Chapter 10 Solutions

1. The wavelength of the peak is

$$\lambda_{pic} = \frac{2.898\times10^{-3}\,mK}{T} = \frac{2.898\times10^{-3}\,mK}{3273K} = 885nm$$

This corresponds to infrared radiation.

2. The temperature is found with

$$\lambda_{pic} = \frac{2.898\times10^{-3}\,mK}{T}$$

$$502\times10^{-9}\,m = \frac{2.898\times10^{-3}\,mK}{T}$$

$$T = 5773K$$

3. The power is

$$\begin{aligned} P &= \sigma A\left(T^4 - T_0^4\right) \\ &= 5.67\times10^{-8}\,\tfrac{W}{m^2K^4}\,4\pi\left(3.2\times10^{10}\,m\right)^2\left(\left(6015K\right)^4 - \left(0K\right)^4\right) \\ &= 9.55\times10^{29}\,W \end{aligned}$$

This is about 2500 times brighter than the Sun.

4. a) The power is

$$\begin{aligned} P &= \sigma A\left(T^4 - T_0^4\right) \\ &= 5.67\times10^{-8}\,\tfrac{W}{m^2K^4}\cdot 1.8m^2\cdot\left(\left(310K\right)^4 - \left(293K\right)^4\right) \\ &= 190.4W \end{aligned}$$

(This is the same power as the power required to go biking with a little effort).

b) The power is

$$\begin{aligned} P &= \sigma A\left(T^4 - T_0^4\right) \\ &= 5.67\times10^{-8} \tfrac{W}{m^2K^4}\cdot 1.8m^2\cdot\left(\left(310K\right)^4 - \left(243K\right)^4\right) \\ &= 586.7W \end{aligned}$$

(This is equivalent to a really strenuous exercising. Go on a stationary bike that shows the power and try to achieve this power...)

5. The temperature will be found from the power with

$$P = \sigma A\left(T^4 - T_0^4\right)$$

To find it, we need the area of the filament.

The filament is a cylinder whose area is

$$\begin{aligned} A &= 2\pi rl \\ &= 2\pi\cdot 0.0005m\cdot 0.1m \\ &= 3.1416\times10^{-4} m^2 \end{aligned}$$

Therefore, the temperature is

$$\begin{aligned} P &= \sigma A\left(T^4 - T_0^4\right) \\ 60W &= 5.67\times10^{-8} \tfrac{W}{m^2K^4}\cdot 3.1416\times10^{-4} m^2\cdot\left(T^4 - \left(293K\right)^4\right) \\ T &= 1355K = 1082°C \end{aligned}$$

6. The energy is

$$\begin{aligned} E &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{550nm} \\ &= 2.25eV \end{aligned}$$

7. The number of photons is given by

$$N = \frac{\text{Total energy emitted}}{\text{Energy of one photon}}$$

The energy of a photon is

$$\begin{aligned} E &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{632nm} \\ &= 1.962eV \\ &= 3.143\times10^{-19}J \end{aligned}$$

The energy emitted per second is

$$\begin{aligned} E &= Pt \\ &= 0.001W\cdot 1s \\ &= 0.001J \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned} N &= \frac{\text{Total energy emitted}}{\text{Energy of one photon}} \\ &= \frac{0.001J}{3.143\times10^{-19}\frac{J}{photons}} \\ &= 3.182\times10^{15}\ photons \end{aligned}$$

8. The number of photons is given by

$$N = \frac{\text{Total energy received}}{\text{Energy of one photon}}$$

The energy of a photon is

$$E = \frac{1240eVnm}{\lambda}$$
$$= \frac{1240eVnm}{585nm}$$
$$= 2.12eV$$
$$= 3.396\times10^{-19}J$$

The energy received in 20 seconds is

$$E = IA_{receiver}t$$
$$= 50\tfrac{W}{m^2}\cdot 3m^2\cdot 20s$$
$$= 3000J$$

Therefore, the number of photons is

$$N = \frac{\text{Total energy received}}{\text{Energy of one photon}}$$
$$= \frac{3000J}{3.396\times10^{-19}\frac{J}{photons}}$$
$$= 8.835\times10^{21}\ photons$$

9. The number of photons is given by

$$N = \frac{\text{Total energy received}}{\text{Energy of one photon}}$$

The energy of a photon is

$$E = \frac{1240eVnm}{\lambda}$$
$$= \frac{1240eVnm}{470nm}$$
$$= 2.638eV$$
$$= 4.227\times10^{-19}J$$

The energy received per second is

$$\begin{aligned} E &= IA_{receiver}t \\ &= 200\tfrac{W}{m^2}\cdot\pi\left(0.0025m\right)^2\cdot 1s \\ &= 0.003927J \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned} N &= \frac{\text{Total energy received}}{\text{Energy of one photon}} \\ &= \frac{0.003927J}{4.227\times10^{-19}\,\frac{J}{photons}} \\ &= 9.291\times10^{15}\ photons \end{aligned}$$

