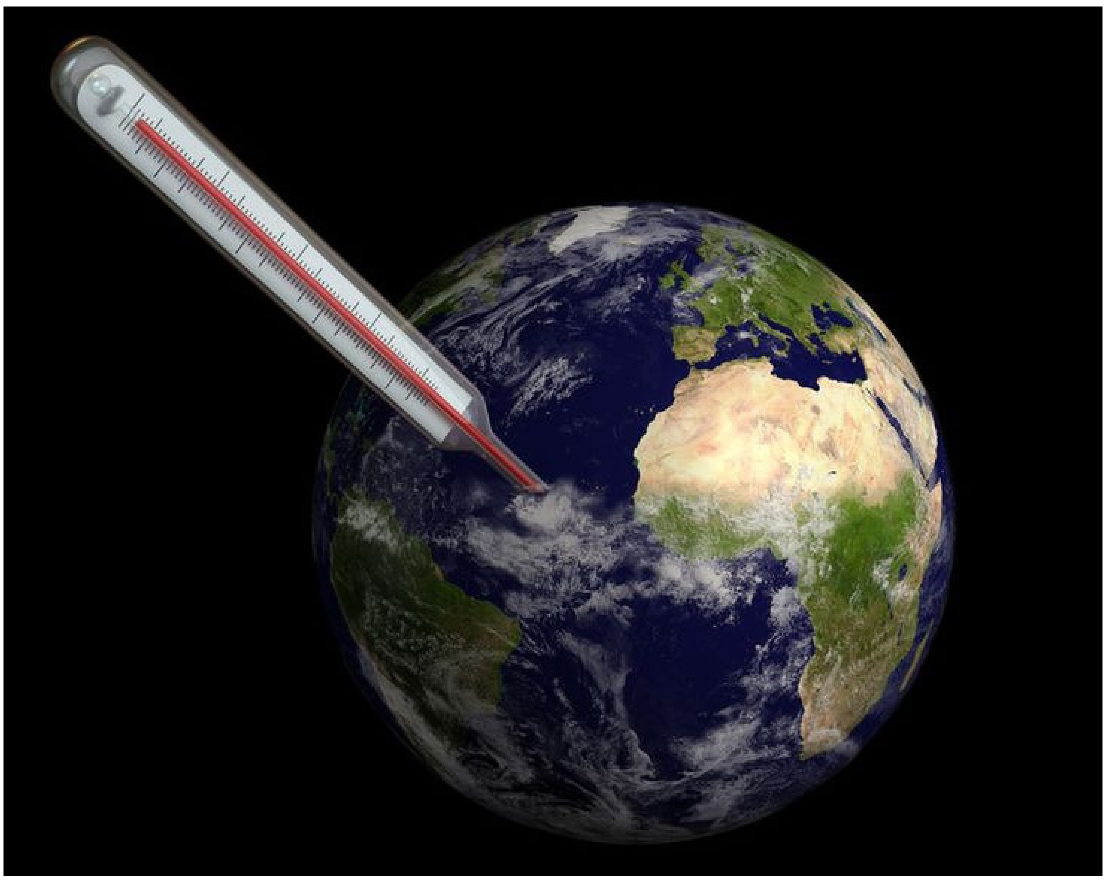
9 HEAT TRANSFER AND EARTH'S TEMPERATURE

Between 1850 and today, about 472 Gt of carbon from fossil fuels have been added in the atmosphere. We will assume that we will go as far as adding 3000 Gt of carbon. What would then be the concentration of CO_2 in the atmosphere if it is assumed that 41% of the carbon emitted remains in the atmosphere and what will be the temperature at the Earth's surface?

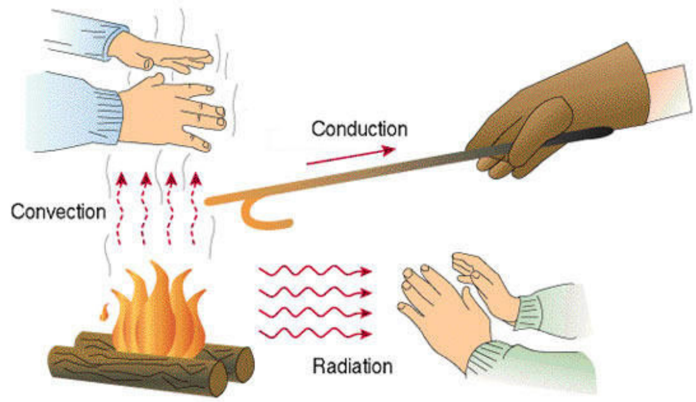


www.pinterest.co.uk/explore/solutions-of-global-warming/?lp=true

Learn how to solve this problem in this chapter.

9.1 HEAT TRANSFER

Heat is a form of energy. This energy can pass from one place to another. There are 3 ways to transport this energy.



https://en.wikipedia.org/wiki/Heat_transfer

1) Conduction

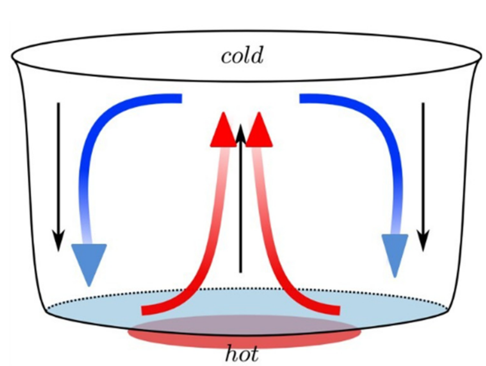
Energy can first be transferred by moving through matter (which remains stationary). In a substance, heat is associated with the average energy of atoms. If the average energy of the atoms is greater in one region of the object (meaning that the temperature is higher at that location), collisions between the atoms will eventually slowly transfer that energy into the substance. The heat thus moves into the object. This is the heat that we would end up feeling in our hand if we held the end of an iron bar whose other end is in a fire. Over time, the heat spreads through the iron rod and this heat is felt at the other end of the rod.

2) Radiation

Energy can also travel in the form of radiation. All hot objects emit energy in the form of electromagnetic radiation. The hot object loses energy, and the surrounding objects receive this energy. If we place our hands near a fire (but not above the fire), the energy received comes from the radiation emitted by the wood and the hot gases of the fire.

3) Convection

With convection, energy is transported by movements of matter in which this energy is located. This mode of transport can be observed in boiling water, for example. In the cauldron, whirlwinds of material form, and these carry the heat to the surface. If we place our hand over a fire, the rising air brings a lot of heat to our hand. In this case, convection is bringing energy to our hand.

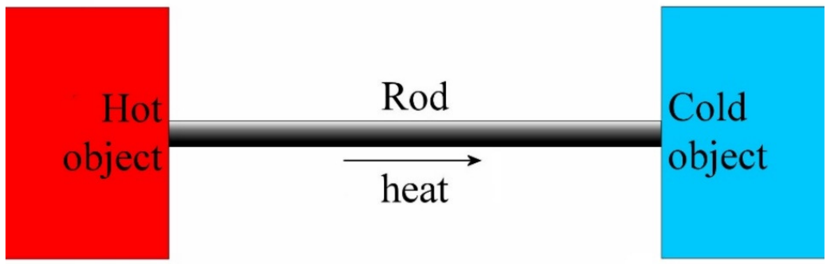


quizlet.com/gb/513798227/convection-diagram/

Calculations are rather difficult to do for convection. However, heat transfers can easily be calculated for conduction and radiation.

9.2 CONDUCTION

If a rod is placed between a hot object and a cold object, the heat will pass from the hot object to the cold object through the rod.



The energy that passes through the rod per second depends on the temperature difference between the ends of the rod. The greater the temperature difference, the more heat energy passes through the rod per second. So, we have

$$P \propto \Delta T$$

We use P because the rate of heat transfer is the power. This power is measured in watts (joules per second).

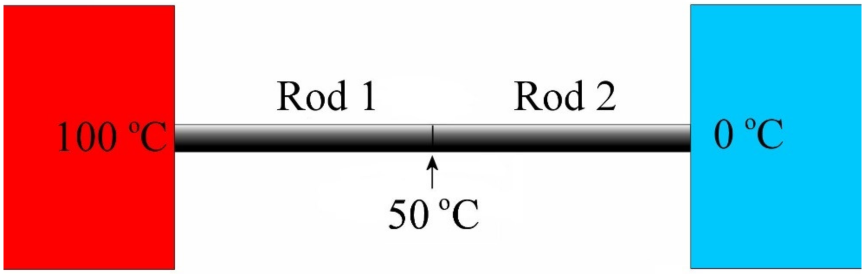
The power that will pass through the rod will also depend on the rod. Some rods will easily let the heat flow through while others will not let the heat flow as easily. Therefore, thermal resistance is defined by the following equation.

$$P = \frac{1}{R}\Delta T$$

This equation simply states that the rate of heat transfer that passes through the rod decreases as the thermal resistance of the rod increases.

It is quite easy to find out how this thermal resistance changes with the dimensions of the rod. First, let's imagine a long cylindrical rod that connects a hot object at 100 °C and a cold object at 0 °C. The heat then passes through the rod with the power P. This power must be identical throughout the rod at equilibrium. With the same P all over the rod, it passes the same amount of heat per second at each point of the rod, and this means that heat does not build up or become scarce in one spot on the rod. If there is no change in the amount of heat, the temperature of each part of the rod remains constant.

Now let's consider that this rod is actually formed of 2 identical shorter rods put end to end.



Since the rods are identical, they must have the same thermal resistance. Since they must also have the same P, ΔT must be the same for both rods. This means that the temperature at the junction of the two rods should be halfway between the temperature of the hot object and the temperature of the cold object. In our example, the temperature of the junction point between the rods must therefore be 50 °C.

For the whole stem (formed by the two shorter rods), we have

$$P = \frac{1}{R}\Delta T$$

For each smaller rod, we have

$$P = \frac{1}{R'}\frac{\Delta T}{2}$$

Since the P are identical, we must have

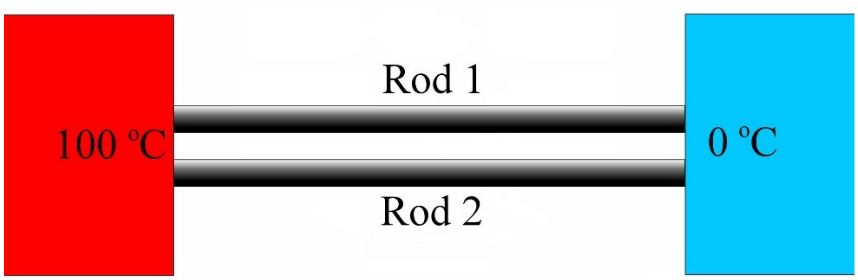
$$\frac{1}{R}\Delta T = \frac{1}{R'}\frac{\Delta T}{2}$$

This leads to

$$R' = \frac{1}{2}R$$

Thus, the rod with half the length has half the resistance. Thermal resistance must therefore be proportional to the length.

Now let's imagine that two identical rods are placed next to each other.



Since there are two rods, the power should be 2 times greater for the same ΔT. Thus, with a single rod, we have

$$P = \frac{1}{R}\Delta T$$

whereas with 2 rods, we have

$$P' = \frac{1}{R'}\Delta T$$

Since $P' = 2P$ (the rate is 2 times higher with 2 rods), we have

$$\frac{1}{R'}\Delta T = 2\frac{1}{R}\Delta T$$

$$R' = \frac{1}{2}R$$

The thermal resistance is now 2 times smaller with 2 rods. Now, these two rods next to each other are actually identical to a single rod whose end area is twice as large. This means that with an end area 2 times larger, the thermal resistance is divided by 2.

It can therefore be concluded that the thermal resistance increases with the length of the rod and decreases with the area of the end of the rod.

$$R \propto \frac{\ell}{A}$$

The proportionality constant is called *thermal conductivity,* and its value depends solely on the substance that makes up the rod.

$$R = \frac{1}{k}\frac{\ell}{A}$$

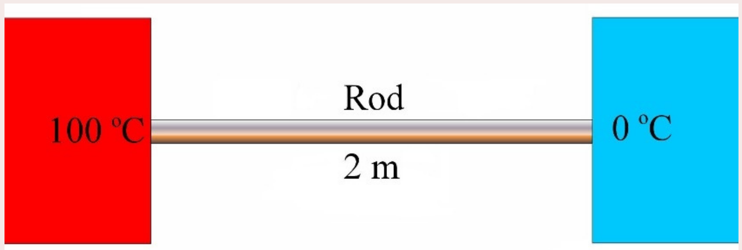
The thermal resistance is inversely proportional to the thermal conductivity because the resistance must increase when the conductivity decreases. The equation of the heat transfer rate therefore becomes

Conductive Heat Transfer

$$P = k\frac{A}{\ell}\Delta T$$

Example 9.2.1

An object at 100 °C is 2 metres from another object at 0 °C. Both objects are huge and perfectly conductive. A copper rod with a 4 cm diametre is then placed between the two objects. What is the rate of transfer of thermal energy in the rod knowing that the thermal conductivity of copper is 398 W/m°C?

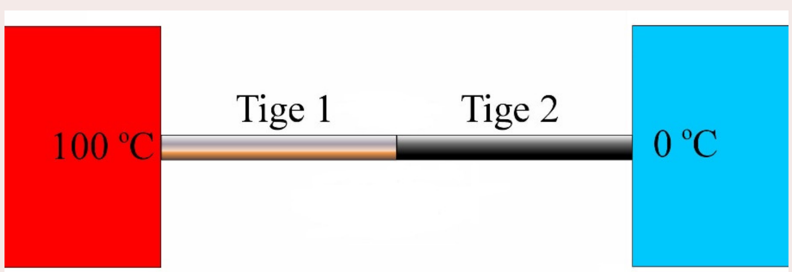


The transfer rate is

$$\begin{aligned} P &= k\frac{A}{\ell}\Delta T \\ &= 398\tfrac{W}{m^\circ C}\cdot\frac{\pi\cdot(0.02m)^2}{2m}\cdot 100^\circ C \\ &= 25.01W \end{aligned}$$

Example 9.2.2

An object at 100 °C is 2 metres from another object at 0 °C. Both objects are huge and perfectly conductive. The two objects are connected with two rods in contact end to end, both of which are 1 m long and have a 4 cm diametre. One rod is made of copper (rod 1) and the other is made of aluminum (rod 2). The thermal conductivity of copper is 398 W/m°C, and the thermal conductivity of aluminum is 239 W/m°C.



a) What is the temperature at the junction of the two rods?

Since the heat that passes through rod 1 must then pass through rod 2, the values of P must be identical for both rods.

For the 1st rod, we have

$$\begin{aligned} P &= k_1\frac{A}{\ell}\Delta T \\ &= k_1\frac{A}{\ell}\left(100^\circ C - T_j\right) \end{aligned}$$

where T_j is the temperature at the junction of the rods. For stem 2, we have

$$\begin{aligned} P &= k_2 \frac{A}{\ell} \Delta T \\ &= k_2 \frac{A}{\ell} \left(T_j - 0°C \right) \end{aligned}$$

Since the P are equal, we have

$$\begin{aligned} k_1 \frac{A}{\ell} \left(100°C - T_j \right) &= k_2 \frac{A}{\ell} \left(T_j - 0°C \right) \\ k_1 \left(100°C - T_j \right) &= k_2 \left(T_j - 0°C \right) \\ 398 \tfrac{W}{m°C} \cdot \left(100°C - T_j \right) &= 239 \tfrac{W}{m°C} \cdot \left(T_j - 0°C \right) \\ 39\,800°C - 398 \cdot T_j &= 239 \cdot T_j \\ 39\,800°C &= 637 \cdot T_j \\ T_j &= 62.48°C \end{aligned}$$

b) What is the rate of heat transfer (in W) in the rods?

The transfer rate can be found by looking at any of the 2 rods. Let's take rod 1.

$$\begin{aligned} P &= k_1 \frac{A}{\ell} \Delta T \\ &= 398 \tfrac{W}{m°C} \cdot \frac{\pi \cdot (0.02m)^2}{1m} \cdot \left(100°C - 62.48°C \right) \\ &= 18.77W \end{aligned}$$

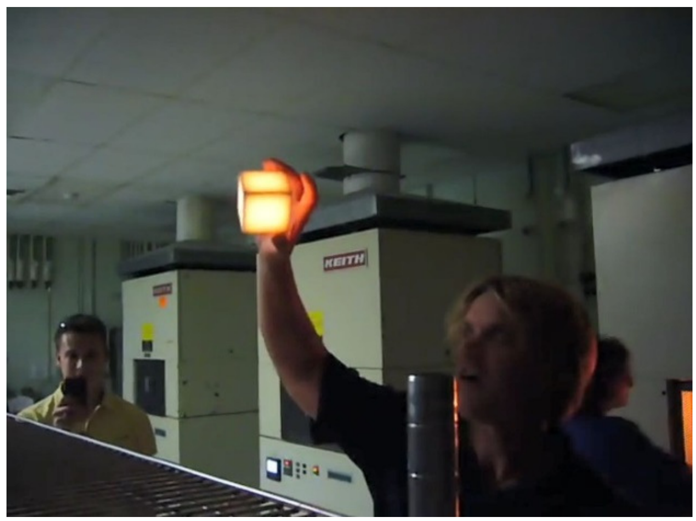
In these examples, it is assumed that heat enters through one end of the rod and exits only from the other end of the rod without any loss through the sides of the rod. In reality, heat could be lost through convection and radiation through the sides.

Generally, metals have a high thermal conduction. Free electrons in metals can efficiently transfer energy from one part of the metal to another.

Gases are not very good conductors of heat. The thermal conductivity of the ambient air is only 0.026 W/m°C. This conductivity is nearly 15 000 times smaller than the conductivity of copper. Their low density means that there are not many collisions between atoms and energy does not easily pass from one place to another. For this reason, air is often used as thermal insulation. Many insulating substances used in construction contain a lot of air, such as the layer of air between the 2 pieces of glass of a double-pane window. The air trapped in certain textiles is also making some clothes particularly insulating. Air insulates well as long as it is trapped and cannot move because there must be no convection that would allow heat to be transmitted more efficiently.

Note that your body's temperature sensors don't really measure the temperature of objects. They rather measure the rate of heat transfer (i.e. the power *P*) between the object you are touching and your body. A ceramic floor looks colder than a carpet at the same temperature because the thermal conductivity of ceramics is much greater. Heat can therefore easily flow from your body to the ceramic and this rapid heat loss is interpreted as a cold temperature by your brain. If you place your frozen hands in hot water, the water appears abnormally hot because the heat transfer is greater compared to the same situation when your hands are not frozen.

