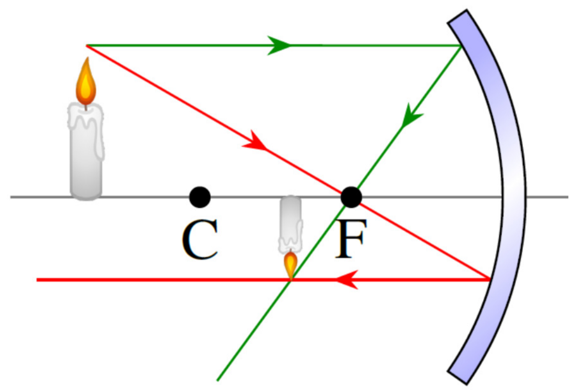
Chapter 5 Solutions

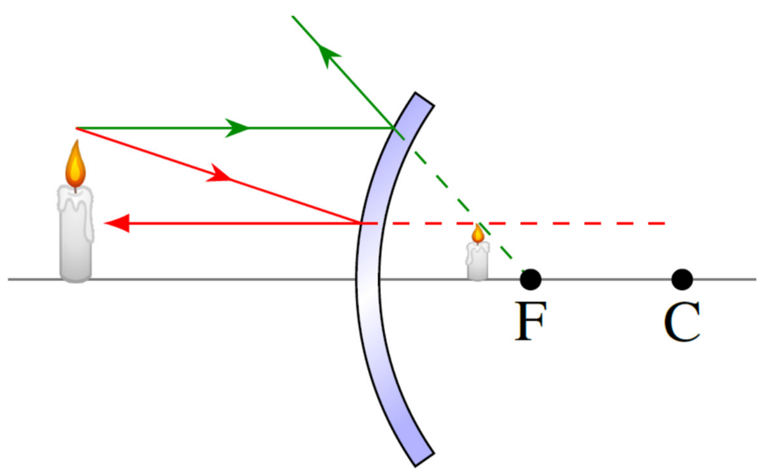
1. The object is at the same distance behind the mirror, so 120 cm behind the mirror. The image has the same height as the object (20 cm).

2. The image of the candle is 1 m behind the mirror. It is therefore 4 m from Anna.

3.



4.



5. a) The position will be found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

but we need the focal length.

The focal length of the mirror is

$$f=\frac{R}{2}=20cm$$

Therefore, the position of the image is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{10cm}+\frac{1}{q}=\frac{1}{20cm}$$
$$q=-20cm$$

Therefore, the image is 20 cm behind the mirror.

b) The position of the image is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{50cm}+\frac{1}{q}=\frac{1}{20cm}$$
$$q=33.3cm$$

Therefore, the image is 33.3 cm in front of the mirror.

6. a) The position will be found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

but we need the focal length.

The focal length of the mirror is

$$f = \frac{R}{2} = \frac{-40cm}{2} = -20cm$$

Therefore, the position of the image is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{10cm} + \frac{1}{q} = \frac{1}{-20cm}$$
$$q = -6.67cm$$

Therefore, the image is 6.67 cm behind the mirror.

b) The position of the image is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{50cm} + \frac{1}{q} = \frac{1}{-20cm}$$
$$q = -14.29cm$$

Therefore, the image is 14.29 cm behind the mirror.

7. The position will be found with

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

but we need the focal length.

The focal length of the mirror is

$$f = \frac{R}{2} = \frac{28cm}{2} = 14cm$$

The position of the image is therefore

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{16cm}+\frac{1}{q}=\frac{1}{14cm}$$
$$q=112cm$$

Therefore, the image is 112 cm in front of the mirror.

The magnification is

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{112cm}{16cm} \\ &= -7 \end{aligned}$$

Therefore, the height of the image is

$$m=\frac{y_i}{y_o}$$
$$-7=\frac{y_i}{3cm}$$
$$y_i=-21cm$$

The image is therefore inverted and has a height of 21 cm.

