5 MIRRORS AND LENSES

A 2 cm tall object is 12 cm in front of a spherical mirror. A 1.2 cm tall erect image is then obtained. What kind of mirror is used (concave, plane or convex) and what is its focal length?

www.totalsafes.co.uk/interior-convex-mirror-900mm.html

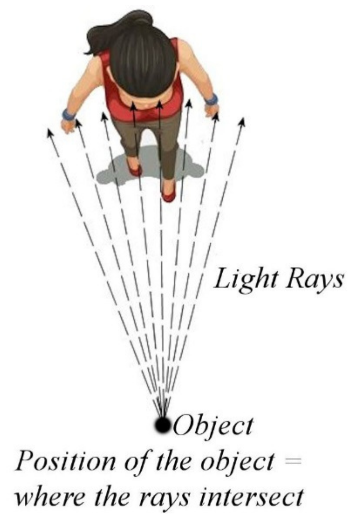
Discover how to solve this problem in this chapter.

In this chapter, we will see that an image of an object can be formed using reflection and refraction with mirrors and lenses. Here, it will be assumed that the only things that can deflect a ray are reflection and refraction. This assumption is the base of ***geometrical optics***.

5.1 OBJECT POSITION

How Can the Position of an Object Be Known?

When a person looks at an object, the position of the object is found from the direction of the light rays. The object is a light source (it directly emits light or simply reflects the light coming from a source) that sends rays in every direction. When one of these rays arrives in one eye, the person knows from what direction this ray is coming. However, the rays arriving in each eye are not coming from the same exact direction. The position of the object is determined by finding where these rays intersect to know where they started their journey. (We are not really aware that this is what we do, but this is how it is done.) In so doing, our brain always assume that the rays are travelling in straight lines.

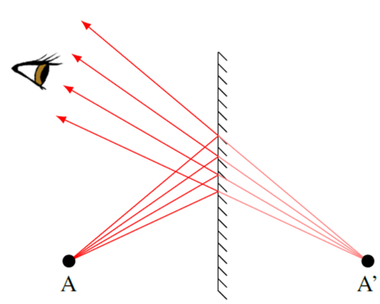


If the rays were deflected (by a mirror or a lens for example), our brain still assumes that the rays are still travelling in straight lines and thus we might be fooled into believing that there is an object in a place where there's actually none. In the following video, there is a small plastic pig, seen at the beginning, and there is an image of the pig, seen at the end. This last pig does not exist. Our eyes receive beams of light which intersect at this place, but this is the result of reflections on mirrors. This is an image of the object.
http://www.youtube.com/watch?v=qNG-L_iWYZo

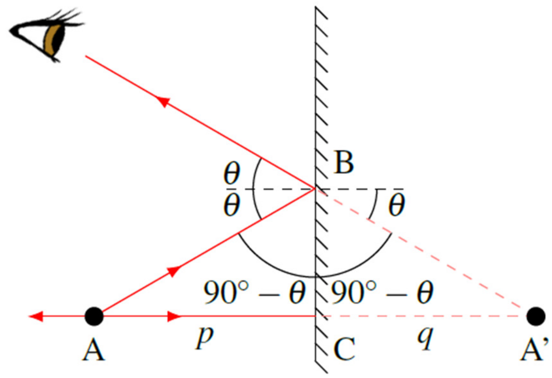
5.2 PLANE MIRRORS

Image Made by Plane Mirror

Rays coming from the object A are reflected on the mirror and are picked up by the eyes. We then determine the point of intersection of these rays by assuming they are travelling in straight lines. We, therefore, think that there is an object at point A'. We then see an image of the object at the position A'. Obviously, we can also see the object by looking at it directly. We can, therefore, see the object and the image of the object at the same time.



Calculation of the Position of the Image



The position of point A' will now be found by determining the point of intersection of two reflected rays. First, there's a ray which starts from A and arrives with a 90° incidence angle on the mirror (point C). This ray is reflected back on the same path so that it passes through point A. A second ray arrives with an incidence angle θ at point B. If the law of reflection is used, it can be shown quite easily that the three angles θ shown in the diagram are equal. This means that triangle ABC is similar to triangle $A'BC$ since all the angles are identical and the side BC is a common side of the two triangles. If triangle ABC is similar to triangle $A'BC$, then the point A' is at the same distance from the surface of the mirror than point A. If p is the distance between the object and the mirror and q is the distance between the image and the mirror, then, this means that

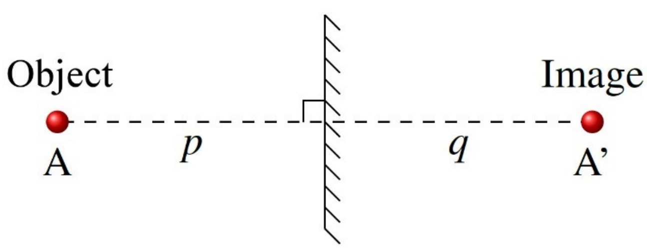
$$q = p$$

This is not the final formula since the following sign convention will be used: the distance between the mirror and an image is positive if the image is in front of the mirror and the distance between the mirror and an image is negative if the image is behind the mirror. The equation must then be

Plane Mirror Law

$$q = -p$$

The image is, therefore, at the same distance from the mirror as the object, but on the other side of the mirror. A line connecting the object and the image is always perpendicular to the mirror.



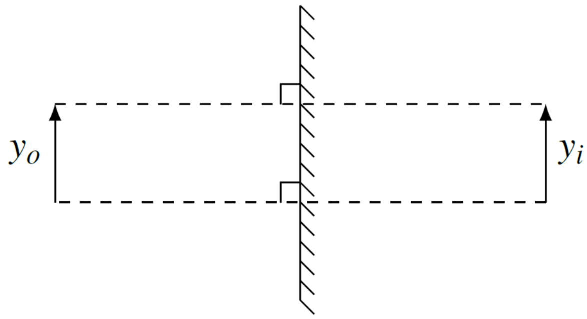
Magnification

The magnification (m) is defined as the height of the image (y_i) divided by the height of the object (y_o).

Definition of Magnification

$$m = \frac{y_i}{y_o}$$

For a plane mirror, we have the situation shown to the right.



The image of the bottom end of the object must be exactly in line with the bottom end of the object and the image of the top end of the object must be exactly in line with the top end of the object. These are two parallel lines connecting the two ends of the object and the image. As these lines are parallel, the image must have exactly the same size as the image. Thus

$$y_i = y_o$$

and

Magnification With a Plane Mirror

$$m = 1$$

en.wikipedia.org/wiki/Mirror

In summary, if a person is looking at its image on a plane mirror, the image must be behind the mirror. If the person is 1.5 m from the mirror, the image is 1.5 m behind the mirror and has the same dimensions as the person.

Here is a nice effect obtained with plane mirrors.
http://www.youtube.com/watch?v=TZkdQeevJu0

Movie Mistake: One-Way Mirror

A mirror that let light pass in one direction and reflects light in the opposite direction does not exist. A mirror can let part of the light pass and reflect the other part, but the percentage of reflected light is the same in both directions. Such a mirror can be used as a one-way mirror provided there is very little light in one of the rooms. Thus, on the side of the lit room, the light reflected from the mirror is much more intense than the light coming from the other room and it is impossible to see in the dark room. On the side of the dark room, there is not much light reflected because there is no light in this dark room and the light passing through the mirror is more intense than the reflected light and it is then possible to see what is happening in the other room.

If a person seeks to remain anonymous behind such a semi-reflective mirror, he must be sure that no one will turn the light on in the dark room. If someone opens the light, the person would become visible to the people in the other room.

If a mirror that lets the light pass in only one direction were to exist, free energy can be obtained by building a box that let the light in, but not out. This box would then constantly accumulate light, thereby accumulating energy without any work done. This is not permitted by the laws of thermodynamics.

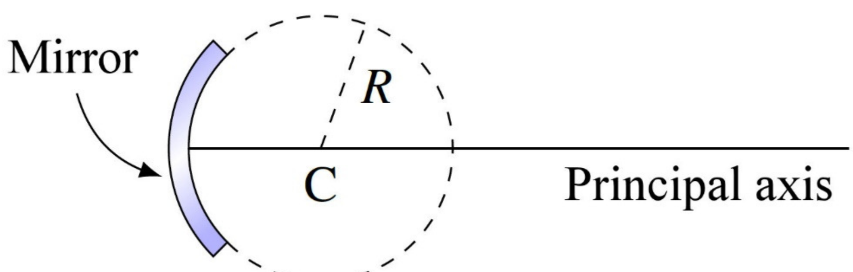
5.3 SPHERICAL MIRRORS

Images formed by a spherical mirror will now be considered. There are several possible shapes for curved mirrors, but only spherical mirrors will be examined here. These mirrors are formed by using a sphere or a portion of a sphere as a reflective surface.

The picture shoes a spherical mirror at Texas Tech University.

arkadiusz.jadczyk.salon24.pl/472422,spojrz-w-lustro-najlepiej-sferyczne

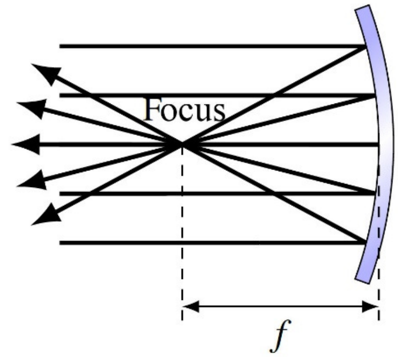
If only a part of a sphere is used, then we have the following situation.



The line starting from the centre of the mirror and passing through the centre of the sphere is the principal axis of the mirror. The radius of curvature is the radius of the sphere. Two situations may arise.

1) The interior of the sphere is reflective: a concave mirror is obtained.
2) The exterior of the sphere is reflective: a convex mirror is obtained.

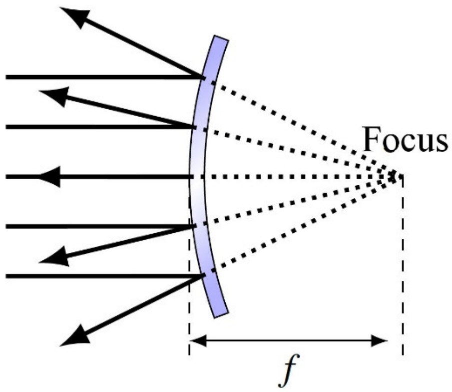
The Focus



Every parallel ray arriving on a concave mirror converges at a point called the *focus* after its reflection on the mirror. The distance between the mirror and the focus, measured along the principal axis, is the focal length *f*.

This phenomenon can be seen in this video.
http://www.youtube.com/watch?v=kqxdWpMOF9c

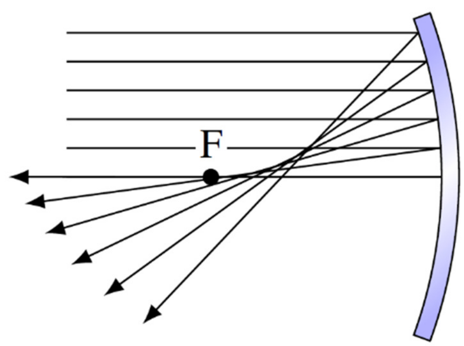
Sunlight can thus be concentrated at the focus of a concave mirror. The amount of energy concentrated at the focus may be large enough to ignite a piece of paper.
https://www.youtube.com/watch?v=btFei_I_R1s



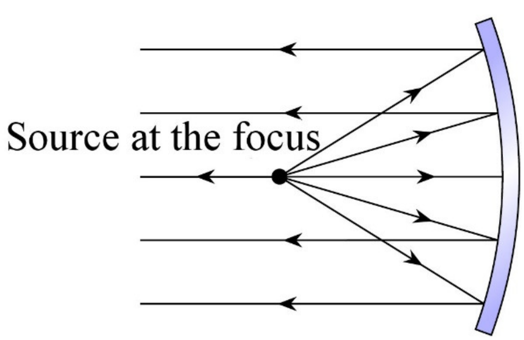
For a convex mirror, parallel rays appear to come from a point behind the mirror after their reflection. This point is also called the focus and the focal length is still the distance between the mirror and the focus.

See the light reflected on a convex mirror in this clip.
http://www.youtube.com/watch?v=5zZNC2YzRDo

Actually, the rays do not all concentrate exactly at the same place with a spherical mirror. Rays that strike the mirror far away from the principal axis pass a little closer to the mirror than the focus. This default is called *spherical aberration*. Here, this aberration will be ignored.



To have all the rays concentrating exactly at the same place, a mirror with a parabolic shape must be used. For those interested, here is the proof.
http://physique.merici.ca/waves/proof-parabola-mirror.pdf



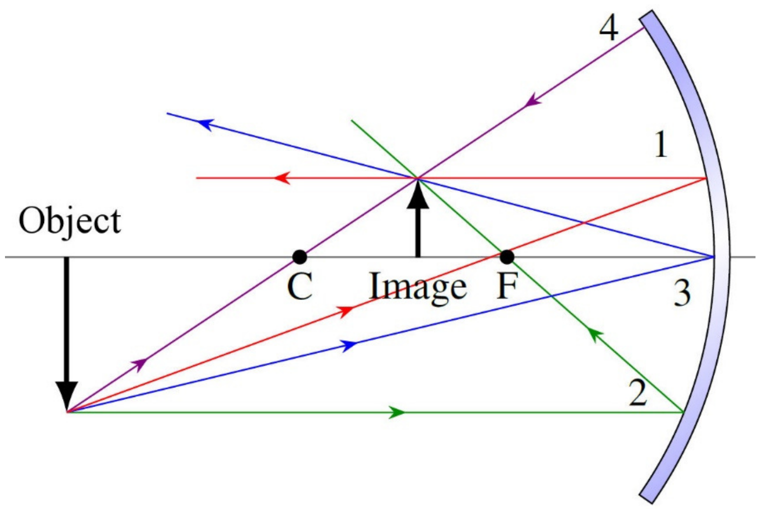
If the directions of propagation of the rays are inverted, the result obtained is always a correct result (in accordance with the laws of physics). For example, let's invert the direction of the parallel rays coming to the focus after their reflections. Then, rays coming from a source placed at the focus will be reflected on the mirror, and will all become parallel to the principal axis after their reflection on the mirror. This is a correct result.

This idea is always valid in geometrical optics: If the directions of the rays are inverted, the result is always a possible situation. This is called ***the principle of reciprocity.***

Graphical Method to Find the Image Position

Four rays whose trajectories are easy to predict can be used to find the position of the image of an object. All these rays start from the object.

1) A ray passing through the focus becomes parallel after its reflection.
2) A ray parallel to the principal axis passes through the focus after its reflection.
3) A ray passing through the centre of the mirror is reflected with the same angle relative to the principal axis.
4) A ray passing through the centre of curvature (C) arrives on the mirror with a 0° incidence angle, and will therefore pass through the centre of curvature again after its reflection.



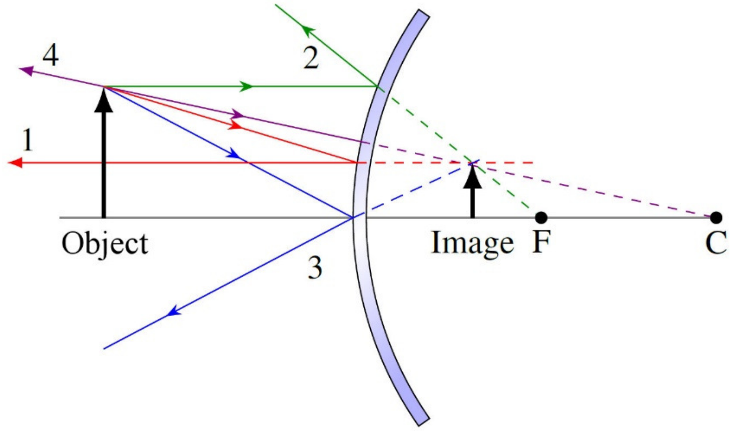
All these rays are going to meet at the same place: at the position of the image. In the diagram, the position of the image of the tip of the arrow was found. Then, each reflected ray seems to come from the position of the image. If a person sees these rays, he will think there's something at this place because this is the point of intersection of the rays.

There is no need to draw all these rays. With just two rays, the point of intersection of the rays, and thus the position of the image, can be found.

The same thing can be done for convex mirrors. The idea is the same, but there are some minor differences because the focus is behind the mirror. The rays are:

1) A ray travelling towards the focus becomes parallel after its reflection.
2) A ray parallel to the principal axis seems to be coming from the focus after its reflection.
3) A ray passing through the centre of the mirror is reflected with the same angle relative to the principal axis.

4) A ray going towards the centre of curvature (*C,* now behind the mirror) arrives on the mirror with a 0° incidence angle and is, therefore, reflected back along the same path it had before the reflection.

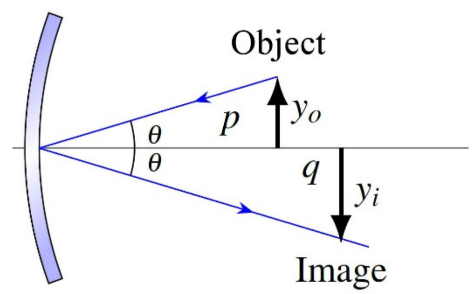


It can be seen that these 4 rays, after reflection, all seem to come from the same place. The point of intersection of these rays is the position of the image.

Obviously, it will be easier to find the position of the image with a formula, but these rays will sometimes be useful to visualize the situation.

Calculation of the Image Position

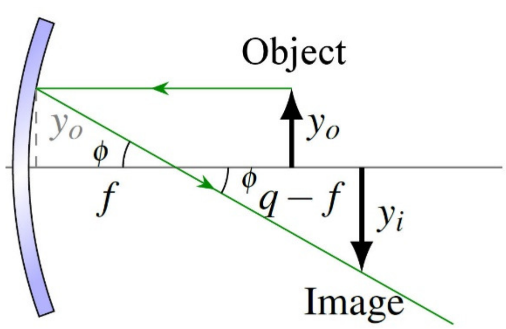
The following notation will be used: p is the distance between the mirror and the object, q is the distance between the mirror and the image, y_o is the height of the object, and y_i is the height of the image.



First, let's consider the ray passing through the centre of the mirror.

Two similar triangles are thus formed. Thus,

$$\frac{y_i}{y_o} = \frac{q}{p}$$



Now, let's consider the triangles with angles ϕ formed by a ray parallel to the principal axis that goes through the focus after the reflection.

