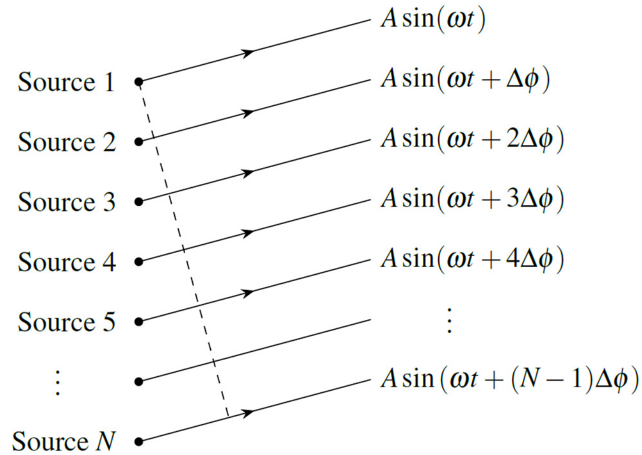
The resulting amplitude of the oscillation at a location on the screen when N waves are superimposed will now be sought.

To find the resulting oscillation, two things must be done: determine the value of $\Delta\phi$ and find the resulting amplitude of the sum of N sine function.

The Phase Shift $\Delta\phi$

All the lines going from a source to the screen are roughly parallel if it is assumed that distance of the screen (L) is much larger than the distance between the sources (d).

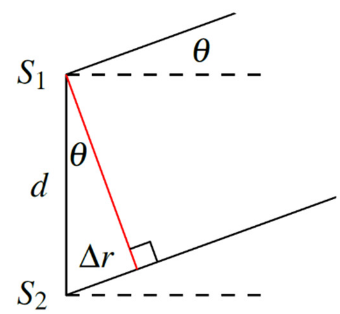
From the dotted line, all the waves have the same distance to travel to reach the point on the screen.



The waves coming from the last sources have a longer distance to travel to reach the observer. (The amplitude would be somewhat smaller for these, but that difference will be neglected here.)

This greater distance to travel means the phase difference $\Delta\phi_T$ is greater for the last sources.

This longer distance to travel for the waves coming from the last sources means that the phase difference $\Delta\phi_T$ is larger for these last sources. Thus, the additional time is the time it takes the wave to reach the dotted line. This time increases linearly passing from one source to the next. This means that if the time to reach the dotted line from source 2 is Δt, then it is $2\Delta t$ for source 3, $3\Delta t$ for source 4, $4\Delta t$ for source 5, and so on. This also means that the time phase difference also increases linearly ($\Delta\phi_T$ for source 2, $2\Delta\phi_T$ for source 3, $3\Delta\phi_T$ for source 4, $4\Delta\phi_T$ for source 5, and so on).



Let's consider the first two sources to find $\Delta\phi_T$. The wave coming from source 2 has a slightly longer path to travel to get to the observer than the wave coming from source 1. This additional distance is Δr (shown in the diagram). According to this diagram, this path-length difference is

$$\Delta r = d\sin\theta$$

Thus, the phase difference due to time between sources 1 and 2 is then

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{d\sin\theta}{\lambda}2\pi\end{aligned}$$

The total phase difference $\Delta\phi$ is simply $\Delta\phi_T$ because all the sources are in phase and there is no reflection. If the minus sign is removed (the sign of $\Delta\phi$ can always be changed), the result is

Phase Difference Between Two Adjacent Slits

$$\Delta\phi = \frac{d\sin\theta}{\lambda}2\pi$$

The Resulting Amplitude

Now, it is time to add all the oscillations created by the waves coming from each source. This sum is

$$y_{tot} = A\sin(\omega t) + A\sin(\omega t + \Delta\phi) + A\sin(\omega t + 2\Delta\phi) + A\sin(\omega t + 3\Delta\phi) + \ldots + A\sin(\omega t + (N-1)\Delta\phi)$$

where A is the amplitude of the wave emitted by each source.

This sum of sine functions can be performed, but it involves slightly higher-level mathematics. It can be done with complex numbers, and some of you will see this at the University. As it would take dozens of pages just to explain properly the mathematics of this addition, and as the only interesting thing here is the physics, we will go directly to the end result. So, by a mathematical miracle, the sum is

$$y_{tot} = \underbrace{A\frac{\sin\left(\frac{N\Delta\phi}{2}\right)}{\sin\left(\frac{\Delta\phi}{2}\right)}}_{Amplitude}\underbrace{\sin\left(\omega t + \frac{(N-1)\Delta\phi}{2}\right)}_{Oscillation}$$

If you really want to see this proof, here it is
http://physique.merici.ca/waves/proofaddsine.pdf

Thus, the amplitude is

$$A_{tot} = A\frac{\sin\left(\frac{N\Delta\phi}{2}\right)}{\sin\left(\frac{\Delta\phi}{2}\right)}$$

Light Intensity

Now, the resulting intensity if the wave is a light wave will be found. In this case, the amplitude is the amplitude of the electric field. As this amplitude is denoted E_0 instead of A, the amplitude of the resulting wave is

$$E_{0tot} = E_0\frac{\sin\left(\frac{N\Delta\phi}{2}\right)}{\sin\left(\frac{\Delta\phi}{2}\right)}$$

Thus, the intensity of the light wave is

$$I_{tot} = \frac{cn\varepsilon_0 E_{0tot}^2}{2}$$

$$= \frac{cn\varepsilon_0}{2}\left(E_0 \frac{\sin\left(\frac{N\Delta\phi}{2}\right)}{\sin\left(\frac{\Delta\phi}{2}\right)} \right)^2$$