8. The focal length will be found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

We have *p*, but not *q*.

q can be found with the magnification formula

$$\frac{y_i}{y_o} = -\frac{q}{p}$$
$$-0.3 = -\frac{q}{p}$$
$$q = 0.3p$$
$$q = 0.3 \cdot 30cm$$
$$q = 9cm$$

Therefore

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{30cm} + \frac{1}{9cm} = \frac{1}{f}$$
$$f = 6.923cm$$

As the value is positive, this is a concave mirror. Its radius of curvature is

$$f = \frac{R}{2}$$
$$6.923cm = \frac{R}{2}$$
$$R = 13.85cm$$

9. The position will be found with

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

We have *p*, but not *f*.

f can be found since we know that q = 20 cm when p = 20 cm. Thus, the focal length of the mirror is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{60cm} + \frac{1}{20cm} = \frac{1}{f}$$
$$f = 15cm$$

Now, if the object is 10 cm from the mirror, the position of the image is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{10cm}+\frac{1}{q}=\frac{1}{15cm}$$
$$q=-30cm$$

Therefore, the image is 30 cm behind the mirror.

10. a) The position *p* will be found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

but we do not *f* and *q*.

The focal length is

$$\begin{aligned} f &= \frac{R}{2} \\ &= \frac{-40cm}{2} \\ &= -20cm \end{aligned}$$

q cannot be found but a relation between *q* and *p* can be found with the magnification formula.

$$\frac{y_i}{y_o}=-\frac{q}{p}$$
$$0.25=-\frac{q}{p}$$
$$q=-0.25p$$

We then have 2 equations and 2 unknowns.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{-20cm} \qquad \text{and} \qquad q=-0.25p$$

The solution is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{-20cm}$$
$$\frac{1}{p}+\frac{1}{-0,25p}=\frac{1}{-20cm}$$
$$\frac{1}{p}\cdot\left(1+\frac{1}{-0,25}\right)=\frac{1}{-20cm}$$
$$\frac{1}{p}(-3)=\frac{1}{-20cm}$$
$$p=60cm$$

b) The position of the image is, therefore,

$$\begin{aligned}q&=-0.25p\\&=-0.25\cdot 60cm\\&=-15cm\end{aligned}$$

Therefore, the image is 15 cm behind the mirror.

11. a) The focal distance will be found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

We have $p = x$ and $q = x + 1$ m. The mirror equation then gives

$$\frac{1}{x}+\frac{1}{x+1m}=\frac{1}{1,2m}$$

This equation gives

$$\frac{x+1m}{x(x+1m)}+\frac{x}{x(x+1m)}=\frac{1}{1,2m}$$
$$\frac{2x+1m}{x(x+1m)}=\frac{1}{1,2m}$$
$$1.2m\cdot(2x+1m)=x(x+1m)$$
$$2.4m\cdot x+1.2m^2=x^2+1m\cdot x$$
$$x^2-1.4m\cdot x-1.2m^2=0$$

The solutions to this equation are

$$\begin{aligned} x &= \frac{1.4m \pm \sqrt{(1.4m)^2 + 4 \cdot 1.2m^2}}{2} \\ &= \frac{1.4m \pm 2.6m}{2} \end{aligned}$$

The only positive answer is x = 2 m.

12. The position of the image is found with

$$\begin{aligned} \frac{n_1}{p} + \frac{n_2}{q} &= \frac{n_2 - n_1}{R} \\ \frac{1.33}{10cm} + \frac{1}{q} &= \frac{1 - 1.33}{-20cm} \\ q &= -8.58cm \end{aligned}$$

Therefore, the image of the fish is 8.58 cm behind the wall of the aquarium.

13. The position of the image is found with

$$\begin{aligned} \frac{n_1}{p} + \frac{n_2}{q} &= \frac{n_2 - n_1}{R} \\ \frac{1.33}{10cm} + \frac{1}{q} &= \frac{1 - 1.33}{40cm} \\ q &= -7.08cm \end{aligned}$$

Therefore, the image of the fish is 7.08 cm behind the wall of the aquarium.