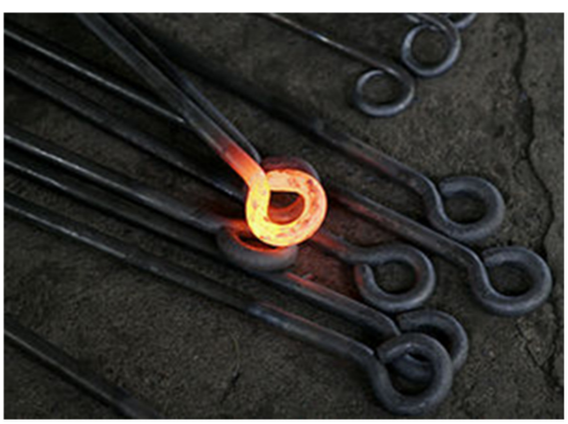
Substances with low conductivity cannot transfer heat quickly to your body. People can walk on hot embers because the thermal conductivity of wood is relatively small. Wood, even if it is very hot, has a lot of difficulty giving heat to your body and that is why you can walk on the embers without getting burned. The image on the right shows a person holding a piece of material once used to form the space shuttle's heat shield. This material can be held in the hands even if its temperature is several hundred degrees Celsius because its thermal conductivity is extremely low. Since the heat cannot leave the material, you don't get burned.



gigazine.net/gsc_news/en/20200324-picking-up-hot-space-shuttle-tiles/

9.3 RADIATION EMITTED BY HOT OBJECTS

19th-century Experimental Results



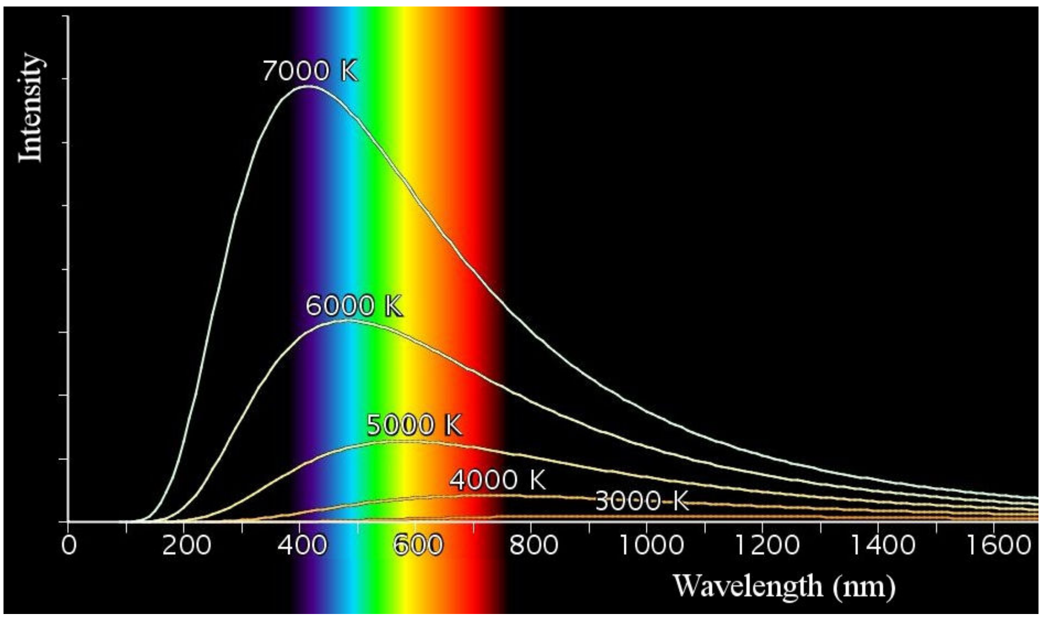
en.wikipedia.org/wiki/Thermal_radiation

Hot objects emit electromagnetic waves. All objects emit radiation if their temperature is above 0 K. This phenomenon has already been discussed in the section on the electromagnetic spectrum. For example, this metal ring heated to several hundred degrees Celsius emits a rather orange-red radiation.

As early as 1792, an English porcelain manufacturer named Thomas Wedgewood was the first to note that there is some kind of relation between colour and temperature. He noticed that his very hot ovens emitted

exactly the same shade of red when they were at the same temperature, regardless of their shape, their size or the way in which they were built. The observations made by many physicists in the following years confirmed that the colour of the light emitted changes with temperature. At room temperature, the radiation is invisible (it is mainly infrared radiation). If the object is heated, it starts to emit red light at about 700 K. Then, as the object is heated even more, its colour changes from red to orange, to yellow, to white, and to blue. Here is an animation showing how the colour of the light emitted by an object changes with the temperature.
http://www.youtube.com/watch?v=jCTmN7HY76k

In fact, this radiation is not monochromatic (a single colour or wavelength). Here's what is obtained when the intensity of the emitted radiation is plotted as a function of the wavelength for objects at 3000 K, 4000 K, 5000 K, 6000 K, and 7000 K.



www.astrosurf.com/spectrodavid/page_resultats_basse_resolution_au_SA100.htm

The graph shows that there is a peak of emissivity, and that the wavelength of this peak depends on the temperature of the object. The warmer an object is, the smaller the wavelength of the maximum is. The wavelength of the maximum is called λ_{max}. From the experimental observations, the following law was discovered in 1893.

Wavelength of the Emission Peak (Wien's Law)

$$\lambda_{\max} = \frac{2.898 \times 10^{-3} m \cdot K}{T}$$

where T is the temperature of the object.

An object at 6000 K therefore emits several wavelengths, and the maximum emission is at 483 nm (which is blue). Since, for visible light, there is a little more blue than other colours, the object emits a slightly bluish radiation.

The area under the curve represents the power radiated by the object per unit area. It is easy to see that the area under the curve increases with the temperature. If the object is warmer, then more power is radiated per unit area. In fact, the radiated power increases very rapidly with the temperature as it follows this law, discovered in 1879.

Power Radiated by a Hot Object (Stephan-Boltzmann Law)

$$P = \varepsilon\sigma A T^4$$

where σ is a constant whose value is 5.67 x 10^{-8} W/m²K^4, A is the area of the object and T is the temperature of the object (in kelvins.) ε is called the *emissivity*. The equilibrium conditions require that this emissivity be identical to the proportion of radiation that is absorbed by the object when light shines on the object. If the object absorbs 80% of the light received, then the emissivity is 0.8. The value of ε is 1 for a perfectly black object that absorbs all the radiation that arrives at the object. The value of ε is 0 for an object that reflects all the radiation that arrives at the object. The value of ε is therefore between 0 and 1.

These results are most valuable in astrophysics because several star features can be calculated with them since stars radiate (almost) like perfect hot objects, which means that ε is practically equal to 1 for stars.

Example 9.3.1

The light emitted by the star Sirius has a power of 1.003 x 10^{28} W (26.2 times more luminous than the Sun) and a peak of emissivity is at 291.5 nm.

a) What is the surface temperature of this star?

The temperature can be found with the formula for the maximum of emissivity.

$$\lambda_{\max} = \frac{2.898\times10^{-3} m\cdot K}{T}$$

$$291{,}5\times10^{-9} m = \frac{2.898\times10^{-3} m\cdot K}{T}$$

$$T = 9940K$$

b) What is the radius of this star?

The area of this star is found with the power formula.

$$P = \varepsilon\sigma A T^4$$

$$1.003\times10^{28} W = 1\cdot 5.67037\times10^{-8} \tfrac{W}{m^2K^4}\cdot A\cdot(9940K)^4$$

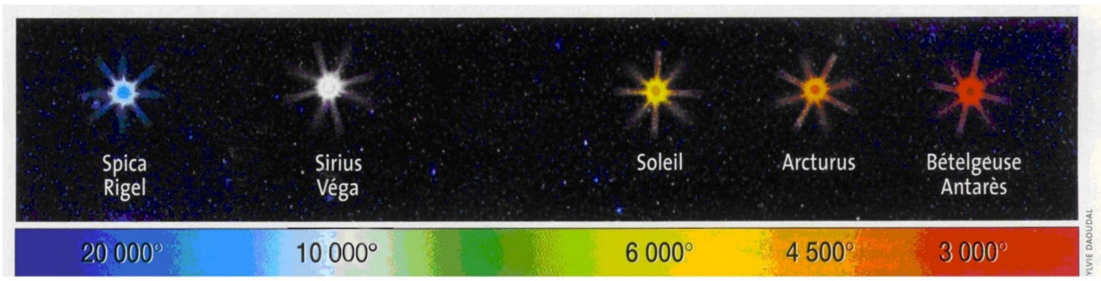
$$A = 1.812\times10^{19} m^2$$

As the area of a sphere is $4\pi r^2$, the radius is

$$4\pi r^2 = 1,812\times 10^{19}\, m^2$$
$$r = 1.201\times 10^{9}\, m$$

(This is 1.73 times the radius of the Sun.)

Since stars radiate like hot objects and since the colour of the light emitted depends on temperature, the colour of the star is a fairly obvious indication of its surface temperature.



physiquechimieedgarpoe.wordpress.com/2010/10/20/20102010/

This difference can easily be seen for Albireo, a double star system (two stars orbiting around their centre of mass).

The orange star (Albireo A) has a temperature of 4383 K while the blue star (Albireo B) has a temperature of 13,200 K.

www.astronomy.com/science/101-must-see-cosmic-objects-albireo/

Measuring the Temperature With Emitted Light

The temperature of an object can be measured from the radiation it emits. Some metallurgists can estimate quite accurately the temperature of an oven only from the colour of the light emitted by the furnace.

Thermometers that determine the temperature with radiation can be bought. The image shows one of these thermometers used to measure body temperature, a kind of thermometer widely used during the covid-19 pandemic.

The thermometer does not measure the entire spectrum emitted by the object. A simple measurement of a part of the infrared radiation is sufficient. The extend of the area examined varies depending on the quality of the device, and if the area is too large, the device will give a kind of average if the temperature is not uniform (in fact, the fourth root of the average of T^4).

www.quirumed.com/uk/contactless-infrared-thermometer.html

This kind of thermometer can have a hard time distinguishing between the light emitted by the hot object and the light simply reflected by the object. The thermometer will work best if the object absorbs practically all the infrared radiation that arrives on it so that there is no reflection. If the object reflects a lot of infrared or is transparent in infrared, the displayed temperature could be very wrong. Since the skin reflects only 2% of infrared light, body temperature measurements made with an optical thermometer are very reliable. On the other hand, measuring the temperature of a metal object, which reflects a lot of light, may give the temperature of the object that is the source of the reflected light. Also, never measure the temperature of a cold object surrounded by hot objects with an optical thermometer. Since the radiation increases very rapidly with temperature, the low radiation of the cold object may be completely overwhelmed by the reflection of the radiation of the surrounding hot objects even if the percentage of light reflected by the object is relatively small.

An Object in a Medium With Temperature T_0

Note that if an object is in a medium that has a temperature T_0, like air for example, then this medium will also emit radiation, and the object in the medium will receive this energy. The energy received by the object will be given by

$$P = \varepsilon\sigma A T_0^4$$

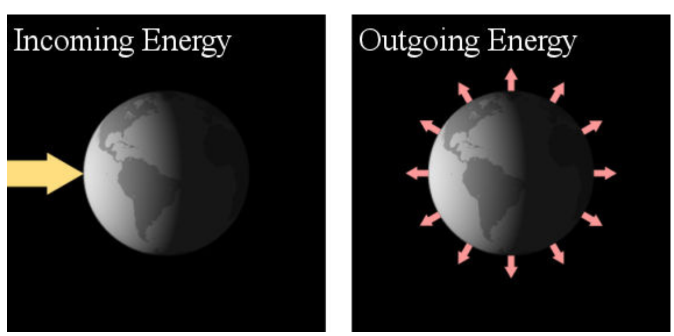
Thus, the net energy emitted by the object will be

$$\begin{aligned} P &= P_{emitted} - P_{received} \\ &= \varepsilon\sigma A T^4 - \varepsilon\sigma A T_0^4 \\ &= \varepsilon\sigma A\left(T^4 - T_0^4\right) \end{aligned}$$

If we wanted to make a very precise calculation, we would take T_0 = 3 K when we make the calculation for the stars since the temperature of the universe is 3 K (but that 3 K doesn't change the result much).

9.4 THE SURFACE TEMPERATURE OF A PLANET

The temperature of a planet is determined by the energy received from the star and the energy emitted by the planet due to hot-object radiation. At equilibrium, the energy received per second from the star must be equal to the energy emitted per second by the planet.



feww.wordpress.com/2009/01/21/earth-s-climate-a-solar-powered-system/

If the planet receives more energy per second than it emits, it accumulates energy, and its temperature rises. If the planet receives less energy per second than it emits, it loses energy and its temperature decreases.

The Power Received

Planets receive light from their star. Since stars are an isotropic source, the intensity of the light received from the star can be found with the following formula.

$$I = \frac{P_{star}}{4\pi D^2}$$

where D is the distance between the planet and the star. For the Earth, the power of the Sun is 3.828 x 10^{26} W and the distance between the Sun and the Earth is 1.496 x 10^{11} m. The intensity of the light received by the Earth is therefore

$$\begin{aligned} I &= \frac{P_{star}}{4\pi D^2} \\ &= \frac{3.828\times 10^{26} W}{4\pi \cdot \left(1.496\times 10^{11} m\right)^2} \\ &= 1361.1 \tfrac{W}{m^2} \end{aligned}$$

The energy received by the planet is calculated by considering the planet as a circular sensor with the same radius as the planet. Indeed, the energy captured is the same with a flat circle, because the same number of light rays is received.

feww.wordpress.com/2009/01/21/earth-s-climate-a-solar-powered-system/

The power received is therefore

$$\begin{aligned} P_{received} &= IA \\ &= I\pi R_{planet}^2 \end{aligned}$$

The value of Q, which is the average power received per unit area on Earth (which is called the average flux) is often used. This flux is

$$Q = \frac{P_{received}}{A_{planet}}$$
$$= \frac{I\pi R_{planet}^2}{4\pi R_{planet}^2}$$
$$= \frac{I}{4}$$

Knowing that $I = P_{star} / 4\pi D^2$, the following formulas is obtained for Q.

Average Flux (Power per Unit Area) Arriving on a Planet

$$Q = \frac{I}{4} = \frac{P_{star}}{16\pi D^2}$$

For Earth, the average flux received is therefore

$$Q = \frac{I}{4}$$
$$= \frac{1361.1 \frac{W}{m^2}}{4}$$
$$= 340.275 \frac{W}{m^2}$$

This is the average power that each square metre of the Earth's surface receives from the Sun. Obviously, there are regions that receive more (near the equator) and others that receive less (near the poles). This value is an average for the entire surface of the Earth.

This would be the average flux received if all the light was absorbed by the planet. That's what we'd have with a perfectly black planet. However, we know that objects do not absorb all the energy received. For a planet, the percentage of light reflected is called the *albedo*. (We could also have used the emissivity ε to take into account the reflection.) For the Earth, the albedo is 30% (A = 0.30). The received power represents only the percentage that remains after reflection. It is therefore necessary to multiply the power received by $1 - A$ to keep only what is actually received. We then have

$$Q_{received} = Q(1 - A)$$

The Power Emitted

According to Stephan-Boltzmann's law, the power emitted by the Earth (which is a sphere) at a temperature T is

$$P_{emitted} = \varepsilon\sigma A T^4$$

In infrared (since this is essentially what a planet will emit), the emissivity ε of the Earth is almost 1. Thus, we're going to simplify a bit by using $\varepsilon = 1$. The average flux emitted is equal to the power per unit area. Therefore, the average flux is

$$\begin{aligned} F_{emitted} &= \frac{P_{emitted}}{A} \\ &= \frac{\varepsilon\sigma A T^4}{A} \\ &= \sigma T^4 \end{aligned}$$

The Surface Temperature

The equilibrium temperature is found by equalizing the average flux received and the average flux emitted.

$$\begin{aligned} Q_{received} &= F_{emitted} \\ Q(1-A) &= \sigma T_e^4 \end{aligned}$$

(where T_e is the equilibrium temperature). From this equation, the temperature is obtained.

Equilibrium Temperature on the Surface of a Planet

$$T_e = \sqrt[4]{\frac{Q(1-A)}{\sigma}}$$

Common Mistake: Calculating the Square Root Rather Than the Fourth Root

Be careful, the root is a fourth root.

Let's see what this means for the Earth.

Example 9.4.1

What is the equilibrium temperature of the Earth if the albedo of the Earth is 0.30?

The temperature is

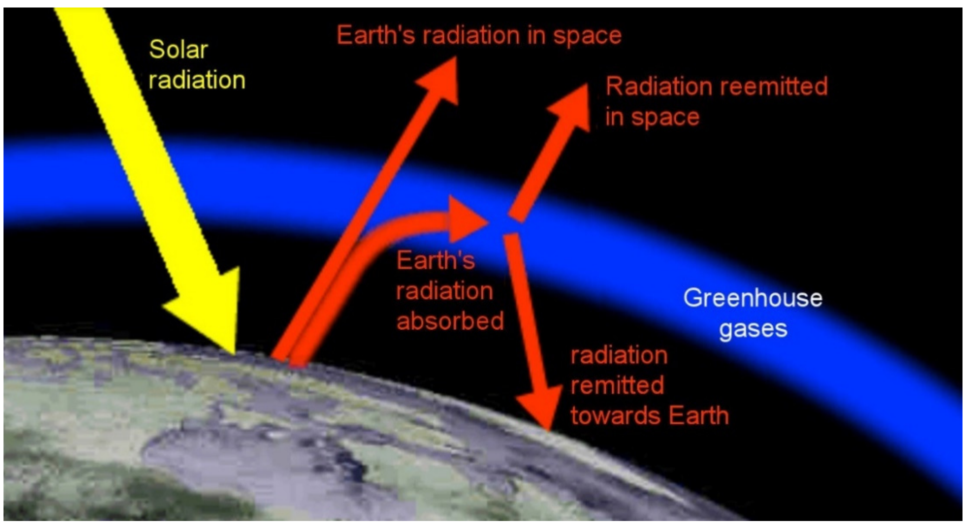
$$\begin{aligned} T_e &= \sqrt[4]{\frac{Q(1-A)}{\sigma}} \\ &= \sqrt[4]{\frac{340.275\frac{W}{m^2}\cdot(1-0.30)}{5.67037\times 10^{-8}\frac{W}{m^2K^4}}} \end{aligned}$$

$$= 254.58K$$
$$= -18.57°C$$

This seems a bit low. This would effectively be the Earth's equilibrium temperature if there were no atmosphere. (Although the Earth's albedo would be smaller if there were no atmosphere since there would be no clouds, which would result in a warmer Earth...)