This intensity will be compared to the intensity that would have been obtained if there were only one source (intensity that will be called I_1). With only one source, the intensity would be

$$I_1 = \frac{cn\varepsilon_0}{2}(E_0)^2$$

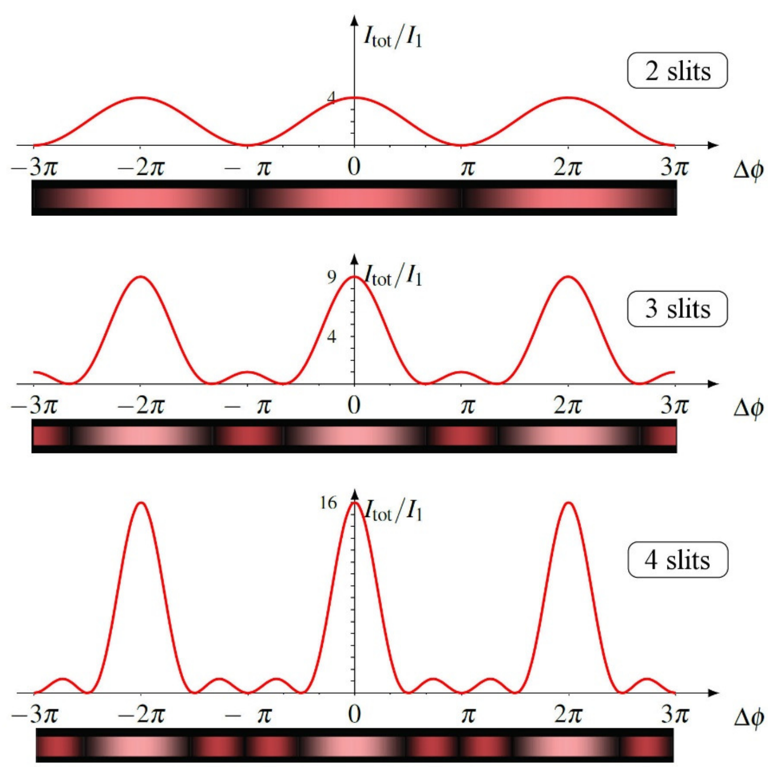
By dividing these two intensities, the result is

$$\frac{I_{tot}}{I_1} = \left(\frac{\sin\left(\frac{N\Delta\phi}{2}\right)}{\sin\left(\frac{\Delta\phi}{2}\right)} \right)^2$$

Thus, the intensity is given by

$$I_N = I_1 \frac{\sin^2\left(\frac{N\Delta\phi}{2}\right)}{\sin^2\left(\frac{\Delta\phi}{2}\right)}$$

Here is the graph of this intensity as a function of $\Delta\phi$ for $N = 2$, 3 and 4. Directly below the graph, an image shows what would be seen on a screen.



We notice that there are large maxima and small maxima.

There are great maxima when the phase shift is an even number of π. They become more and more intense as more slits are added ($4I_1$ for 2 slits, $9I_1$ for 3 slits, and $16I_1$ for 4 slits). These intense maxima are also getting thinner as more slits are added.

Small maxima also appear between these large maxima. The number of small maxima is always equal to the number of slits minus 2. Thus, there is 1 small maximum with 3 slits, 2 with 4 slits, and so on. Note that these small maxima are becoming less and less important compared to the large maximum as the number of slits increases.

Bright Maxima

There are bright maxima when the phase difference is an even number of π, thus when

$$\Delta\phi = 2m\pi$$

$$\frac{d\sin\theta}{\lambda}2\pi = 2m\pi$$

(where m is an integer.) If this result is simplified, it becomes

Angle of the Bright Maxima With Many Slits

$$d\sin\theta = m\lambda$$

This is exactly the same result as the maxima with 2 slits.

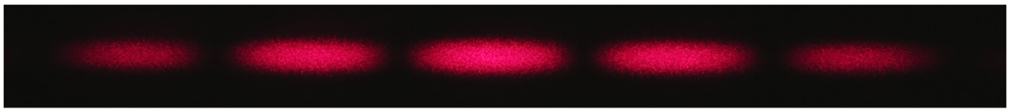
There are such bright maxima when the denominator in the intensity equation becomes zero. Despite the division by 0, the intensity is not infinitely large since the numerator is also 0 when the denominator is 0. In fact, the intensity is

$$\begin{aligned}
I_{tot} &= \lim_{\Delta\phi\to 2m\pi} I_1 \frac{\sin^2\left(\frac{N\Delta\phi}{2}\right)}{\sin^2\left(\frac{\Delta\phi}{2}\right)} \\
&= I_1\left(\lim_{\Delta\phi\to 2m\pi} \frac{\sin\left(\frac{N\Delta\phi}{2}\right)}{\sin\left(\frac{\Delta\phi}{2}\right)}\right)^2 \\
&= I_1\left(\lim_{\Delta\phi\to 2m\pi} \frac{\frac{N}{2}\cos\left(\frac{N\Delta\phi}{2}\right)}{\frac{1}{2}\cos\left(\frac{\Delta\phi}{2}\right)}\right)^2 \\
&= I_1\left(\frac{\frac{N}{2}\cdot 1}{\frac{1}{2}\cdot 1}\right)^2 \\
&= N^2 I_1
\end{aligned}$$

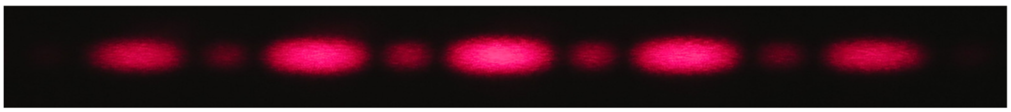
(We used L'Hospital's rule on the 3rd line.) This result is consistent with what can be seen on the graphs ($4I_1$ for 2 slits, $9I_1$ for 3 slits, and $16I_1$ for 4 slits).