14. a) The position of the image is found with

$$\begin{aligned} \frac{n_1}{p} + \frac{n_2}{q} &= \frac{n_2 - n_1}{R} \\ \frac{1.5}{25cm} + \frac{1}{q} &= \frac{1 - 1.5}{-40cm} \\ q &= -21.05cm \end{aligned}$$

Therefore, the image of the spot is 21.05 cm underneath the top of the glass dome. Thus, the distance between the image and the observer is 45cm + 21.05cm = 66.05 cm.

b) The magnification is

$$\begin{aligned} m &= -\frac{n_1 q}{n_2 p} \\ &= -\frac{1.5 \cdot (-21.05cm)}{1 \cdot 25cm} \\ &= 1.263 \end{aligned}$$

The radius of the image is 1.263 times larger than the radius of the object. The radius of the image is thus

$$\begin{aligned} r &= 1.263 \cdot 2cm \\ &= 2.53cm \end{aligned}$$

15. There are two surfaces.

First surface (curved surface)

The position of the image is found with

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{5cm} + \frac{1.5}{q} = \frac{1.5 - 1}{4cm}$$

$$q = -20cm$$

The magnification is

$$\begin{aligned} m &= -\frac{n_1 q}{n_2 p} \\ &= -\frac{1 \cdot (-20cm)}{1.5 \cdot 5cm} \\ &= 2.667 \end{aligned}$$

Second surface (flat surface)

The image of the first surface is used as the object for the second surface. As the image is 20 cm to the left of the curved surface, it is 36 cm from the flat surface. So, we have p = 36 cm. Therefore, the position of the final image is

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$
$$\frac{1.5}{36cm}+\frac{1}{q}=\frac{1-1.5}{\infty}$$
$$q=-24cm$$

Therefore, the image is 24 cm to the left of the flat surface. For the observer, the image is 24 cm behind the flat surface.

The magnification is

$$\begin{aligned} m &= -\frac{n_1 q}{n_2 p} \\ &= -\frac{1.5\cdot(-24cm)}{1\cdot 36cm} \\ &= 1 \end{aligned}$$

The total magnification is

$$m_{tot}=m_1\cdot m_2=2.667\cdot 1=2.667$$

The final image is 2.667 times larger than the object. Its diameter is thus

$$y_i = 2.667 \cdot 1 \text{ cm} = 2.667 \text{ cm}$$

16. There are two surfaces.

First surface (water-glass interface)

The position of the image is found with

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$
$$\frac{1.33}{4m}+\frac{1.5}{q}=\frac{1.5-1.33}{\infty}$$
$$q=-4.511m$$

Second surface (glass-air interface)

The image of the first surface is used as the object for the second surface. As the image is 4.511 m under the water-glass interface, it is 7.511 m under the glass-air interface. So, we have p = 7.511 cm. Therefore, the position of the final image is

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$
$$\frac{1.5}{7.511m}+\frac{1}{q}=\frac{1-1.5}{\infty}$$
$$q=-5.01m$$

The image is therefore to 5.01 m under the glass-air interface. For the observer, the image is therefore 5.01 m below the top of the glass surface.

17. a) The position of the image is given by

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{4m}+\frac{1}{q}=\frac{1}{0.5m}$$
$$q=57.1cm$$

b) The magnification is

$$m=\frac{-q}{p}$$
$$=-\frac{0.571m}{4m}$$
$$=-0.143$$

Therefore, the height of the image is

$$\begin{aligned} y_i &= m \cdot y_0 \\ &= -0.143 \cdot 1cm \\ &= -0.143cm \end{aligned}$$

A 0.143 cm high inverted image is then obtained.

18. a) The magnification is

$$\begin{aligned} m &= \frac{y_i}{y_o} \\ &= \frac{-0.5cm}{1cm} \\ &= -0.5 \end{aligned}$$

Therefore,

$$\begin{aligned} m &= \frac{-q}{p} \\ -0.5 &= -\frac{q}{2m} \\ q &= 1m \end{aligned}$$

b) The focal length is

$$\begin{aligned} \frac{1}{p} + \frac{1}{q} &= \frac{1}{f} \\ \frac{1}{2m} + \frac{1}{1m} &= \frac{1}{f} \\ f &= 66.6cm \end{aligned}$$

19. a)

Firstly, it should be noted that we don't know if the image is virtual or real. Let's try both possibilities. Since the magnification is + 4, we have (with a real image)

$$m = \frac{-q}{p}$$
$$4 = -\frac{20cm}{p}$$
$$p = -5cm$$

This answer is not possible since p cannot be negative in this situation.