9.5 THE GREENHOUSE EFFECT

The temperature at the Earth's surface is higher than the temperature formula predicts because there is a greenhouse effect. This greenhouse effect is caused by the atmosphere absorbing part of the radiation emitted by the Earth (and some of the radiation arriving from the Sun). This warms the atmosphere, which then begins to emit hot object radiation that warms the Earth's surface.



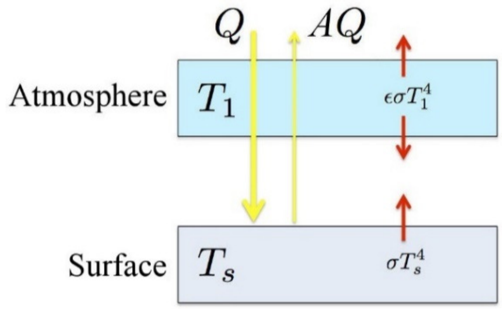
arxiv.org/pdf/2305.14433.pdf

The idea is not new. It was formulated as early as 1824 by Joseph Fourier, but he had no idea what could absorb the radiation in the atmosphere, and he did not make any calculations to determine the warming that such a mechanism could generate. The mechanism began to become clearer with the measurements of the absorption of infrared radiation by gases by John Tyndall in 1859 and with temperature calculations by Svante Arrhenius in 1896.

Single-Layer Model

We will start with a relatively simple model by assuming that the atmosphere is a simple layer that absorbs radiation. In this simple model, the layer only absorbs the radiation emitted by the Earth, while it does not absorb the radiation from the Sun at all. This approximation is not completely insane because the atmosphere absorbs very little radiation in the visible part of the spectrum (which the Sun emits) while it absorbs infrared radiation (which the Earth emits).

In this single-layer model, the surface of the planet is at a temperature T_S, and the atmosphere is at a temperature T_1. The flux Q arrives from the Sun, and the part AQ is reflected by the surface. The flux emitted by the surface is σT_S^4.



www.atmos.albany.edu/facstaff/brose/classes/ATM623_Spring2015/Notes/Lectures/Lecture06%20--%20Elementary%20greenhouse%20models.html

The atmosphere layer does not absorb all the radiation emitted by the Earth. It absorbs a proportion ε of the radiation emitted by the Earth. For example, if ε = 0.5, then the atmospheric layer absorbs 50% of the radiation emitted by the Earth. This ε is the emissivity, and this means that the power emitted by the atmosphere is

$$P = \varepsilon\sigma A T_1^4$$

and that the average flux emitted by the atmosphere is

$$F = \varepsilon\sigma T_1^4$$

Equation for the Surface

At equilibrium, the average flux received by the surface must be equal to the average flux emitted by the surface so that the surface does not accumulate or lose energy.

The equilibrium equation is therefore

Flux received from the Sun + flux received from the atmosphere =
flux emitted by the surface

$$Q(1-A) + \varepsilon\sigma T_1^4 = \sigma T_s^4$$

Without the greenhouse effect, we had

$$T_e = \sqrt[4]{\frac{Q(1-A)}{\sigma}}$$

Which means that

$$Q(1-A) = \sigma T_e^4$$

We can therefore write the flux equation for the surface in the following form.

$$\sigma T_e^4 + \varepsilon\sigma T_1^4 = \sigma T_s^4$$

$$T_e^4 + \varepsilon T_1^4 = T_s^4$$

Equation for the Atmosphere

At equilibrium, the flux of energy received by the atmosphere must be equal to the flux of energy emitted by the atmosphere so that the atmosphere does not accumulate or lose energy.

The equilibrium equation is therefore

Flux received from the surface = flux emitted by the atmosphere

$$\varepsilon\sigma T_s^4 = 2\varepsilon\sigma T_1^4$$

The received flux is $\varepsilon\sigma T_s^4$, because the atmosphere absorbs only the fraction ε of the energy emitted by the surface. Simplifying the equation, we arrive at

$$T_s^4 = 2T_1^4$$

Solving the Equations

The following 2 equations were obtained.

$$T_e^4 + \varepsilon T_1^4 = T_s^4$$
$$T_s^4 = 2T_1^4$$

Using the 2nd equation in the 1st equation, the result is

$$T_e^4 + \varepsilon T_1^4 = T_s^4$$
$$T_e^4 + \tfrac{1}{2}\varepsilon T_s^4 = T_s^4$$
$$2T_e^4 + \varepsilon T_s^4 = 2T_s^4$$
$$2T_e^4 = 2T_s^4 - \varepsilon T_s^4$$
$$2T_e^4 = (2-\varepsilon)T_s^4$$
$$T_s^4 = \frac{2}{2-\varepsilon}T_e^4$$

This leads to

$$T_s = T_e\sqrt[4]{\frac{2}{2-\varepsilon}}$$

Then, the temperature of the atmosphere can be found.

$$T_s^4 = 2T_1^4$$
$$\frac{2}{2-\varepsilon}T_e^4 = 2T_1^4$$
$$T_1^4 = \frac{1}{2-\varepsilon}T_e^4$$

The end result is thus

$$T_1 = T_e \sqrt[4]{\frac{1}{2-\varepsilon}}$$

Example 9.5.1

Measurements show that the atmosphere absorbs 71% of the infrared radiation emitted by the Earth, which means that $\varepsilon = 0.71$. The equilibrium temperature without the greenhouse effect is 254.59 K.

a) What should the soil temperature be according to the one-layer model?

The soil temperature should be

$$\begin{aligned} T_s &= T_e \sqrt[4]{\frac{2}{2-\varepsilon}} \\ &= 254.58K \cdot \sqrt[4]{\frac{2}{2-0.71}} \\ &= 284.08K \\ &= 10.93^\circ C \end{aligned}$$

b) What should the temperature of the atmosphere be according to the one-layer model?

The temperature of the atmosphere should be

$$\begin{aligned} T_1 &= T_e \sqrt[4]{\frac{1}{2-\varepsilon}} \\ &= 254.58K \cdot \sqrt[4]{\frac{1}{2-0.71}} \\ &= 238.88K \\ &= -34.27^\circ C \end{aligned}$$

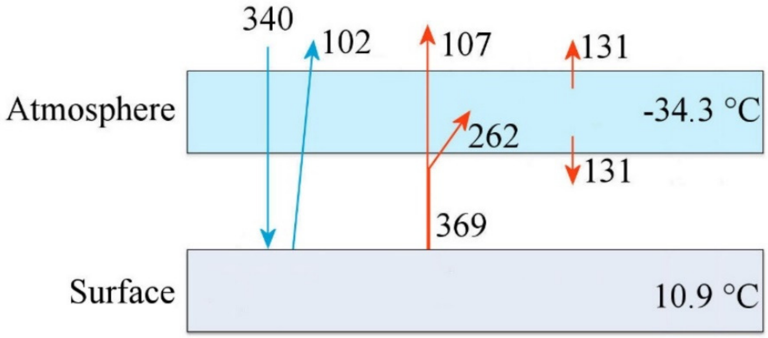
The average fluxes can also be calculated. The flux emitted by the surface would be

$$\begin{aligned} \sigma T_s^4 &= 5.67037 \times 10^{-8} \tfrac{W}{m^2K^4} \cdot (284.08K)^4 \\ &= 369.30 \tfrac{W}{m^2} \end{aligned}$$

and the flux emitted by the atmosphere would be

$$\begin{aligned} \varepsilon\sigma T_1^4 &= 0.71 \cdot 5.67037 \times 10^{-8} \tfrac{W}{m^2K^4} \cdot (238.88K)^4 \\ &= 131.10 \tfrac{W}{m^2} \end{aligned}$$

The following image shows the radiation balance of the Earth and its atmosphere (the fluxes are in W/m²).



We see that there is an equilibrium for the surface, the atmosphere and the radiation above the atmosphere. The flux emitted into space above the atmosphere is the sum of the reflected flux (102 W/m²), the energy radiated from the surface that is not absorbed by the atmosphere (107 W/m²) and the flux emitted from the atmosphere into space (131 W/m²). The sum of these three fluxes is 340 W/m², the same as the received flux. It can also be seen that the flux received by the surface (340 W/m² – 102 W/m² + 131 W/m²) is also equal to the flux emitted by the surface (369 W/m²), and that the flux received by the atmosphere (369 W/m² – 107 W/m² = 262 W/m²) is also equal to the flux emitted by the atmosphere (2 times 131 W/m²).

Multi-Layer Model

This model can be improved by separating the atmosphere into several layers. This can be done with 2, 3, 4... layers. For example, the following formulas are obtained with a 2-layer model.

$$T_s^4 = \frac{2+\varepsilon_c}{2-\varepsilon_c} T_e^4 \qquad T_1^4 = \frac{1}{2-\varepsilon_c} T_e^4 \qquad T_2^4 = \frac{1+\varepsilon_c}{2-\varepsilon_c} T_e^4$$

In these formulas, ε_c is the absorption of each layer, not the absorption of all layers taken together. Index 1 refers to the 1st layer of the atmosphere (the one with the highest altitude) and index 2 refers to the 2nd layer of the atmosphere.

(The proof for these formulas and those for the other multi-layer model can be seen in this document
https://physique.merici.ca/waves/multilayer.pdf.)

If we want the total absorption of the atmosphere to be 0.71 (as we had with a single layer), then we must have $\varepsilon_c = 0.4615$ each layer. This value is found from the transmission (which is $t = 1 - \varepsilon$) of each layer. When a layer allows a proportion t_c of the radiation to pass through, then n layers will allow a proportion of the radiation equal to t_c^n. Since the entire atmosphere allows 29% of the light to pass through, we must have, for 2 layers,

$$t_c^2 = 0.29$$

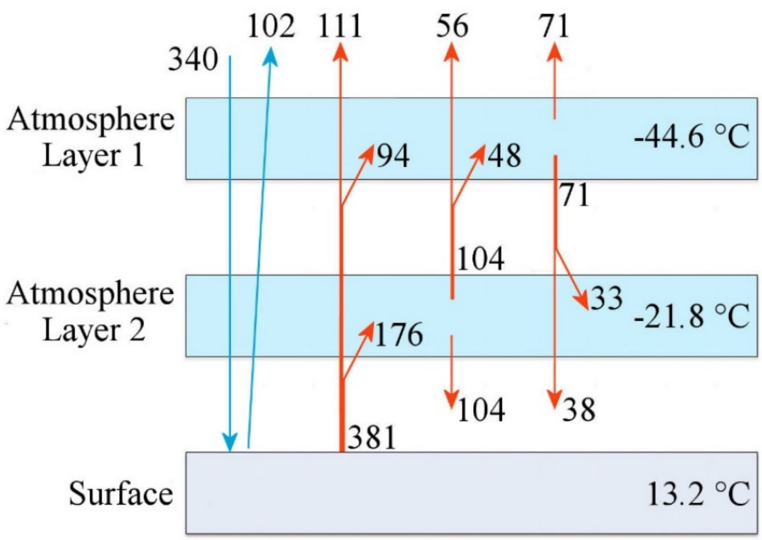
This leads to

$$t_c = 0.5385$$

Thus, the absorption made by each layer must be

$$\begin{aligned}\varepsilon_c &= 1 - t_c \\ &= 1 - 0.5385 \\ &= 0.4615\end{aligned}$$

With this value, the following temperatures and flux are obtained (in W/m²).



Note that the atmosphere is not separated into layers of equal thickness here. It is separated into layers of equal absorption. Each layer has the same absorption coefficient.

It can be seen that the surface temperature increases slightly with a 2-layer model compared to the single-layer model (13.2 °C instead of 10.9 °C).

Now let's see what is obtained if the number of layers is increased. With more layers, the becomes more realistic. There is no need to know the temperature of all the layers, only the temperature at the Earth's surface is needed. Here's how the temperature of the surface changes as the number of layers is increased.

3 layers

$$T_s^4 = \frac{2 + 2\varepsilon_c}{2 - \varepsilon_c} T_e^4$$

4 layers

$$T_s^4 = \frac{2 + 3\varepsilon_c}{2 - \varepsilon_c} T_e^4$$

5 layers

$$T_s^4 = \frac{2+4\varepsilon_c}{2-\varepsilon_c} T_e^4$$

With N layers, the result is

$$T_s^4 = \frac{2+(N-1)\varepsilon_c}{2-\varepsilon_c} T_e^4$$

We can then make the limit for an infinity of thin layers. Note that with an infinite number of layers, the absorption coefficient ε_c of each layer tends towards 0. The result of this limit is

$$T_s^4 = \left(1 - \ln\sqrt{1-\varepsilon}\right) T_e^4$$

where ε is the coefficient of total absorption of the atmosphere. Click here for a proof.
https://physique.merici.ca/waves/limitinfinitelayers.pdf

We therefore obtain

Temperature at the Surface of a Planet (With the Greenhouse Effect)

$$T_s = T_e \sqrt[4]{1 - \ln\sqrt{1-\varepsilon}}$$

Example 9.5.2

What should be the average temperature at the Earth's surface if the absorption coefficient ε is 0.71?

The temperature is

$$\begin{aligned} T_s &= T_e \sqrt[4]{1 - \ln\sqrt{1-\varepsilon}} \\ &= 254.58K \cdot \sqrt[4]{1 - \ln\sqrt{1-0.71}} \\ &= 254.58K \cdot 1.1280 \\ &= 287.17K \\ &= 14.02°C \end{aligned}$$

This result corresponds quite well to the value of the current average temperature at the surface of the Earth.

Note that, no matter how many layers is used, the temperature of the atmosphere layer at the highest altitude is always

$$T_1^4 = \frac{1}{2-\varepsilon_c} T_e^4$$

When the number of layers tends to infinity, ε_c tends towards 0, and the result is

$$T_1^4 = \frac{1}{2} T_e^4$$

For Earth, this formula gives 214.1 K, i.e. -59.1 °C. This is not very far from the temperature in the stratosphere, which is -55 °C. (The temperature of the atmosphere above the stratosphere then rises as ozone strongly absorbs ultraviolet rays from the Sun.)

The model with an infinite number of layers gives results quite close to the observed values, but keep in mind that this model uses several approximations and simplifications. Here are a few things that could be added to improve this model.

1) The fact that part of the sun's radiation is absorbed by the atmosphere could be taken into account. It has been assumed here that all the solar radiation reaches the surface without being absorbed by the atmosphere. In fact, 23% of this radiation is absorbed by passing through the atmosphere. That's almost half of the radiation that is not reflected.

2) The model treats the atmosphere as a series of layers that always stay in place and exchange heat only by radiation. However, there are vertical air movements (convection) and water evaporation that carry heat into the atmosphere (about 10% of the energy received is transported by convection and about 25% of the energy received is transported by evaporation).

3) It was assumed that the surface temperature of the Earth had the same value everywhere. A more sophisticated model would separate the Earth's surface into small units and consider the radiation balance on each of these units before making an overall balance.

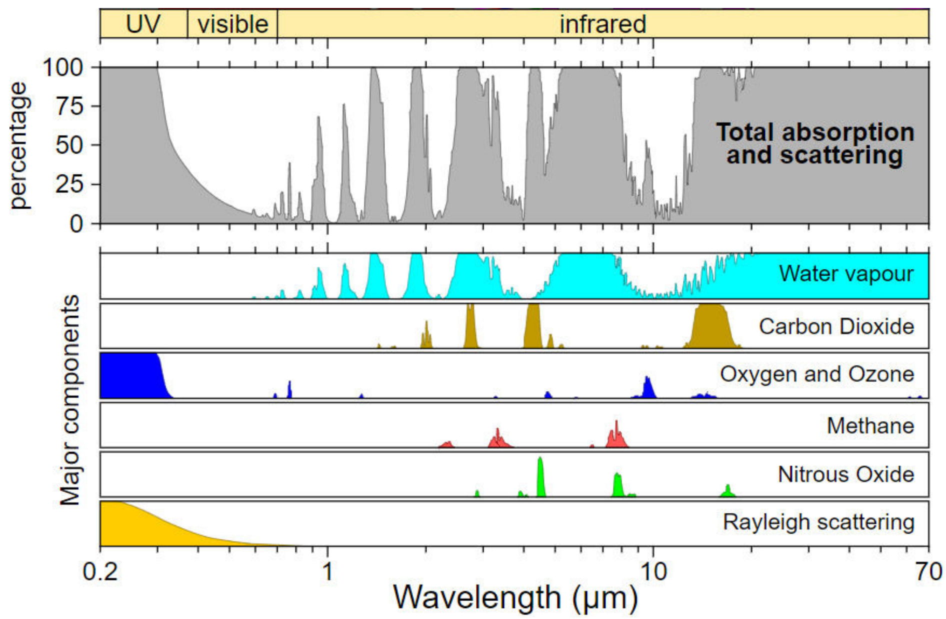
4) Our model gives the equilibrium temperature of the Earth. However, it is important to remember that it will take some time reach a new equilibrium if there is a change. If a change ever occurs, we should not think that the change in temperature would be instantaneous. For Earth, this time is essentially determined by the time it takes to increase or decrease the temperature of the oceans.

Gas Absorption

The atmosphere seems quite transparent and doesn't really seem to be absorbing light. In fact, the atmosphere absorbs very little visible light, and it appears completely transparent for that specific kind of light. On the other hand, several gases in the atmosphere absorb infrared quite strongly. It is precisely this type of radiation that is emitted by the Earth.

In these short videos, it is shown that carbon dioxide (CO_2) does indeed block infrared radiation.
https://www.youtube.com/watch?v=kGaV3PiobYk
https://www.youtube.com/watch?v=Ge0jhYDcazY

The following graph shows the absorption made by different gases in the atmosphere.



commons.wikimedia.org/wiki/File:Radiation_transmise.png

This graph shows that the atmosphere is quite transparent in visible light, but that there are large areas of the infrared spectrum that are absorbed. Thus, a good part of the energy coming from the Sun can reach the Earth's surface, because this radiation is mainly composed of visible light. Only a small part of the Sun's radiation is made up of infrared rays, and this is why 23% of the Sun's radiation is absorbed by the atmosphere (19% by gases and 4% by clouds).