Here are real images of interference pattern obtained on a screen with multiple slits. From one figure to the next, slits are added while keeping the same distance between slits.

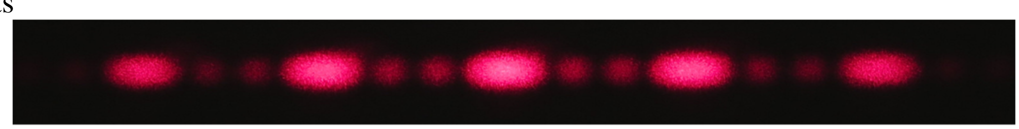
2 slits



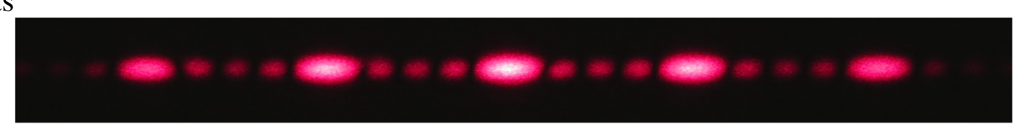
3 slits



4 slits

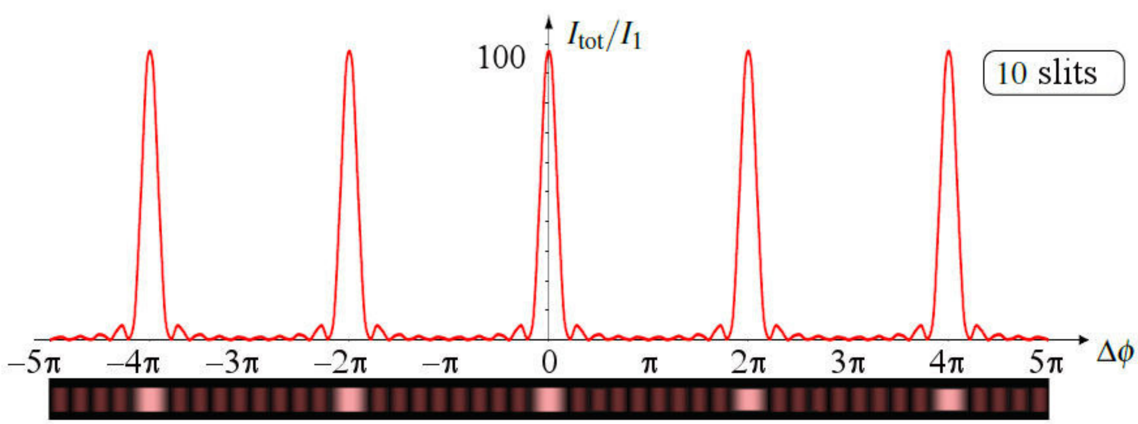


5 slits

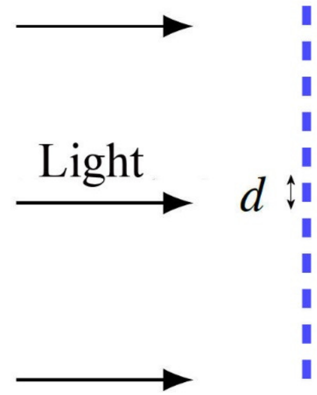


You surely notice that all the bright maxima do not have the same intensity while the formula predicts the same intensity for all these maxima. The intensity formula obtained here is not quite perfect. This issue will be resolved in the next chapter.

If the number of slits continue to increase, the large maxima become thinner and brighter and there are more and more small maxima that are less and less important compared to the big maxima. With 10 slits, the intensity is

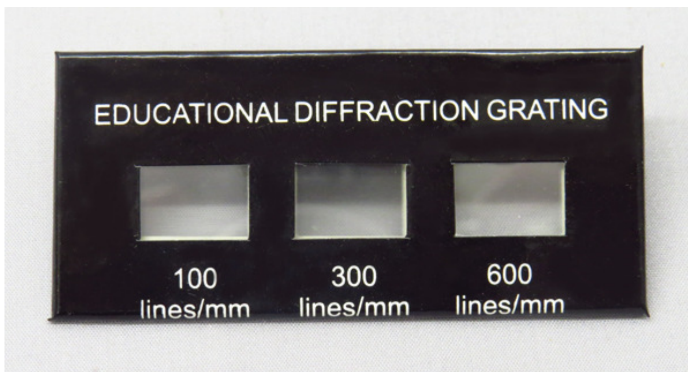


7.4 GRATINGS



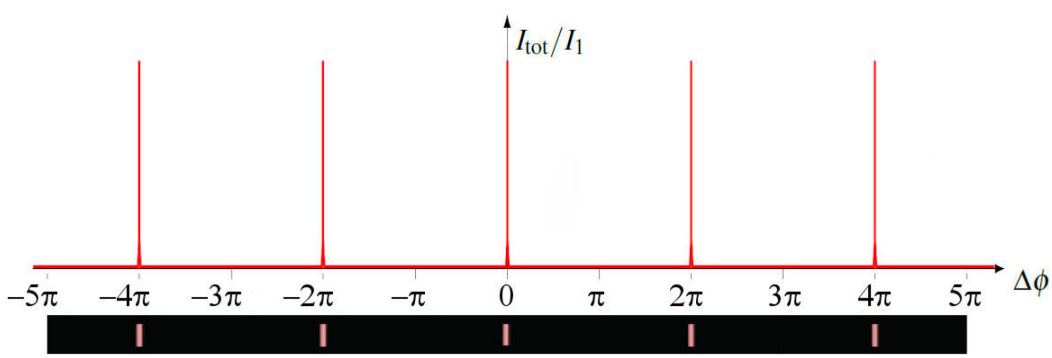
A grating is simply a long series of numerous slits. We are talking about several hundred slits.