With a virtual image, we have

$$m = \frac{-q}{p}$$
$$4 = -\frac{-20cm}{p}$$
$$p = 5cm$$

This is an acceptable answer. The object is 5 cm from the lens.

b) The focal length is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{5cm} + \frac{1}{-20cm} = \frac{1}{f}$$
$$f = 6.66cm$$

20. a) With a magnification of -3, we have

$$m = \frac{-q}{p}$$
$$-3 = -\frac{q}{p}$$
$$q = 3p$$

Therefore,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{p}+\frac{1}{3p}=\frac{1}{25cm}$$
$$\frac{1}{p}\cdot\left(1+\frac{1}{3}\right)=\frac{1}{25cm}$$
$$p=33.3cm$$

b) With a magnification of +3, we have

$$m=\frac{-q}{p}$$
$$3=-\frac{q}{p}$$
$$q=-3p$$

Therefore,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{p}+\frac{1}{-3p}=\frac{1}{25cm}$$
$$\frac{1}{p}\cdot\left(1-\frac{1}{3}\right)=\frac{1}{25cm}$$
$$p=16.7cm$$

21. a) With a magnification of –0.4, we have

$$m=\frac{-q}{p}$$
$$-0.4=-\frac{q}{p}$$
$$q=0.4p$$

Therefore,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{p}+\frac{1}{0.4p}=\frac{1}{-25cm}$$
$$\frac{1}{p}\cdot\left(1+\frac{1}{0.4}\right)=\frac{1}{-25cm}$$
$$p=-87.5cm$$

Since p cannot be negative here, this answer is impossible.

b) With a magnification of +0.4, we have

$$m=\frac{-q}{p}$$
$$0.4=-\frac{q}{p}$$
$$q=-0.4p$$

Therefore,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{p}+\frac{1}{-0.4p}=\frac{1}{-25cm}$$
$$\frac{1}{p}\cdot\left(1-\frac{1}{0.4}\right)=\frac{1}{-25cm}$$
$$p=37.5cm$$

22. The focal length will be found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

However, p and q are not known.

p and q can be found with 2 equations.

First, we know that

$$-2=-\frac{q}{p}$$
$$2p=q$$

And we also know that

$$p+q=36cm$$

With these 2 equations, we have

$$p+2p=36cm$$
$$3p=36cm$$
$$p=12cm$$

Therefore, the focal length is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{12cm}+\frac{1}{24cm}=\frac{1}{f}$$
$$f=8cm$$

23. The thin lens equation gives

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{x}+\frac{1}{2m-x}=\frac{1}{0.4m}$$

The solution of this equation is found with

$$\frac{1}{x}+\frac{1}{2m-x}=\frac{1}{0.4m}$$
$$\frac{2m-x}{x(2m-x)}+\frac{x}{x(2m-x)}=\frac{1}{0.4m}$$
$$\frac{2m-x+x}{x(2m-x)}=\frac{1}{0.4m}$$
$$\frac{2m}{x(2m-x)}=\frac{1}{0.4m}$$
$$2m\cdot 0.4m=x(2m-x)$$
$$0.8m^2=2m\cdot x-x^2$$
$$x^2-2m\cdot x+0.8m^2=0$$

The solutions of this quadratic equation are x = 1.4472 m and x = 0.5528 m.

24. We will deal with one lens at the same time.

First Lens

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{10cm}+\frac{1}{q}=\frac{1}{12cm}$$
$$q=-60cm$$

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{-60cm}{10cm} \\ &= 6 \end{aligned}$$

Second Lens

The image of the first lens is used as an object for the second lens. With an image 60 cm to the left of the first lens, the distance between this image and the second lens is 90 cm. So, we have p = 90 cm.

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{90cm}+\frac{1}{q}=\frac{1}{20cm}$$
$$q=25.7cm$$

The final image is 25.7 cm to the right of the lens to the right.

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{25.7cm}{90cm} \\ &= -0.286 \end{aligned}$$

The total magnification is thus

$$\begin{aligned} m_{tot} &= m_1 \cdot m_2 \\ &= 6\cdot(-0.286) \\ &= -1.714 \end{aligned}$$

The height of the image is

$$\begin{aligned} y_i &= my_0 \\ &= (-1.714)\cdot 1cm \\ &= -1.714cm \end{aligned}$$

Therefore, the final image is a 1.714 high inverted image.