As these are also similar triangles, the following equation must be true.

$$\frac{y_i}{y_o} = \frac{q-f}{f}$$

With these two equations, the following result is obtained.

$$\frac{q}{p} = \frac{q-f}{f}$$

$$\frac{1}{p} = \frac{q-f}{qf}$$

$$\frac{1}{p} = \frac{q}{qf} - \frac{f}{qf}$$

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{q}$$

This finally gives

Mirror Equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Although this equation was proven for a concave mirror, this formula is also valid for convex mirrors, provided the following sign convention is respected.

Sign Convention for Mirrors

For a concave mirror: R and f are positive.

For a convex mirror: R and f are negative.

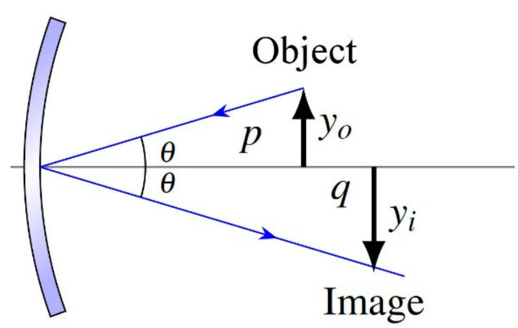
If q is positive: the image is in front of the mirror.

If q is negative: the image is behind the mirror.

Note that this formula also works with plane mirrors. With a plane mirror, R is infinite, and the mirror formula becomes $p = -q$ which is exactly what we had for this type of mirror.

Magnification

To find the magnification, the ray passing through the centre of the mirror is considered.



With this ray, the following result was previously obtained.

$$\frac{y_i}{y_0} = \frac{q}{p}$$

This ratio of the height of the object and the image is the magnification. However, to take into account the fact that the image is inverted, a negative sign is added.

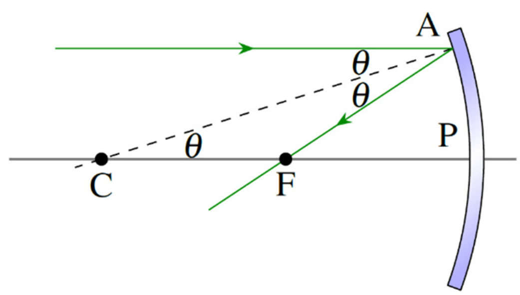
Magnification for Mirrors

$$m = -\frac{q}{p}$$

If the magnification is positive, the image has the same orientation as the object (it is then said that the image is *erect* ☺). If the magnification is negative, then the image is inverted relative to the object.

Calculation of the Focal Length

To find the focal length of the mirror, a ray parallel to the principal axis will be considered to determine where it intersects with the principal axis after its reflection on the mirror.



The two angles at point A are equal because of the law of reflection.

The angle at point C is also equal to θ, because this angle and the top angle at point A are alternate interior angles.

As there are two similar angles in the triangle CFA, it is an isosceles triangle, and the following sides are equal.

$$CF = FA$$

If the distance between the parallel ray and the principal axis is small, the following sides are almost equal.

$$FA \approx FP$$

Therefore (since $CF = FA$),

$$CF \approx FP$$

However, the radius of curvature (R) is the distance between points C and P. Thus

$$R = CP$$
$$R = CF + FP$$
$$R \approx FP + FP$$
$$R \approx 2FP$$

As FP is the distance between the focus and the mirror, its length is f. Therefore,

$$R \approx 2f$$

Forgetting that this is an approximation, the formula for the focal length is obtained.

Focal Length of a Mirror

$$f = \frac{R}{2}$$

Even if it will not be demonstrated, this result is also valid for convex mirrors.

Example 5.3.1

A 1 cm tall object is placed 6 cm in front of a concave mirror having a radius of 8 cm.

a) Where is the image?

The position of the image is found with

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

To find it, the focal length of the mirror is needed. If the radius of curvature is 8 cm, then the focal length is 4 cm. Therefore,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{6cm} + \frac{1}{q} = \frac{1}{4cm}$$
$$q = 12cm$$

b) What is the height of the image?

The height is found with

$$m = \frac{y_i}{y_o}$$

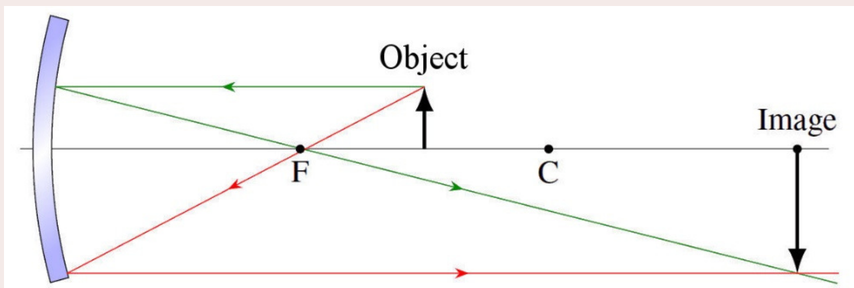
To find y_i, the magnification is needed. This magnification is

$$m = -\frac{q}{p} = -\frac{12cm}{6cm} = -2$$

Therefore, the height of the image is

$$m = \frac{y_i}{y_o}$$
$$y_i = m y_o$$
$$y_i = -2 \cdot 1cm$$
$$y_i = -2cm$$

The image is, therefore, 12 cm in front of the mirror, is inverted and is twice as tall as the object. This answer can be checked by looking at the ray tracing diagram of this problem.



Example 5.3.2

A 1 cm tall object is placed 2 cm in front of a concave mirror having a radius of 8 cm.

a) Where is the image?

The position of the image is found with

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

To find it, the focal length of the mirror is needed. If the radius of curvature is 8 cm, then the focal length is 4 cm. Therefore,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{2cm} + \frac{1}{q} = \frac{1}{4cm}$$
$$q = -4cm$$

b) What is the height of the image?

The height is found with

$$m = \frac{y_i}{y_o}$$

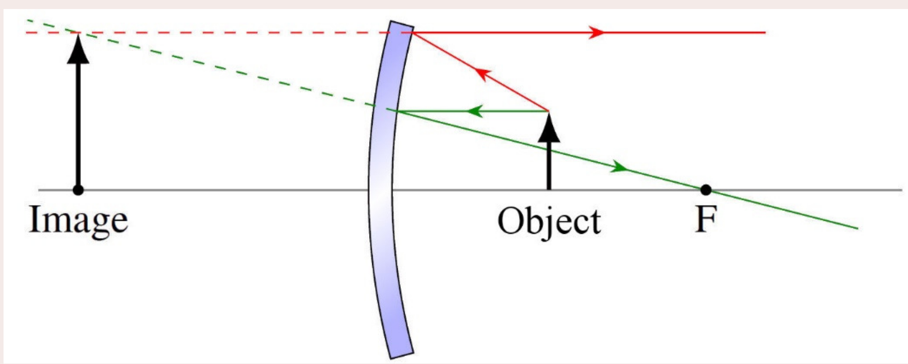
To find y_i, the magnification is needed. This magnification is

$$m = -\frac{q}{p} = -\frac{-4cm}{2cm} = 2$$

Therefore, the height of the image is

$$m = \frac{y_i}{y_o}$$
$$y_i = my_o$$
$$y_i = 2 \cdot 1cm$$
$$y_i = 2cm$$

The image is, therefore, 4 cm behind the mirror, is erect and is twice as tall as the object. This answer can be checked by looking at the ray tracing diagram of this problem.



Two types of image were encountered in these two examples.

Types of Images

Real Image: The rays actually pass at the position of the image after their reflection. (The image is, therefore, in front of the mirror and q is positive.)

Virtual Image: The rays do not actually pass at the position of the image after their reflection, they only seem to come from the position of the image. (The image is, therefore, behind the mirror and q is negative.)

Everyone knows what a virtual image is because that is what we see when we look in the mirror. It is less frequent to see a real image. Maybe this video will help you visualize this concept.
http://www.youtube.com/watch?v=KVpSCICCD9A

(The effect would have been better with a higher-quality mirror.)

Example 5.3.3

A 2 cm tall object is 12 cm in front of a spherical mirror. A 1.2 cm tall erect image is then obtained. What kind of mirror is used (concave, plane or convex) and what is its focal length?

The focal length of the mirror is sought (the sign will indicate the type of mirror used). Because the radius of curvature is not known, the focal length must be found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

The value of p is known, but not the value of q. However, since the height of the image and of the object are known, q can be calculated with the magnification formula.

$$m=\frac{y_i}{y_o}=\frac{-q}{p}$$

$$\frac{1.2cm}{2cm}=-\frac{q}{12cm}$$

$$q=-7.2cm$$

A virtual image located 7.2 cm behind the mirror image is thus obtained. Therefore, the value of f is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{12cm}+\frac{1}{-7.2cm}=\frac{1}{f}$$

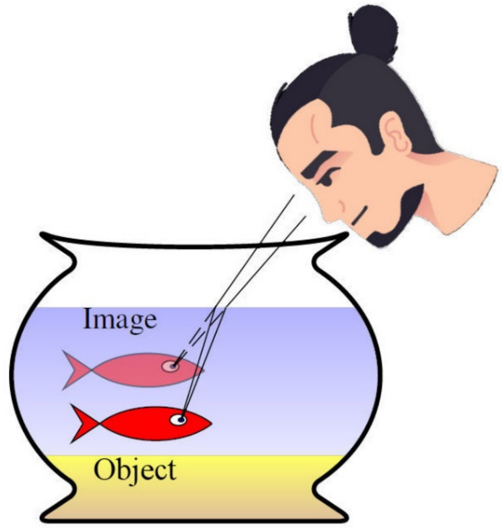
$$f=-18cm$$

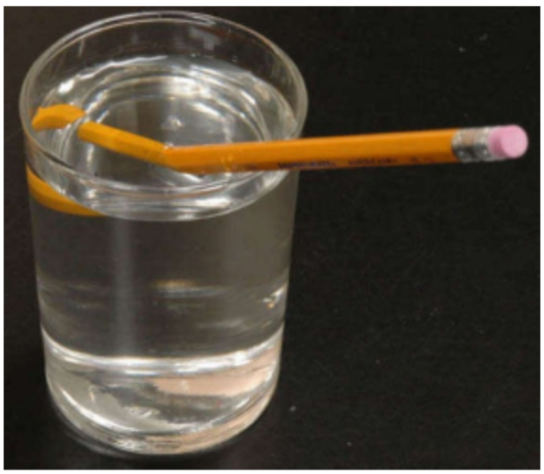
A convex mirror (because the focal distance is negative) with an 18 cm focal length was then used.

5.4 SPHERICAL DIOPTRES

An Image With Refraction

An image of an object can be formed with refraction since light changes direction when the medium changes. As the brain assumes that light rays travel in straight lines, we can think that the light is coming from a location different from the place where the object is truly located. An image of the object is, therefore, seen. For example, in the diagram, the image of the fish is seen at a location different from the true position of the fish.





wiki.metropolia.fi/display/Physics/Reflection+and+Refraction

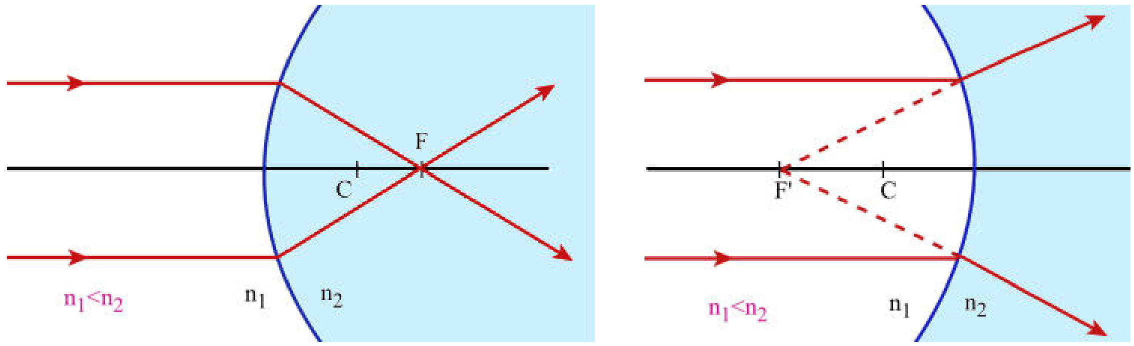
This effect can be seen in this picture. Only the image of the pencil can be seen for the part of the pencil in water. As the image is closer to the surface than the object (as it will be shown later), it seems that the pencil is bent.

In this clip, it can be seen that the position of the image depends on the substance since two substances are used: water and oil.
http://www.youtube.com/watch?v=FM1g1zNuCM0

The surface separating the two media is called a ***dioptre***. A more general case where the surface separating the two media is curved will be considered here. These are ***spherical dioptres*.**

The Focus

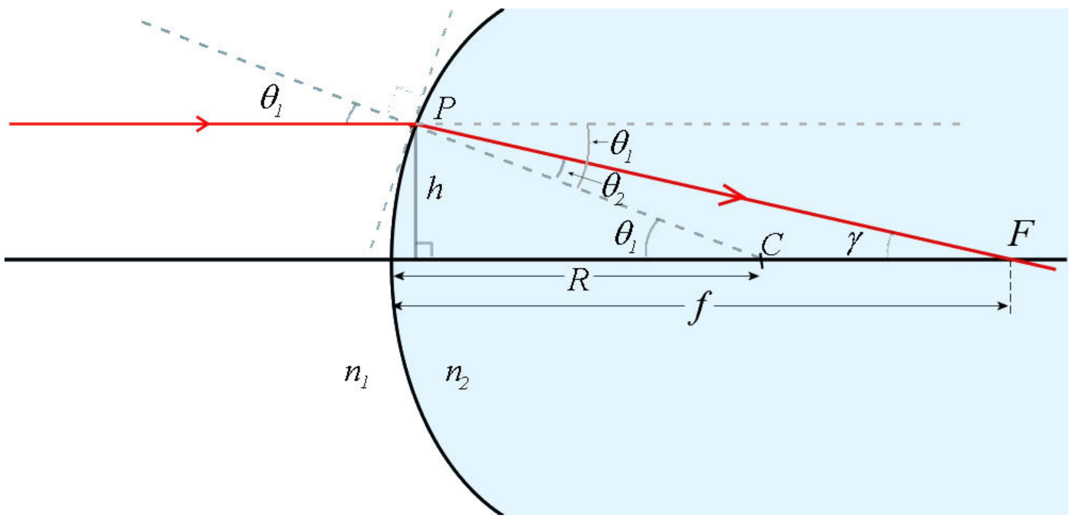
Parallel rays deflected by a spherical dioptre will concentrate at one point or will appear to come from a point. This point is the focus.



uel.unisciel.fr/physique/optigeo/optigeo_ch04/co/apprendre_ch04_15.html

Let's start by calculating the focal distance of the dioptre. Here we will consider the case shown on the left of the diagram and the result will be generalized later to encompass all possible cases.

We're going to work with the following angles.



fr.wikiversity.org/wiki/Lentilles_en_optique_géométrique/Dioptre_sphérique

The law of refraction for small angles is

$$n_1\theta_1 = n_2\theta_2$$

(since $\sin\theta \approx \theta$ for small angles in radians.)

For the triangle *FPC*, the sum of the angles, in radians, is

$$\gamma + \theta_2 + (\pi - \theta_1) = \pi$$
$$\theta_2 = \theta_1 - \gamma$$

Thus, the law of refraction becomes

$$n_1\theta_1 = n_2(\theta_1 - \gamma)$$

Then, those angles are

$$\tan\theta_1 \approx \frac{h}{R} \qquad \tan\gamma \approx \frac{h}{f}$$

To understand why these are approximations, consider the first of these equations. For this equation to be exact, the divisor should have been a little smaller than *R* because the height is a bit further closer than the dioptre. A correction is needed, but it is very small if point *P* is close to the principal axis. This small correction will be neglected here.

Since the angles are small, the following relation can also be used.

$$\tan x \approx x$$

Thus, the angles are

$$\theta_1 \approx \frac{h}{R} \qquad \gamma \approx \frac{h}{f}$$

Using these values, the equation becomes

$$n_1\theta_1 = n_2(\theta_1 - \gamma)$$

$$n_1\frac{h}{R} = n_2\left(\frac{h}{R} - \frac{h}{f}\right)$$

$$n_1\frac{1}{R} = n_2\left(\frac{1}{R} - \frac{1}{f}\right)$$

$$\frac{n_2}{f} = \frac{n_2}{R} - \frac{n_1}{R}$$

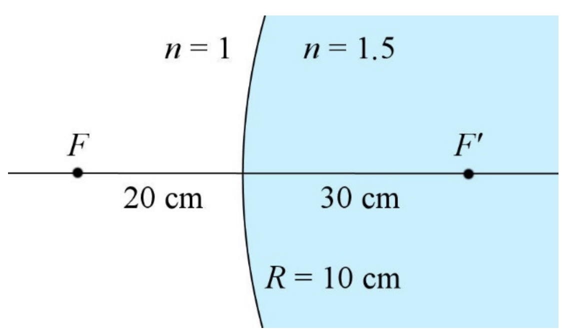
Solving this equation for *R*, the result is

Focal Length of a Spherical Dioptre

$$f = \frac{n_2}{n_2 - n_1}R$$

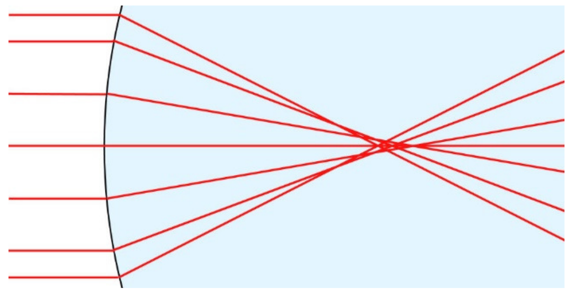
To include all possible cases, the following sign convention must be followed: If the centre of curvature is on the side where the light is going, *R* is positive. If the centre of curvature is on the side where the light is coming, *R* is negative.

Beware, the formula shows that the focus is not at the same distance on either side of the dioptre. For example, if parallel rays arrive from the left on the dioptre shown on the diagram, they concentrate at 30 cm from the dioptre (so the focal distance is 30 cm). If the

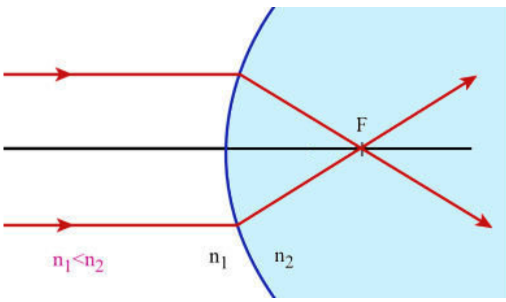


rays come from the right, they concentrate 20 cm from the dioptre (so the focal distance is 20 cm on this side.