The grating often looks like a simple glass plate. (In fact, they manage to do something that is equivalent to slits with variations in the thickness of the glass plate.) The following figure shows gratings with 100 slits per mm, 300 slits per mm and 600 slits per mm. As each grating is about 1 cm wide, there are about 1000 slits on the first grating, 3000 on the second and 6000 on the third.



www.sciencelabsupplies.com/Educational_Diffraction_Grating.html

To find out what is obtained when light passes through a grating, the logic of the previous section must be pushed to the extreme. It has been said that large maxima are becoming thinner and that smaller maxima between large maxima are becoming less and less important as the number of slits increases. Thus, with a large number of slits, there would be only large, very thin maxima. The graph of the intensity of the light on the screen would then looks like this.



The following interference pattern was obtained with a grating comprising nearly 1000 slits (however, the laser was too thin to pass through all these slits, it passed through about a hundred slits.)

There are actually very thin maxima and that the small maxima between these great maxima are gone.

(Perhaps you think the maxima should have been very intense with hundreds of slits since the intensity is given by N^2I_1 but the intensity of each source is very small since the slits are very thin.)

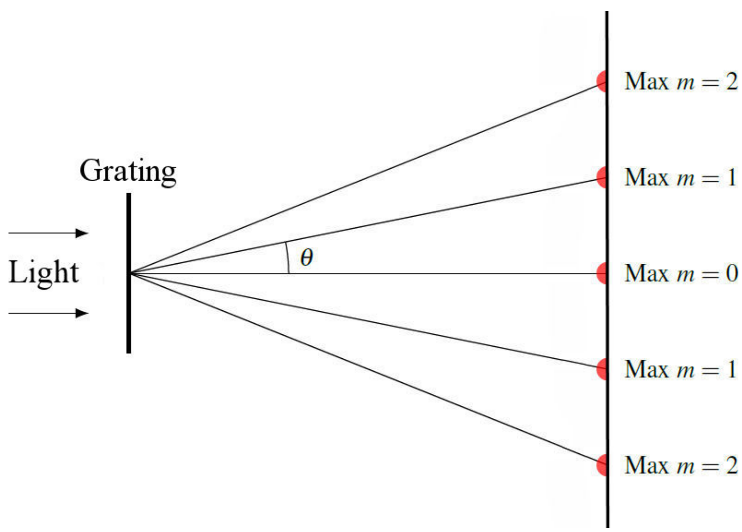
Position of the Maxima on the Screen

The angles of these maxima are still given by

$$d\sin\theta = m\lambda$$

And the link between y and θ is still

$$\tan\theta = \frac{y}{L}$$



(The angle is similarly defined as the angle when there were only 2 slits. In this next diagram, θ is the angle of the first-order maximum.)

The following image shows the m values of each maximum on the screen.

Here is a video showing the maxima obtained when a laser passes through a grating.
http://www.youtube.com/watch?v=1IxfuHI_UN0

Maximum value of *m* on the Screen

The number of maxima is limited. Since the angles of the maxima are given by

$$\frac{m\lambda}{d} = \sin\theta$$

and since the value of a sine function cannot exceed 1, the following condition must be obeyed.

$$\frac{m\lambda}{d} \leq 1$$

Solving for *m*, the maximum value of *m* is obtained.

Maximum Value of *m*

$$m \leq \frac{d}{\lambda}$$

Example 7.4.1

Light having a 500 nm wavelength passes through a grating with 600 slits/mm. What are the angles of every maximum?

The angles of the maxima are given by

$$d\sin\theta = m\lambda$$

The distance between the slits must be found first. If there are 600 mm slits per mm, then the distance between the slits must be 1/600 mm.

Let's now find the maximum value of *m*. This will indicate the number of angles to calculate. The maximum value of *m* is

$$m \leq \frac{d}{\lambda}$$

$$m \leq \frac{\frac{1}{600}\times 10^{-3} m}{500\times 10^{-9} m}$$

$$m \leq 3.33$$

The maximum value of *m* is thus 3.

Obviously, the central maximum is at $\theta = 0°$.

The angle of the first-order maximum is

$$d\sin\theta = \lambda$$
$$\sin\theta = \frac{\lambda}{d}$$
$$\sin\theta = \frac{500\times 10^{-9}m}{\frac{1}{600}\times 10^{-3}m}$$
$$\sin\theta = 0.3$$
$$\theta = 17.45°$$

The angle of the second-order maximum is

$$d\sin\theta = 2\lambda$$
$$\sin\theta = \frac{2\lambda}{d}$$
$$\sin\theta = \frac{2\cdot 500\times 10^{-9}m}{\frac{1}{600}\times 10^{-3}m}$$
$$\sin\theta = 0.6$$
$$\theta = 36.86°$$

The angle of the third-order maximum is

$$d\sin\theta = 3\lambda$$
$$\sin\theta = \frac{3\lambda}{d}$$
$$\sin\theta = \frac{3\cdot 500\times 10^{-9}m}{\frac{1}{600}\times 10^{-3}m}$$
$$\sin\theta = 0.9$$
$$\theta = 64.15°$$