The graph also shows that the infrared radiation emitted by the Earth is strongly absorbed by the atmosphere, because there are several parts of the spectrum that are intensely absorbed for this radiation. There are still portions of the infrared spectrum that are not absorbed, and this is what ensures that 29% of infrared radiation still manages to pass through the atmosphere.

The graph also shows that several gases contribute to the absorption of infrared radiation. This absorption is not very important for nitrogen, oxygen or argon, the 3 main components of the atmosphere (these gases represent 99.971% of the volume of the atmosphere if water vapour, whose concentration is extremely variable, is excluded).

The main gases that absorb radiation are water vapour, CO_2, ozone, methane and nitrous oxides. Here is the average contribution for the absorption of each of these gases (called *greenhouse gases*).

1) Water vapours 60 %
2) Carbon dioxide 26 %
3) Ozone 8 %
4) Methane and nitrous oxides 6 %

Some gases are much more effective than others at absorbing infrared radiation. CO_2 is responsible for 26% of the absorption despite a concentration of barely 0.04% in the atmosphere (while water can sometimes reach a concentration of 7%). Other gases are even more efficient than CO_2. The following list shows the absorption efficiency of certain gases using CO_2 as a reference.

Gas	Abs/mol
CO_2	1
CH_4	23
N_2O	296
O_3	2000

Methane thus absorbs 23 times more infrared radiation than CO_2, but since the concentration of methane is only 0.0017%, CO_2 absorbs more infrared overall.

Factors That Can Cause the Earth's Surface Temperature to Change

According to our equilibrium temperature and greenhouse effect formulas,

$$T_s = T_e \sqrt[4]{1 - \ln\sqrt{1-\varepsilon}}$$

$$T_e = \sqrt[4]{\frac{Q(1-A)}{\sigma}} \qquad\qquad Q = \frac{I}{4} = \frac{P_{star}}{16\pi D^2}$$

There are not many elements that can change the surface temperature of a planet. There is only Q (related to the intensity of the radiation coming from the star), A (the albedo) and ε (the proportion of infrared absorbed by the atmosphere). If any one of these elements change, the temperature on the Earth's surface can change.

Example 9.5.3

How much would the average temperature at the Earth's surface change if the intensity of radiation I increases to 1370 W/m² (instead of 1361.1 W/m²) and if the values of ε and A remain the same ($\varepsilon = 0.71$ and $A = 0.30$)?

With this intensity, the value of Q would be

$$\begin{aligned} Q &= \frac{I}{4} \\ &= \frac{1370 \frac{W}{m^2}}{4} \\ &= 342.5 \frac{W}{m^2} \end{aligned}$$

The equilibrium temperature would then be

$$\begin{aligned} T_e &= \sqrt[4]{\frac{Q(1-A)}{\sigma}} \\ &= \sqrt[4]{\frac{342.5 \frac{W}{m^2} \cdot (1-0.3)}{5.67037 \times 10^{-8} \frac{W}{m^2 K^4}}} \\ &= 255.00 K \end{aligned}$$

With the greenhouse effect, we would then have

$$\begin{aligned} T_s &= T_e \sqrt[4]{1 - \ln \sqrt{1-\varepsilon}} \\ &= 255.00 K \cdot \sqrt[4]{1 - \ln \sqrt{1-0.71}} \\ &= 255.00 K \cdot 1.1280 \\ &= 287.64 K \\ &= 14.49 °C \end{aligned}$$

Since the temperature obtained previously was 14.02 °C, this would correspond to a temperature increase of 0.47 °C.

The surface temperature therefore increases if the intensity of the solar radiation that arrives on Earth increases. Let's admit that this is not so surprising.

Example 9.5.4

How much would the average temperature at the Earth's surface change if the albedo increases to A = 0.31 (instead of 0.30) and if the values of Q and ε remain the same (Q = 340.275 W/m² and ε = 0.71)?

With this albedo, the equilibrium temperature would be

$$\begin{aligned} T_e &= \sqrt[4]{\frac{Q(1-A)}{\sigma}} \\ &= \sqrt[4]{\frac{340.275 \frac{W}{m^2} \cdot (1-0.31)}{5.67037 \times 10^{-8} \frac{W}{m^2 K^4}}} \\ &= 253.67 K \end{aligned}$$

With the greenhouse effect, we would then have

$$\begin{aligned}T_s &= T_e\sqrt[4]{1-\ln\sqrt{1-\varepsilon}}\\ &= 253.67K\cdot\sqrt[4]{1-\ln\sqrt{1-0.71}}\\ &= 253.67K\cdot 1.1280\\ &= 286.14K\\ &= 12.99°C\end{aligned}$$

Since the temperature obtained previously was 14.02 °C, this would correspond to a temperature drop of 1.03 °C.

The surface temperature therefore decreases if the albedo increases.

In 1815, the eruption of the volcano Tambora sent a phenomenal amount of dust and sulphide aerosols into the upper atmosphere. Because these aerosols increase the Earth's albedo and also because the intensity of sunlight was a bit lower in the early 19th century (a period called the *Dalton minimum*), the months following the explosion were abnormally cold. Actually, the year 1816 is called *the year without a summer*. In July, some lakes in northwestern Quebec were still frozen enough to walk across the ice. The cold was not constant. The temperature has only dropped by 0.5 to 1.5 °C in the northern hemisphere, but this drop has completely disrupted the atmospheric circulation (particularly jet streams) and, as a result, generated very rapid and large temperature variations. During this summer, there were relatively hot periods, interspersed with periods of intense cold. At the beginning of June, a terrible cold wind began to blow over Quebec City. Many birds were found dead, and several freshly shorn sheep succumbed to the cold. Finally, on June 6th, 30 cm of snow fell. Still in Quebec City, the temperature drops below 0 °C on July 18th. Sometimes, in July and August, ice was found on lakes and rivers as far south as Pennsylvania. Harvests were catastrophic throughout the northern hemisphere, and there were many famines and revolts.

Example 9.5.5

How much would the average temperature at the Earth's surface change if the absorption coefficient increased to $\varepsilon = 0.72$ (instead of 0.71) and if the values of Q and A remain the same (Q = 340.275 W/m² and A = 0.30)?

If I and A do not change, the equilibrium temperature T_e does not change and remains at 254.58 K.

With an absorption coefficient of 0.72, the surface temperature would be

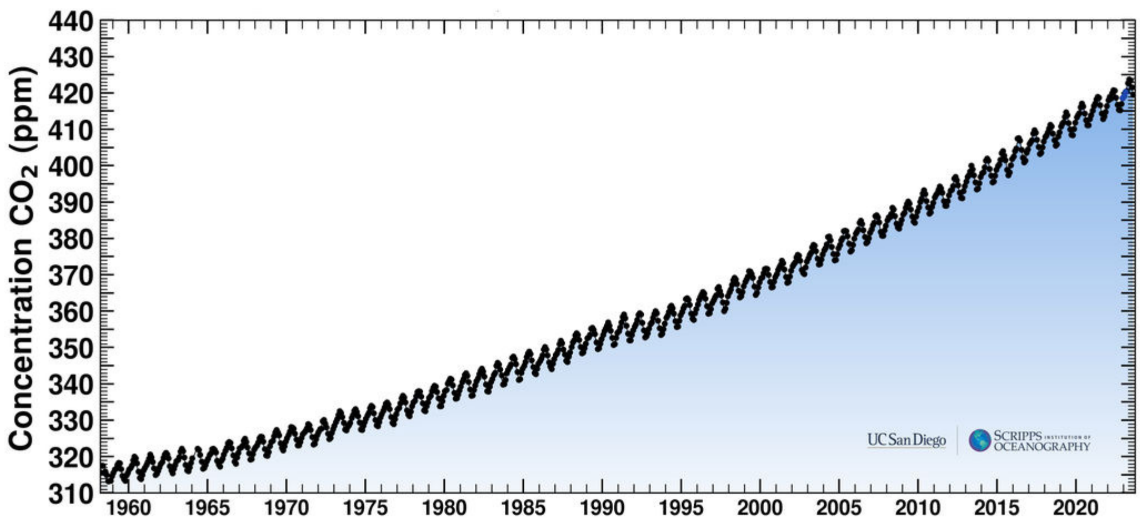
$$\begin{aligned}T_s &= T_e\sqrt[4]{1-\ln\sqrt{1-\varepsilon}}\\ &= 254.58K\cdot\sqrt[4]{1-\ln\sqrt{1-0.72}}\end{aligned}$$

$$= 254.58K \cdot 1.1310$$
$$= 287.94K$$
$$= 14.79°C$$

Since the temperature obtained previously was 14.02 °C, this corresponds to a temperature increase of 0.77 °C.

The temperature on the ground therefore increases if the absorption coefficient of the atmosphere increases.

The absorption coefficient depends on the concentration of greenhouse gases in the atmosphere. The problem is that new CO_2 and methane are constantly added in the atmosphere. The following graph shows the variations in CO_2 concentrations in the atmosphere since 1958.



keelingcurve.ucsd.edu/

This additional CO_2 comes from the combustion of fossil fuels such as coal and oil. Since the beginning of the industrial era (i.e. since 1850), the average concentration of CO_2 in the atmosphere has risen from 278 ppm to 420 ppm, which is 0.042%.

The increase in temperature that comes with the increase in the concentration of greenhouse gases has been known for a long time. As early as 1896, the Swedish scientist Svante Arrhenius predicted that the temperature at the Earth's surface would increase as the use of fossil fuels would add a lot of CO_2 in the atmosphere.

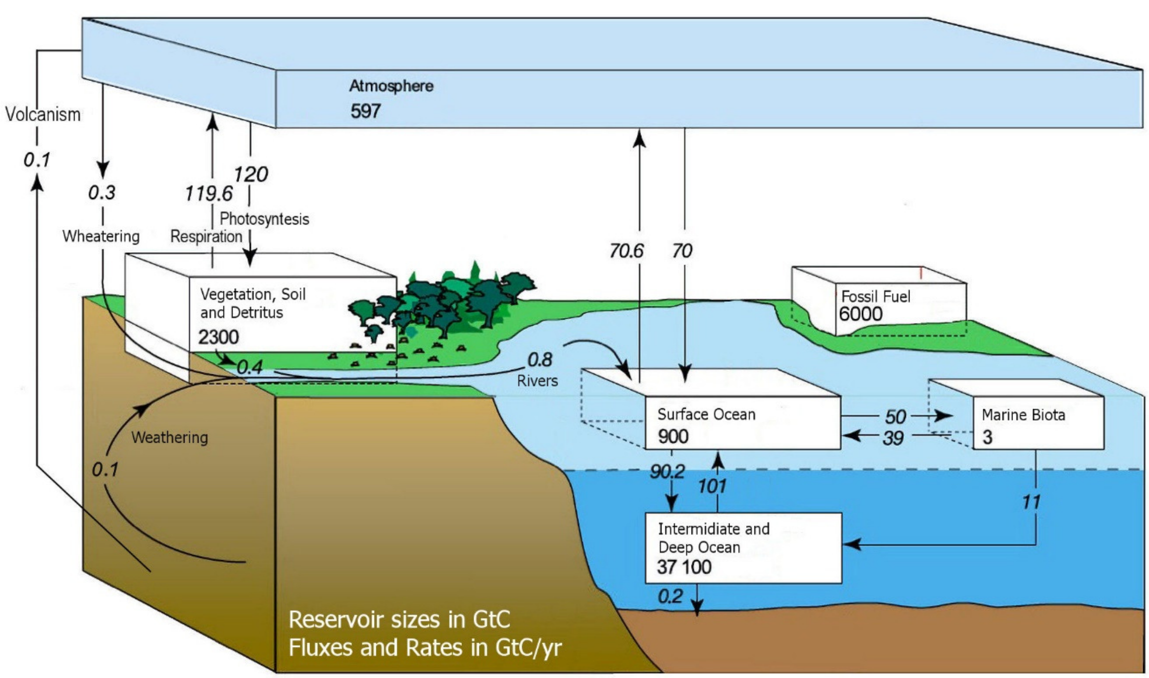
Notice the small annual variation of about 6 ppm on the graph. This variation comes from the imbalance between the size of the continents in the northern and southern hemisphere. The surface area of the continents is much larger in the northern hemisphere than in the southern hemisphere. Thus, the amount of CO_2 is modulated by the seasons in the northern hemisphere. During the summer in the Northern Hemisphere, plant growth in the boreal forest decreases the amount of carbon dioxide in the atmosphere (since plants absorb carbon from the atmosphere as they grow). This absorption of about 13 billion tonnes

(13 Gt) of carbon reduces the concentration of CO_2 in the atmosphere. When carbon absorption stops during the winter, the concentration of CO_2 rises again. Only the boreal forest generates these variations since the tropical forests absorb carbon throughout the year, not just in summer.

9.6 THE CARBON CYCLE

Humans add CO_2 to the atmosphere, but they are not the only source of CO_2. There are several mechanisms that add carbon to the atmosphere (called *carbon sources*) and several mechanisms that remove carbon from the atmosphere (called *carbon sinks*). In fact, the carbon emitted by humans accounts for about 5% of all the carbon emitted into the atmosphere. That doesn't seem like much, and this figure is often quoted by those who oppose climate action. But that extra 5% makes a huge difference. To fully understand what is going on, we need to know the broad outlines of the carbon cycle.

Let's start by looking at the situation before the industrial era. At that time, there was a balance between carbon sources and carbon sinks. (Note that some of the values in this diagram are estimates, and it should not be surprising if other sources give slightly different values.)



jancovici.com/changement-climatique/gaz-a-effet-de-serre-et-cycle-du-carbone/les-puits-de-carbone-ne-vont-ils-pas-absorber-le-surplus-de-co2/

In this diagram, the amount of carbon (not CO_2, carbon) is given in gigatons of carbon, which is written as GtC. The arrows show the carbon exchanges between the different elements of the cycle in gigatons of carbon per year, which are written as GtC/year.

It can be seen that the amount of carbon present in each element must remain the same. Let's take the atmosphere, for example. Volcanism adds 0.1 Gt of carbon per year, vegetation (I'll simply write *vegetation* for the category *vegetation, soils and detritus* (which also includes animals by the way)) adds 119.6 Gt of carbon per year and the oceans add 70.6 Gt of carbon per year, for a total addition of 190.3 Gt of carbon per year. On the other hand, weathering (which we will come back to later) removes 0.3 Gt of carbon per year, vegetation eliminates 120 Gt per year, and oceans eliminate 70 Gt per year. Thus, a total of 190.3 Gt of carbon is removed from the atmosphere. There is as much carbon entering as carbon leaving the atmosphere. If we look at the diagram carefully, we see that this is also the case for vegetation, oceans and the Earth. With this equilibrium, the amounts of carbon remain fixed in each element.

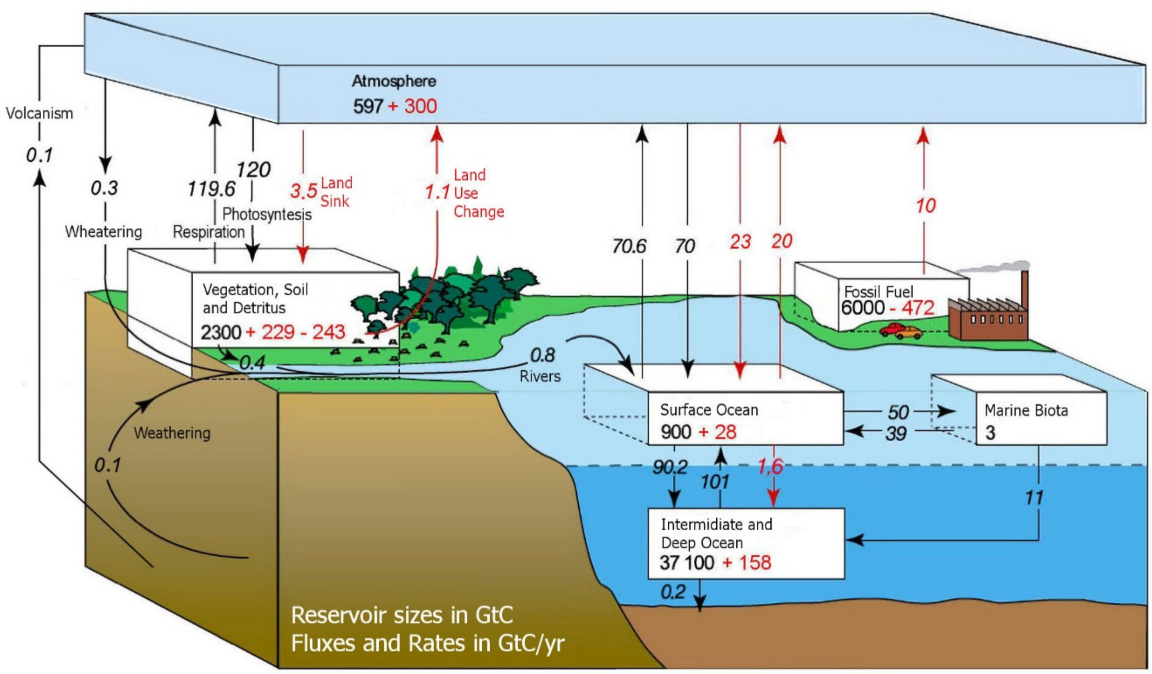
It can also be seen that vegetation removes almost no carbon. Vegetation absorbs almost as much carbon as it puts back into the atmosphere. Vegetation absorbs carbon through photosynthesis, but this carbon eventually returns to the atmosphere through respiration and decomposition (only *respiration* was written on the diagram, but it's actually *respiration and decomposition).* Forests are therefore not the lungs of the Earth as we often hear. They produce almost as much carbon as they absorb. Vegetation still removes 0.4 Gt of carbon per year from the atmosphere, which then flows into the oceans. This doesn't remove the carbon from the cycle; however, it just moves to another place.