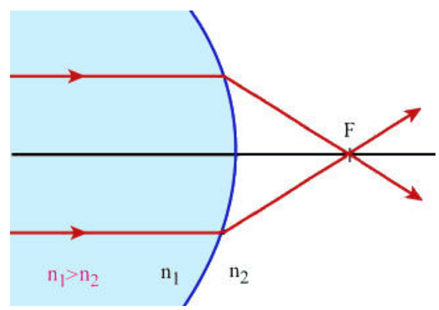
Spherical Aberration



In fact, it is not entirely accurate to say that all the rays meet at the focus. If the surface of the dioptre is spherical, only the rays near the principal axis will meet at the focus. The rays passing far away from the principal axis will focus closer to the dioptre than the focal point in the situation shown on the diagram. This is called *spherical aberration*.



If $n_1 < n_2$ (as on the diagram to the left) the aberration can be corrected by taking a dioptre whose surface is shaped like an ellipse.

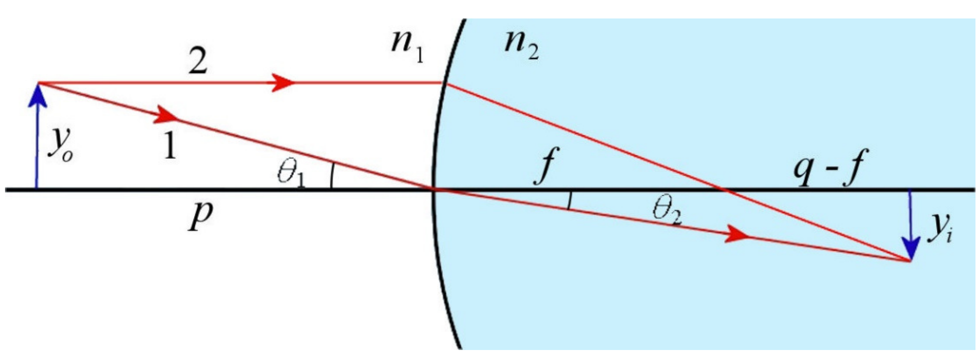


If $n_1 > n_2$ (as on the diagram to the right) the aberration can be corrected by taking a dioptre whose surface is shaped like a hyperbola.

For those interested, here's the proof.
https://physique.merici.ca/waves/proof-dioptre.pdf

Calculation of the Image Position

With the focus, the position of the image formed by a diopter can be found. The following diagram shows two rays forming the image.



fr.wikiversity.org/wiki/Lentilles_en_optique_géométrique/Dioptre_sphérique

Let's look at the path of the ray passing through the center of the dioptre (ray 1). For small angles, we have

$$\theta_1 \approx \frac{y_o}{p} \qquad \theta_2 \approx \frac{y_i}{q}$$

Also with small angles, the law of refraction is

$$n_1\theta_1 \approx n_2\theta_2$$

Using the formulas for the angle, the result is

$$n_1\frac{y_o}{p} = n_2\frac{y_i}{q}$$

$$\frac{y_i}{y_o} = \frac{n_1 q}{n_2 p}$$

To the right of the dioptre, two similar triangles were formed with ray 2. Thus

$$\frac{y_i}{y_o} = \frac{q-f}{f}$$

By combining these last two equations, the result is

$$\frac{n_1 q}{n_2 p} = \frac{q-f}{f}$$

Thus

$$\frac{n_1 q}{n_2 p} = \frac{q-f}{f}$$

$$\frac{n_1}{n_2 p} = \frac{q-f}{qf}$$

$$\frac{n_1}{n_2 p} = \frac{q}{qf} - \frac{f}{qf}$$

$$\frac{n_1}{n_2 p} = \frac{1}{f} - \frac{1}{q}$$

$$\frac{n_1}{n_2 p} + \frac{1}{q} = \frac{1}{f}$$

Since the focal length is

$$f = \frac{n_2}{n_2 - n_1}R$$

The equation becomes

$$\frac{n_1}{n_2 p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{n_1}{n_2 p} + \frac{1}{q} = \frac{n_2 - n_1}{n_2} \frac{1}{R}$$

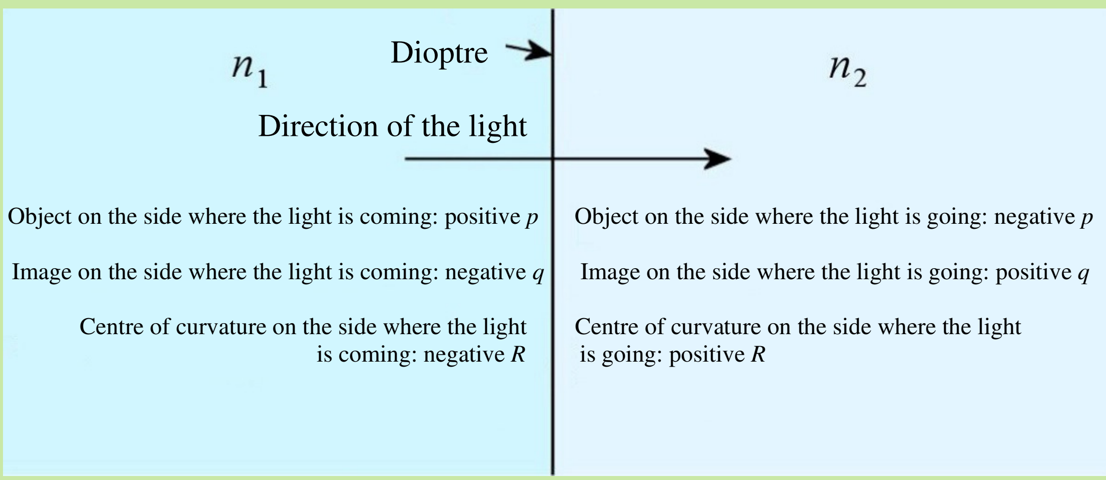
Multiplying by n_2, the result is a

Spherical Dioptre Formula

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

Although this formula has been shown for a dioptre curved in the direction shown in the diagram, and for $n_1 < n_2$, it is valid for any dioptre, provided the following sign convention is respected.

Sign Convention for Spherical Dioptre



(As it is the object that emits light, it may seem odd to have an object on the side where the light goes. We will see later how this is possible.)

Magnification

While we were working to obtain the formula of the position of the image, we obtained, by considering the light ray passing through the center of the dioptre, the following formula.

$$\frac{y_i}{y_o} = \frac{n_1 q}{n_2 p}$$

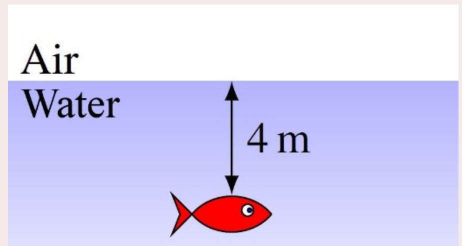
which is the magnification. A minus sign is added to indicate that the image is inverted. The final formula is thus

Magnification With a Spherical Dioptre

$$m = \frac{y_i}{y_o} = -\frac{n_1 q}{n_2 p}$$

Example 5.4.1

A 50 cm long fish is 4 m below the surface of a lake.



a) Where is the image of the fish for an observer in air?

The light passes from water to air. Therefore, n_1 = 1.33, n_2 = 1 and p = 4 m. As the surface of the lake is not curved, R is infinite. The equation thus gives

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

$$\frac{1.33}{4m} + \frac{1}{q} = \frac{1-1.33}{\infty}$$

$$\frac{1.33}{4m} + \frac{1}{q} = 0$$

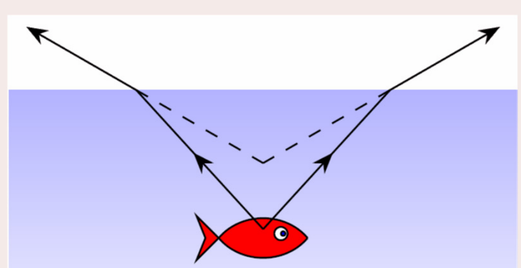
$$q = -\frac{4m}{1.33}$$

$$q = -3m$$

For an observer in air, the fish seems to be in the water, 3 m below the surface. (Note that the radius of the Earth could have been used for R because the surface of a lake follows the curvature of the Earth. The answer would then virtually be the same.)

The following diagram explains why, with refraction, the fish seems to be closer to the surface.

The end result for the position of the image is $q = -p/1.33$. Objects in water always appear to be 1.33 times closer to the surface than they are in reality. The part of the pencil in water shown previously thus seems to be closer to the surface, giving the impression that the pencil is bent.



b) What is the length of the image?

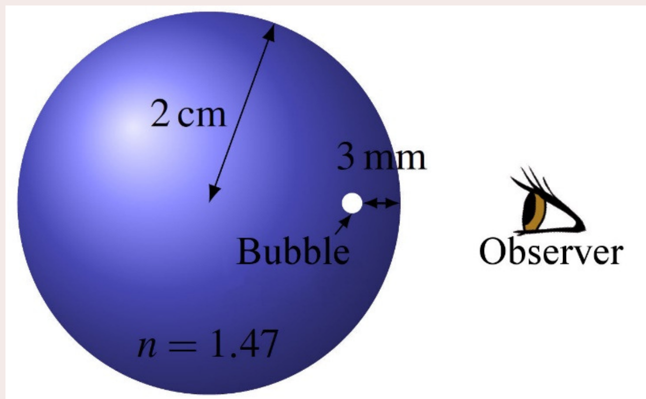
The magnification is

$$\begin{aligned} m &= -\frac{n_1 q}{n_2 p} \\ &= -\frac{1.33 \cdot (-3m)}{1 \cdot 4m} \\ &= 1 \end{aligned}$$

The fish thus seems to have the same length as the real fish.

Example 5.4.2

A bubble with a 1 mm diameter is inside a glass sphere (n = 1.47) with a 2 cm radius. The bubble is 3 mm from the edge. The observer views the bubble from the side where it is closest to the edge.



a) Where is the image of the bubble for an observer positioned as shown in the diagram?

The position is found with

$$\begin{aligned} \frac{n_1}{p} + \frac{n_2}{q} &= \frac{n_2 - n_1}{R} \\ \frac{1.47}{3mm} + \frac{1}{q} &= \frac{1 - 1.47}{-20mm} \\ q &= -2.14mm \end{aligned}$$

The bubble thus seems to be 2.14 mm from the edge, inside the glass sphere.

b) What is the diameter of the image of the bubble?

The magnification is

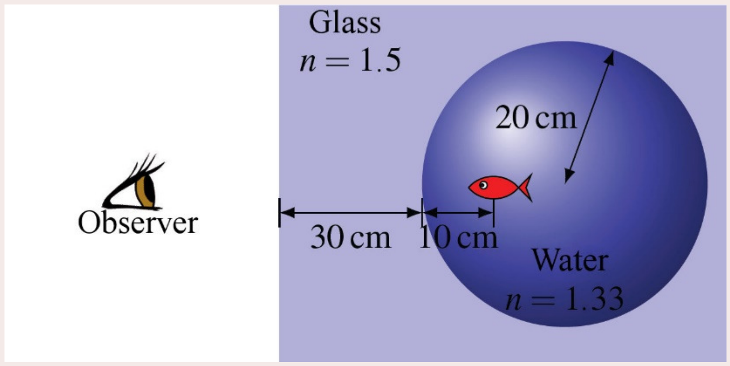
$$m = -\frac{n_1 q}{n_2 p}$$

$$= -\frac{1.47 \cdot (-2.14mm)}{1 \cdot 3mm}$$
$$= 1.0504$$

The image of the bubble is 1.0504 times greater than the object. Therefore, its diameter is 1.0504 mm.

Example 5.4.3

In the situation shown in the diagram, where is the final image of the object and what is the final magnification?



When there are several dioptres one after the other like here, you must:

1) Do the calculation one dioptre at a time.
2) Use the image of the first dioptre as the object for the second dioptre, and then use the image of the second dioptre as the object for the third dioptre, and so forth.
3) Calculate the total magnification by multiplying all the magnifications made by each dioptre.

This last rule is simply common sense. If the first dioptre creates an image 2 times larger than the object, and if the second dioptre takes this image as the object and enlarges it 10 times again, then the final image will be 20 times larger than the original object.

1st dioptre

The distance between the object and the first dioptre is

$$p_1 = 10 \text{ cm}$$

The position of the image is

$$\frac{n_1}{p_1} + \frac{n_2}{q_1} = \frac{n_2 - n_1}{R_1}$$
$$\frac{1.33}{10cm} + \frac{1.5}{q_1} = \frac{1.5 - 1.33}{-20cm}$$
$$q_1 = -10.6cm$$

The magnification is

$$m_1 = -\frac{n_1 q_1}{n_2 p_1} = -\frac{1.33 \cdot (-10.6cm)}{1.5 \cdot 10cm} = 0.9399$$

2nd dioptre

The distance between the image of the first dioptre and the second dioptre is

$$p_2 = d_1 - q_1 = 30cm - -10.6cm = 40.6cm$$

This equation for the position of the object (the distance between the dioptres minus the position of the previous image, $p_n = d_{n-1} - q_{n-1}$) is always true.

The position of the image is

$$\frac{n_1}{p_2} + \frac{n_2}{q_2} = \frac{n_2 - n_1}{R_2}$$
$$\frac{1.5}{40.6cm} + \frac{1}{q_2} = \frac{1-1.5}{\infty}$$
$$q_2 = -27.07cm$$

The magnification is

$$m_2 = -\frac{n_1 q_2}{n_2 p_2} = -\frac{-1.5 \cdot 27.07cm}{40.6cm} = 1$$

Therefore, the image of the fish is 27.07 cm behind the air-glass interface according to the observer. The total magnification is

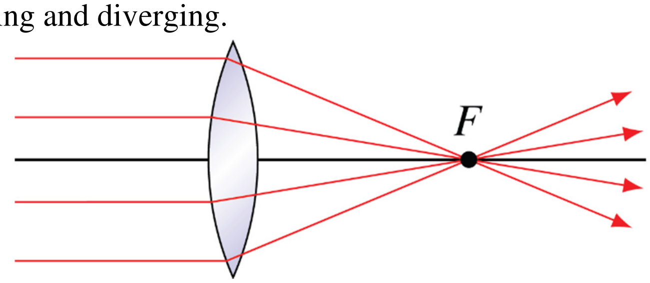
$$m_{total} = m_1 \cdot m_2 = 0.9399 \cdot 1 = 0.9399$$

5.5 THIN LENSES

Converging and Diverging Lens

There are two types of lenses: converging and diverging.

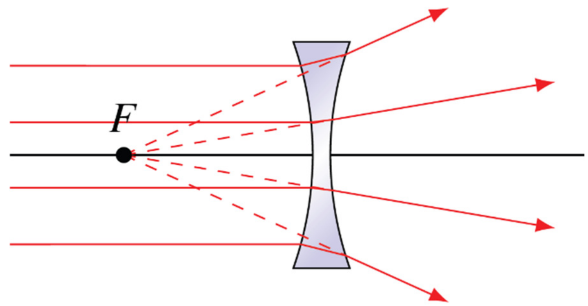
With a converging lens, parallel rays are deviated so that they meet at a point called the focus (F).



A converging lens is always thicker at its centre than at its edges (when it is in air).

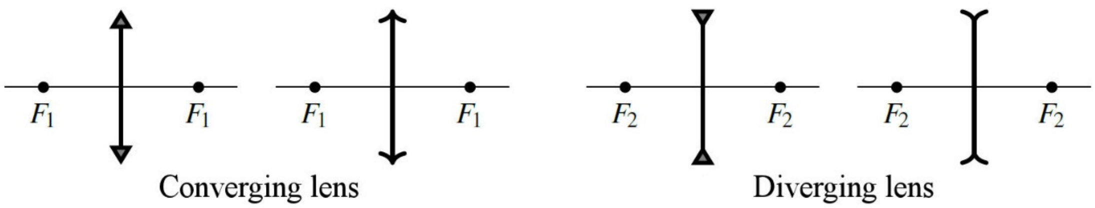
With a diverging lens, parallel rays are deviated so that they seem to come from a point called the focus (F).

A diverging lens is always thinner at its centre than at its edges (when it is in air).



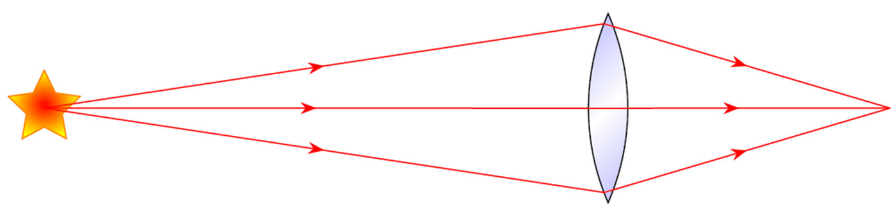
In both cases, the distance between the lens and the focus is the focal length (f). There are actually two focuses on each side of the lens and it will be proven later that, regardless of the shape of the lens, the two focuses are at the same distance from the lens on each side.

There are symbols to represent these lenses. They are shown in this diagram.



The Lens According to Fermat's Principle

According to Fermat's principle, light follows the path that takes the least time to travel from one place to another. However, with a lens, all the rays travel from the same point (object) to another (the image) following different paths.

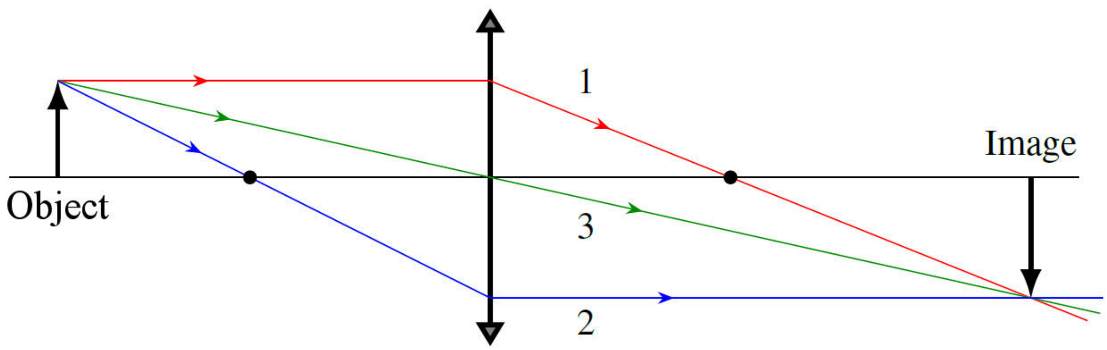


According to Fermat's principle, this simply means that all the rays leaving the object to reach the image take exactly the same time to accomplish their journey. Remember that light travelling in glass is going slower than light travelling in air. Thus, the ray passing in the centre of the lens loses a lot of time because the lens is thicker there. The ray passing near the edge of the lens has a longer path to travel, but it passes through a smaller glass thickness, which slows light for a smaller time than for the ray passing at the centre. The time lost to travel a longer path is exactly offset by a shorter slowdown in a thinner piece of glass.