Even though we know that there is no fourth-order maximum, let's do the math to see what happens. The calculation of the angle of the fourth-order maximum gives

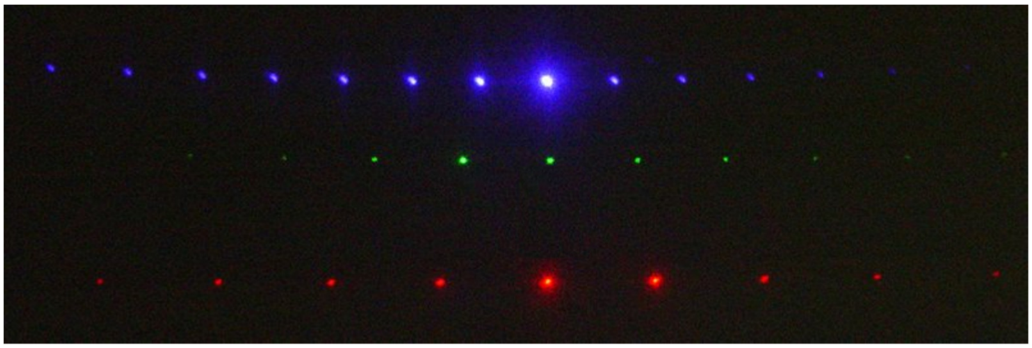
$$d\sin\theta = 4\lambda$$
$$\sin\theta = \frac{4\lambda}{d}$$
$$\sin\theta = \frac{4\cdot 500\times 10^{-9}m}{\frac{1}{600}\times 10^{-3}m}$$
$$\sin\theta = 1.2$$

There is no solution to this equation. Therefore, the is no 4^{th}-order maximum.

Gratings are a great tool to separate the different colours of visible light. Since the angle of the first-order maximum is

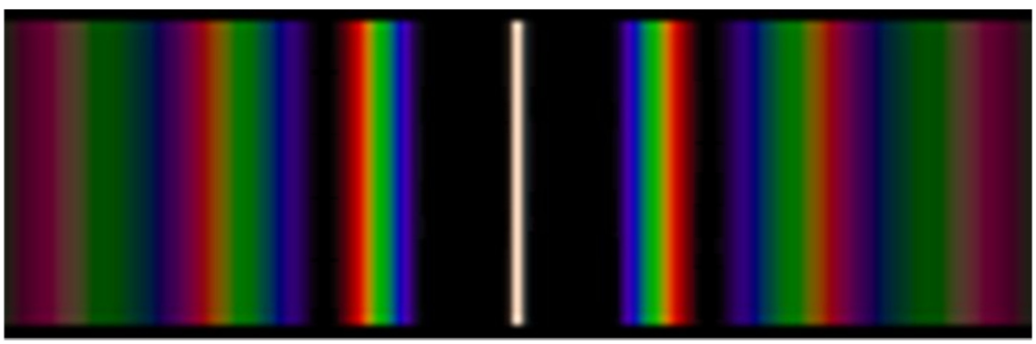
$$d \sin \theta = \lambda$$

the angle of the maximum is smaller if the wavelength is smaller. In the next picture, different patterns obtained with a grating are shown. It is obvious that the angles of the maxima are different if the colour of the light that passes through the grating is changed. It is clear on the image that the maxima are further away from the central maximum if the wavelength is increased.



www.wired.com/wiredscience/2011/10/diffraction-with-infrared-light/

If white light passes through a grating, the following interference pattern is obtained.

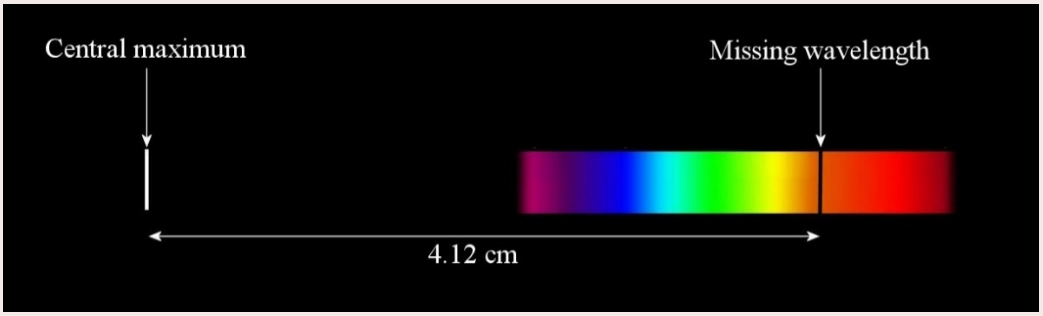


www.itp.uni-hannover.de/~zawischa/ITP/multibeam.html

A separation of the colours of the spectrum is obtained. At the first-order maximum, the violet maximum is closer to the central maximum than the red maximum since the wavelength of violet light is smaller. As each colour has a maximum at a different angle, a spectrum is obtained. The more slits there are, the narrower the maxima are. Thus, the quality of the spectrum increases with the number of slits.

Example 7.4.2

Here is the spectrum obtained on a screen when light passes through a grating with 100 slits/mm. In the first-order spectrum, a colour is missing, 4.12 cm from the central maximum, which means that this colour is absent from the light emitted by the source. What is the wavelength of the missing light knowing that the distance between the grating and the screen is 70 cm?



The wavelength of this light will be found with the following formula.

$$d \sin\theta = m\lambda$$

We already know that m = 1 and that the distance between the slits is 0.01 mm. In order to calculate the wavelength, the only thing missing is the angle.