25. We will deal with one lens at the same time.

First Lens

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{24cm}+\frac{1}{q}=\frac{1}{14cm}$$
$$q=33.6cm$$

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{-33.6cm}{24cm} \\ &= -1.4 \end{aligned}$$

Second Lens

The image of the first lens is used as an object for the second lens. With an image 33.6 cm to the right of the first lens, the distance between this image and the second lens is 8.6 cm. As the object is on the side where the light goes, the value of p is negative. So, we have p = -8.6 cm.

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{-8.6cm}+\frac{1}{q}=\frac{1}{-7cm}$$
$$q=-37.625cm$$

Therefore, the final image is 37.625 cm to the left of the lens to the right.

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{-37.625cm}{-8.6cm} \\ &= -4.375 \end{aligned}$$

Therefore, the total magnification is

$$\begin{aligned} m_{tot} &= m_1 \cdot m_2 \\ &= (-1.4)\cdot(-4.375) \\ &= 6.125 \end{aligned}$$

The height of the image is thus

$$\begin{aligned} y_i &= m y_0 \\ &= 6.125 \cdot 2cm \\ &= 12.25cm \end{aligned}$$

Therefore, the final image is erect and is 12.25 cm high.

26. We will deal with one item (lens or mirror) at the same time.

First passage through the lens (the light is travelling towards the right)

Position of the image

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{20cm} + \frac{1}{q} = \frac{1}{15cm}$$

$$q = 60cm$$

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{60cm}{20cm} \\ &= -3 \end{aligned}$$

Mirror

The image of the first lens is used as an object for the mirror. With an image 60 cm to the right of the first lens, the distance between this image and the mirror is 15 cm. As the object behind the mirror, the value of *p* is negative. So, we have *p* = -15 cm.

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{-15cm}+\frac{1}{q}=\frac{1}{10cm}$$
$$q=6cm$$

Therefore, the image is 6 cm in front of the mirror.

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{6cm}{-15cm} \\ &= 0.4 \end{aligned}$$

Second passage through the lens (the light is travelling towards the left)

The image of the mirror is used as an object for the lens. With an image 6 cm in front of the mirror, the distance between this image and the lens is 39 cm. So, we have p = 39 cm.

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{39cm}+\frac{1}{q}=\frac{1}{15cm}$$
$$q=24.375cm$$

Therefore, the final image is 24.375 cm to the left of the lens.

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{24.375cm}{39cm} \\ &= -0.625 \end{aligned}$$

Therefore, the total magnification is

$$\begin{aligned} m_{tot} &= m_1 \cdot m_2 \cdot m_3 \\ &= (-3) \cdot 0.4(\cdot -0.625) \\ &= 0.75 \end{aligned}$$

The height of the image is then

$$\begin{aligned} y_i &= m y_0 \\ &= 0.75 \cdot 2cm \\ &= 1.5cm \end{aligned}$$

The final image is erect and is 1.5 cm high.

27. a) The focal length is (assuming that light passes from left to right through the lens)

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \frac{1.6-1}{1} \cdot \left(\frac{1}{0.1m} - \frac{1}{-0.15m}\right)$$

$$f = 10cm$$

This is a converging lens whose focal length is 10 cm.

b) The focal length is (assuming that light passes from left to right through the lens)

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \frac{1.6-1}{1} \cdot \left(\frac{1}{0.1m} - \frac{1}{\infty}\right)$$

$$f = 16.7cm$$

This is a converging lens whose focal length is 16.7 cm.

c) The focal length is (assuming that light passes from left to right through the lens)

$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
$$\frac{1}{f}=\frac{1.6-1}{1}\cdot\left(\frac{1}{-0.1m}-\frac{1}{0.15m}\right)$$
$$f=-10cm$$

This is a diverging lens whose focal length is 10 cm.

d) The focal length is (assuming that light passes from left to right through the lens)

$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
$$\frac{1}{f}=\frac{1.6-1}{1}\cdot\left(\frac{1}{-0.1m}-\frac{1}{-0.15m}\right)$$
$$f=-50cm$$

This is a diverging lens whose focal length is 50 cm.