Graphical Method to Find the Position of the Image

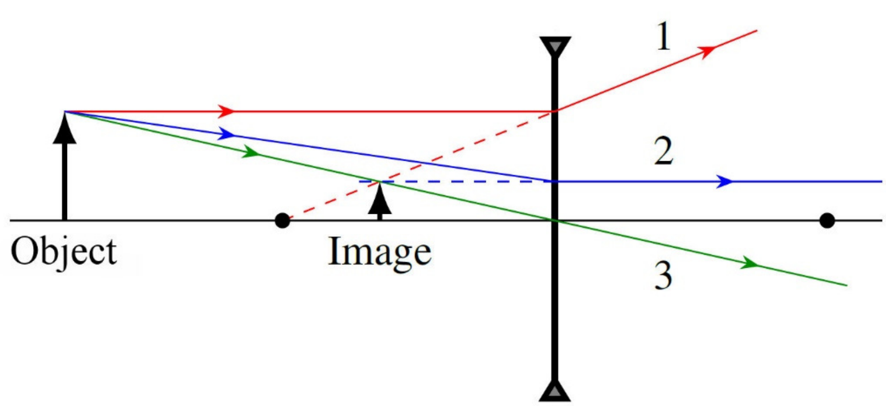
Three rays can be used to easily find the position of the image of an object. For a converging lens, these three rays are:

1) A ray parallel to the principal axis passes through the focus on the other side the lens after passing through the lens.
2) A ray passing at the focus becomes parallel to the principal axis after passing through the lens.
3) A ray passing through the centre of the lens is not deflected.



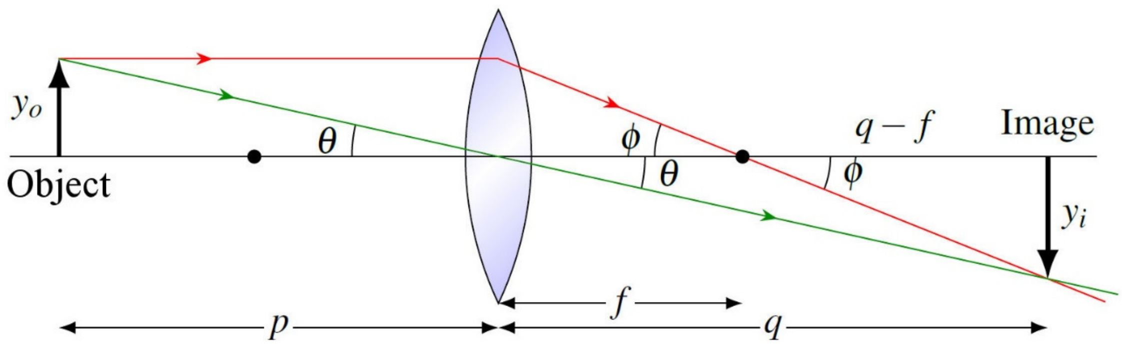
For the diverging lens, there are slight differences. The three rays are:

1) A ray parallel to the principal axis seems to come from the focus on the side where the ray originated after passing through the lens.
2) A ray travelling towards the focus on the other side the lens becomes parallel to the principal axis after passing through the lens.
3) A ray passing through the centre of the lens is not deflected.



Calculation of the Position of the Image

To find the lens equation, two principal rays are used.



The triangles with the angles θ are similar. This means that

$$\frac{y_i}{y_o}=\frac{q}{p}$$

The triangles with the angles ϕ are similar. This means that

$$\frac{y_i}{y_o}=\frac{q-f}{f}$$

Using those two equations, the following result is obtained.

$$\frac{q}{p}=\frac{q-f}{f}$$

$$\frac{1}{p}=\frac{q-f}{qf}$$

$$\frac{1}{p}=\frac{q}{qf}-\frac{f}{qf}$$

$$\frac{1}{p}=\frac{1}{f}-\frac{1}{q}$$

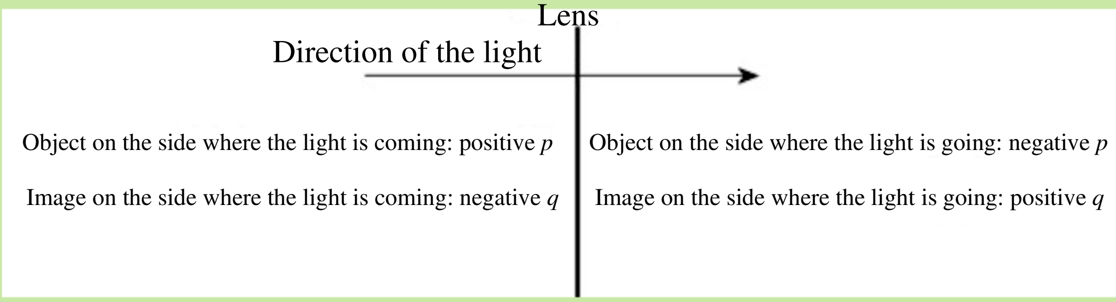
This finally gives

Thin Lens Equation

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

It is also possible to demonstrate this formula for divergent lenses or with virtual image or object and the same formula is always obtained, provided the following sign convention is respected.

Sign Convention for Lenses

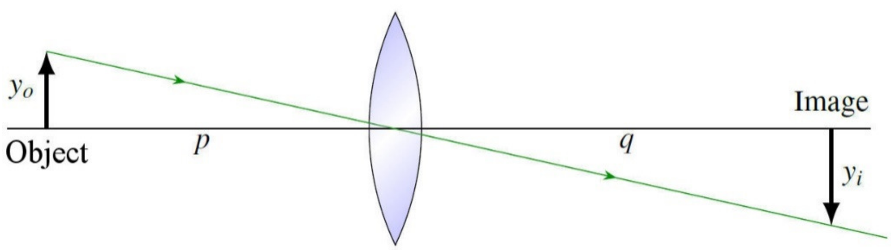


Converging lens: f is positive.

Diverging lens: f is negative.

Magnification

To find the magnification, let's consider the ray passing through the centre of the lens, i.e. the ray that is not deflected.



With these two similar triangles, the following equation is obtained

$$\frac{y_i}{y_o} = \frac{q}{p}$$

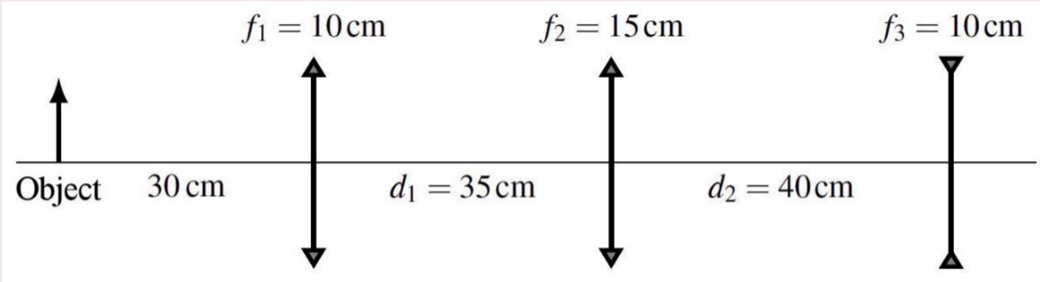
As done previously, a negative sign is added to indicate that the image is inverted. The final result is thus

Magnification With a Lens

$$m = -\frac{q}{p}$$

Example 5.5.1

In the situation shown in the diagram, where is the final image of the object and what is the final magnification?



When there are several lenses one after another, you must:

1) Do the calculation one lens at a time.
2) Take the image of the first lens as the object for the second lens, and then take the image of the second lens as the object for the third lens, and so forth.
3) Calculate the total magnification by multiplying all magnifications made by each lens.

This last rule is simply a question of common sense. If the first lens creates an image 2 times larger than the object, and if the second lens takes this image as the object and enlarges it 10 times again, then the final image will be 20 times larger than the original object.

1st lens

The distance between the object and the first lens is

$$p_1 = 30 \text{ cm}$$

The position of the image is

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}$$

$$\frac{1}{30cm} + \frac{1}{q_1} = \frac{1}{10cm}$$

$$q_1 = 15cm$$

The magnification is

$$m_1 = -\frac{q_1}{p_1} = -\frac{15cm}{30cm} = -\frac{1}{2}$$

2nd lens

The distance between the image of the first lens and the second lens is

$$p_2 = d_1 - q_1 = 35cm - 15cm = 20cm$$

This equation for the position of the object (the distance between the lens minus the position of the previous image, $p_n = d_{n-1} - q_{n-1}$) is always true.

The position of the image is

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$
$$\frac{1}{20cm} + \frac{1}{q_2} = \frac{1}{15cm}$$
$$q_2 = 60cm$$

The magnification is

$$m_2 = -\frac{q_2}{p_2} = -\frac{60cm}{20cm} = -3$$

3rd lens

The distance between the image of the second lens and the third lens is

$$p_3 = d_2 - q_2 = 40cm - 60cm = -20cm$$

The negative sign is correct because the object for the third lens is on the side where the light goes since the image of the second lens is formed at a greater distance from the second lens than the distance between the lenses. (Actually, the image is never formed because the rays will be deflected by the third lens before they meet.) So, for those who wondered how a negative *p* was possible, here is the answer: *p* is negative when an image is formed further away than the next lens or mirror.

The position of the image is

$$\frac{1}{p_3} + \frac{1}{q_3} = \frac{1}{f_3}$$
$$\frac{1}{-20cm} + \frac{1}{q_3} = \frac{1}{-10cm}$$
$$q_3 = -20cm$$

The magnification is

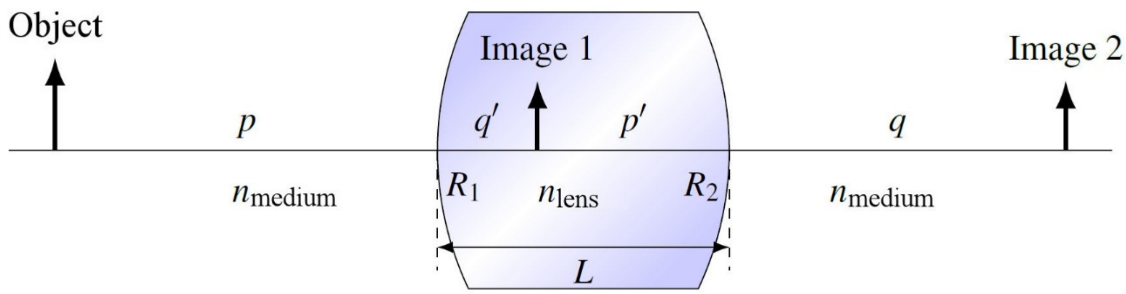
$$m_3 = -\frac{q_3}{p_3} = -\frac{-20cm}{-20cm} = -1$$

Therefore, the image is 20 cm to the left of the third lens (it is therefore located halfway between the second and the third lens). The total magnification is

$$m_{total} = m_1 m_2 m_3 = \left(-\frac{1}{2}\right)\cdot(-3)\cdot(-1) = -\frac{3}{2}$$

Calculation of the Focal Length of a Lens

Lenses are in fact two dioptres one after another. Therefore, to find the focal length of a lens, the dioptre equation will be applied twice. When doing this, the image obtained with the first dioptre (image 1) is used as the object for the second dioptre. The final image (Image 2) is the image formed by the lens.



For the first dioptre, the equation is

$$\frac{n_m}{p} + \frac{n_l}{q'} = \frac{n_l - n_m}{R_1}$$

where n_m in the refractive index of the medium where the lens is, and n_l is the refractive index of the substance composing the lens.

For the second dioptre, the equation is

$$\frac{n_l}{p'} + \frac{n_m}{q} = \frac{n_m - n_l}{R_2}$$

$$\frac{n_l}{L - q'} + \frac{n_m}{q} = \frac{n_m - n_l}{R_2}$$

Here, only thin lenses are considered. Actually, they will be considered so thin that their thickness will be neglected. This means that $L = 0$ will be used in the equation.

The equation for the second dioptre then becomes

$$\frac{n_l}{-q'} + \frac{n_m}{q} = \frac{n_m - n_l}{R_2}$$

The two dioptre equations are then added to obtain

$$\frac{n_m}{p}+\frac{n_1}{q'}+\frac{n_1}{-q'}+\frac{n_m}{q}=\frac{n_l-n_m}{R_1}+\frac{n_m-n_l}{R_2}$$

Two terms cancel each other to give

$$\frac{n_m}{p}+\frac{n_m}{q}=\frac{n_l-n_m}{R_1}+\frac{n_m-n_l}{R_2}$$

$$\frac{1}{p}+\frac{1}{q}=\left(\frac{n_l-n_m}{n_m}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

But since

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

The equation becomes

$$\frac{1}{f}=\left(\frac{n_l-n_m}{n_m}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

This equation is called the *lens maker's equation*. Since the power of a lens (*P*, in dioptres (D, which are m^{-1})) is defined as the inverse of the focal length, we have

Focal Length and Power of a Lens (Lens Maker's Equation)

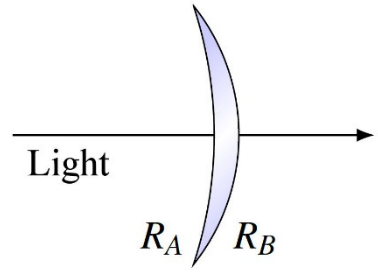
$$P=\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

where n_m in the refractive index of the medium where the lens is, n_l is the refractive index of the substance composing the lens, R_1 is the radius of curvature of the first surface encountered by the light, and R_2 is the radius of curvature of the second surface encountered by the light. The signs of the radii of curvature are important, and they follow the same sign convention as for dioptres.

Proof That the Focal Distance of the Two Focuses is the Same

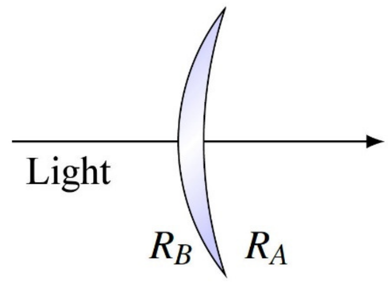
It can now be demonstrated that the two focuses of a lens are at the same distance on each side of the lens. Let's consider a lens and imagine that light passes in one direction.

The light first encounters a surface with a radius of curvature R_A and then a surface with a radius of curvature R_B. Therefore, the focal distance for light travelling in this direction is



$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_A}-\frac{1}{R_B}\right)$$

If the lens is now inverted, the light first meets the surface with a radius of curvature R_B and then the surface with a radius of curvature R_A. In addition, by reversing the lens, the signs of the radii of curvature are inverted, as the centres of curvature of the surfaces change sides. In our example, they went from the side where the light is coming to the side where the light goes. Thus, the focal distance for light passing in this direction is



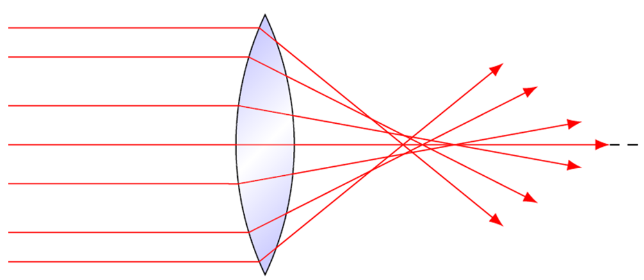
$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{(-R_B)}-\frac{1}{(-R_A)}\right)$$

$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_A}-\frac{1}{R_B}\right)$$

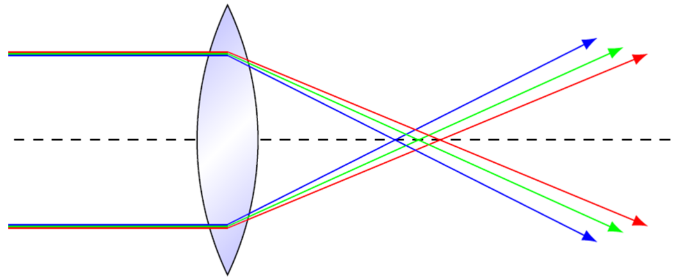
which is the same value as for the light passing in the other direction. This proves that the focal lengths are the same on each side.

Aberrations

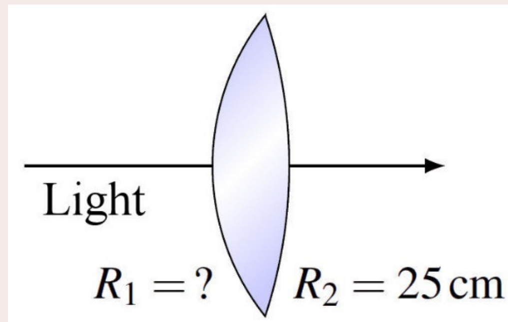
Because spherical lenses are made up of two dioptres, they suffer from the same problem as the dioptres. This means that there is also spherical aberration with spherical lenses.



Dispersion is also a problem. Different colours make slightly different refraction so that the position of the focus is different for each colour. The refractive index for blue light being greater than for red light (for most material), the focus for blue light is closer to the lens than the focus for red light. This is called *chromatic aberration*. Note that diopters are also affected by chromatic aberration.



Example 5.5.2



A lens in air has a power of 10 D and is made of a material with a refractive index of 1.5.

a) If one surface has the radius of curvature shown in the diagram, what is the radius of curvature of the other surface?

If it is assumed that the light passes from the left to the right in the lens, the lens maker's equation gives

$$P = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$10D = \frac{1.5-1}{1}\cdot\left(\frac{1}{R_1} - \frac{1}{-0.25m}\right)$$
$$R_1 = 0.0625m = 6.25cm$$

A positive value means that the centre of curvature is on the side where the light goes, therefore to the right of the lens in the diagram. The diagram then shows correctly what this lens looks like.

b) What is the power of this lens in water?