The angle of the missing maximum is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{4.12cm}{70cm}$$
$$\theta = 3.3684°$$

Thus, the wavelength is

$$d \sin\theta = m\lambda$$
$$0.01 \cdot 10^{-3} m \cdot \sin 3.3684° = 1 \cdot \lambda$$
$$\lambda = 587.55nm$$

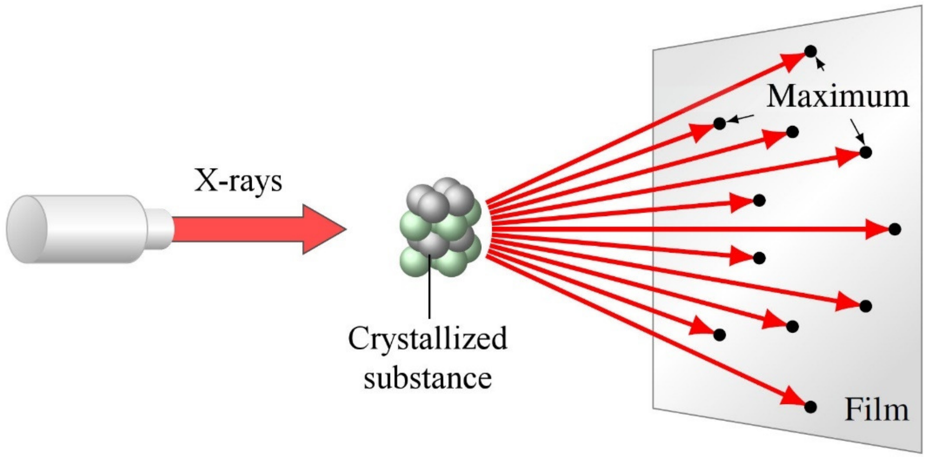
Most of the time, this is the technic used (not a prism) to separate the colour to study the spectrum of a source. With a lot of slits, the result is excellent and it's relatively easy to calculate the wavelength from the angle of the maximum. It is much more difficult to calculate the wavelength if the colours were separated with a prism.

CDs also act as gratings. The disc reflects light except at the places where the information is engraved (the tracks). As the tracks are evenly spaced, the places that reflect light are also evenly spaced. These places, therefore, act exactly like a grating, except that the light is reflected on the disc rather than passing through a grating. However, the result is the same: there are evenly spaced sources emitting light. This is why there is a separation of colours on the surface of a CD. If a place on the CD looks red, it's because there is a maximum for red light made by the sources in this direction.

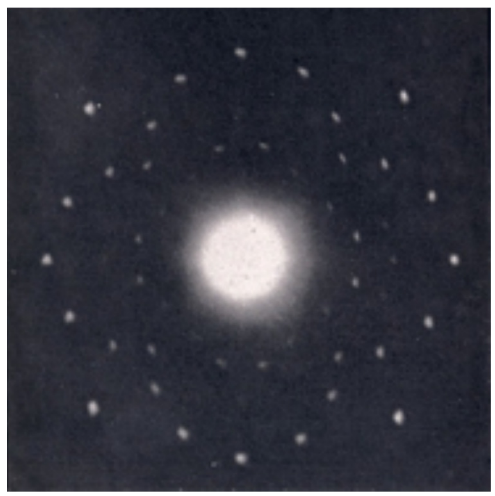
bullmurph.com/tag/compact-disc/

The Study of Crystals with X-rays

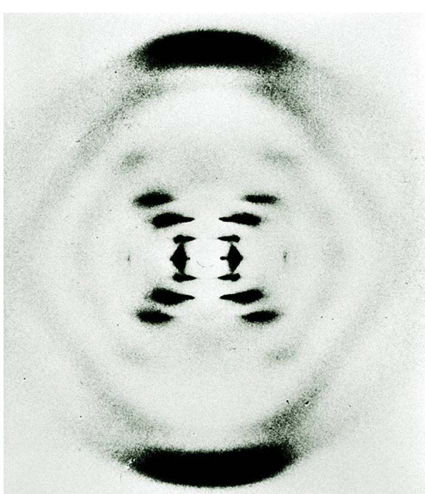
When X-rays pass through a crystal, the atoms, evenly spaced, become X-ray sources themselves. Then, there are several evenly spaced sources, which will form an interference pattern on a screen sensitive to the X-ray.



By observing the interference pattern, scientists can infer the crystal structure, exactly as the distance between the slits in a grating can be calculated from an interference pattern. It is a little more complex here because there are several sources in 3 dimensions, but the idea is basically the same. For example, this picture is obtained if X-rays pass through a salt crystal. From this picture, the structure of salt crystals and the distances between the atoms of the crystal can be calculated.



www.auntminnieeurope.com/index.aspx?sec=ser&sub=def&pag=dis&ItemID=606329

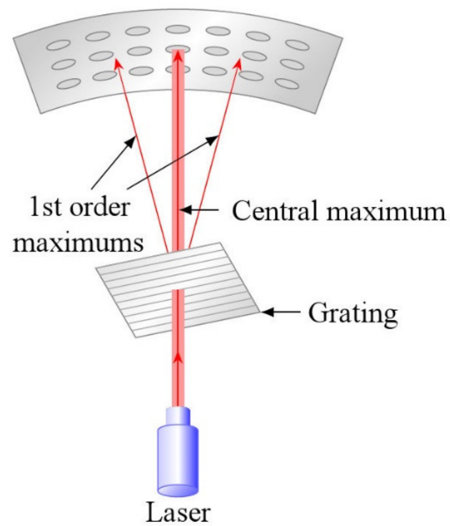


This interference pattern, obtained in 1952 by Rosalind Franklin, allowed Crick and Watson to discover the double helix structure of DNA.

www.bbc.co.uk/schools/gcsebitesize/science/add_edexcel/cells/dnarev3.shtml

Guides for Reading a CD

To guide the laser in CD, DVD and Blu-ray players, two other laser beams are used to follow nearby tracks. Actually, it is not necessary to install new lasers as a grating is used to split the laser beam. The first-order maxima generated by the grating then follow the nearby tracks to guide the central maximum.