28. We have (assuming that light passes from left to right through the lens)

$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
$$\frac{1}{-30cm}=\frac{1.62-1.33}{1.33}\cdot\left(\frac{1}{-10cm}-\frac{1}{R}\right)$$
$$R=18.9cm$$

29. For red light, the focal length is (assuming that light passes from left to right through the lens)

$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
$$\frac{1}{f}=\frac{1.62-1}{1}\cdot\left(\frac{1}{30cm}-\frac{1}{-20cm}\right)$$
$$f=19.35cm$$

For violet light, the focal length is (assuming that light passes from left to right through the lens)

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{f} = \frac{1.67 - 1}{1} \cdot \left(\frac{1}{30cm} - \frac{1}{-20cm}\right)$$
$$f = 17.91cm$$

The distance between the focuses is then 1.44 cm.

30. In water, the focal length is

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{f} = \frac{1,6 - 1,33}{1,33} \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

We need the radiuses of curvature but the value of each radius cannot be found. However, the value of the whole parenthesis can be found from the focal length in air.

In air, we have

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{15cm} = \frac{1.6 - 1}{1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{9}cm^{-1}$$

In water, we then have

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{f} = \frac{1.6 - 1.33}{1.33} \cdot \frac{1}{9}cm^{-1}$$
$$f = 44.33cm$$

31. The position of the image will be found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

but we need the focal length.

The focal length of the lens is

$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
$$\frac{1}{f}=\frac{1.5-1}{1}\cdot\left(\frac{1}{30cm}-\frac{1}{-25cm}\right)$$
$$f=27.3cm$$

The position of the image is then found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{40cm}+\frac{1}{q}=\frac{1}{27.3cm}$$
$$q=85.7cm$$

32. The position of the image will be found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

but we need the focal length.

The equivalent focal length of the two lenses is

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$
$$\frac{1}{f_{eq}}=\frac{1}{10cm}+\frac{1}{15cm}$$
$$f_{eq}=6cm$$

The position of the image is then found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{40cm}+\frac{1}{q}=\frac{1}{6cm}$$
$$q=7.06cm$$

The magnification is

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{7.06cm}{40cm} \\ &= -0.176 \end{aligned}$$

The height of the image is thus

$$\begin{aligned} y_i &= my_0 \\ &= (-0.176)\cdot 3cm \\ &= -0.529cm \end{aligned}$$

Therefore, the image is inverted and is 0.529 cm high.

33. a) The distance is

$$\begin{aligned} p &= f \\ &= 3cm \end{aligned}$$

b) The maximum angular magnification is

$$\begin{aligned} G_{\min} &= \frac{d_{pp}}{f} \\ &= \frac{20cm}{3cm} \\ &= 6.67 \end{aligned}$$

c) The distance is

$$\begin{aligned} p &= \frac{d_{pp} f}{d_{pp} + f} \\ &= \frac{20cm \cdot 3cm}{20cm + 3cm} \\ &= 2.61cm \end{aligned}$$

d) The maximum angular magnification is

$$\begin{aligned} G_{\max} &= 1 + \frac{d_{pp}}{f} \\ &= 1 + \frac{20cm}{3cm} \\ &= 7.67 \end{aligned}$$

34. The angular magnification is

$$\begin{aligned} G &= \frac{d_{pp}}{p} \\ &= \frac{20cm}{1.9cm} \\ &= 10.53 \end{aligned}$$

35. a) The d_{pp} is

$$\begin{aligned} G &= \frac{d_{pp}}{p} \\ 15 &= \frac{d_{pp}}{1.4cm} \\ d_{pp} &= 21cm \end{aligned}$$

b) The focal length is

$$G_{\max} = \frac{d_{pp}}{f} + 1$$
$$15 = \frac{21cm}{f} + 1$$
$$f = 1.5cm$$

c) The angular magnification is

$$G = \frac{d_{pp}}{p} = \frac{21cm}{1.45cm} = 14.48$$

36. The power of the glasses is

$$P_{gla} = -\frac{1}{d_{pr}} = -\frac{1}{5m} = -0.2D$$

37. The power of the glasses is

$$P_{gla} = \frac{1}{d'_{pp}} - \frac{1}{d_{pp}} = \frac{1}{0.2m} - \frac{1}{0.45m} = 2.78D$$