In water, the power of this lens is

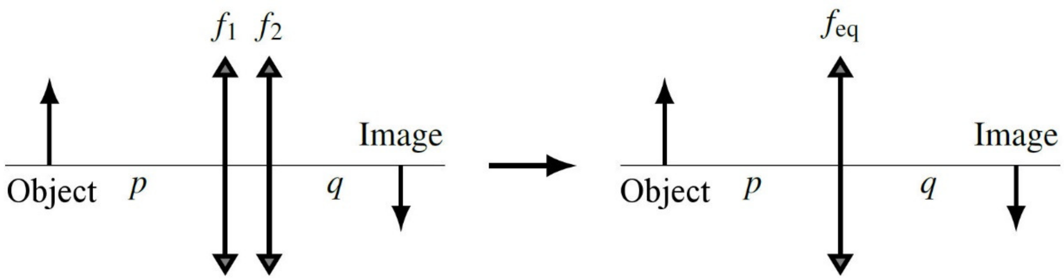
$$P = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$= \frac{1.5-1.33}{1.33}\cdot\left(\frac{1}{0.0625m} - \frac{1}{-0.25m}\right)$$
$$= 2.56D$$

This power corresponds to a 39.11 cm focal length.

If one day you want to take into account the thickness of the lens, you can check out this document on the thick lenses equation.
http://physique.merici.ca/waves/thick_lens.pdf

Compound Lens

The equivalent power of two lenses placed in contact, one after the other will now be sought. In other words, the equivalent power of the lens that would make an image exactly at the same place as the final position of the image with both lenses will be calculated.



For the compound lens (to the left), the position of the image is calculated by considering one lens at a time. For the first lens

$$\frac{1}{p}+\frac{1}{q_1}=\frac{1}{f_1}$$

For the second lens, the position of the image is

$$p_2=d-q_1$$

Therefore,

$$\frac{1}{p_2}+\frac{1}{q}=\frac{1}{f_2}$$
$$\frac{1}{d-q_1}+\frac{1}{q}=\frac{1}{f_2}$$

As the lenses are touching, the distance d is $d = 0$. The equation for the second lens then becomes

$$\frac{1}{-q_1}+\frac{1}{q}=\frac{1}{f_2}$$

Adding those two lens equation,

$$\left(\frac{1}{p}+\frac{1}{q_1}\right)+\left(\frac{1}{-q_1}+\frac{1}{q}\right)=\frac{1}{f_1}+\frac{1}{f_2}$$

the result is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f_1}+\frac{1}{f_2}$$

For the equivalent lens, the equation is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f_{eq}}$$

By comparing these last two results, the following result is obtained.

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$

Generalizing, the final result is

Compound Lens

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\frac{1}{f_4}+\dots \qquad P_{eq}=P_1+P_2+P_3+P_4+\dots$$

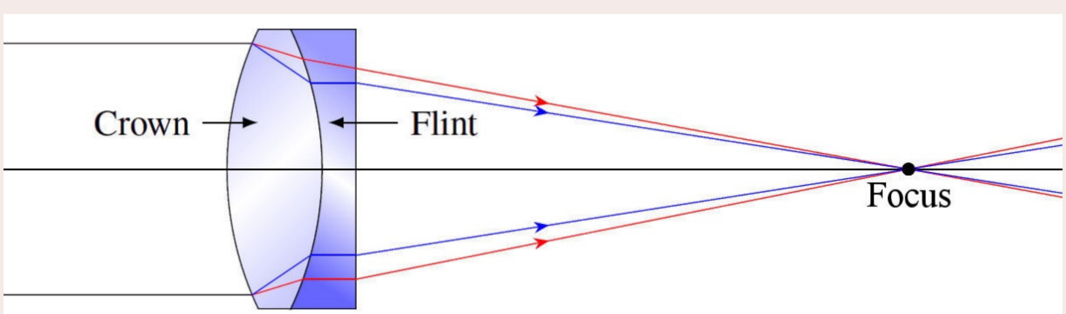
Example 5.5.3

It is possible to make a lens that does not have any chromatic aberration by using a compound lens made of different types of glass. This is what we'll try to do here with a compound lens made of Flint glass and Crown glass. Here are the properties of these 2 types of glass.

Wavelength	***Crown***	***Flint***
759.370 nm	1.5089	1.6391
396.847 nm	1.5314	1.6886

It can be noted that for these types of glass, the indices are greater for smaller wavelengths, as is often the case.

The compound lens will have the following shape.



The contact between the two lenses is perfect (the radius of curvature of both lenses is the same at this place) and the right side of the Flint glass lens is flat

What must be the radii of curvature of the lens made of Crown glass so that the focal length is 25 cm for both the 759.370 nm wavelength and the 396.847 nm wavelength?

A 25 cm focal length corresponds to a 4 D power. As the power of the compound lens is the sum of the two powers of each lens, the following equation is obtained.

$$\begin{aligned} 4D &= P_{crown} + P_{flint} \\ &= (n_C - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_F - 1)\left(\frac{1}{R_3} - \frac{1}{R_4}\right) \end{aligned}$$

Since $R_3 = R_2$ and $R_4 = \infty$, this equation is

$$\begin{aligned} 4D &= P_{crown} + P_{flint} \\ &= (n_C - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + (n_F - 1)\left(\frac{1}{R_2}\right) \\ &= (n_C - 1)\frac{1}{R_1} + (n_F - n_C)\frac{1}{R_2} \end{aligned}$$

At 759.370 nm, this equation gives

$$4D = (1.5089 - 1)\cdot\frac{1}{R_1} + (1.6931 - 1.5089)\cdot\frac{1}{R_2}$$

$$4D = 0.5089\cdot\frac{1}{R_1} + 0.1302\cdot\frac{1}{R_2}$$

At 396.847 nm, the equation gives

$$4D = (1.5314 - 1)\cdot\frac{1}{R_1} + (1.6886 - 1.5314)\cdot\frac{1}{R_2}$$

$$4D = 0.5341\cdot\frac{1}{R_1} + 0.1572\cdot\frac{1}{R_2}$$

So, we have 2 equations and 2 unknowns. We first solve for $1/R_2$ in the first equation

$$\frac{1}{R_2} = 30.722D - 3.909\cdot\frac{1}{R_1}$$

and substitute in the second equation.

$$4D = 0.5341 \cdot \frac{1}{R_1} + 0.1572 \cdot \left(30.722D - 3.909 \cdot \frac{1}{R_1} \right)$$

Now, this equation can be solved for R_1.

$$4D = 0.5341 \cdot \frac{1}{R_1} + 4.8295D - 0.6144 \cdot \frac{1}{R_1}$$

$$0.08033 \cdot \frac{1}{R_1} = 0.8295D$$

$$R_1 = 9.68cm$$

Finally, R_2 can be found.

$$\frac{1}{R_2} = 30.722D - 3.909 \cdot \frac{1}{0.09686m}$$

$$R_2 = -10.38cm$$

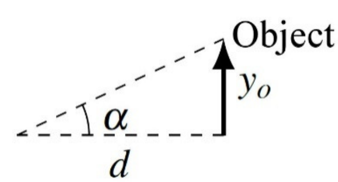
5.6 MAGNIFYING GLASSES, MICROSCOPES, AND TELESCOPES

Formula for the Angular Magnification of a Magnifying Glass

Two elements must be taken into account to determine if it is easier to see the details of an object.

1) The size of the object: If there are two identical objects at the same distance but one of them is twice as large as the other, the details on the largest object are easier to see.
2) The distance of the object: If there are two identical objects but one is closer than the other, the details of the object which is closer are easier to see.

In fact, the angle subtended by the object combines these two elements. If the object gets larger, then the angle increases and if the object gets closer, then the angle increases.



Therefore, to see more details, the subtended angle must be increased. With a physical object, there is not much choice: the object must be brought closer to the eyes (since the size of the object cannot be changed). However, there is a maximum angle since the human eye cannot clearly see objects if they are too close. The smallest distance that there can be between an object and an eye while the object is still clearly seen is called d_{pp}, where *pp* stands for punctum proximum or near point. Therefore, the maximum angle is

$$\tan \alpha = \frac{y_o}{d_{pp}}$$

Small details on the object interest us, and so the angle subtended by these small details is never great. Thus, the angle is small and the maximum angle can be written as

$$\alpha = \frac{y_o}{d_{pp}}$$

The angle can be increased by using a lens. By looking at the image instead of the object, the subtended angle can be increased and more details can be seen. The angle subtended by the image is noted β.

1st Possibility: Using a Real Image

A larger real image can be formed with a lens. If an observer looks at this image, projected onto a screen, then he will see more details. This is what happens when an overhead projector, a slide projector, or a computer screen projector is used.

Again, the subtended angle can be increased by looking at the image more closely but without being closer than d_{pp}. Thus, the maximum subtended angle that can be obtained in this way is

$$\beta = \frac{y_i}{d_{pp}}$$

The angular magnification is defined as the ratio between the subtended angle with a lens and the maximum subtended angle without a lens.

Angular Magnification

$$G = \frac{\beta}{\alpha}$$

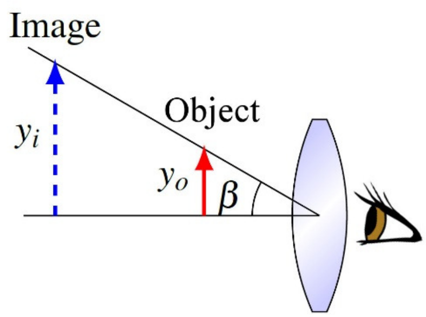
With a real image, the maximum angular magnification is, therefore,

$$G_{\max} = \frac{\beta}{\alpha} = \frac{y_i / d_{pp}}{y_o / d_{pp}} = \frac{y_i}{y_o}$$

which is the same thing as the absolute value of the magnification.

2nd Possibility: Using a Virtual Image

A virtual image can also be used. In this case, the observer looks at the image through the lens, which then acts as a magnifying glass. Only the case of a person looking at the image with its eyes close to the lens will be considered here. There is little doubt that more details will be seen then because the eyes are closest to the image, which makes the subtended angle as large as possible.



Using the ray passing through the centre of the lens (which is not deflected) the subtended angle is

$$\beta = \frac{y_i}{-q} = \frac{y_o}{p}$$

The object is at a distance *p* and the image at a distance -*q* (a minus sign is needed to change the sign of *q* since *q* is negative for a virtual image).

The angular magnification is thus

$$G = \frac{\beta}{\alpha} = \frac{y_0 / p}{y_0 / d_{pp}}$$

This leads to

Angular Magnification of a Magnifying Glass

$$G = \frac{d_{pp}}{p}$$

There are two limit cases since the image must be between q = - infinity and $q = -d_{pp}$ (so that it can be seen clearly by the observer).

If the image is at an infinite distance, the position of the object is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{-\infty} = \frac{1}{f}$$

$$p = f$$

The object is, therefore, at the focus. Substituting in the formula of the angular magnification, the result is

Minimum Angular Magnification of a Magnifying Glass

$$G_{\min} = \frac{d_{pp}}{f}$$

The object must be at the focus ($p = f$) to obtain this angular magnification.

If the image is as close as possible to the eye of the observer ($q = -d_{pp}$), then

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{-d_{pp}} = \frac{1}{f}$$

$$\frac{1}{p} = \frac{1}{f} + \frac{1}{d_{pp}}$$

$$\frac{1}{p} = \frac{d_{pp} + f}{d_{pp} f}$$

The angular magnification is then

$$\begin{aligned} G &= \frac{d_{pp}}{p} = d_{pp} \cdot \frac{1}{p} \\ &= d_{pp} \frac{d_{pp} + f}{d_{pp} f} \\ &= \frac{d_{pp} + f}{f} \end{aligned}$$

The result is

Maximum Angular Magnification of a Magnifying Glass

$$G_{\max} = \frac{d_{pp}}{f} + 1$$

The object must be closer than the focus $\left(p = \dfrac{d_{pp} f}{d_{pp} + f} \right)$ to have this magnification.

Example 5.6.1

A person having a 24 cm d_{pp} uses a lens whose focal length is 4 cm to examine the details on an ancient coin. What are the minimum and maximum angular magnifications that can be achieved and where should he place the coin to get these magnifications?

The minimum angular magnification is

$$G_{\min} = \frac{d_{pp}}{f}$$

$$= \frac{24cm}{4cm}$$
$$= 6$$

and it is obtained by placing the object at the focus, so 4 cm from the lens.

The maximum angular magnification is

$$G_{\max} = \frac{d_{pp}}{f} + 1$$
$$= \frac{24cm}{4cm} + 1$$
$$= 7$$

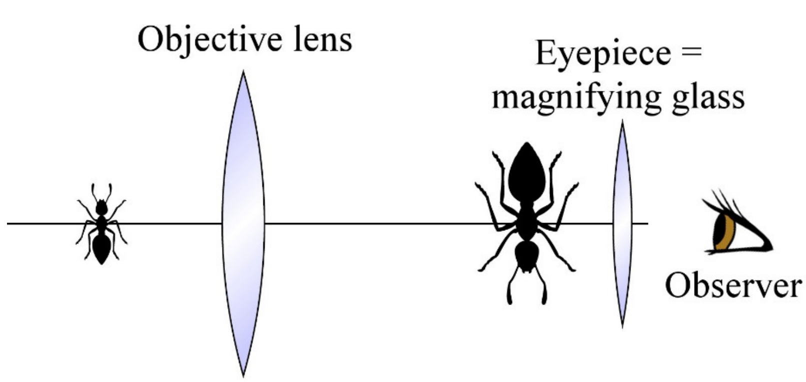
and it is obtained by placing the object at the position given by

$$p = \frac{d_{pp} f}{d_{pp} + f}$$
$$= \frac{24cm \cdot 4cm}{4cm + 24cm}$$
$$= 3.429cm$$

The coin can then be anywhere between 3.429 cm and 4 cm from the lens. He then gets an angular magnification between 6 and 7, according to the position.

Microscope

It is possible to do even better than with a simple magnifying glass by using the following trick. A lens (objective lens) is used to form a real image larger than the object and a magnifying glass (the eyepiece) is then used to examine this image. Thus, the angular magnification obtained is a little better than with a simple magnifying glass because an image larger than the object is examined with the magnifying glass. This is the basic principle of the microscope.

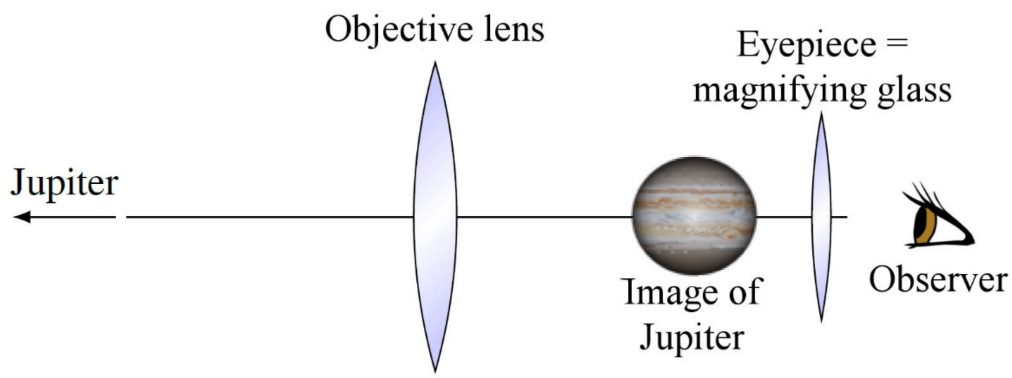


Note that in this case, the image observed is inverted relative to the object. Microscopes that correct this inversion with another lens or a mirror exist.

Telescopes

It is useless to observe a celestial object with a magnifying glass. Remember that the object observed must be at the focus or a little closer to the magnifying glass than the focus. If an astronomer wants to observe Jupiter with a magnifying glass, he would then have to use a lens with a focal length equal to the distance between Earth and Jupiter. The formula of the maximum magnification then indicates that the magnification is 1 with a very large focal length. Not great…

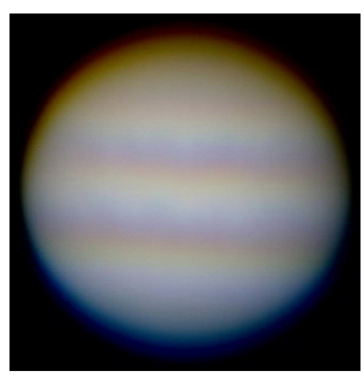
It is possible to do better with the following trick. Using a lens (objective) or a mirror, a real image of a celestial object is created. Then, this image is looked at with a magnifying glass.



The image is much smaller than the object, but so much angular magnification can be obtained with the magnifying glass that, ultimately, more details can be seen.

With a telescope, a large focal distance is better for the objective lens. Then, a larger image can be obtained. That is why telescopes are usually very long. On the other hand, a very small focal distance is better for the eyepiece to obtain a larger magnification.

For large telescopes, a mirror is always used to make the real image of the celestial body because there is no chromatic aberration with a mirror. With a lens, there is a slight colour separation when the light is refracted, which gives the appearance shown in the image. It can be seen that there is a slight colour separation.



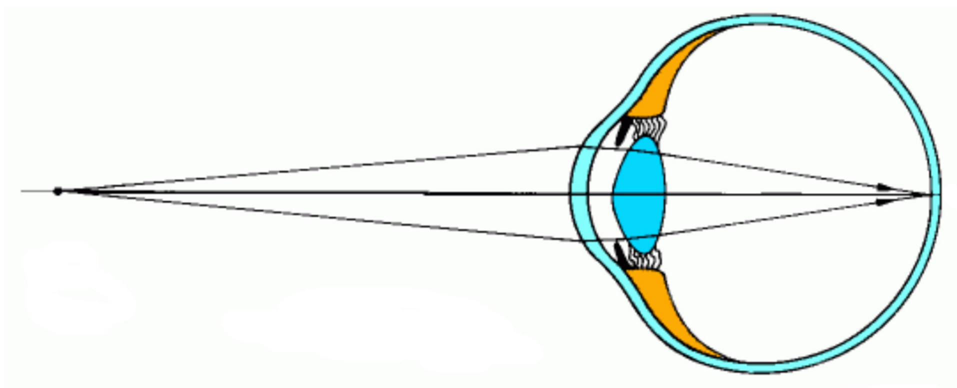
www.astrosystems.nl/projects_products/accessories/dispersion%20corrector/dispersie_correct.htm

Once again, the image observed is inverted relative to the object.