SUMMARY OF EQUATIONS

Position From the Angle in Young's Experiment

$$\tan\theta = \frac{y}{L}$$

Path-Length Difference in Young's Experiment

$$\Delta r = d\sin\theta$$

Phase Difference Between the Two Waves in Young's Experiment

$$\Delta\phi = \frac{d\sin\theta}{\lambda}2\pi$$

Angles of the Bright Fringes in Young's Experiment

$$d\sin\theta = m\lambda$$ m is the order of the maximum.

Angles of the Dark Fringes in Young's Experiment

$$d\sin\theta = \left(m+\tfrac{1}{2}\right)\lambda$$ m is the order of the maximum.

Phase Difference Between Two Adjacent Slits

$$\Delta\phi = \frac{d\sin\theta}{\lambda}2\pi$$

Angle of the Bright Maxima With Many Slits (This Includes Grating)

$$d\sin\theta = m\lambda$$

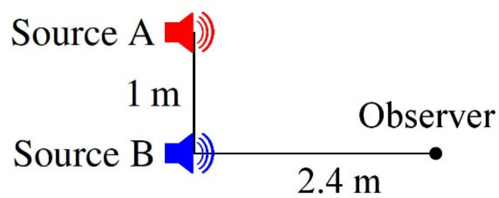
Maximum Value of *m*

$$m \leq \frac{d}{\lambda}$$

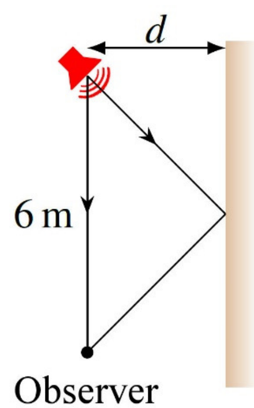
EXERCISES

7.1 Superposition of Two Waves

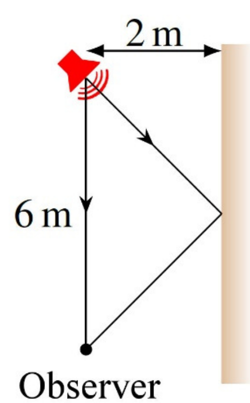
1. Two in-phase loudspeakers emit sound waves as shown in the diagram. What is the minimum frequency that produces destructive interference at the place where the observer is located? (Use 340 m/s for the speed of sound.)



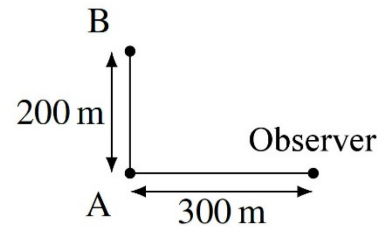
2. The speaker in the diagram emits a 490 Hz sound wave. An observer receives the sound coming directly from the speaker and the sound reflected on a wall that can be moved. What is the smallest distance *d* that will produce constructive interference at the place where the observer stands? (Use 343 m/s for the speed of sound.)



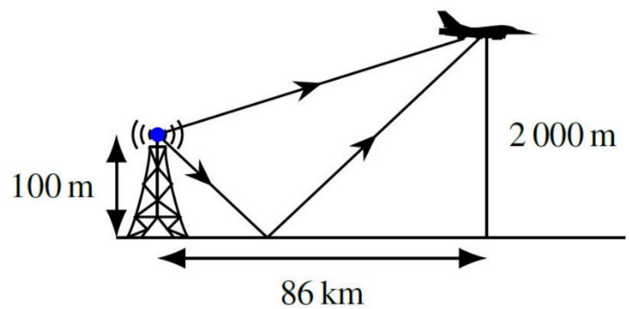
3. The source in the diagram emits an electromagnetic wave with a 1.4 m wavelength. An observer receives the wave coming directly from the source and the wave reflected on a wall. What is the intensity of the wave received by the observer compared to the intensity that would be received if there were no wall if the reflected wave has an amplitude equal to 70% of the amplitude of the wave coming directly from the source? (The reflected wave is inverted, and it is assumed that the polarization of the wave is in the right direction so that the formula for the total amplitude is valid.)



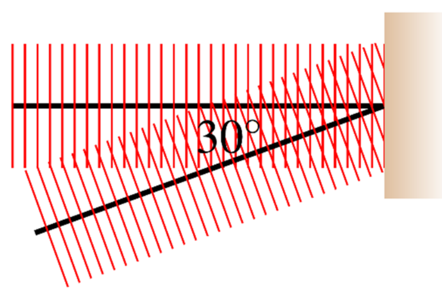
4. Two isotropic radio wave sources having the same power emit (in phase) 100 MHz waves. The sources and the observer positions are shown in the diagram. When source A is the only source emitting, the intensity of the wave received by the observer is 0.001 W/m². What will the intensity of the total wave received by the observer be if source B starts to emit? (Reminder: the intensity of the wave emitted by an isotropic source decreases with the square of the distance.)



5. A transmission tower emits a radio signal with a 120 MHz frequency. A plane receives two signals from this tower. One wave is coming directly from the tower, and the other wave is being reflected by the ground. What is the phase difference between these two waves received by the plane if the reflected wave is inverted by the reflection?