38. a) As this person is myopic, the power of the glasses is

$$\begin{aligned} P_{gla} &= -\frac{1}{d_{pr}} \\ &= -\frac{1}{2,4m} \\ &= -0.417D \end{aligned}$$

b) Without glasses, the power of accommodation is

$$\begin{aligned} P_{acc} &= \frac{1}{d_{pp}} - \frac{1}{d_{pr}} \\ &= \frac{1}{0.18m} - \frac{1}{2.4m} \\ &= 5.139D \end{aligned}$$

This power remains the same with glasses. So we have

$$\begin{aligned} P_{acc} &= \frac{1}{d'_{pp}} - \frac{1}{d'_{pr}} \\ 5.139D &= \frac{1}{d'_{pp}} - \frac{1}{\infty} \\ d'_{pp} &= 0.1946m = 19.46cm \end{aligned}$$

With his glasses, this person sees clearly from 19.46 cm to infinity.

39. a) As this person is farsighted, the power of the glasses is

$$\begin{aligned} P_{gla} &= \frac{1}{d'_{pp}} - \frac{1}{d_{pp}} \\ &= \frac{1}{0.25m} - \frac{1}{0.5m} \\ &= 2D \end{aligned}$$

b) As the power of accommodation is 3 D, we have

$$P_{acc} = \frac{1}{d'_{pp}} - \frac{1}{d'_{pr}}$$
$$3D = \frac{1}{0.25m} - \frac{1}{d'_{pr}}$$
$$d'_{pr} = 1m$$

With his glasses, this person sees clearly from 25 cm to 1 m.

40. With his 2 D glasses, the punctum proximum is at 45 cm. Let's find where the punctum proximum without glasses is.

$$P_{gla} = \frac{1}{d'_{pp}} - \frac{1}{d_{pp}}$$
$$2D = \frac{1}{0.45m} - \frac{1}{d_{pp}}$$
$$d_{pp} = 4.5m$$

To bring back the d_{pp} at 25 cm, he needs glasses with a power of

$$\begin{aligned} P_{gla} &= \frac{1}{d'_{pp}} - \frac{1}{d_{pp}} \\ &= \frac{1}{0.25m} - \frac{1}{4.5m} \\ &= 3.78D \end{aligned}$$

41. The first distance of the object that gives a real image is p_1 and the second distance of the object that gives a virtual image is p_2. We know that

$$p_1 - p_2 = 1.2m$$

If the object forms a 3 times larger real image, we have

$$-3 = -\frac{q_1}{p_1}$$
$$q_1 = 3p_1$$

(The magnification is negative for a real image.) The lens equation then gives

$$\frac{1}{p_1}+\frac{1}{3p_1}=\frac{1}{f}$$

Solving this equation for p_1, we obtain

$$\frac{1}{p_1}\cdot\left(1+\frac{1}{3}\right)=\frac{1}{f}$$
$$\frac{1}{p_1}\cdot\left(\frac{4}{3}\right)=\frac{1}{f}$$
$$p_1=\frac{4f}{3}$$

If the object forms a 3 times larger virtual image, we have

$$3=-\frac{q_2}{p_2}$$
$$q_2=-3p_2$$

(The magnification is positive for a virtual image.) The lens equation then gives

$$\frac{1}{p_2}+\frac{1}{-3p_2}=\frac{1}{f}$$

Solving this equation for p_2, we obtain

$$\frac{1}{p_2}\cdot\left(1-\frac{1}{3}\right)=\frac{1}{f}$$
$$\frac{1}{p_1}\cdot\left(\frac{2}{3}\right)=\frac{1}{f}$$
$$p_1=\frac{2f}{3}$$

Therefore,

$$p_1-p_2=1.2m$$
$$\frac{4f}{3}-\frac{2f}{3}=1.2m$$
$$\frac{2f}{3}=1.2m$$
$$f=1.8m$$