5.7 THE EYE

Image Formation

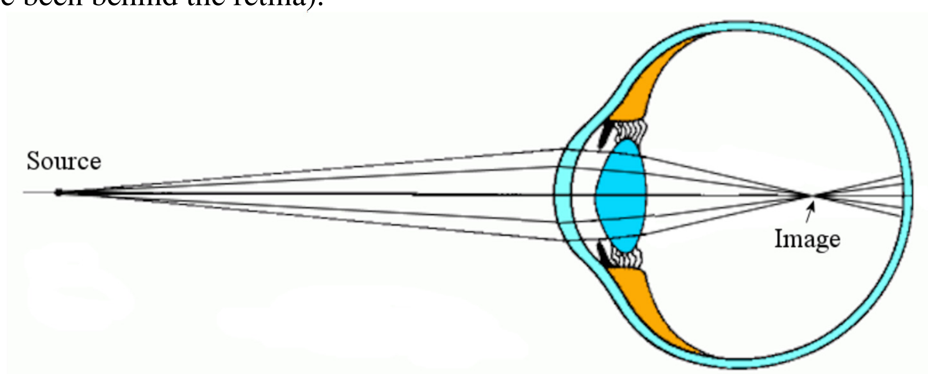
To clearly see an object, its image must be formed exactly on the retina in the back of the eye. In this way, all the rays coming from a bright point are all acting on the same receptor cell in the retina.



www.lhup.edu/~dsimanek/scenario/miscon.htm

If a single cell receives the light, a small bright spot is seen. It's like if all the white light coming from a very small source arrive on the same pixel in a camera. The resulting photo would then be a simple white pixel, and there would be only one point.

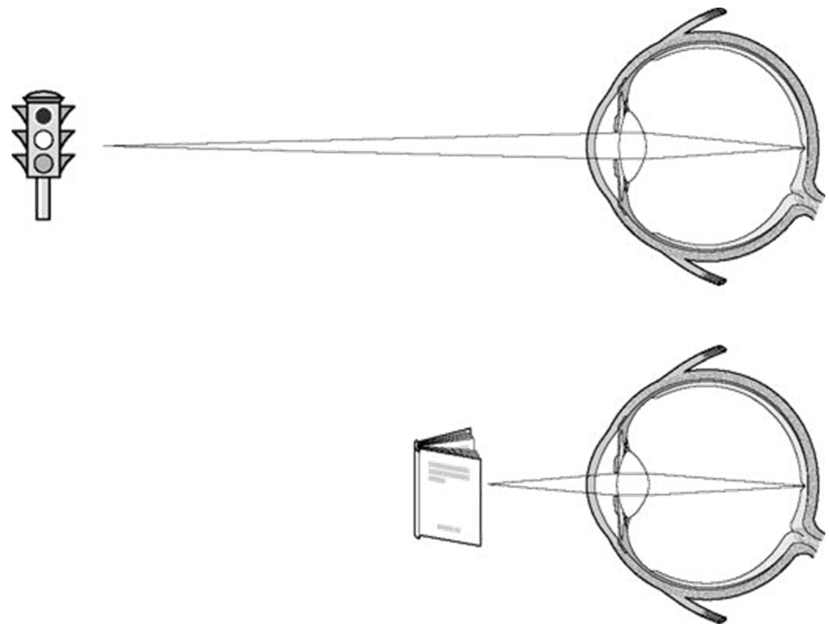
If the image is not formed on the retina, the following situation occurs (the image could also have been behind the retina).



www.lhup.edu/~dsimanek/scenario/miscon.htm

In this case, several light receptor cells receive the rays coming from a single point, and a fuzzy bright spot is seen. It's like if white light coming from a very small source arrive on several pixels in a camera. Then, there would be many white pixels in the photo and a white spot would be seen instead of a single point.

Refraction concentrates the rays on the retina. A large part of the refraction occurs when the light enters the eyeball. The eye is actually a dioptre (roughly spherical). A power of about 44 dioptres comes from this dioptre. Then comes the crystalline lens (dark blue part in the last diagram) acting like a lens. This lens can change its shape by becoming more or less curved, thereby changing its power. The power must change so that objects can be seen clearly at different distances. As the position of the image is fixed (on the retina), the focal length of the lens must change if the position of the object changes. In the next diagram, you can see how the curvature of the crystalline lens changes when an object gets closer to the eye. It is clear that the radius of curvature of the lens decreases a bit if the person looks at something closer. This change allows the image to always be focused on the retina.



www.eyecairo.net/accommodation.html

However, there is a limit to the change in the shape of the lens and this is why a person cannot see objects clearly when they are too close to the eye. The power of the crystalline lens can vary between 8 and 14 dioptres (for young people).

Power of Accommodation

The power of accommodation of the eye indicates by how much the power of the eye can change. This change of power comes from the change in the shape of the crystalline lens. To know the power of accommodation of the eye, let's first consider what happens when a person looks at an object when it is as close as possible to the eye, without any blur. This minimum distance between the eye and the object is d_{pp}. In this case, the eye has its maximum power, and the lens equation gives

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{d_{pp}}+\frac{1}{q}=\frac{1}{f_{\min}}=P_{\max}$$

Let's now consider what happens when the person looks at an object as far as possible from the eye without any blur. For many people, there is no limit: they can watch a star hundreds of light years away and it is not blurred. On the other hand, there is a limit for shortsighted people. For them, everything is blurry beyond a certain distance. The maximum distance between the eye and the object is called d_{pr}, where *pr* stands for punctum remotum, which translates as far point. In this case, the eye has its minimum power, and the lens equation is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{d_{pr}}+\frac{1}{q}=\frac{1}{f_{\max}}=P_{\min}$$

By subtracting these powers, the power of accommodation of the eye, i.e. the change of power, is obtained

$$P_{acc}=P_{\max}-P_{\min}$$
$$P_{acc}=\left(\frac{1}{d_{pp}}+\frac{1}{q}\right)-\left(\frac{1}{d_{pr}}+\frac{1}{q}\right)$$

The result is

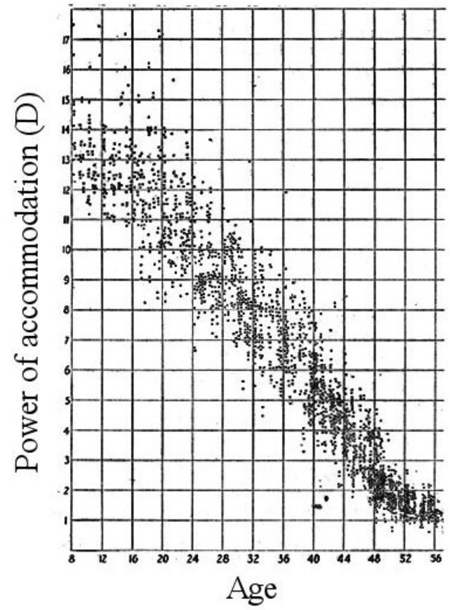
Power of Accommodation of the Eye

$$P_{acc}=P_{\max}-P_{\min}=\frac{1}{d_{pp}}-\frac{1}{d_{pr}}$$

This capacity represents the variation of the power of the eye due to the change in the shape of the crystalline lens. As the crystalline lens tends to become stiffer with age, the power of accommodation of the eye decreases with age. The following values are typical.

Age	Power of Accommodation (D)	d_{pp}
10	13	7.5 cm
20	11	9.1 cm
30	9	11 cm
40	6	18 cm
50	2	50 cm
60	1	90 cm

In other words, the power of accommodation decreases slowly with age, thereby increasing the d_{pp}. Reading glasses are thus required from a certain age, usually between 40 and 50.

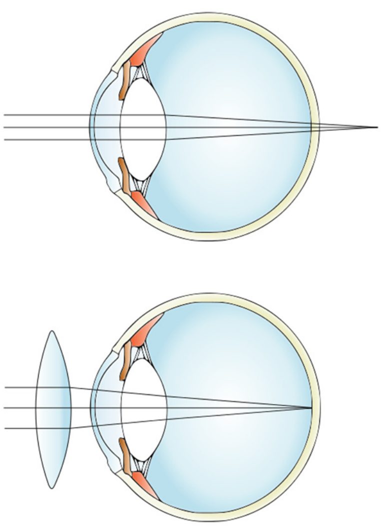


web.ncf.ca/aa456/misc/cataracts/

Vision Problems: Near Objects Are Not Seen Clearly

Some people see clearly objects far away from their eyes, but not the objects that are near them. Obviously, everyone has a limit (d_{pp}) but this limit should be smaller than 25 cm. A person with a d_{pp} greater than 25 cm suffers from farsightedness.

visianinfo.com/myopia-nearsightedness/



en.wikipedia.org/wiki/Hyperopia

This problem can be fixed with a lens. A blurred near object means that the eye does not have enough power and that the image is formed behind the retina. A converging lens that increases the power of the eye will help to form the image on the retina (image to the left). Let's find the power of the lens needed to correct the problem.

When a farsighted person looks at an object as close as possible without it being blurry, without glasses, the equation is,

$$\frac{1}{d_{pp}}+\frac{1}{q}=P_{\max}$$

With glasses, the equation is

$$\frac{1}{d'_{pp}}+\frac{1}{q}=P_{\max}+P_{gla}$$

where the compound lens approximation was used to calculate the resulting power of the eye with glasses. The new d_{pp} with glasses is noted d'_{pp}. If the two equations are subtracted (the second minus the first), the result is

Power of the Glasses Needed to Correct Farsightedness

$$P_{gla}=\frac{1}{d'_{pp}}-\frac{1}{d_{pp}}$$

Example 5.7.1

Anthony cannot see clearly objects closer than 40 cm. He wants to see clearly objects as close as 20 cm. What is the power of the glasses that he must wear?

The power of the glasses is

$$\begin{aligned}P_{gla}&=\frac{1}{d'_{pp}}-\frac{1}{d_{pp}}\\&=\frac{1}{0.2m}-\frac{1}{0.4m}\\&=2.5D\end{aligned}$$

The positive power tells us that these are converging lenses. These lenses are quite easily spotted since they produce a larger image of the eyes (see the picture).

forum.bodybuilding.com/showthread.php?t=109576461

There are two main causes of farsightedness problems: hyperopia or presbyopia.

Hyperopia occurs when the front of the eye has not enough curvature compared to the depth of the eye. The power of the cornea is thus reduced, and the image is formed behind the retina. Sometimes, even with a great power of accommodation, the crystalline lens is not powerful enough to compensate for the small curvature of the cornea.

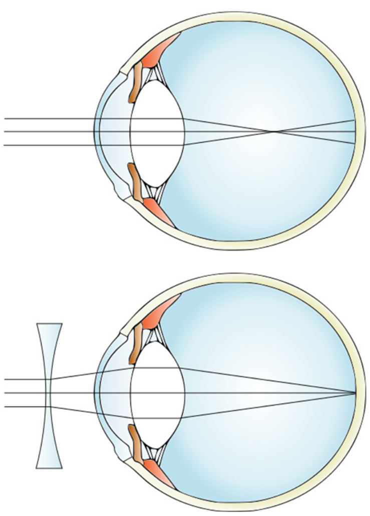
Presbyopia is the result of the loss of power of accommodation of the crystalline lens with age. With age, the power decreases and the d_{pp} increases. Generally, the d_{pp} becomes too large for 40-year-old people (approximately) and the problem must be corrected with glasses. A person is considered to have presbyopia if the power of accommodation is less than 4 D.

Vision Problems: Far Objects Are Not Seen Clearly

In this situation, people clearly see objects not too far from them, but cannot clearly see objects located beyond a certain distance (d_{pr}). They suffer from myopia or nearsightedness.

In this case, the curvature of the eye is too great relative to the depth of the eye. Thus, the cornea has too much power and the image is created in front of the retina, even with a crystalline lens at minimum power.

www.atheistrev.com/2012/12/obstacles-to-atheist-activism-myopia.html



en.wikipedia.org/wiki/Myopia

This problem can also be fixed with a lens. This time, the power of the eye must be reduced, and this can be done with a diverging lens (whose power is negative). Let's find the power of the lens needed to correct the problem.

When a myopic person looks at an object as far as possible without it being blurry, without glasses, the equation is,

$$\frac{1}{d_{pr}} + \frac{1}{q} = P_{\min}$$

With glasses, the equation is

$$\frac{1}{d'_{pr}} + \frac{1}{q} = P_{\min} + P_{gla}$$

The new d_{pr} with glasses is noted d'_{pr}. If the two equations are subtracted (the second minus the first), the result is

$$P_{gla} = \frac{1}{d'_{pr}} - \frac{1}{d_{pr}}$$

Most of the time, people want to see clearly objects that are very far away, and so d'_{pr} must be equal to infinity. The end result is

Power of the Glasses Needed to Correct Nearsightedness

$$P_{gla} = -\frac{1}{d_{pr}}$$

Example 5.7.2

Gordon cannot see clearly objects that are farther than 2 m from him. What is the power of the glasses that will enable him to see very distant objects clearly?

The power of the glasses is

$$\begin{aligned} P_{gla} &= -\frac{1}{d_{pr}} \\ &= -\frac{1}{2m} \\ &= -0.5D \end{aligned}$$

The negative power obtained confirms that these are diverging lenses. There are quite easily spotted since these lenses create a smaller image of the eyes. The person in this picture has glasses whose power is -26,5 D!

public.fotki.com/Russian-GWG/06---glasses-and-lenses/minus-glasses-by-dioptrs/minus-26-glasses/minus-26-5-planet.html

d_{pp} and d_{pr} with Glasses

Note that the power of accommodation remains the same with glasses since it represents the change in power of the crystalline lens. Wearing glasses or not does not make the crystalline lens less powerful or more powerful.

With glasses, d_{pp} and d_{pr} change. When a person looks at an object at d'_{pp} with glasses, the lens equation gives

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{d'_{pp}}+\frac{1}{q}=P_{\max}+P_{gla}$$

When a person looks at an object at d'_{pr} with glasses, the lens equation is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{d'_{pr}}+\frac{1}{q}=P_{\min}+P_{gla}$$

If these two equations are subtracted (the first minus the second), the result is

Power of Accommodation of the Eye With Glasses

$$P_{acc}=\frac{1}{d'_{pp}}-\frac{1}{d'_{pr}}$$

Example 5.7.3

Ossa-Alizée sees clearly between 10 cm and 40 cm.

a) What is the power of the glasses that she must wear?

Ossa is obviously nearsighted (myopic) because she does not see clearly objects farther than 40 cm. Let's correct this problem.

$$\begin{aligned}P_{gla}&=-\frac{1}{d_{pr}}\\&=-\frac{1}{0.4m}\\&=-2.5D\end{aligned}$$

b) What will the new d_{pp} and d_{pr} be with glasses?

The power of accommodation is

$$\begin{aligned} P_{acc} &= \frac{1}{d_{pp}} - \frac{1}{d_{pr}} \\ &= \frac{1}{0.1m} - \frac{1}{0.4m} \\ &= 7.5D \end{aligned}$$

(Ossa-Alizée is not presbytic, she is just myopic since the power of accommodation is larger than 4 D.)

With glasses to correct myopia, the d_{pr} is now at infinity and the power of accommodation of the eye remains the same. Therefore,

$$\begin{aligned} P_{acc} &= \frac{1}{d'_{pp}} - \frac{1}{d'_{pr}} \\ 7.5D &= \frac{1}{d'_{pp}} - \frac{1}{\infty} \\ d'_{pp} &= 13.3cm \end{aligned}$$

With her glasses, Ossa-Alizée then sees clearly between 13.3 cm and infinity.

Example 5.7.4

Edmond sees clearly between 40 cm and infinity. Tests have shown that the power of accommodation of his eyes is 3 D.

a) What must be the power of his glasses if he wants to have a d_{pp} at 20 cm?

Edmond cannot see near objects. To correct this problem, the power of the glasses needed is

$$\begin{aligned} P_{gla} &= \frac{1}{d'_{pp}} - \frac{1}{d_{pp}} \\ &= \frac{1}{0.2m} - \frac{1}{0.4m} \\ &= +2.5D \end{aligned}$$

b) What will the new d_{pp} and d_{pr} be with glasses?

With his glasses, the power of accommodation remains the same. Then

$$P_{acc} = \frac{1}{d'_{pp}} - \frac{1}{d'_{pr}}$$

$$3D = \frac{1}{0.2m} - \frac{1}{d'_{pr}}$$

$$d'_{pr} = 50cm$$

With his glasses, Edmond can see clearly between 20 cm and 50 cm. If he wants to see objects farther away than 50 cm clearly, he must remove his glasses.

Note: a Negative d_{pr}?

Sometimes, it is possible to obtain a negative d_{pr} with a calculation. In the last example, Edmond has a d_{pp} at 40 cm and a power of accommodation of 3 D. If the value of d_{pr} without glasses is calculated, the result is

$$P_{acc} = \frac{1}{d_{pp}} - \frac{1}{d_{pr}}$$

$$3D = \frac{1}{0.4m} - \frac{1}{d_{pr}}$$

$$d_{pr} = -2m$$

This seems ridiculous but it actually means that when Edmond looks at objects far away, the crystalline lens is not at its minimum power. The power of the eye could still decrease, but it would be useless to do so. For Edmond, the lens is 0.5 D above its minimum power when he looks at objects far away (the power remaining is always opposite of the absolute value of $1/d_{pr}$. For example, if d_{pr} = -0.4 m, then the eye is 2.5 D above the minimum power when the person looks at objects infinitely far.)

The power of the crystalline lens of people with a negative d_{pr} can still go to its minimum value. When that happens (they do this to relax their eyes), everything is blurry, regardless of the distance of the object.

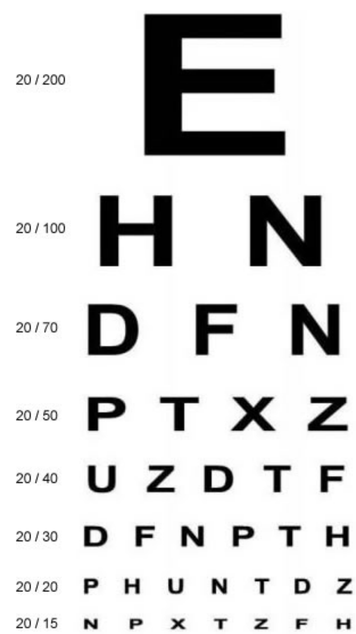
Some Differences in Nature

Power of Accommodation

The power of accommodation is not the same for every species. It is estimated that dogs and cats have a power of accommodation around 2 or 3 D. They do not see clearly objects closer than 40 cm from their eyes. Rodents, rabbits, and horses have practically no power of accommodation. They only see clearly at a certain distance. Animals that need to see clearly in water and in air have enormous powers of accommodation because they have to compensate for the almost total loss of the power of the cornea when the eye is in water. Thus, cormorants have a power of accommodation as high as 50 D.