There is also a lot of carbon exchange between the atmosphere and the oceans. The oceans contain a lot of carbon, mainly in the form of dissolved CO_2 (1%), HCO_3^- ions (90%) and CO_2- carbonate ions (9%). The ocean can be divided into 2 parts: the surface oceans and the deep ocean. In the surface ocean, water makes fairly frequent contact with the atmosphere, which allows exchanges between water and air. This part of the ocean is a few hundred metres thick (the thickness is not always the same depending on the place). Carbon exchanges between the atmosphere and the deep ocean are more difficult. Marine organisms help a little to transfer carbon into the deep sea thanks to the waste produced and the dead bodies that sink to the bottom (this is the *biological pump*). It still takes hundreds of years to reach equilibrium between the atmosphere and the deep ocean.

This whole cycle is a gigantic balancing act.

If carbon is added or removed in this cycle, the amounts of carbon in each component (atmosphere, vegetation and oceans) change. In nature, there are a few mechanisms that can add carbon. Volcanoes and weathering are the main sources of new carbon. The average annual flux from volcanoes is relatively small, but volcanoes slowly add new carbon. Weathering occurs when rainwater interacts with the soil. When rainwater passes through the atmosphere, it picks up CO_2 as it passes, which then forms carbonic acid. If the H^+ in this carbonic acid reacts with a carbonate ($CaCO_3$) from a rock on the ground, it will capture the carbon from that rock and add it to the cycle (this adds about 0.1 Gt of carbon per year). There are also two natural mechanisms that remove carbon from the cycle. Vegetation does not remove carbon, as all living things eventually decompose, which recycles the carbon. However, some plants or animals escape decomposition by ending up buried deep in the soil, which removes carbon from the cycle. This mechanism probably removed a total of 6000 Gt of carbon from the cycle, and this buried carbon eventually became coal or oil. Actually, it's essentially rain that slowly removes carbon from the cycle. If the rain that arrives on the ground encounters rocks containing silicates (which are much more abundant than carbonates), the carbonic acid in the rain reacts with the silicates to form new compounds, including carbonate, which are carried into rivers and eventually into the oceans. Carbonate can be deposited directly on the ocean floor, but it can also be used by marine organisms to make their shells. When they die, the shell settles on the bottom of the ocean. These carbon-containing compounds therefore become sediments on the bottom of oceans, which removes part of the carbon from the cycle (about 0.3 Gt of carbon/year). It is essentially the small difference between adding and removing carbon that determines whether the amount of carbon in the cycle increases or decreases. This is why the concentration of CO_2 in the atmosphere has not always been the same in the history of our planet.

Now let's see what we have right now. The changes compared to the situation in 1850 are indicated in red.



Since 1850, human activities have added 715 Gt of carbon to the atmosphere.

Of these 715 Gt added since 1850, 472 Gt (66%) come from the combustion of fossil fuels. At this moment, this combustion adds 10 Gt of additional carbon to the cycle each year. This carbon is new carbon in the cycle (it was already part of the cycle a long time ago and now it returns to it millions of years after being eliminated). 243 Gt (34%) comes from land use. This category includes all human-induced carbon fluxes related to vegetation cover and soil organic matter whose carbon content depends on the use made of them. This is the flux of carbon that comes from all land "managed" by humans, even if the fluxes are of natural origin. This includes forest land and farmland. The main fluxes come from land-use changes, such as reforestation, afforestation (planting trees in a place where there has been no forest for a long time), deforestation or urbanization of agricultural land, and changes in practices, such as no-till farming and forest growth. Currently, these activities add an additional 1.1 Gt of carbon per year. This carbon is not new in the cycle, it was already part of the cycle.

Of these 715 Gt added to the atmosphere, the oceans have absorbed 183 Gt (26%). The increase in the concentration of CO_2 in the atmosphere increases the partial pressure of this gas and this increases the dissolution of the gas in the oceans. At this moment, the oceans absorb 3 Gt of the 11.1 Gt emitted each year by human activities. This represents 27% of emissions.

Of these 715 Gt added since 1850, 229 Gt (32%) were absorbed by the land sink. The land sink is the flux of carbon that comes from all land "not managed" by humans. This is due to the combined effect of fertilization caused by the increase in atmospheric CO_2 and the lengthening of the growing season generated by the increase in temperature. Since CO_2 is practically the food of plants that do photosynthesize, an increase in the concentration of this gas allows plants to accelerate their growth and absorb more carbon. This 229 Gt of absorbed carbon therefore corresponds to an increase in the total mass of plants in unmanaged land. In the past, emissions from vegetation were greater than absorption, but now the trend is towards an increase in carbon sequestered in vegetation since human land use currently passes 1.1 Gt/year of carbon into the atmosphere while vegetation and soils absorb 3.5 Gt/year. Currently, the net flux is causing vegetation to absorb 2.4 Gt of carbon each year. This represents 22% of carbon emissions.

The remaining 300 Gt of the 715 Gt emitted remained in the atmosphere. This means that 42% of the carbon emitted since 1850 has remained in the atmosphere and has made the concentration of CO_2 change from 278 ppm to 420 ppm. Currently, carbon emissions from human activities are 11.1 Gt/year. If the 3 Gt absorbed by the oceans and the 3.5 Gt absorbed by vegetation and soil are subtracted, 4.6 Gt of carbon remains to stay in the atmosphere each year (41% of emissions). Every time 2.124 Gt of carbon is added to the atmosphere, the concentration of CO_2 increases by 1 ppm.

Relationship between the amount of carbon and the concentration of CO_2 in the atmosphere

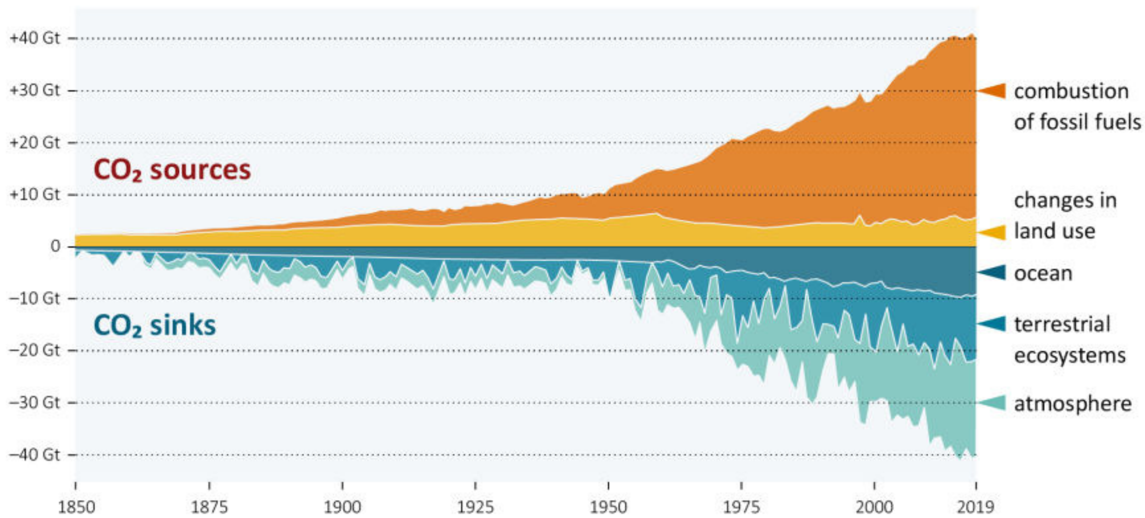
$$2,214 Gt_C = 1 ppm_{CO_2}$$

The 4.6 Gt that is added annually therefore represents an increase of just over 2 ppm per year.

The following graph shows the evolution of these carbon sources and sinks since 1850. The values are in Gt of CO_2 and not in Gt of carbon. We can easily convert since the proportion of mass of carbon in a CO_2 molecule is

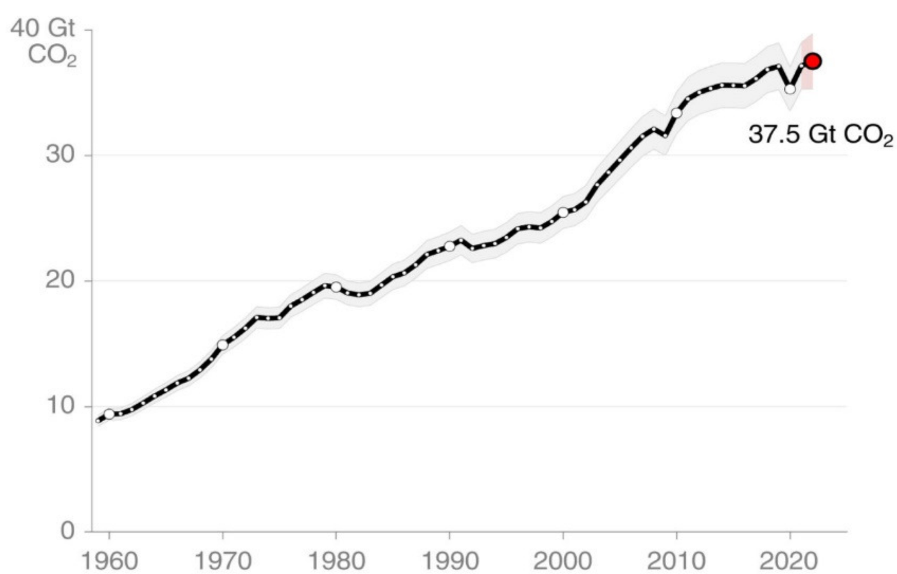
$$\frac{12}{44} = 0.273$$

(The molar mass of carbon is 12 g/mol and the molar mass of CO_2 is 44 g/mol.) Thus, the total emissions of about 40 Gt of CO_2 in 2019 correspond to 10.9 Gt of carbon (27.3% of 40).



www.leopoldina.org/en/publications/detailview/publication/climate-change-causes-consequences-and-possible-actions-2021/

The following graph shows with a little more detail the evolution of emissions from fossil fuels since 1960.



global-climat.com/2022/11/12/les-emissions-de-co2-au-plus-haut-en-2022/

The carbon cycle diagram shows that human-generated emissions account for only 5% of total emissions (11.1 Gt out of the 221.4 Gt emitted into the atmosphere each year), but this does not matter. The important thing is that human emissions add carbon to the cycle and have thus upset the balance between the flow of carbon in and out of the atmosphere. The new carbon that is added causes a new equilibrium to be established, one in which the concentration of atmospheric CO2 is higher. This slight imbalance between the addition and removal of carbon, generated by the combustion of fossil fuels, has caused the concentration of CO_2 in the atmosphere to increase from 278 ppm to 420 ppm.

For the moment, the system has not reached its new equilibrium and, on top of that, we continue to add carbon. Even if we were to stop adding more CO_2 today, it would take hundreds of years for this new equilibrium to be established since the exchange of carbon between the deep oceans and the atmosphere is very slow. Maybe, in the end, almost all of the carbon will end up in the deep oceans, which are the largest reservoir of carbon. Maybe not.

9.7 GLOBAL WARMING

As early as 1896, Arrhenuis predicted that the consumption of fossil fuels would cause global warming. This warming would have had rather beneficial consequences according to him (perhaps because he lived in Sweden, a relatively cold country). Let's try to calculate this warming and understand the consequences of such a warming.

Temperature Increase

The effect of all the factors that contribute to warming is measured by quantities called *energy imbalance* and *radiative forcing*. At equilibrium, the total radiation received from the Sun by the Earth is equal to the radiation emitted by the Earth. When there is any change, the flows may not be equal until the balance is restored.

The energy imbalance (called EEI for *Earth's Energy Imbalance*) measures this difference between the emitted flow and the received flow. If the imbalance is positive, the Earth receives more energy than it emits, the Earth is warming. If the imbalance is negative, the Earth receives less energy than it emits, the Earth is cooling. Now, this imbalance is about 1 W/m².

The radiative forcing (ΔF) corresponds to the difference that there would be between the received flux and the emitted flux if the Earth had kept the same temperature as it had in 1850. Now, this radiative forcing is estimated at 2.7 ± 1.0 W/m².

This means that the atmosphere would let through 2.7 W/m² less than in 1850 if the Earth had not changed its temperature. However, the Earth has warmed since that time so that the radiation emitted by the Earth has increased. It has actually increased by about 1.7 W/m². This is why the current energy imbalance is 1 W/m². As the imbalance is not zero, the Earth is warming.

This also means that even if we were to suddenly stop adding greenhouse gases, the Earth would continue to warm until the energy imbalance became zero.

There are several factors that contribute to radiative forcing. The following table shows the contribution of key elements.

	Radiative forcing (W/m²)
Carbon dioxide (CO_2)	2.2 ± 0.3
Methane (CH_4)	0.56 ± 0.11
Ozone (O_3)	0.51 ± 0.25
Nitrous oxides (NO et NO_2)	0.22 ± 0.03
Halogenated compounds	0.41 ± 0.08
Aerosols	-1.2 ± 1.0
Total	2.7 ± 1.0

Aerosols have a negative forcing because they contribute to an increase in albedo, which decreases the flow received.

Obviously, CO_2 has a positive forcing since the increase in the amount of CO_2 in the atmosphere blocks the radiation emitted by the Earth, which decreases the flux emitted into space.

It has been discovered that the radiative forcing due to the increase in CO_2 concentration can be approximated by the following formula.

Radiative Forcing Due to Increased CO_2 Concentration

$$\Delta F_{CO2} \approx 5.35\tfrac{W}{m^2} \cdot \ln\left(\frac{C}{278\,ppm}\right)$$

(Note that there is an uncertainty of 0.5 on the value of 5.35 W/m².) This radiative forcing must lead to an increase in temperature in order to increase the fluxes emitted by the Earth so that the radiation received and emitted becomes equal again. If the value of the radiative forcing is not too large (and it always will be), the link between the radiative forcing and the temperature increase is

$$\Delta T = \tilde{\lambda} \cdot \Delta F_{CO2}$$

The constant $\tilde{\lambda}$ in this formula is called the *climate sensitivity parameter*. An approximation of the value of this parameter can be calculated. As the link between the average emitted fluxes and the temperature is

$$F = \varepsilon \sigma T^4$$

we must have

$$\Delta F = \varepsilon\sigma \frac{dT^4}{dT}\Delta T$$
$$= \varepsilon\sigma 4T^3 \Delta T$$

If we divide by the flux, we get

$$\frac{\Delta F}{F} = \frac{\varepsilon\sigma 4T^3 \Delta T}{F}$$
$$\frac{\Delta F}{F} = \frac{\varepsilon\sigma 4T^3 \Delta T}{\varepsilon\sigma T^4}$$
$$\frac{\Delta F}{F} = \frac{4\Delta T}{T}$$
$$\frac{\Delta T}{T} = \frac{1}{4}\frac{1}{F}\Delta F$$

As the flux of radiation into space is F = 238.2 W/m², we arrive at

$$\frac{\Delta T}{T} = 0.00105 \tfrac{m^2}{W} \cdot \Delta F$$

Basically, this would mean that the temperature of all the layers would have to increase by this amount to restore the balance. For the surface, which is at 287 K, we arrive at

$$\frac{\Delta T}{287K} = 0.00105 \tfrac{m^2}{W} \cdot \Delta F$$
$$\Delta T \approx 0.3 \tfrac{Km^2}{W} \cdot \Delta F$$

This value gives the link between the forcing and the temperature variation. This is the total forcing, which is the sum of the forcing due to the increase in CO_2 and other forcings.

$$\Delta T \approx 0.3 \tfrac{Km^2}{W} \cdot \left(\Delta F_{CO2} + \Delta F_{others}\right)$$

But then, there is a complication: an increase in the amount of CO_2 leads to changes that can in turn generate additional forcings. For example, the addition of CO_2 leads to a warming that will lead to a decrease in the size of the polar ice caps, which leads to a decrease in the Earth's albedo (because ice reflects a lot of sunlight), which means that the temperature increases. So, an additional forcing is added. Also, the addition of CO_2 leads to a warming that will lead to an increase in the amount of water vapour in the atmosphere. Since water vapour is also a greenhouse gas, the temperature will rise. Again, another forcing is added. At the same time, increased evaporation would increase the cloud cover of the Earth. Because these white clouds reflect light well, they increase the Earth's albedo, which lowers the temperature. This time, a negative forcing is added.

It is quite difficult to precisely quantify all the effects related to the addition of CO_2 in the atmosphere. By how much will the ice caps melt? How will the amount of water vapour in the atmosphere change? How will cloud cover change? Since these changes will not be the same everywhere in every place on Earth, a detailed model measuring the change at each place on Earth must be made. In this case, currents, prevailing winds, the presence of nearby oceans and several other factors must be taken into account. At the end, a kind of average is calculated. In addition, it may well be that the climate sensitivity parameter is not a constant. Maybe the temperature starts to rise faster or slower above a certain temperature.

Ultimately, it is necessary to be able to find the link between CO_2 forcing and all other secondary forcings. This includes forcings induced by the increase in CO_2, but it also includes all other forcings that vary with the concentration of CO_2 even if they are not directly induced by the increase in CO_2. For example, part of the increase in methane concentration comes from emissions made directly by humans. In this case, the increase in methane is not caused by the increase in CO_2. However, it can be argued that there should be a link between the two since CO_2 and methane emissions increase in a similar way as a society grows. This all means that, for small changes, there should be a relationship of this type.

$$\Delta F_{others} = k \Delta F_{CO2}$$

where k is a constant of proportionality. As the calculation is not easy, the values obtained for k vary between 0.3 and 3 depending on the study. Often, they arrive at values close to 0.8, which means that the warming caused by all the side effects generated by the increase in CO_2 has almost as much effect as the warming directly caused by the increase in CO_2. If this value of 0.8 is used, the result is

$$\begin{aligned} \Delta T &\approx 0.3 \tfrac{Km^2}{W} \cdot \left(\Delta F_{CO2} + \Delta F_{autres} \right) \\ &\approx 0.3 \tfrac{Km^2}{W} \cdot \left(\Delta F_{CO2} + 0.8 \cdot \Delta F_{CO2} \right) \end{aligned}$$

Which leads to

Temperature Increase Related to CO_2 Radiative Forcing

$$\Delta T \approx 0.55 \tfrac{°Cm^2}{W} \cdot \Delta F_{CO2}$$

Example 9.7.1

Between 1850 and today, about 472 Gt of carbon from fossil fuels has been used. We will assume that we will go as far as adding 3000 Gt of carbon (which probably corresponds to half of the total fossil fuels in existence, but this is difficult to assess).

a) What would then be the concentration of CO_2 in the atmosphere if we assume that 41% of the carbon emitted remains in the atmosphere? (Remember that the concentration of CO_2 in the atmosphere was 278 ppm in 1850.)