10. The maximal energy of the electrons is found with

$$E_{k\max} = hf - \phi$$

The photon energy is

$$\begin{aligned} E &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{150nm} \\ &= 8.267eV \end{aligned}$$

The maximum energy of the ejected electrons is, therefore,

$$\begin{aligned} E_{k\max} &= hf - \phi \\ &= 8.267eV - 4.5eV \\ &= 3.767eV \end{aligned}$$

11. The maximal energy of the electrons is found with

$$E_{k\max} = hf - \phi$$

The work function of cesium is

$$\begin{aligned}\phi &= \frac{1240eVnm}{\lambda_0} \\ &= \frac{1240eVnm}{686nm} \\ &= 1.808eV\end{aligned}$$

a) With a wavelength of 690 nm, the energy of the photons is

$$\begin{aligned}E &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{690nm} \\ &= 1.797eV\end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned}E_{k\max} &= hf - \phi \\ &= 1.797eV - 1.808eV \\ &= -0.011eV\end{aligned}$$

This means that there are no electrons ejected since a negative kinetic energy is impossible. Photons don't have enough energy to eject electrons.

b) With a wavelength of 450 nm, the energy of the photons is

$$\begin{aligned}E &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{450nm} \\ &= 2.756eV\end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned}E_{k\max} &= hf - \phi \\ &= 2.756eV - 1.808eV \\ &= 0.948eV\end{aligned}$$

12. a) The threshold wavelength is

$$\phi = \frac{1240eVnm}{\lambda_0}$$
$$3.2eV = \frac{1240eVnm}{\lambda_0}$$
$$\lambda_0 = 387.5nm$$

b) The maximal speed is found with the maximum energy of the electrons, which is found with

$$E_{k\max} = hf - \phi$$

With a wavelength of 250 nm, the energy of the photons is

$$\begin{aligned} E &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{250nm} \\ &= 4.96eV \end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned} E_{k\max} &= hf - \phi \\ &= 4.96eV - 3.2eV \\ &= 1.76eV \\ &= 2.82\times10^{-19}J \end{aligned}$$

Therefore, the speed of the electrons is

$$E_{k\max} = \frac{1}{2}mv_{\max}^2$$
$$2.82\times10^{-19}J = \frac{1}{2}\cdot 9.1094\times10^{-31}kg \cdot v_{\max}^2$$
$$v_{\max} = 7.868\times10^5 \tfrac{m}{s}$$

13. The threshold wavelength id found with the work function, and this work function is found with

$$E_{k\max} = hf - \phi$$

The maximum kinetic energy of the electrons is

$$\begin{aligned}E_{k\max} &= \frac{1}{2}mv_{\max}^2 \\ &= \frac{1}{2}\cdot 9.1094\times10^{-31}kg\cdot\left(5\times10^5\tfrac{m}{s}\right)^2 \\ &= 1.139\times10^{-19}J \\ &= 0.711eV\end{aligned}$$

The energy of the photons is

$$\begin{aligned}E &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{400nm} \\ &= 3.1eV\end{aligned}$$

The work function is then found with

$$\begin{aligned}E_{k\max} &= hf - \phi \\ 0.711eV &= 3.1eV - \phi \\ \phi &= 2.389eV\end{aligned}$$

Therefore, the threshold wavelength is

$$\begin{aligned}\phi &= \frac{1240eVnm}{\lambda_0} \\ 2,389eV &= \frac{1240eVnm}{\lambda_0} \\ \lambda_0 &= 519nm\end{aligned}$$

14. Since 3 % of the photons eject electrons, the number of ejected electrons is

$$N_{electrons} = 0.03\cdot N_{photons}$$

The energy of a photon received is given by

$$N_{photons} = \frac{\text{Total energy}}{\text{Energy of one photon}}$$

$$\begin{aligned} E &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{450nm} \\ &= 2.756eV \\ &= 4.414\times10^{-19} J \end{aligned}$$

The energy received per second per square centimetre is

$$\begin{aligned} E &= IA_{receiver}t \\ &= 40\tfrac{W}{m^2}\cdot 0.0001m^2\cdot 1s \\ &= 0.004J \end{aligned}$$

Therefore, the number of photons received is

$$\begin{aligned} N &= \frac{\text{Total energy}}{\text{Energy of one photon}} \\ &= \frac{0.004J}{4.414\times10^{-19}\,\frac{J}{photons}} \\ &= 9.091\times10^{15}\ photons \end{aligned}$$