Visual Acuity

Acuity is related to the image quality, and it depends on the number of photoreceptors in the eye, exactly like the quality of the photo depends on the number of megapixels in the camera. More cells per mm² of retina mean that the image is sharper. The acuity can be measured with the charter on the right.

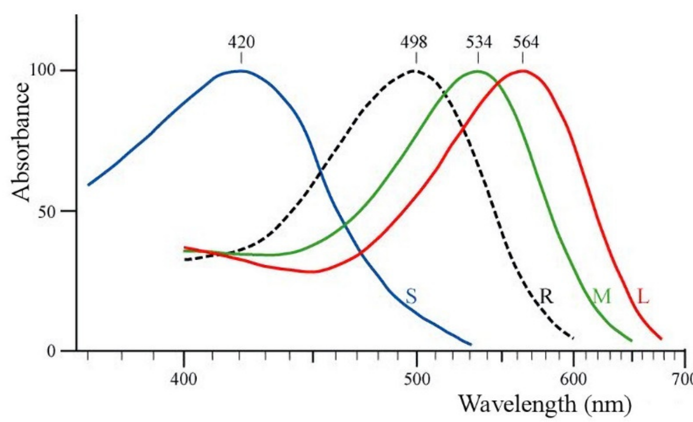


3-squeezes.blogspot.ca/2013/08/diy-eye-chart-love-note.html

For humans, the maximum density of cells on the retina is 160,000 receivers/mm². Eagles have 1 million receivers/mm². The resolution of an eagle eye is much better than ours. On the other hand, the visual acuity of the dog is not very good with about 10,000 receivers/mm². Tests made with a German poodle showed an acuity of around 20/75. This means that something that a human can perceive from 75 feet away will be seen clearly by the dog if it is 20 feet away. Note that a human with a 20/40 vision (which is better than 20/75) cannot have a driver's licence because he is considered legally blind. It would then be impossible for a German poodle to get a driver's licence. The acuity of cats is even worst (20/100 or so) while the acuity of rats and cows is simply catastrophic (20/300).

Colour

There are four types of photoreceptor cells: three types look like cones and one type looks like a rod. Each of these types of cells captures light in a certain part of the visible spectrum as shown on this graph.



biology.stackexchange.com/questions/1446/why-can-cones-detect-colour-but-rods-cant

Rods (R) are more sensitive to light than the cones (S, M, and L). Thus, in low-intensity light (such as during the night), rods are the only photoreceptor capturing light. As the information is coming from only one type of cell, our vision is in black-and-white. With a little more light, the cones come into action and the combined information of four different cell types allows us to see the colour of objects. We have about 20 times more rods than cones in our eye.

The part of the retina directly behind the crystalline lens has fewer rods and more cones than elsewhere on the retina. This is also the region where the cells are more closely packed. If an image is formed on this part of the retina, the object will be seen with much more details and colours, as long as the light is intense enough. Around this area, the density of receptor cells decreases on the retina, the proportion of cones decreases and the proportion of rods increases. Our peripheral vision is less detailed and the colours are not as defined. However, this region is more sensitive to light. For example, if you try to look at the Andromeda Galaxy with the naked eye, you won't see much if you look at it directly because the image, not bright enough to excite the cones, is formed in a region of the retina where there are few rods. On the other hand, if you don't look at the galaxy directly, the image is formed on a part of the retina where there are more rods and you can see it better.

With the exception of primates, mammals have only two types of cones (S and L). Therefore, they do not have the M cone and they do not perceive the colours as we do. The following image shows you what they see.

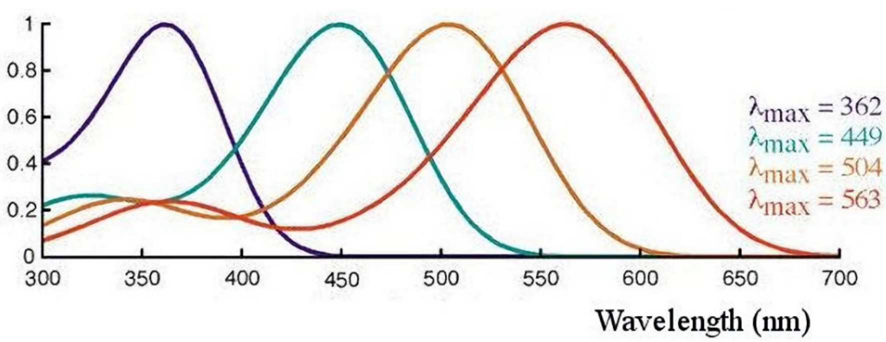
The way a mammalian trichromat (three cones) would see a scene

The way a mammalian dichromat (two cones) would see the same scene

www.retrieverpro.com/dog-health-eyes/

This is also what happens with colour blindness. A genetic defect prevents the formation of M or L cones (or even these two sometimes) and colour vision is impaired.

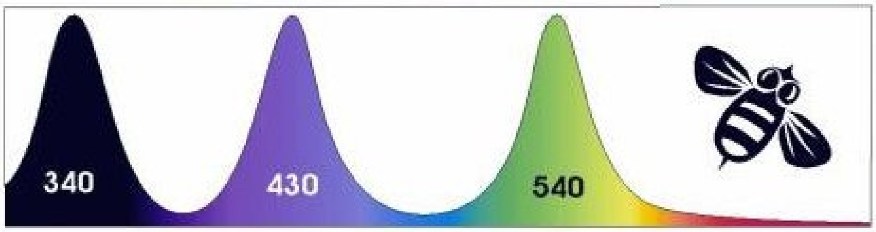
Birds have four types of cones, including one capturing ultraviolet light. Here's the graph of the sensitivity of the cones of the common starling.



www.webexhibits.org/causesofcolour/17B.html

Surely, they can see colours that are completely unnoticed by us.

Bees have three types of cones, but the colours related to the maximum of sensitivity are different. They have a cone that can receive light further away in the ultraviolet than birds. On the other hand, bees cannot see red light.



fieldguidetohummingbirds.wordpress.com/2008/11/11/do-we-see-what-bees-see/

Many mammals have a vision adapted to low-light intensity. They have few cones and a large number of rods to increase the sensitivity of the eye. This lower proportion of cones results in several mammals having a black-and-white vision, slightly tinted with the colours captured by two types of cones.

Some species have a reflective retina in the back of the eye to make it more sensitive. If the back of the eye reflects light, photoreceptor cells have two chances to capture the light: before and after the reflection. As a result, the eyes of these animals are very bright when we light them at night (see picture). This strategy is perhaps good for the brightness of the image, but it is not very good for the sharpness of the image. Cats have this kind of reflective surface in the back of their eyes, which allows them to have eyes 6 times more sensitive to light than ours, but their visual acuity is not very good.

en.wikipedia.org/wiki/Tapetum_lucidum

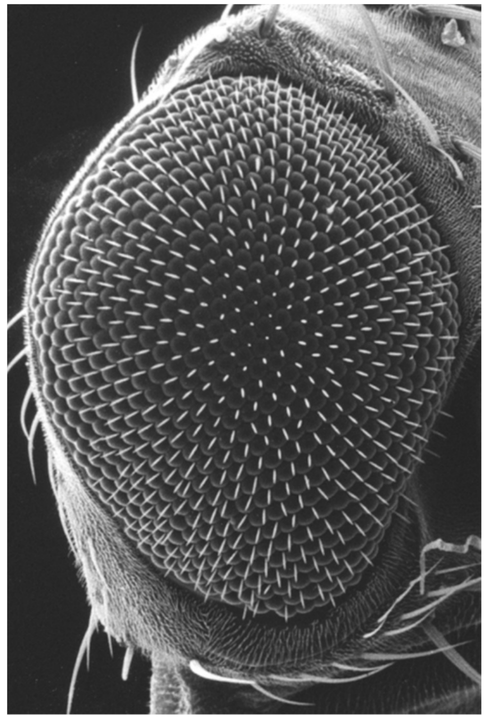
Let's summarize for a dog (for example): they have a less detailed, black-and-white vision, slightly tinted with altered colours. The end result must be something like what is shown in the picture.

Humans Dogs

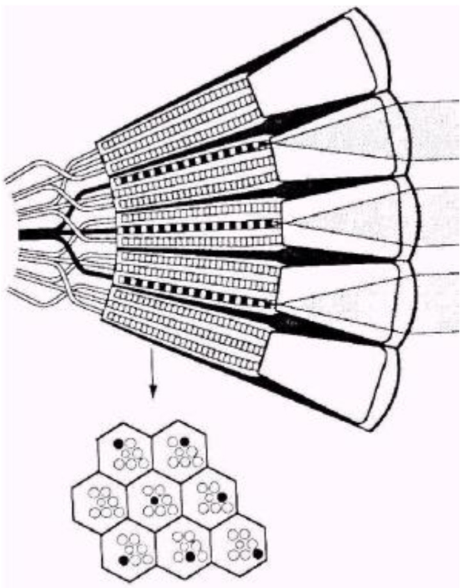
dopdogimaging.net/blog/dog-facts

Eye of a Fly

The functioning of the eye of a fly is very different from that of the human eye. It is an eye composed of long tubes. Looking at the eye, the end of each of these tubes can be seen. At the bottom of each of these tubes, there are photoreceptor cells. There is a small lens at the end of each tube that sends light on those receptors at the other end of the tube.



edwardduca.wordpress.com/2011/12/



www.optics.rochester.edu/workgroups/cml/opt307/spr04/greg/

SUMMARY OF EQUATIONS

Plane Mirror Law

$$q = -p$$

Definition of Magnification

$$m = \frac{y_i}{y_o}$$

Magnification for a Plane Mirrors

$$m = 1$$

Mirror Formula

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

Sign Convention for Mirrors

For a concave mirror: R and f are positive.

For a convex mirror: R and f are negative.

If q is positive: the image is in front of the mirror.

If q is negative: the image is behind the mirror.

Magnification for Mirrors

$$m=-\frac{q}{p}$$

Focal Length of a Mirror

$$f=\frac{R}{2}$$

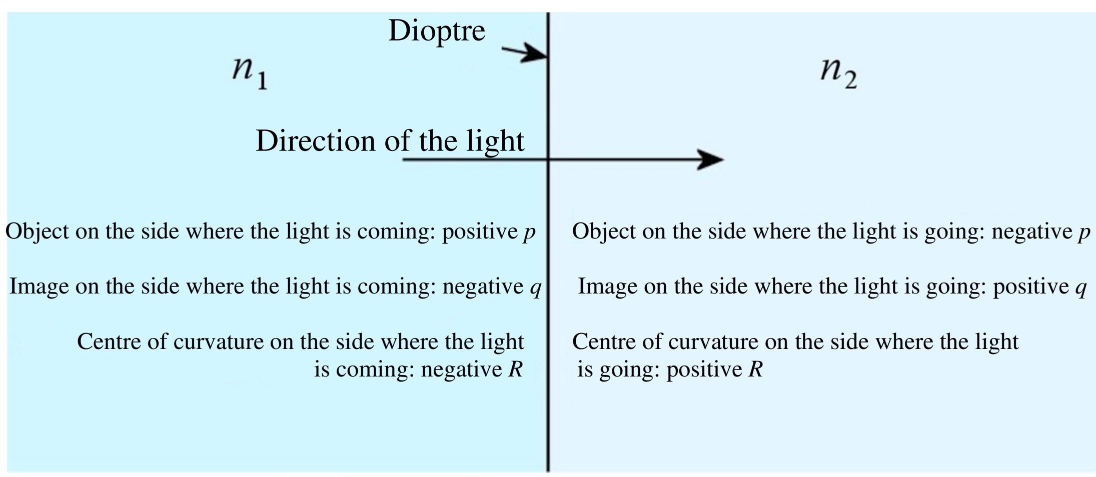
Focal Length of a Spherical Dioptre

$$f=\frac{n_2}{n_2-n_1}R$$

Spherical Dioptre Formula

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$

Sign Convention for Spherical Dioptre



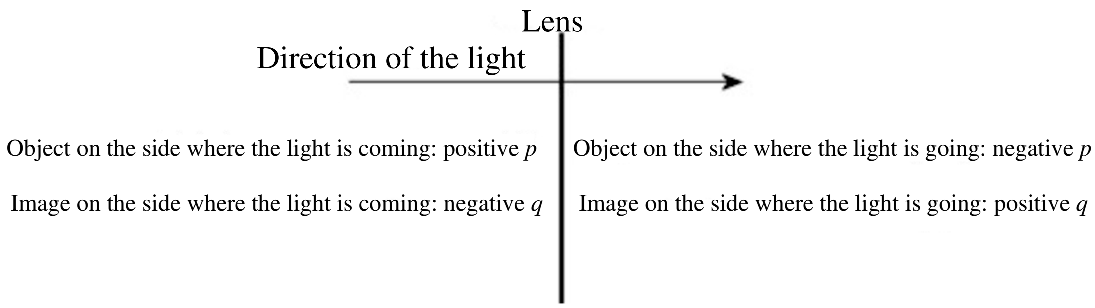
Magnification With a Spherical Dioptre

$$m = \frac{y_i}{y_o} = -\frac{n_1 q}{n_2 p}$$

Thin Lens Equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Sign Convention for Lenses



Converging lens: f is positive.

Diverging lens: f is negative.

Magnification With a Lens

$$m = -\frac{q}{p}$$

Focal Length and Power of a Lens (Lens's Maker Equation)

$$P = \frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Compound Lens

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \frac{1}{f_4} + \dots$$

$$P_{eq} = P_1 + P_2 + P_3 + P_4 + \dots$$

Angular Magnification

$$G = \frac{\beta}{\alpha}$$

Angular Magnification of a Magnifying Glass

$$G = \frac{d_{pp}}{p}$$

Minimum Angular Magnification of a Magnifying Glass

$$G_{\min} = \frac{d_{pp}}{f}$$

The object must be at the focus ($p = f$) to obtain this angular magnification.

Maximum Angular Magnification of a Magnifying Glass

$$G_{\max} = \frac{d_{pp}}{f} + 1$$

The object must be closer than the focus $\left(p = \frac{d_{pp} f}{d_{pp} + f} \right)$ to have this magnification.

Accommodation Power of the Eye

$$P_{acc} = P_{\max} - P_{\min} = \frac{1}{d_{pp}} - \frac{1}{d_{pr}} = \frac{1}{d'_{pp}} - \frac{1}{d'_{pr}}$$

Power of the Glasses Needed to Correct Farsightedness

$$P_{gla} = \frac{1}{d'_{pp}} - \frac{1}{d_{pp}}$$

Power of the Glasses Needed to Correct Nearsightedness

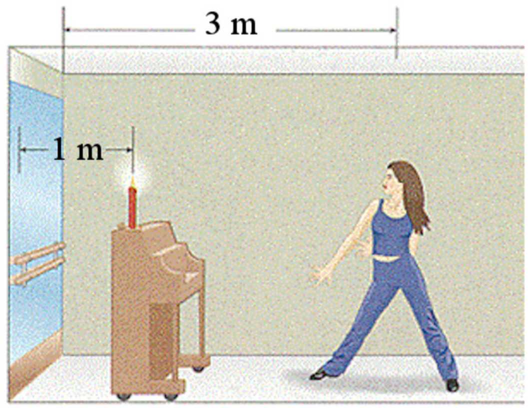
$$P_{gla} = -\frac{1}{d_{pr}}$$

EXERCISES

5.2 Plane Mirrors

1. A 20 cm tall object is 120 cm in front of a mirror. Where is the image and what is its height?

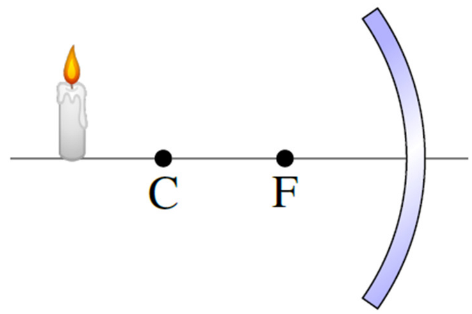
2. In the situation shown in the diagram, what is the distance between Anna and the image of the candle?



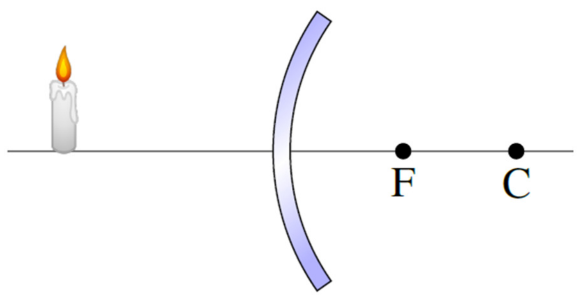
www.chegg.com/homework-help/questions-and-answers/hannah-standing-middle-room-opposite-walls-separated-d-105-m-covered-plane-mirrors-candle--q1598802

5.3 Spherical Mirrors

3. Use ray tracing to find the position of the image of the candle.



4. Use ray tracing to find the position of the image of the candle.



5. An object is in front of a concave mirror having a radius of 40 cm. What is the position of the image if the object is…

a) 10 cm in front of the mirror?
b) 50 cm in front of the mirror?

6. An object is in front of a convex mirror having a radius of 40 cm. What is the position of the image if the object is…

a) 10 cm in front of the mirror?
b) 50 cm in front of the mirror?

7. A 3 cm tall object is 16 cm in front of a concave mirror having a 28 cm radius of curvature. Where is the image and what is its height?

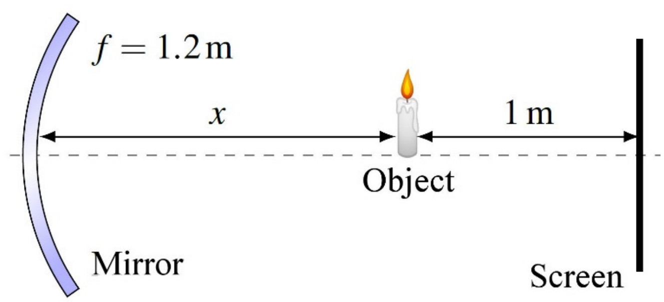
8. A 30 cm tall object is 30 cm in front of a spherical mirror. An inverted image whose height is equal to 30% of the height of the object is obtained. What type of mirror is used (concave, convex or flat) and what is its radius of curvature?

9. An image located 20 cm in front of a mirror is obtained when the object is 60 cm in front of the mirror. Where will the image be if the object is placed 10 cm in front of the mirror?