As 41% of emissions remain in the atmosphere, the amount of carbon added will be

$$0.41 \cdot 3000Gt = 1230Gt$$

Such a quantity of carbon corresponds to an increase in the concentration of

$$\frac{1230Gt}{2.214 \frac{Gt}{ppm}} = 556\,ppm$$

The concentration would then be

$$278\,ppm + 556\,ppm = 834\,ppm$$

The concentration of CO_2 in the atmosphere would thus have tripled compared to the pre-industrial concentration.

b) What would then be the radiative forcing?

The radiative forcing would be

$$\begin{aligned}\Delta F_{CO2} &\approx 5.35 \tfrac{W}{m^2} \cdot \ln\left(\frac{C}{278\,ppm}\right) \\ &\approx 5.35 \tfrac{W}{m^2} \cdot \ln\left(\frac{834\,ppm}{278\,ppm}\right) \\ &\approx 5.88 \tfrac{W}{m^2}\end{aligned}$$

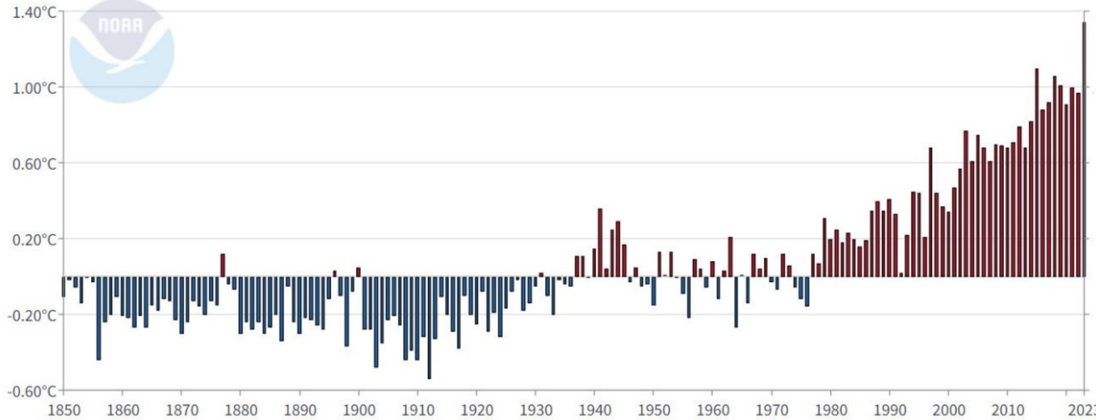
c) What would then be the increase in the average temperature?

The temperature increase would be

$$\begin{aligned}\Delta T &\approx 0.55 \tfrac{°Cm^2}{W} \cdot \Delta F_{CO2} \\ &\approx 0.55 \tfrac{°Cm^2}{W} \cdot 5.88 \tfrac{W}{m^2} \\ &\approx 3,2°C\end{aligned}$$

With a current concentration of 420 ppm, the radiative forcing is 2.20 W/m² and the temperature increase is 1.2 °C since 1850.

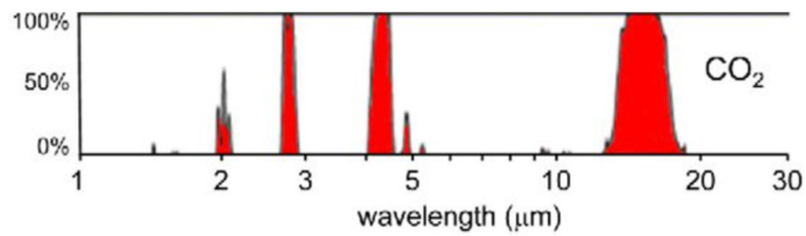
The following graph shows the variations in average temperature since 1850. This graph shows the difference between the average temperature of a specific year and the average temperature between 1900 and 2000. We can see that there was a variation of about 1.2°C during this period. We went from about 0.2°C below average to about 1°C above average around 2020.



www.ncei.noaa.gov/access/monitoring/climate-at-a-glance/global/time-series

Has the Maximum of Warming Been Reached?

Some have argued that CO_2 cannot cause global warming since this gas already absorbs 100% of the radiation and cannot absorb more. Indeed, we can see on the graph of radiation absorption in the atmosphere that the CO_2 absorption curve reaches almost 100% absorption for certain wavelengths.



www.quora.com/Does-CO2-absorb-all-infrared-frequencies-near-infrared-far-infrared-etc-or-does-it-just-absorb-one-frequency

In fact, the absorption can never be 100%. Theoretically, the percentage can be close to 100%, but it can never reach it. It can be at 99.999%, but not at 100%.

How could the situation get worse if CO_2 is already absorbing the maximum amount of radiation? Here are two explanations that show that the temperature will increase even if the absorption almost at its maximum.

First, the graph shows that absorption is not 100% for all wavelengths. By increasing the concentration of CO_2, the absorption will increase for these other wavelengths, and this will increase the total absorption.

Second, the temperature would increase even if the absorption were at almost 100% for all wavelengths. To illustrate this, let's go back to our multilayered model of the greenhouse effect. We will assume that the total absorption of the atmosphere is $\varepsilon = 0.9999$. With this value, the Earth's temperature would be

$$\begin{aligned} T_s &= T_e \sqrt[4]{1 - \ln \sqrt{1-\varepsilon}} \\ &= 254.58K \cdot \sqrt[4]{1 - \ln \sqrt{1 - 0.9999}} \end{aligned}$$

$$\begin{aligned}&= 254.58K \cdot 1.538\\&= 391.72K\\&= 118.57°C\end{aligned}$$

It seems that the situation cannot get worse than that, since the absorption is already at its maximum value. Let's see that. With $\varepsilon = 0.9999$, the proportion of radiation that manages to exit the atmosphere is 0.0001. If we divide the atmosphere into 1000 layers, then this would mean that the percentage of light that can pass through each layer is

$$\sqrt[1000]{0.0001} = 0.99083$$

Each of the layers absorbs only about 0.917% of the radiation. Obviously, this percentage can increase. Suppose enough absorbing gas is added to raise the absorption of each layer to 1%. This corresponds to a transmission rate of 99% for each layer. The transmission through 1000 layers (i.e. the entire atmosphere) would then be

$$0.99^{1000} = 0.000\,043\,17$$

This means that the absorption coefficient would now be

$$\begin{aligned}\varepsilon &= 1 - 0.000\,043\,17\\&= 0.999\,956\,83\end{aligned}$$

The coefficient ε is thus a bit closer to 100%. The temperature would then be

$$\begin{aligned}T_s &= T_e \sqrt[4]{1 - \ln\sqrt{1-\varepsilon}}\\&= 254.58K \cdot \sqrt[4]{1 - \ln\sqrt{1 - 0.999\,956\,83}}\\&= 254.58K \cdot 1.56672\\&= 398.86K\\&= 125.71°C\end{aligned}$$

Obviously, the small increase of 0.917% to 1% for the absorption rate of each layer has a huge impact. Even though we were close to 100% absorption, this small change brought us even closer to 100% and the temperature increased considerably from 118.6°C to 125.7°C. Essentially, the temperature goes up very quickly when ε is close to 1 because the temperature tends to infinity when ε tends to 1.

By the way, there is a rather impressive case of this effect in the Solar System: Venus. Venus is 108,000,000 km from the Sun, which means that the intensity of radiation received from the Sun is

$$\begin{aligned}I &= \frac{P_{star}}{4\pi D^2}\\&= \frac{3.828\times10^{26}W}{4\pi\cdot\left(1.082\times10^{11}m\right)^2}\end{aligned}$$

$$= 2602.0 \tfrac{W}{m^2}$$

The average energy flux received is therefore

$$\begin{aligned} Q &= \frac{I}{4} \\ &= \frac{2602.0 \tfrac{W}{m^2}}{4} \\ &= 650.5 \tfrac{W}{m^2} \end{aligned}$$

This is almost twice as much as the flux received on Earth. With an albedo of 0.77 (Venus reflects light remarkably well thanks to a phenomenal number of clouds), the equilibrium temperature is

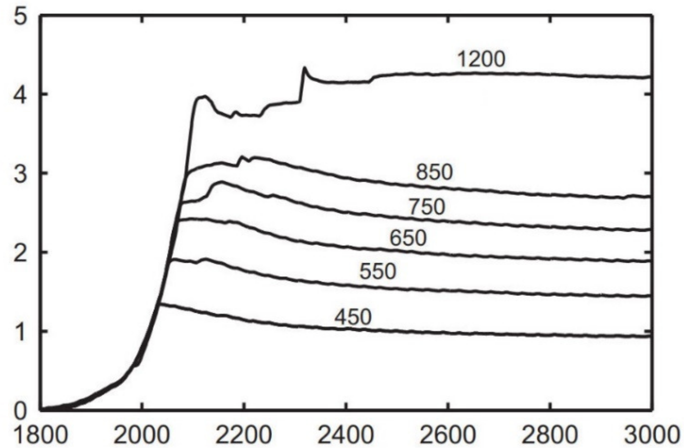
$$\begin{aligned} T_e &= \sqrt[4]{\frac{Q(1-A)}{\sigma}} \\ &= \sqrt[4]{\frac{650.5 \tfrac{W}{m^2} \cdot (1-0.77)}{5.67037\times 10^{-8} \tfrac{W}{m^2K^4}}} \\ &= 226.62K \\ &= -46.53°C \end{aligned}$$

It's rather cold. However, Venus has an atmosphere composed of 95% CO_2 and has a surface air pressure 92 times greater than the pressure on Earth's surface. This is a phenomenal amount of CO_2, and this CO_2 generates a greenhouse effect that raises the temperature to 464°C. Such a temperature is possible if the transmission coefficient of the entire atmosphere is 10^{-96} (it is 0.29 for the Earth). If the atmosphere of Venus is divided into 1000 layers, each layer would then absorb about 20% of the radiation that passes through it (whereas it is 0.12% in the Earth's atmosphere). Clearly, we are far from saturation on Earth.

How Far Will We Go?

The concentration of CO_2 in the atmosphere is now 420 ppm and it is still increasing by about 2 ppm per year. What value will this concentration reach before stabilizing? Consuming every 6000 Gt of carbon from fossil fuels (assuming this value has been correctly estimated) would mean that the concentration would reach about 1450 ppm and the warming would be of nearly 5 °C. We probably won't go that far, but it's very difficult at this instant to predict up to what value the concentration of CO_2 will increase.

The following graph shows how the temperature will change over the years depending on the maximum concentration value reached. For example, the highest curve (identified 1200) shows how the Earth's temperature will change over the years if a maximum concentration of 1200 ppm of CO_2 in the atmosphere is reached.



courses.washington.edu/pcc588/readings/PNAS-2009-Solomon-0812721106.pdf

The bad news is that the temperature decreases very slowly after the emissions stop... Even if we were to severely limit emissions now to limit the increase to 450 ppm, the temperature would remain stuck at the 1°C warming for at least 1000 years!

But why doesn't the temperature drop after the emissions stop? Essentially, the Earth remains warm because the oceans have warmed. Nearly 90% of the heat accumulated with the increase in temperature ends up in the oceans. It is estimated that since 1955, the oceans have absorbed about 4×10^{23} J, which corresponds to slightly more than the energy of the Hiroshima atomic bomb absorbed every second by the oceans since 1955. At this instant, 75% of this heat is still less than 700 m from the surface.

So, if we were to stop producing CO_2 right now, the temperature would not drop immediately, because the oceans will slowly return this accumulated heat to the atmosphere. Thus, the atmosphere will stay practically at the same temperature for hundreds of years.

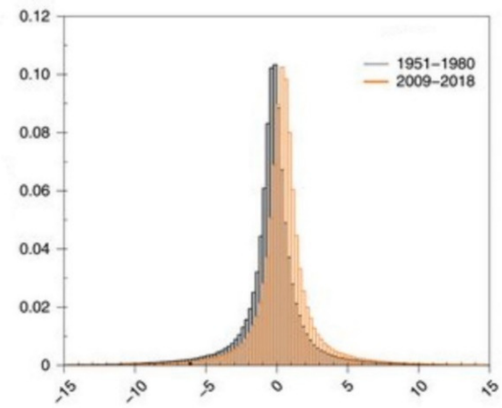
The Consequences

Rising temperatures

You may be thinking that a 1.2°C increase is not a disaster. It's going to be a little warmer and that's good, especially if you live in a rather cold country like Canada. Such a warming would not lead to huge changes if the warming would simply mean an increase in average temperature everywhere on Earth.

If a histogram for a city is made with the number of days with a given maximum temperature for a particular date of the year (e.g. every June 15 of each year), a Gaussian-shaped diagram is usually obtained. On this histogram, there are very few days that are much colder than average, very few days that are much warmer than average, and many days that have a temperature close to the average. The temperature differences between hot and cold days are much greater than the 1.2°C increase due to warming.

The warming is just going to move this graph a little bit towards the higher temperatures. Now, the average has simply shifted by 1.2°C towards the higher temperature compared to 1850. There will always be much colder days and much warmer days than the average. There will be a few more hot days and a little less cold days than before. This is not a huge change, and it is hard to perceive.



www.researchgate.net/publication/338345425_Climate_change_now_detectable_from_any_single_day_of_global_weather/figures?lo=1

So, a 40°C in June is not a sign that the Earth is warming at all and a -40°C in January is not a sign that the Earth's warming is not true. The temperature of a specific day in a particular place means absolutely nothing about climate change since the differences due to the local weather are much greater than the temperature increase. A complete statistical study is needed to eliminate these variations due to the local weather to see the warming trend. When they tell you on the television news that a heat wave is due to global warming, they are speaking nonsense. There is a much greater chance that the heat wave is linked to particular weather conditions than to climate change. There are climatic cycles, such as El Niño and the North Atlantic Oscillation, for example, which lead to much greater temperature variations than the 1.2°C caused by the increase in CO_2 concentration.

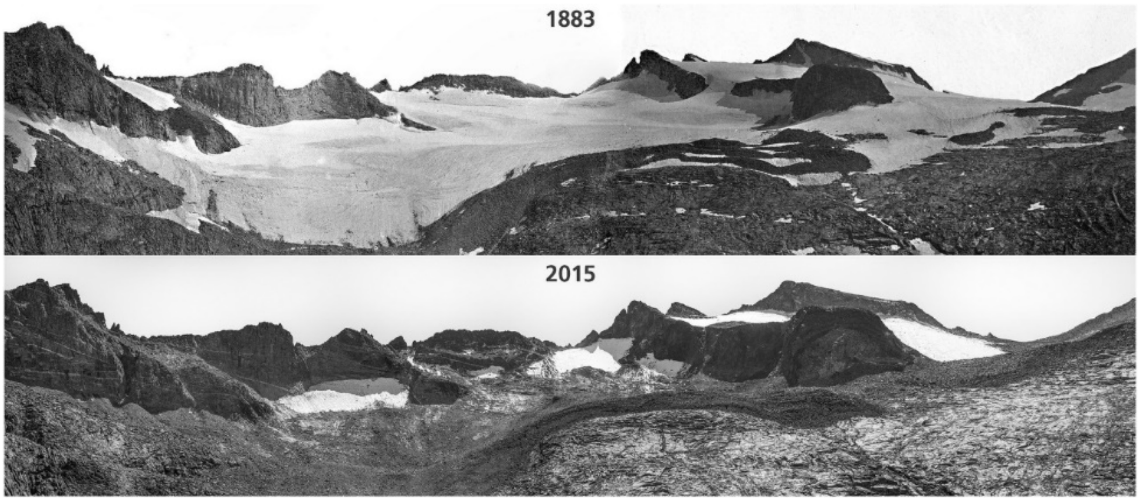
What climate change means is that hot days are expected to become a little more frequent in the future. Just a little more frequent. A warming of a few degrees will not transform a month of July, whose average temperature is 25 °C, into a suffocating July, whose average temperature is 35 °C.

The warming does not mean that there will be no more snow in Quebec in winter in 20 years. Very cold days are expected to become a little less frequent in the future, but it would take a quite impossible warming for the average maximum temperature in January (currently at -7°C) to be high enough so that this month would be snow free (unless there is a very particular localized effect).

If there were only this small change of temperature, it wouldn't be so bad, right?

Melting Ice Caps

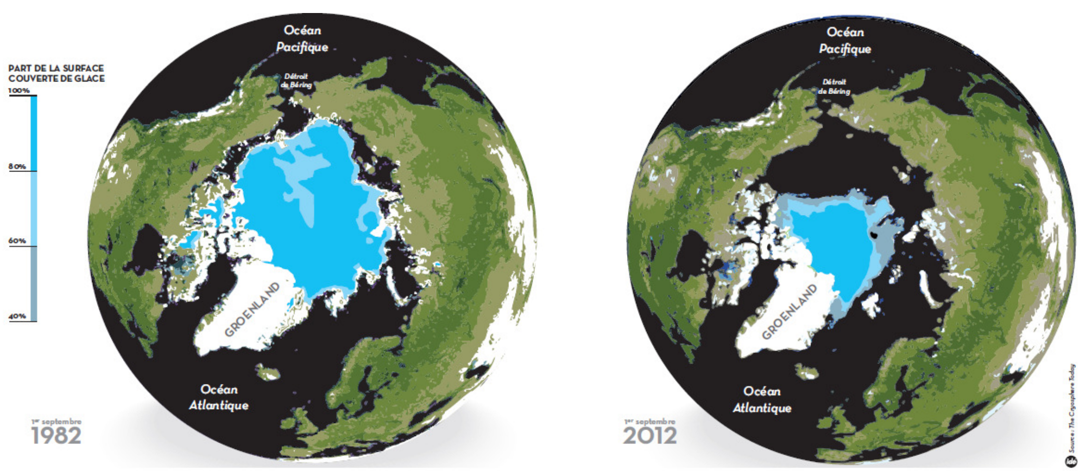
The effects of climate change are particularly visible with glaciers and polar ice caps. Their size depends on the average temperature in a very sensitive way. A small increase in average temperature can cause a glacier to disappear in a few decades. It has been observed that the size of several glaciers has decreased sharply since 1850. The following image shows the melting of the Sierra Nevada glacier between 1883 and 2015.



fr.wikipedia.org/wiki/%C3%89l%C3%A9vation_du_niveau_de_la_mer

With a warming of at least 1°C that will last at least 1000 years, mountain glaciers will almost all disappear. Some scientists estimate that half of the mountain glaciers will have completely disappeared by 2100.