If only 3% of the photons eject an electron, then the number of ejected electrons is

$$\begin{aligned} N_{electrons} &= 0.03\cdot N_{photons} \\ &= 0.03\cdot 9.091\times10^{15} \\ &= 2.718\times10^{15} \end{aligned}$$

15. a) The wavelength shift is

$$\begin{aligned} \Delta\lambda &= 2.4263\times10^{-3}nm\cdot(1-\cos\theta) \\ &= 2.4263\times10^{-3}nm\cdot(1-\cos 45°) \\ &= 0.0007106nm \end{aligned}$$

b) The wavelength of the incident photon is

$$E = \frac{1240eVnm}{\lambda}$$
$$62,000eV = \frac{1240eVnm}{\lambda}$$
$$\lambda = 0.02nm$$

The new wavelength is thus

$$\begin{aligned}\lambda' &= \lambda + \Delta\lambda \\ &= 0.02nm + 0.0007106nm \\ &= 0.0207106nm\end{aligned}$$

c) The new energy of the photon is

$$\begin{aligned}E' &= \frac{1240eVnm}{\lambda'} \\ &= \frac{1240eVnm}{0.0207106nm} \\ &= 59,873eV\end{aligned}$$

d) The kinetic energy of the electron is

$$E_\gamma = E'_\gamma + E_{k\,e}$$
$$62,000eV = 59,873eV + E_{k\,e}$$
$$E_{k\,e} = 2127eV$$

e) The angle with the conservation of *y*-component of the momentum.

$$0 = p'_\gamma \sin\theta - p'_e \sin\phi$$

The momentum of the photon is found with

$$E' = p'_\gamma c$$
$$59,873 \cdot 1.602\times10^{-19} J = p'_\gamma \cdot 3\times10^8 \tfrac{m}{s}$$
$$p'_\gamma = 3.197\times10^{-23} \tfrac{kgm}{s}$$

The momentum of the electron is found with

$$E_e = \frac{p^2}{2m}$$
$$2127 \cdot 1.602 \times 10^{-19} J = \frac{p_e'^2}{2 \cdot 9.1094 \times 10^{-31} kg}$$
$$p_e' = 2.491 \times 10^{-23} \tfrac{kgm}{s}$$

The conservation equation then becomes

$$0 = p_\gamma' \sin\theta - p_e' \sin\phi$$
$$0 = 3.197 \times 10^{-23} \tfrac{kgm}{s} \cdot \sin 45° - 2.491 \times 10^{-23} \tfrac{kgm}{s} \cdot \sin\phi$$
$$0 = 3.197 \cdot \sin 45° - 2.491 \cdot \sin\phi$$
$$\phi = 65.1°$$

16. The diffusion angle is found with

$$\Delta\lambda = 2.4263 \times 10^{-3} nm \cdot (1 - \cos\theta)$$

To find the angle, we need the wavelength shift. This shift is found with the wavelengths before and after the collision.

The initial wavelength is

$$E = \frac{1240eVnm}{\lambda}$$
$$50,000eV = \frac{1240eVnm}{\lambda}$$
$$\lambda = 0.0248nm$$

The wavelength after the scattering is

$$E' = \frac{1240eVnm}{\lambda'}$$
$$49,500eV = \frac{1240eVnm}{\lambda'}$$
$$\lambda' = 0.02505nm$$

So, the wavelength shift is

$$\begin{aligned}\Delta\lambda &= \lambda' - \lambda \\ &= 0.02505nm - 0.0248nm \\ &= 0.00025nm\end{aligned}$$

Therefore, the angle is

$$\begin{aligned}\Delta\lambda &= 2.4263\times10^{-3}nm\cdot(1-\cos\theta) \\ 0.00025nm &= 2.4263\times10^{-3}nm\cdot(1-\cos\theta) \\ \theta &= 26.3°\end{aligned}$$

17. The wavelength is

$$\begin{aligned}\lambda &= \frac{h}{p} \\ &= \frac{h}{mv} \\ &= \frac{6.626\times10^{-34}Js}{1.6726\times10^{-27}kg\cdot10^{4}\frac{m}{s}} \\ &= 3.96\times10^{-11}m = 0.0396nm\end{aligned}$$

18. As the speed is close to the speed of light, the relativistic momentum formula must be used. The wavelength is, therefore,

$$\begin{aligned}\lambda &= \frac{h}{p} \\ &= \frac{h}{\gamma mv} \\ &= \frac{6.626\times10^{-34}Js}{\frac{1}{\sqrt{1-\left(\frac{2\times10^{8}\frac{m}{s}}{3\times10^{8}\frac{m}{s}}\right)^2}}\cdot1.6726\times10^{-27}kg\cdot2\times10^{8}\frac{m}{s}} \\ &= 1.476\times10^{-15}m\end{aligned}$$

19. The wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

The speed will be found from the kinetic energy.