10. An object is in front of a convex mirror whose radius is 40 cm. An erect image whose height is equal to 25% of the height of the object is obtained.

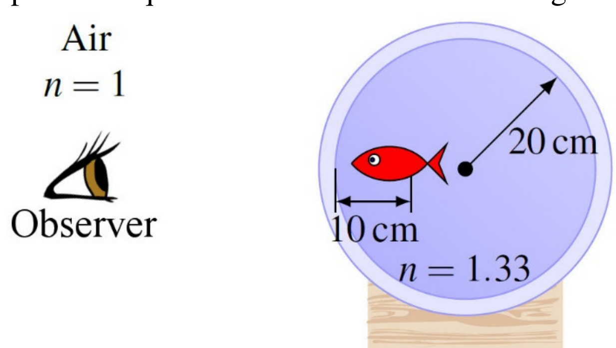
a) What is the distance between the object and the mirror?
b) What is the position of the image?

11. In the situation shown in the diagram, the image is exactly on the screen. What is the value of x?

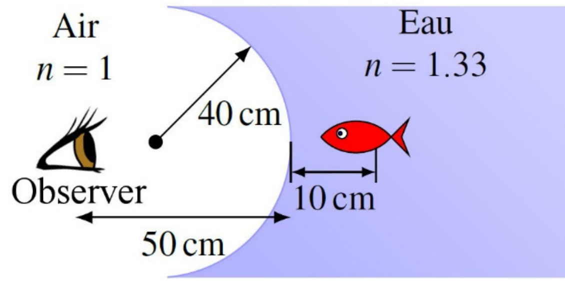


5.6 Spherical Dioptre

12. A person is looking at a fish in a spherical aquarium as illustrated in the diagram. How far from the edge of the aquarium seems to be the fish for this person? (Neglect the glass wall of the aquarium.)

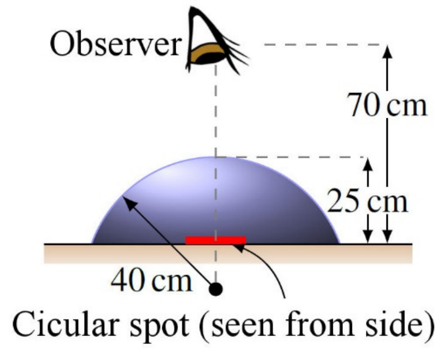


13. Yvette is in a small round room surrounded by water. There is a fish 10 cm behind the glass of the aquarium. Where is the image of the fish according to Yvette? (Neglect the glass wall of the aquarium.)



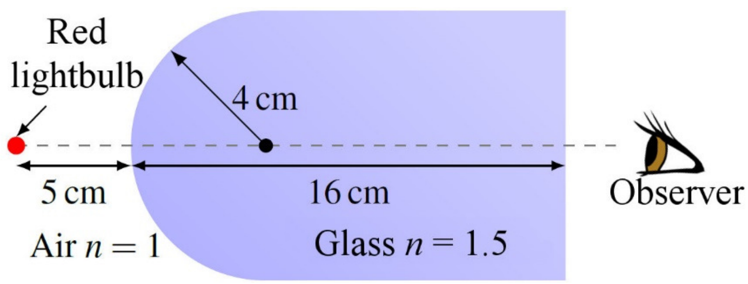
14. On the surface of a table, there is a circular red spot whose radius is 2 cm. Over the spot, there is a piece of glass having a refractive index of 1.5 as shown in the diagram.

a) What is the distance between the observer and the image of the spot?
b) What is the radius of the image of the spot according to the observer?

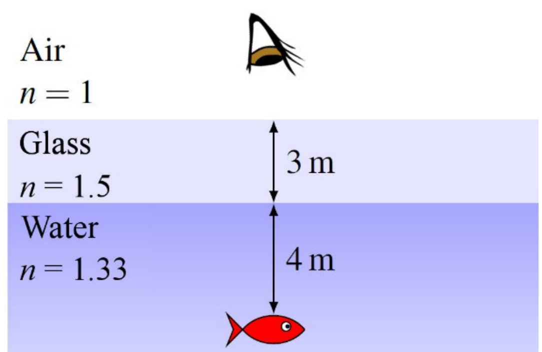


15. The light coming from a red light bulb (whose diameter is 1 cm) passes through a piece of glass having the shape shown in the diagram to finally arrive at an observer.

a) Where is the image of the light bulb, according to the observer?
b) What is the diameter of the image of the light bulb?



16. A 3 m thick layer of glass is placed over a lake. An observer looks at a fish that is 4 m below the glass layer. Where is the image of the fish according to the observer?



5.5 Thin Lenses

17. A 1 cm high object is 4 m from a converging lens whose focal length is 50 cm. The image is then projected on a screen.

    a) What is the distance between the lens and the screen if the image is clear?
    b) What is the height of the image if the image is clear?

18. A 1 cm high object is 2 m from a converging lens. The image is then inverted and has a height of 0.5 cm.

    a) What is the distance between the lens and the image?
    b) What is the focal length of the lens?

19. An image 4 times larger than the object is obtained with a lens. The distance between the lens and the image is 20 cm and the image is not inverted.

    a) What is the distance between the object and the lens?
    b) What is the focal length of the lens?

20. A convergent lens has a 25 cm focal length. Where should the object be placed to obtain an image which is 3 times larger than the object if the image…

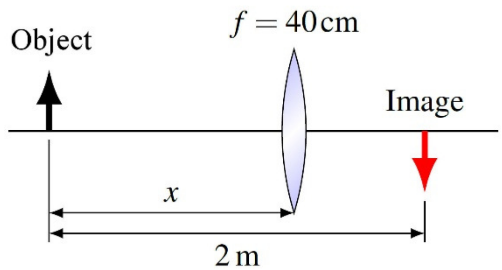
    a) is inverted?
    b) is erect?

21. A diverging lens has a 25 cm focal length. Where should the object be placed to obtain an image whose height is equal to 40% of the height of the object if the image…

    a) is inverted?
    b) is erect?

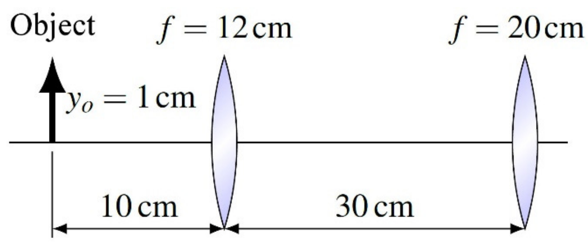
22. An object and a screen are 36 cm apart. With a convergent lens, an inverted image twice as large as the object is obtained on the screen. What is the focal distance of the lens?

23. In the situation shown in the diagram, what should be the distance between the object and the lens (x in the diagram) if the lens has a 40 cm focal length? (Be careful, there are two possible answers.)



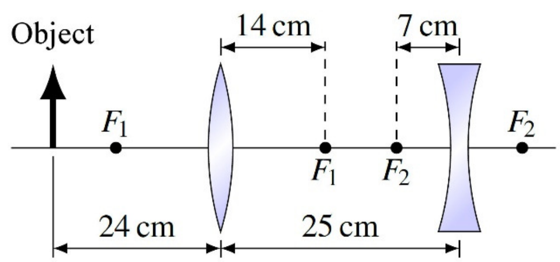
24. Here is an object with two lenses.

a) What is the position of the final image?
b) What is the height of the final image?



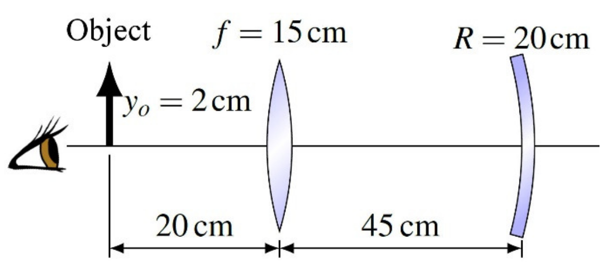
25. A 2 cm high object is placed as shown in the diagram.

a) What is the position of the final image?
b) What is the height of the final image?

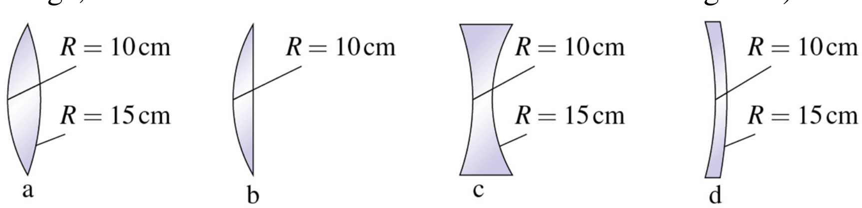


26. In the situation shown in the diagram, Nestor sees the object, but he also sees the image of the object after the light has passed through the lens, has been reflected by the mirror and has passed again through the lens.

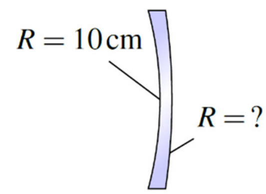
a) What is the position of this image?
b) What is the height of this image?



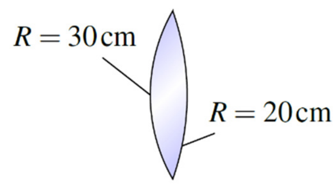
27. All these lenses are made of glass having a refractive index of 1.6. Calculate the focal lengths of all these lenses and determine if the lens is convergent or divergent. (On the image, the absolute values of the radii of curvature are given.)



28. This diverging lens is made of a material having an index of refraction of 1.62. According to the diagram, what must be the unknown radius of curvature so that the focal length of the lens is 30 cm when this lens is in water (refractive index of 1.33)? (The surface is perhaps curved the opposite direction compared to what is shown in the diagram.)

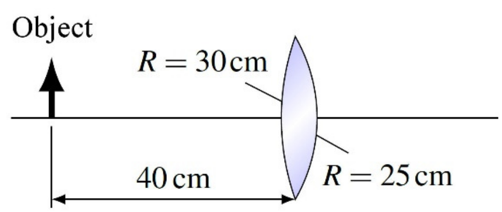


29. This lens is made of Flint glass whose index of refraction is 1.62 for red light and 1.67 for violet light. What is the distance between the focus of red light and the focus of violet light for this lens?

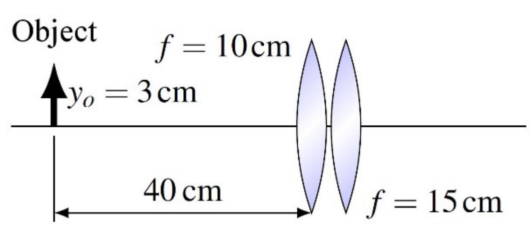


30. A lens made of a material whose refractive index is 1.6 has a 15 cm focal length when it is in the air. What is its focal distance if this lens is put in water?

31. Where is the image of this object if the lens is made of a material whose refractive index is 1.5? (On the image, the absolute values of the radii of curvature are given.)



32. Two lenses are used to form the image of an object. In the situation shown in the diagram, find the image position and height.



5.6 Magnifying Glasses, Microscope, and Telescopes

33. A person with a 20 cm d_{pp} uses a lens with a 3 cm focal length to examine a stamp.

a) What should be the distance between the stamp and magnifying glass to obtain the minimum angular magnification?
b) What is the minimum angular magnification?
c) What should be the distance between the stamp and magnifying glass to obtain the maximum angular magnification?
d) What is the maximum angular magnification?

34. Subrahmanyan, who has a 20 cm d_{pp}, uses a lens with a 2 cm focal distance to examine a coin. In these conditions, he gets the minimum angular magnification (10) if the distance between the lens and the coin is 2 cm and he gets the maximum angular magnification (11) if the distance between the lens and the coin is 1.818 cm. What angular magnification does he obtain when the distance between the coin and the magnifying glass is 1.9 cm?

35. Ronan uses a lens to examine a coin. In these conditions, he obtains the maximum angular magnification (15) if the distance between the lens and the coin is 1.4 cm.

a) What is Ronan's d_{pp}?
b) What is the focal length of the lens?
c) What angular magnification does he obtain when the distance between the coin and the magnifying glass is 1.45 cm?

5.7 The Eye

36. Elodie does not see clearly objects that are farther away than 5 m from her. What is the power of the glasses to be prescribed?

37. Maurice has difficulty reading his newspaper because he does not see clearly objects that are less than 45 cm from his eyes. What should be the power of the glasses that must be given to him if he wants to bring his punctum proximum at 20 cm?

38. Frida has a punctum proximum at 18 cm and a punctum remotum at 2.4 m.

a) What is the power of the glasses that must be given to her?
b) What will the distances of the punctum proximum and the punctum remotum be when she wears her glasses?

39. Georgette cannot see clearly objects closer than 50 cm. Tests showed that her power of accommodation is only 3 D.

a) What is the power of the glasses that must be given to her to bring back the punctum proximum at 25 cm?
b) What will the distances of the punctum proximum and the punctum remotum be when she wears her glasses?

40. Germain realizes that he ages. Even with his 2 D glasses that he's been using for a few years, he can't see clearly objects that are closer than 45 cm anymore. So, he goes to see the optometrist to bring back his punctum proximum at 25 cm. Then, what will the power of his new glasses be?

Challenges

(Questions more difficult than the exam questions.)

41. An object is placed in front of a concave mirror so that a real image that is three times larger than the object is obtained. When the object is moved 1.2 m towards the mirror, a virtual image that is three times larger than the object is now obtained. What is the focal length of the mirror?

42. An object forms a real image when the distance between the lens and the object is 36.8 cm. When the distance between the object and the lens is decreased to 36 cm, the image moves away from the lens by 3 cm. What is the focal length of the lens?

43. The thin lens equation can also be written in the form

$$x \cdot x' = f^2$$

where x is the distance between the object and the focus and x' is the distance between the image and the other focus of the lens. This form was used by Newton. Demonstrate this law from the thin lens equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

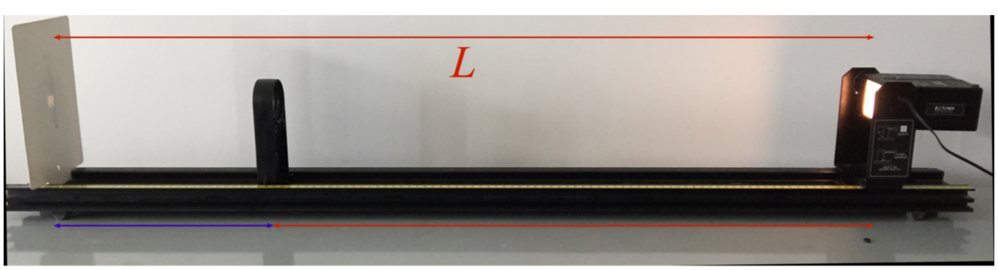
44. The dioptre equation can also be written in the form

$$\frac{f_2}{p} + \frac{f_1}{q} = 1$$

Where the f are the focci of the dioptre (f_1 when the rays are parallel in medium 1 and f_2 when the rays are parallel in medium 2). Demonstrate this law from the dioptre equation

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

45. A light source and a screen are at a distance L from each other. A lens is then placed between the two to form a sharp image on the screen. Show that L must be greater than or equal to $4f$ to have a sharp image on the screen.



www.ipls.gatech.edu/labs/lab-9-lenses-and-magnification/

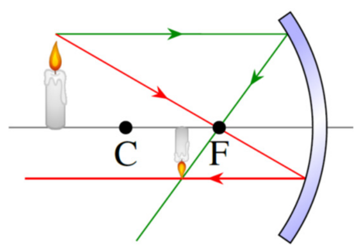
ANSWERS

5.2 Plane Mirrors

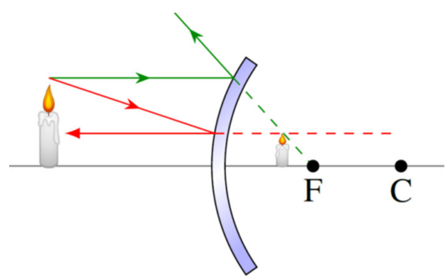
1. 20 cm tall located 120 cm behind the mirror
2. 4 m

5.3 Spherical Mirrors

3.



4.



5. a) 20 cm behind the mirror. b) 33.3 cm in front of the mirror.
6. a) 6.67 cm behind the mirror. b) 14.29 cm behind the mirror.
7. The image is 112 cm in front of the mirror, it is inverted and 21 cm tall.
8. A concave mirror whose radius is 13.85 cm.
9. 30 cm behind the mirror.
10. a) 60 cm b) 15 cm behind the mirror.
11. $x = 2$ m

5.4 Spherical Dioptre

12. The image of the fish is 8.58 cm behind the wall of the aquarium.
13. The image of the fish is 7.08 cm behind the wall of the aquarium.
14. a) 66.05 cm b) 2.53 cm
15. a) For the observer, the image is 24 cm behind the flat surface.
    b) 2.667 cm
16. For the observer, the image is 5.01 m underneath the top of the glass surface.

5.5 Thin Lenses

17. a) 57.1 cm from the lens. b) An inverted image whose height is 0.143 cm.
18. a) 1 m b) 66.6 cm
19. a) 5 cm b) 6.66 cm
20. a) 33.3 cm b) 16.7 cm
21. a) impossible b) 37.5 cm
22. 8 cm
23. $x = 1.4472$ m and $x = 0.5528$ m
24. a) 25.7 cm to the right of the lens to the right. b) The final image is inverted and has a height of 1.714 cm.
25. a) 37.625 cm to the left of the lens to the right. b) The final image is not inverted and has a height of 12.25 cm.
26. a) 24.375 cm to the left of the lens. b) The final image is not inverted and has a height of 1.5 cm.
27. a) converging $f = 10$ cm b) converging $f = 16.7$ cm
    c) diverging $f = 10$ cm d) diverging $f = 50$ cm
28. 18.9 cm (the surface is curved in the opposite direction from what is shown in the diagram.)
29. 1.44 cm
30. 44.3 cm
31. 85.7 cm to the right of the lens.
32. 7.06 cm to the right of the lens to the right. The image is inverted and has a height of 0.529 cm.

5.6 Magnifying Glasses, Microscopes, and Telescopes

33. a) 3 cm b) 6.67 c) 2.61 cm d) 7.67
34. 10.53
35. a) 21 cm b) 1.5 cm c) 14.48

5.7 The Eye

36. -0.2 D
37. 2.78 D
38. a) -0.417 D b) With his glasses, this person sees clearly from 19.46 cm to infinity.
39. a) 2 D b) With his glasses, this person sees clearly from 25 cm to 1 m.
40. 3.78 D

Challenges

41. $f = 1.8$ m
42. $f = 24$ cm