6. Two waves, both having a 500 nm wavelength arrive on a wall, as shown. On the wall, what is the distance between the interference maxima?



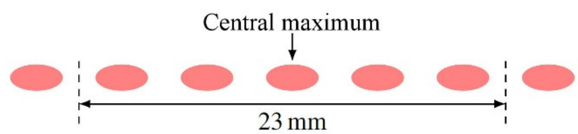
7.2 Young's Experiment

7. In Young's experiment, the wavelength of the light is 600 nm. The 4th maximum is 1 cm away from the central maximum on a screen located 2 m away from the slits. What is the distance between the slits?

8. In Young's experiment, the wavelength of the light is 500 nm. The distance between the slits is 0.1 mm and the interference pattern is located on a screen located 1.6 m away from the slits. What is the distance between the central maximum to the fifth-order maximum?

9. Young's experiment is done using light composed of two colours. One colour, green, has a 550 nm wavelength and the other colour corresponds to an unknown wavelength. On the screen, the 5th maximum of the green light wave is located at the same place as the 4th maximum of the other colour. What is the wavelength of the second colour?

10. In Young's experiment, the wavelength of the light is 632 nm and the distance between the slits is 0.2 mm. The diagram shows the interference pattern observed on a screen. How far away is the screen from the slits?

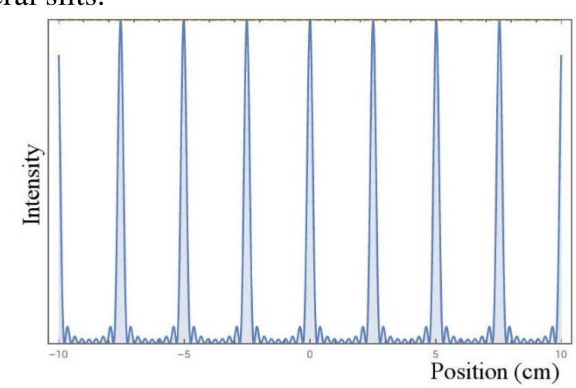


11. In Young's experiment, the wavelength of light is 450 nm and the distance between the slits is 0.2 mm. The interference pattern is on a screen located 2.4 m away from the slits. How far away from the central maximum is the phase difference between the two waves equal to 2 radians (in absolute value)?

7.3 Interference with Many Slits

12. Here is the graph of the intensity of light on a screen obtained by passing light with a wavelength of 500 nm through several slits.

    a) How many slits are there?
    b) What is the distance between the slits if the screen was 30 cm from the slits?
    c) What is the intensity of the maxima in relation to the intensity that we would have with a single source?



demonstrations.wolfram.com/MultipleSlitDiffractionPattern

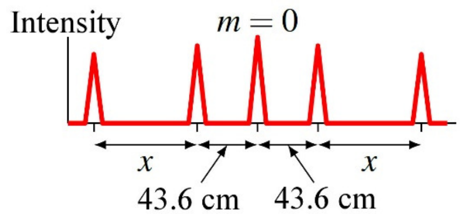
7.4 Gratings

13. A grating has 300 slits/mm. Red light with a 650 nm wavelength passes through the grating and the maxima are observed on a screen located 2.4 m away from the grating.

    a) How many maxima will be observed on the screen?
    b) What is the distance between the first-order maximum and the central maximum on the screen?

14. Here is the graph of the light intensity obtained on a screen located 1 m away from a grating when light passes through the grating having 800 slits/mm.

a) What is the wavelength of the light?
b) What is the distance between the first-order maximum and the second-order maximum (x in the diagram)?
c) How many maxima are visible on the screen?



15. Light coming from a sodium light bulb passes through a grating with 300 slits/mm. The maxima are observed on a screen located 2 m away from the grating. Among others, the lamp produces waves whose wavelengths are 589.0 nm and 589.6 nm (called the sodium doublet). What is the distance, on the screen, between the first-order maxima of these two waves of different wavelengths?

16. A beam of light passes through in a grating and the maxima are observed on a screen located 1 m away from the grating. Then, the distance between the central maximum and the first-order maximum is 35 cm. What is the distance between the central maximum and the second-order maximum?

Challenges

(Questions more difficult than the exam questions.)

17. We have seen that the interference maxima are thinner if there are more slits.

a) Knowing that a maximum starts and ends at the minima on each side of the maximum, show that the angular width of a maximum is given by the following formula.

$$\Delta\theta = \frac{2\lambda}{Nd\cos\theta}$$

b) If an observer wants to see two maxima having different wavelengths (but close to each other) separately, the separation between the two maxima must (approximately) be greater than or equal to half the value of $\Delta\theta$ given in a). Knowing this, how many slits must a grating have to allow the observer to see the two spectral lines of the sodium doublet separately at the first order if the wavelengths of these maxima are 589.00 nm and 589.59 nm?

ANSWERS

7.1 Superposition of Two Waves

1. 850 Hz
2. 1.0395 m
3. The intensity is 56.37% of the intensity there would be if there were only the wave coming directly from the source.
4. 0.002353 W/m²
5. -8.545 rad
6. 1 µm

7.2 Young's Experiment

7. 0.48 mm
8. 4.001 cm
9. 687.5 nm
10. 145.6 cm
11. 0.1719 cm

7.3 Interference with Many Slits

12. a) 9 b) 6.083 µm c) $81I_1$

7.4 Gratings

13. a) 11 b) 47.7 cm
14. a) 499.6 nm b) 89.4 cm c) 5
15. 0.377 mm
16. 88.02 cm

Challenges

17. b) 1000 slits or more