42. When p = 36.8 cm, the distance of the image is q. Thus

$$\frac{1}{36.8cm}+\frac{1}{q}=\frac{1}{f}$$

When p = 36 cm, the distance of the image is q + 3 cm. thus

$$\frac{1}{36cm}+\frac{1}{q+3cm}=\frac{1}{f}$$

We have two equations and two unknowns. Then, q is found with

$$\frac{1}{36.8cm}+\frac{1}{q}=\frac{1}{36cm}+\frac{1}{q+3cm}$$

If this equation is solved for q, we obtain

$$\frac{1}{q}-\frac{1}{q+3cm}=\frac{1}{36cm}-\frac{1}{36.8cm}$$

$$\frac{1}{q}-\frac{1}{q+3cm}=\frac{1}{1656cm}$$

$$\frac{q+3cm}{q(q+3cm)}-\frac{q}{q(q+3cm)}=\frac{1}{1656cm}$$

$$\frac{3cm}{q(q+3cm)}=\frac{1}{1656cm}$$

$$4968cm^2=q(q+3cm)$$

$$4968cm^2=q^2+q\cdot 3cm$$

$$q^2+q\cdot 3cm-4968cm^2=0$$

The solutions of this equation are

$$q = \frac{-3cm \pm \sqrt{(3cm)^2 + 4 \cdot 4968cm^2}}{2}$$
$$q = \frac{-3cm \pm \sqrt{19\,881cm^2}}{2}$$
$$q = \frac{-3cm \pm 141cm}{2}$$

Since the image is real, q must be positive. The only positive solution is

$$q = \frac{-3cm + 141cm}{2}$$
$$= 69cm$$

Thus, the focal distance is

$$\frac{1}{36.8cm} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{36.8cm} + \frac{1}{69cm} = \frac{1}{f}$$
$$f = 24cm$$

43. If x is the distance between the object and the focus, then the distance between the object and the lens is

$$p = f + x$$

(If the object is between the focus and the lens, x is negative.)

If x' is the distance between the image and the other focus, then the distance between the lens and the image is

$$q = f + x'$$

(If the image is between the focus and the lens, x' is negative.)

Therefore,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{f+x}+\frac{1}{f+x'}=\frac{1}{f}$$

$$\frac{f+x'}{(f+x)(f+x')}+\frac{f+x}{(f+x)(f+x')}=\frac{1}{f}$$

$$\frac{2f+x+x'}{(f+x)(f+x')}=\frac{1}{f}$$

$$f(2f+x+x')=(f+x)(f+x')$$

$$2f^2+fx+fx'=f^2+fx+fx'+x\cdot x'$$

$$2f^2=f^2+x\cdot x'$$

$$f^2=x\cdot x'$$

which is the desired result.

44. When the rays are parallel in medium 1, we have $p=\infty$ and $q=f_1$. thus

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$

$$\frac{n_1}{\infty}+\frac{n_2}{f_1}=\frac{n_2-n_1}{R}$$

$$f_1=\frac{n_2}{n_2-n_1}R$$

When the rays are parallel in medium 2, we have $p=\infty$ and $q=f_1$. thus

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$

$$\frac{n_1}{f_2}+\frac{n_2}{\infty}=\frac{n_2-n_1}{R}$$

$$f_2=\frac{n_1}{n_2-n_1}R$$

Therefore

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$

$$\frac{n_1\frac{R}{n_2-n_1}}{p}+\frac{n_2\frac{R}{n_2-n_1}}{q}=1$$

Using the values of the foci, the result is

$$\frac{f_2}{p}+\frac{f_1}{q}=1$$

which is the desired result.

45. If x is the distance between the object and the lens, then the distance between the lens and the image is

$$q=L-x$$

The lens equation is, therefore,

$$\frac{1}{x}+\frac{1}{L-x}=\frac{1}{f}$$

Let's solve this equation for x.

$$\frac{L-x}{x(L-x)}+\frac{x}{x(L-x)}=\frac{1}{f}$$

$$\frac{L}{x(L-x)}=\frac{1}{f}$$

$$Lf=x(L-x)$$

$$Lf=xL-x^2$$

$$x^2-xL+Lf=0$$

The solution of this equation is

$$x=\frac{L\pm\sqrt{L^2-4Lf}}{2}$$

To have a solution, the expression inside the square root must be positive. Thus,

$$L^2 - 4Lf \geq 0$$
$$L^2 \geq 4Lf$$
$$L \geq 4f$$

which is the desired result.