The amount of floating Arctic ice is also decreasing. The area covered by sea ice area has decreased by 40% since 1980.



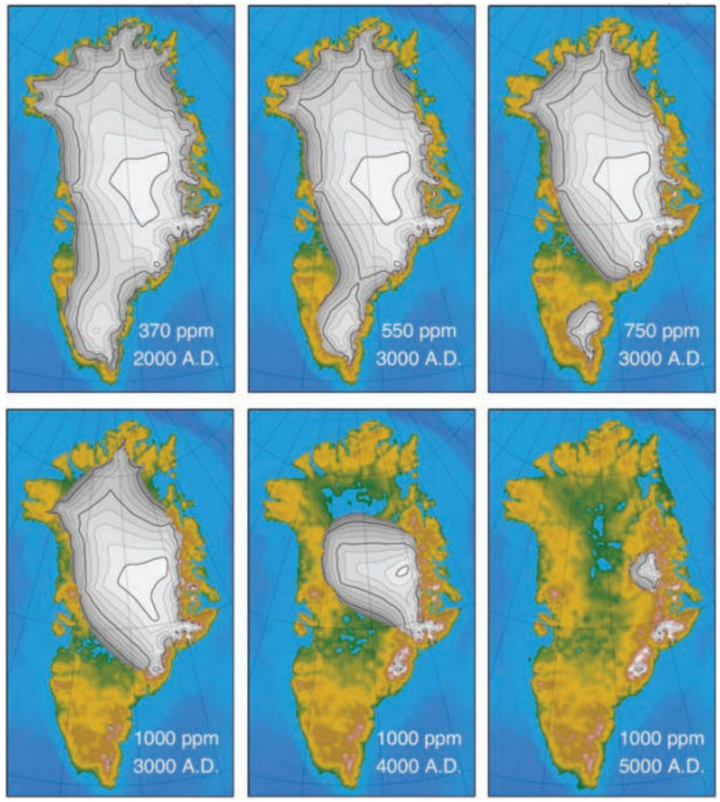
leclimatdanstoussesetats.wordpress.com/2015/02/02/les-impacts-du-rechauffement-climatique-en-arctique/comment-page-1/

The area covered by ice in September (when it reaches its minimum size) decreased from 7.67 million km² in 1980 to 3.92 million km² in 2020.

The Greenland polar ice sheet is also melting. The amount of ice that covers Greenland is strongly related to the average local temperature. 400 000 years ago, there was no ice at all in Greenland and the local temperature was 2 to 4 °C higher. It does not take a very large increase in average temperature to make the Greenland ice sheet disappear. With a warming of at least 1°C that will last at least 1000 years, a significant proportion of Greenland's ice will melt.

The map on the right shows the size of the polar ice sheet now and the remaining size in the years 3000, 4000 and 5000 as a function of the concentration of CO_2 in the atmosphere.

It can be seen that the ice sheet disappears almost completely in the year 5000 with a concentration of 1000 ppm (which would correspond to a temperature increase of about 3.5 °C).



citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=300c4e8e1b0bdccb1b9965b98769c8fa6c1b9305

There are 2 700 000 Gt (2 900 000 m³) of ice in Greenland. At this instant, 270 Gt are lost each year (average since 2002). At this rate, it would take 10 000 years to melt all the ice left (but this rate is expected to increase with warming).

In fact, the disappearance of much of Greenland's ice sheet is inevitable since there is no way to swiftly reverse the warming that has already occurred. This disappearance will take place over several centuries.

The melting of the ice sheet is not as significant in Antarctica. There are nearly 24 380 000 Gt of ice in Antarctica, and an average of 146 Gt per year has been lost since 2002. In fact, some parts of the Antarctic ice sheet are losing mass, while others are gaining mass. In regions where the ice sheet is gaining mass, the melting is more than offset by an increase in snowfall (which adds mass to the glacier) induced by climate change.

Rising Sea Levels

Sea levels are rising firstly because the water is warming. As for all substances, the volume of water increases with temperature (water has some particularities, but its volume increases with temperature when its temperature is above 4 °C). This increase in volume is causing the level of the oceans to rise.

Water coming from melting glaciers and ice caps will also make the sea levels rise. Floating Arctic ice does not cause sea levels to rise by melting at all, because they occupy exactly the same volume of water in the ocean as the amount of water produced by melting ice. It is the ice that rests on the continents (i.e. glaciers and ice caps) that will make the sea levels rise. The level increase can easily be calculated.

Example 9.7.2

How much will sea levels rise if the entire Greenland ice sheet melts, knowing that there are 2 700 000 Gt of ice and that the surface of the oceans is 361 million km²?

We know that water has a density of 1000 kg/m³. Let us transform this density into Gt per km³.

$$1000\frac{kg}{m^3}\cdot\left(\frac{1000m}{1km}\right)^3\cdot\frac{1Gt}{10^9t}\cdot\frac{1t}{1000kg}=1\frac{Gt}{km^3}$$

So, the 2 700 000 Gt of ice will become 2 700 000 km³ of water. Since this water will be distributed over the entire surface of the oceans, the volume is equal to the area of the oceans multiplied by the thickness of the water layer. So, we have

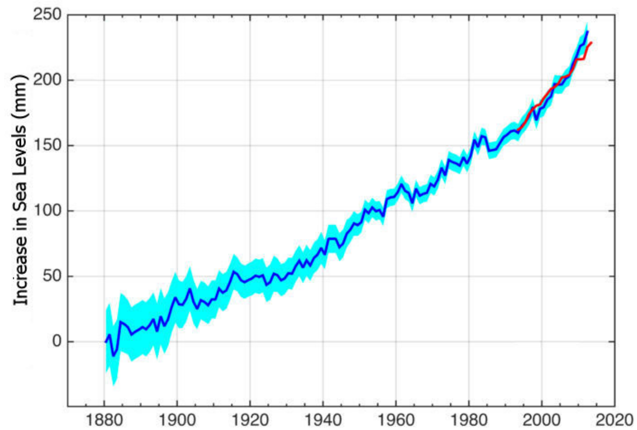
$$vol = h\cdot A_{oceans}$$
$$2\,700\,000km^3 = h\cdot 361\,000\,000km^2$$
$$h = 0,00748km$$
$$h = 7.48m$$

The complete melting of Greenland's ice would therefore increase the sea levels by 7.5 m. A similar calculation shows that the complete melting of Antarctica's ice would cause the oceans to rise by 67.5 m.

However, these ice caps, especially those in Antarctica, will not melt so quickly. You are not going to see such a rise in sea levels in your lifetime. One might therefore wonder how this level will change between now and 2100.

In the worst-case scenario, the thermal expansion of the water is expected to raise the sea level by 23 ± 9 cm. Greenland's melting is expected to add 16 cm, Antarctica's melting is expected to add another 12 cm, and the melting of other mountain glaciers is expected to add 37 ± 2 cm. In total, there should be an increase of 88 ± 11 cm in 2100. If things go a little better (the melting of Greenland is not accelerating and the accumulation of ice in East Antarctica compensates for the melting in West Antarctica), the increase could be a bit smaller at 65 ± 12 cm. Thus, the rise in sea level in 2100 is expected to be somewhere between 50 cm and 1 m above the level of 1850.

Note that sea levels have already risen by 25 cm since 1880. Between 2013 and 2022, the average rate of rise was 4.62 mm/year. If this rate is maintained until 2100, we will add 37 cm to the 25 cm already present for a total increase of 62 cm.



research.csiro.au/slrwavescoast/sea-level/sea-level-changes/

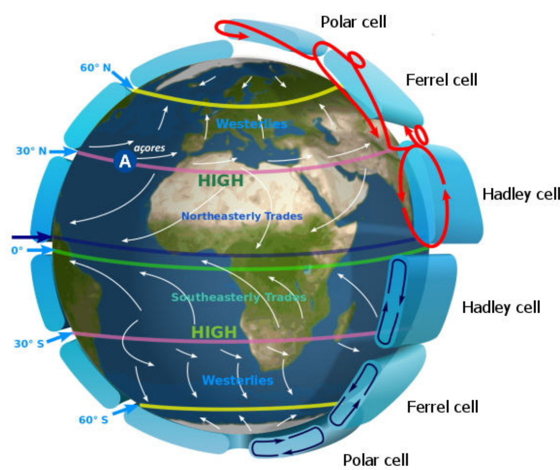
This increase of 1 m at most in 2100 does not seem too catastrophic, but it will already be a big problem for the Netherlands and several other states made up of islands with little elevation in the Pacific and Indian Oceans. Part of Belgium, Italy (Venice region), Louisiana, Bangladesh and Vietnam will be flooded. In some regions, rising oceans will allow the saltwater of the oceans to enter a little more inland in the estuaries of certain rivers, which will lead to major changes in the fauna and flora of these rivers. The Mekong Delta in Vietnam, a very fertile agricultural area, is expected to be particularly affected by this phenomenon.

This link takes you to an interactive map that shows the flooded areas based on the chosen sea level rise.
https://flood.firetree.net/embed.php?w=1200&h=700&ll=46.227638,2.213749000000007&zoom=5&m=13

A Change in Atmospheric Circulation

Global warming doesn't just increase the average temperature of every place on Earth. The increase in temperature can change the circulation of air in the atmosphere, which can profoundly change the weather conditions of certain places.

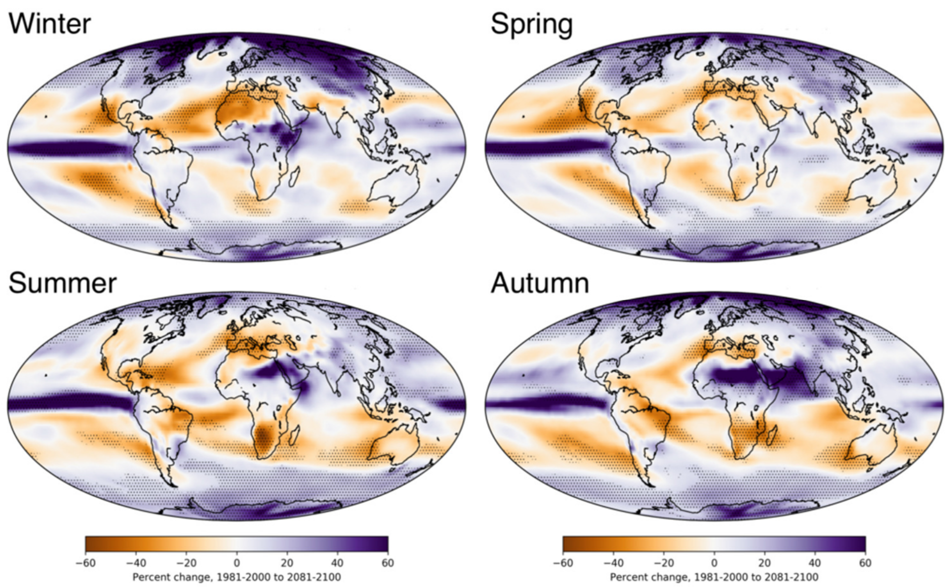
For example, the warming is predicted to cause a northward expansion of the Hadley cells, which are large convection zones between the equator and a latitude of 30° in each hemisphere. At these latitudes, the air returns to the ground following the rotational movement of the cell. This air from a high altitude is very dry and that's why there are a lot of deserts at these latitudes.



www.meteo-paris.com/actualites/le-changement-climatique-egalement-synonyme-de-temps-plus-calme

If the Hadley cells expand northwards, desert areas are expected to move northwards. For example, the Mediterranean region and the southwestern United States (New Mexico, Arizona and California) are expected to go through some desertification with the warming.

Changes in atmospheric circulation can also change the amount of rain that falls. Some places will have more precipitation, and others will receive less. Here is a map showing the change in precipitation predicted by the RPC 8.5 model, a rather pessimistic model that predicts that CO_2 concentration will reach 1350 ppm. We may not reach such a large value, but this model makes it easy to see the trends.

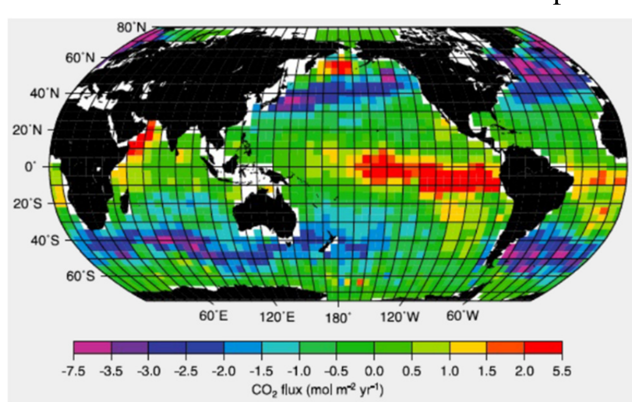


www.carbonbrief.org/explainer-what-climate-models-tell-us-about-future-rainfall/

It is clear on these maps that the Mediterranean region and the southwestern United States will receive less precipitation. We also see that precipitation will increase in Quebec, especially in winter...

Ocean Acidification

We have seen that the oceans have absorbed 183 Gt of carbon since 1850. When the partial pressure of CO_2 in the atmosphere increases, more CO_2 can dissolve in the water. This absorption takes place at high latitudes. Since the amount of CO_2 that can dissolve in water decreases with temperature, CO_2 dissolves well in cold water. Near the equator, warm water rather tends to transfer some of its CO_2 back to the atmosphere.



jancovici.com/changement-climatique/gaz-a-effet-de-serre-et-cycle-du-carbone/les-puits-de-carbone-ne-vont-ils-pas-absorber-le-surplus-de-co2/

Once in the water, the CO_2 reacts with the water to form carbonic acid.

$$CO_2 + H_2O \rightleftharpoons H_2CO_3$$

This acid can then dissociate into bicarbonate and hydrogen ions.

$$H_2CO_3 \rightleftharpoons HCO_3^- + H^+$$

The H+ ions can then react with the carbonate ions in the water to form even more bicarbonate.

$$H^+ + CO_3^{2-} \rightleftharpoons HCO_3^-$$

When CO_2 is added to water, the balance of all these reactions is shifted to the right. The concentrations of H+, H_2CO_3 and HCO_3^- increase while the concentration of CO_3^{2-} decreases.

These extra H+ cause the acidity to increase. In 1850, the pH of the oceans was 8.2. Today, it is 8.1. It is estimated that the pH could drop to 7.8 by 2100.

The decrease in CO_3^{2-} concentration is not good news for some organisms, such as oysters, crabs, sea urchins, lobsters, coral and certain types of plankton, which use these ions to develop and maintain their shells and skeletons. Studies also indicate that the shells and skeletons of these organisms break down more easily when acidity increases. These organisms could therefore have difficulty surviving in the oceans of the future.

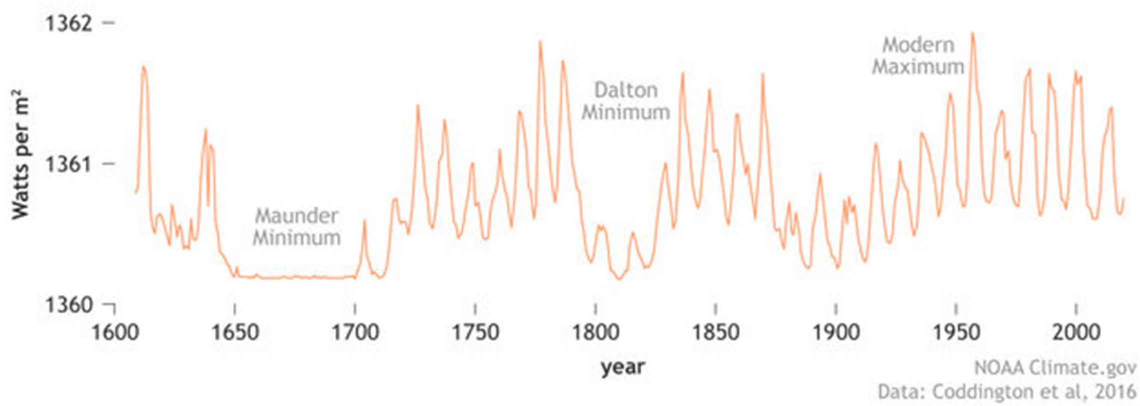
9.8 NATURAL VARIATIONS

Do not believe that the climate has always been the same throughout Earth's history, and that humans have suddenly upset a perfect balance that had lasted for billions of years. The climate has varied in the past. The average temperature of the Earth has been higher than today before (in fact, it has almost always been higher than today), and it has been lower than today before.

The Earth's surface temperature formulas seen before show that there are not a lot of elements that influence the temperature of a planet. There is the average flux received from the star, the albedo and the greenhouse effect. Since these quantities can vary naturally, the climate can vary without human influence. Here we will explore these natural variations.

Variations in the Sun's Brightness

The Sun does not always have the same luminosity. Sometimes it's a little bit brighter and sometimes it's a little bit dimmer. The intensity of radiation varies over the long term and follows an 11-year cycle (11 years on average: some cycles are as short as 7 years and others are as long as 15 years). The following graph shows the intensity of solar radiation received on Earth since 1600. An 11-year cycle can be clearly very clearly.



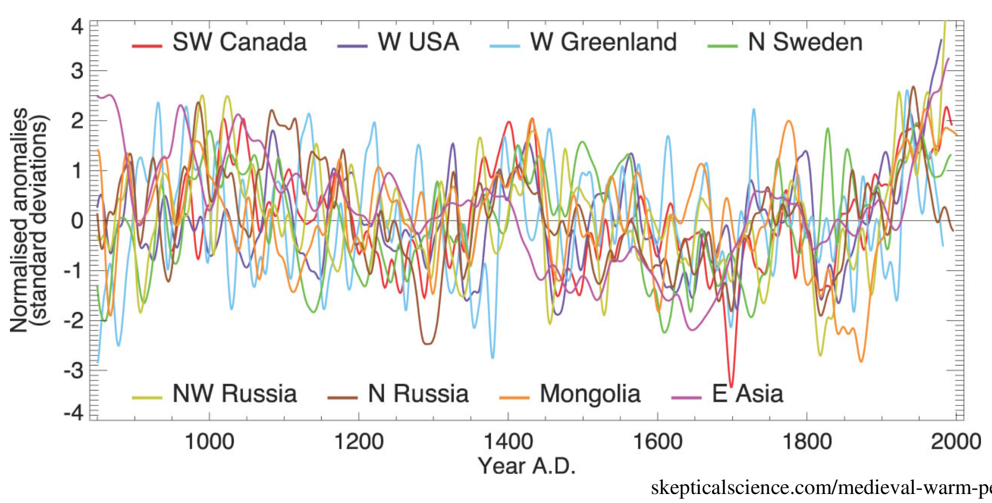
www.climate.gov/news-features/understanding-climate/climate-change-incoming-sunlight

A drop in radiation corresponds to a drop in temperature. Based on what we can see on the graph, we should therefore expect that the period between 1650 and 1700 was colder than it is today. It was indeed colder, and this period has been called *the Little Ice Age*. At that time, the Thames often froze in winter in London (something it hasn't done since 1814). Some winters were very harsh in France. Between 1.5 and 2 million French people froze to death during the winters of 1693 and 1694.