With a 10 eV kinetic energy, the speed of the electron is

$$E_k = \frac{1}{2}mv^2$$
$$10 \cdot 1.602 \times 10^{-19} J = \frac{1}{2} \cdot 9.1094 \times 10^{-31} kg \cdot v^2$$
$$v = 1.875 \times 10^6 \tfrac{m}{s}$$

Thus, the wavelength is

$$\begin{aligned}\lambda &= \frac{h}{mv} \\ &= \frac{6.626 \times 10^{-34} Js}{9.1094 \times 10^{-31} kg \cdot 1.875 \times 10^6 \tfrac{m}{s}} \\ &= 3.879 \times 10^{-10} m = 0.3879 nm\end{aligned}$$

20. With a 6 eV kinetic energy, the speed of the electron is

$$E_k = \frac{1}{2}mv^2$$
$$6 \cdot 1.602 \times 10^{-19} J = \frac{1}{2} \cdot 9{,}1094 \times 10^{-31} kg \cdot v^2$$
$$v = 1.453 \times 10^6 \tfrac{m}{s}$$

Thus, the wavelength is

$$\begin{aligned}\lambda &= \frac{h}{p} \\ &= \frac{h}{mv} \\ &= \frac{6.626 \times 10^{-34} Js}{9.1094 \times 10^{-31} kg \cdot 1.453 \times 10^6 \tfrac{m}{s}} \\ &= 5.007 \times 10^{-10} m = 0.5007 nm\end{aligned}$$

When U increases to 2 eV, the kinetic energy decreases to 4 eV. The speed of the electron is then

$$E_k = \frac{1}{2}mv^2$$
$$4 \cdot 1.602\times10^{-19} J = \frac{1}{2}\cdot 9.1094\times10^{-31} kg \cdot v^2$$
$$v = 1.186\times10^{6} \tfrac{m}{s}$$

And the wavelength is

$$\begin{aligned}\lambda &= \frac{h}{p}\\ &= \frac{h}{mv}\\ &= \frac{6.626\times10^{-34} Js}{9.1094\times10^{-31} kg \cdot 1.186\times10^{6} \tfrac{m}{s}}\\ &= 6.132\times10^{-10} m = 0.6132 nm\end{aligned}$$

The change in wavelength is, therefore,

$$\begin{aligned}\Delta\lambda &= \lambda' - \lambda\\ &= 0.6132nm - 0.5007nm\\ &= 0.1125nm\end{aligned}$$

21. The distance *x* is the distance between the order-2- maxima. The position of these maxima will be found with

$$d\sin\theta = m\lambda$$

We have *d* but not λ. We will find this wavelength with *h/p*.

With a 2 eV kinetic energy, the speed of the electron is

$$E_k = \frac{1}{2}mv^2$$
$$2 \cdot 1.602\times10^{-19} J = \frac{1}{2}\cdot 9.1094\times10^{-31} kg \cdot v^2$$
$$v = 8.3877\times10^{5} \tfrac{m}{s}$$

The wavelength is

$$\begin{aligned}\lambda &= \frac{h}{p} \\ &= \frac{h}{mv} \\ &= \frac{6.626\times10^{-34}Js}{9.1094\times10^{-31}kg\cdot 8.3877\times10^{5}\frac{m}{s}} \\ &= 8.672\times10^{-10}m = 0.8672nm\end{aligned}$$

Therefore, the angle of the order-2 maximum is

$$d\sin\theta = m\lambda$$
$$0.1\times10^{-6}m\cdot\sin\theta = 2\cdot 8.672\times10^{-10}m$$
$$\theta = 0.9938°$$

The distance from the central maximum to the order 2 maximum is, therefore,

$$\tan\theta = \frac{y}{L}$$
$$\tan(0.9938°) = \frac{y}{300cm}$$
$$y = 5.204cm$$

The distance between the two order 2 maxima is twice as big. Therefore, it is 10.408 cm.

22. The uncertainty of the momentum is

$$\begin{aligned}\Delta p &= p_{max} - p_{min} \\ &= 2.05\times10^{-23}\frac{kgm}{s} - 2\times10^{-23}\frac{kgm}{s} \\ &= 5\times10^{-25}\frac{kgm}{s}\end{aligned}$$

Therefore, the uncertainty of the position is

$$\Delta x\Delta p = h$$
$$\Delta x\cdot 5\times10^{-25}\frac{kgm}{s} = 6.626\times10^{-34}Js$$
$$\Delta x = 1.325\times10^{-9}m = 1.325nm$$

23. The uncertainty of the energy is

$$\Delta E \Delta t = h$$
$$\Delta E \cdot 10^{-8} s = 6.626 \times 10^{-34} Js$$
$$\Delta E = 6.626 \times 10^{-26} J = 4.136 \times 10^{-7} eV$$