However, the variations in the intensity of the radiation are not so great. At least from the 1650s to 1700s, the radiation received was just over 1360 W/m² rather than the current average of 1361.1 W/m². This would correspond to a drop of less than 0.05 °C according to the temperature formulas. This is far from enough to make the Thames freeze.

Actually, one must be very careful with this kind of historical narrative. It could well be that there was only a local cooling (like the North Atlantic) and not a generalized cooling. Since most of the sources are European, it's easy to get carried away by exaggerations. To ensure that there was a widespread cooling, we need to study the climate in several places on Earth.

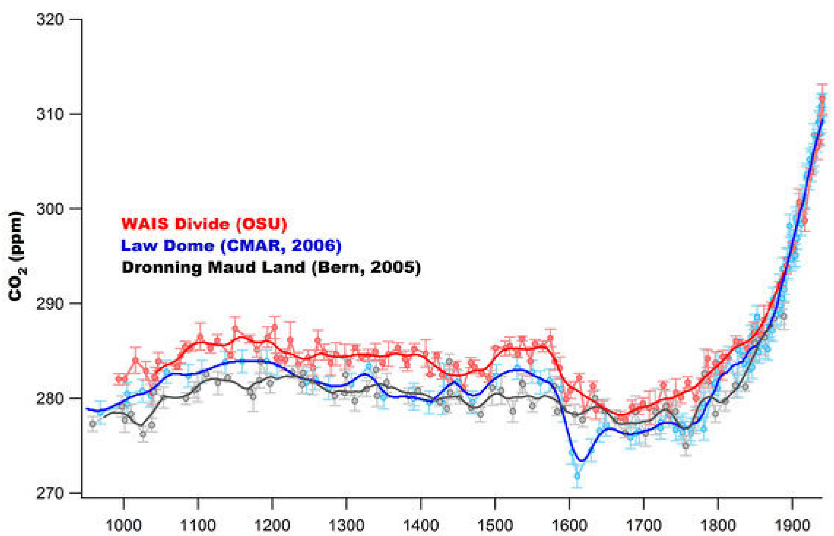
The following graph shows the variations in average temperatures at different locations on Earth.



skepticalscience.com/medieval-warm-period.htm

It’s not easy to get a clear idea with so many curves, but we can see that several curves were below average during the period between 1650 and 1700. The trend appears to be a cooling of 1°C compared to the average. Note that several curves are above average for much of the period between 1000 and 1400. This corresponds to a slightly warmer period, often referred to as the *Warm Medieval Period*.

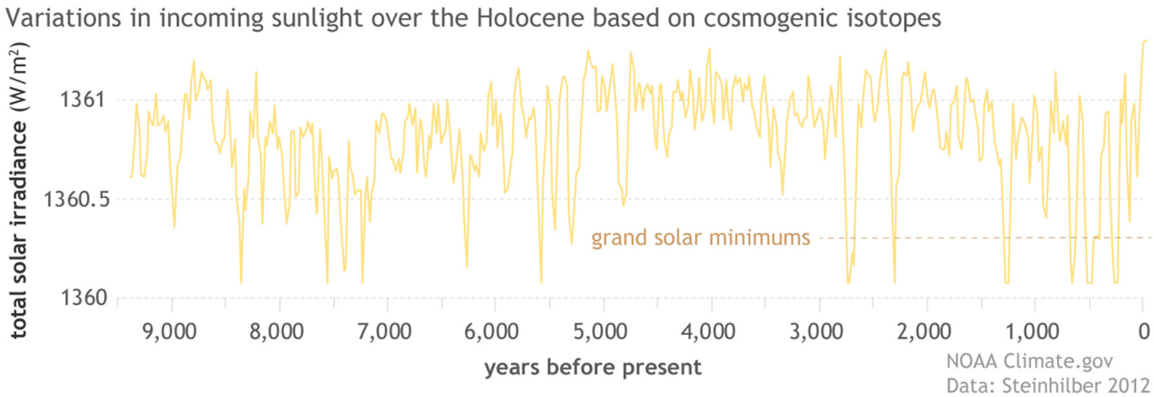
How then could a very small variation in solar radiation have caused such a cooling? This is because variations in solar radiation are amplified by other feedback phenomena, including CO_2. Temperature and CO_2 influence each other. A rise in the amount of CO_2 causes the temperature to rise, and a rise in temperature causes the level of CO_2 to rise. Very often, with natural cycles, everything starts with a slight variation in temperature and this variation in temperature generates a release of CO_2. For example, oceans release CO_2 when they warm up because warm water can contain less CO_2 than cold water. This extra CO_2 released into the atmosphere then raises the temperature, which releases even more CO_2 from the oceans and so on. In fact, the following graph shows that the CO_2 level was a little lower (only slightly lower) during the so-called Little Ice Age (between about 1600 and 1800). (We have the curves for 3 different places on Earth.)



agupubs.onlinelibrary.wiley.com/doi/full/10.1029/2011GB004247

The variation in the amount of CO_2 is not the only thing that can amplify temperature variations. An increase in temperature also increases the evaporation of water. Since water is also a greenhouse gas, this increases the temperature. An increase in temperature can cause the size of ice sheets to decrease, which decreases the albedo and, therefore, increases the temperature. With these amplification mechanisms, small variations in solar brightness can generate quite large temperature variations.

Let’s now look at a slightly longer period. The curve of solar radiation over the last 9000 years allows us to conclude that the millennium from the year 1000 to the year 2000 was particularly cold. Between the years 500 and 1800, there were 4 exceptionally cold periods, while these cold periods were much rarer between 5500 years before today and 1500 years before today. 4000 years ago, the climate would have been particularly mild.

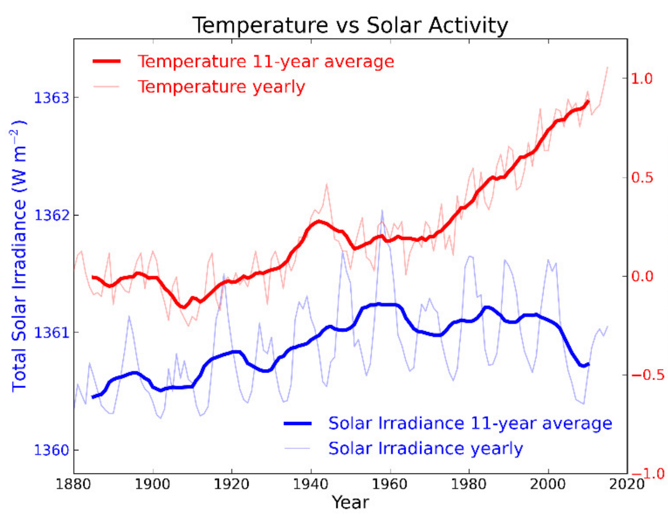


www.climate.gov/news-features/understanding-climate/climate-change-incoming-sunlight

The whole period of the Roman Empire would have been warmer. In fact, the Alpine glaciers were even smaller than they are today during the Roman period. They then grew strongly between the years 500 and 1850.

The end of the graph shows that solar radiation is particularly large today. This is the maximum of the curve in the last 9000 years (and maybe even more)! We would therefore be in a particularly hot period according to the solar radiation curve. No wonder glaciers and permafrost are melting. This obviously leads to the following question: Could the current warming be only the result of a natural variation in solar radiation?

The idea is not completely wrong. Glaciers began to melt in the mid-19th century, when humans had not yet added significant amounts of CO_2 to the atmosphere. In reality, variations in solar radiation are largely responsible for the first phases of warming that began in 1850. The amount of CO_2 sent into the atmosphere before 1940 was not very large (we had 310 ppm in 1940 compared to 278 in 1850 and 420 now) and yet a warming of nearly 1 °C between 1910 and 1940 was observed. During this period, the solar radiation and temperature curve follow each other quite closely, as shown in the graph on the right.



skepticalscience.com/solar-activity-sunspots-global-warming.htm

It is therefore possible to attribute this phase of the warming to the Sun. Until around 1980, it was even rather difficult to be certain that CO_2 of human origin had an influence on the climate, because the curve followed the curve of the increase in solar radiation quite well. However, the graph makes it quite clear that the Sun is not responsible for the current warming (as claimed by those who say that humans have nothing to do with the current warming). While solar radiation decreases from 2000 onwards, the temperature continues to rise. The Sun's brightness variation is no longer the most important element that determines temperature variations on Earth. The effect of anthropogenic CO_2 then becomes evident.

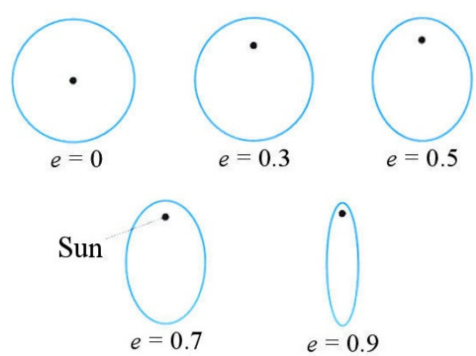
Milankovitch Cycles and Glaciations

Even if the Sun would keep exactly the same brightness, the average intensity of radiation received on Earth would vary slightly. These variations are generated by small changes in the Earth's orbit and tilt.

If the Earth and the Sun were alone in the universe, the shape of the Earth's orbit around the Sun would always remain exactly the same. However, the gravitational pull of the other planets (especially Jupiter) creates small perturbations that very slowly change the configuration of the Earth-Sun system, and these changes affect the climate over a longer period of time.

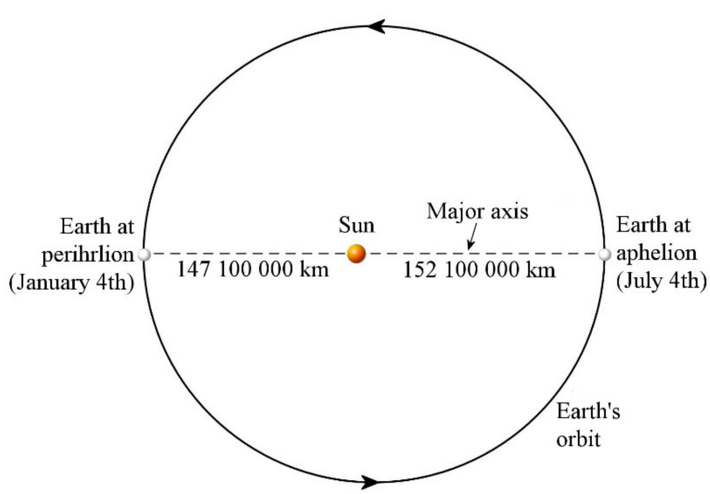
Eccentricity Variations

These disturbances modify the shape of Earth's orbit. This elongation is measured with the eccentricity of the orbit (denoted e). Without going into detail, an orbit that has an eccentricity of 0 is perfectly circular, and the elongation increases as the eccentricity increases. The maximum value of e is 1.



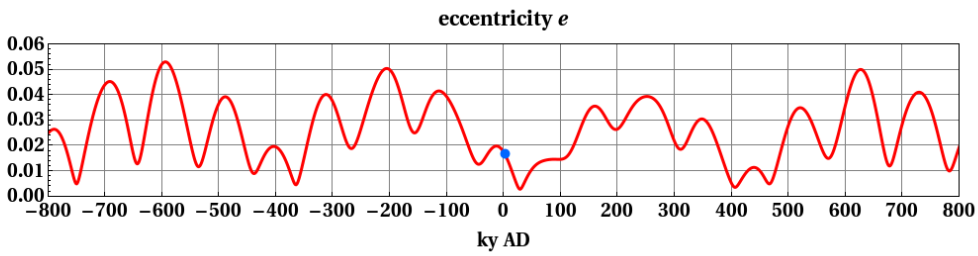
astro.wsu.edu/worthey/astro/html/lec-ellipse.html

The eccentricity of Earth's orbit is never very great (which is a good thing because there would be huge temperature variations during the year if the orbit were very eccentric). The current eccentricity of Earth's orbit is 0.017, which means that at its closest point to the Sun (called the *perihelion*), the Earth is 147 100 000 km from the Sun and at the farthest point from the Sun (called *aphelion*), the Earth is 152 100 000 km from the Sun.



Note that the line that connects perihelion to aphelion via the Sun is called *the major axis of the orbit* or the *apse line*.

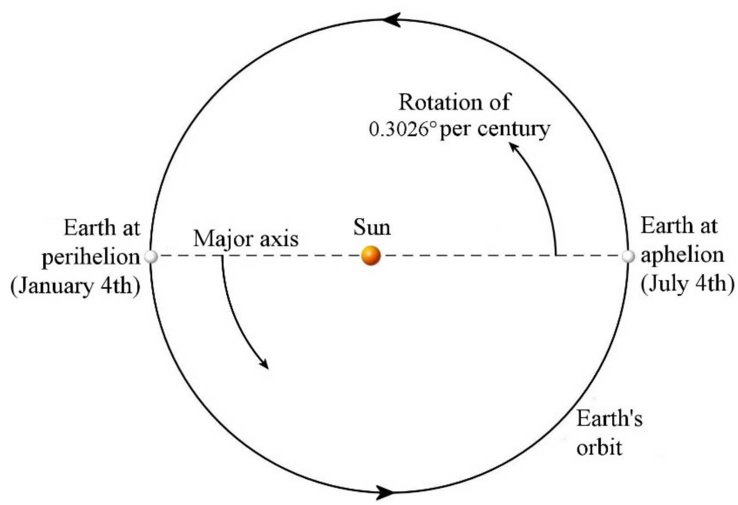
However, perturbations cause this value of e to vary between 0.005 and 0.058. The following graph shows the changes in eccentricity over time.



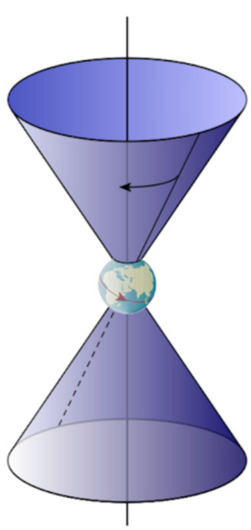
en.wikipedia.org/wiki/User:Incredio/Drafts

Perihelion Displacement

Perturbations also make the major axis of the Earth's orbit (apse line in the figure) rotate in space, but only by 0.30264° per century in the same direction as the Earth's rotation around the Sun. At this rate, it takes nearly 118 950 years for the major axis of the Earth's ellipse to make a complete revolution. This rotation slowly changes the date of aphelion and perihelion.



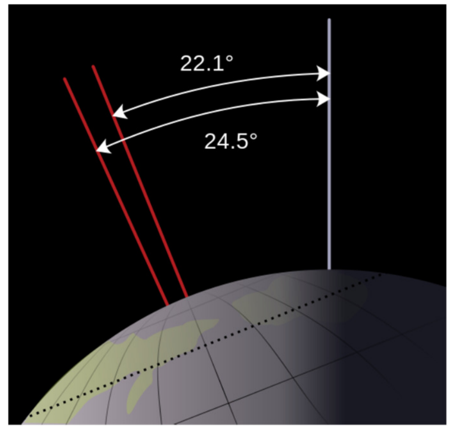
Earth Axis Orientation Variation



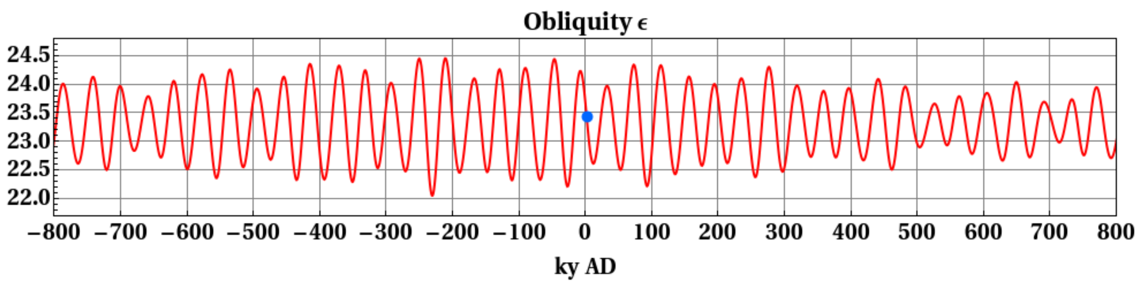
Perturbations also cause the Earth to rotate like a spinning top. This means that at the same time as the Earth rotates on its axis, the Earth's axis of rotation describes a cone. This movement along the cone, called *precession,* is very slow, however. It takes 25 770 years for the axis to make a complete rotation around the cone.

This change of direction can be seen in this clip.
http://www.youtube.com/watch?v=Dw4Xhw4q4ec
In this clip, the precession is very exaggerated compared to the rotation of the Earth. To be correct, the Earth would have to rotate with a period of 24 hours and the precession motion would have to have a period of 25 770 years!

Perturbations also slightly alter the tilt of Earth's axis relative to the plane of Earth's orbit so that the tilting angle (also called the *obliquity*) can vary between 22.1° and 24.5°. At this instant, the angle is 23.45°. On the following graph, the variations of the tilting angle over time can be seen.



wikipedia.qwika.com/en2fr/Milankovitch_cycles



en.wikipedia.org/wiki/User:Incredio/Drafts

Note that the Moon greatly stabilizes the tilting of the Earth's axis. Without the Moon, the variations could be much greater and could even reach 180°!

<u>*Effects of These Perturbations: The Sahara*</u>

The weather conditions in the Sahara clearly show the effects of the perturbations. The Sahara alternates between dry and wet periods with a period of about 30 000 years, and there is evidence that there are at least 230 cycles in the last 7 to 8 million years! 20 000 years ago, the Sahara was a drier and larger desert than the present-day Sahara. 8000 years ago, it was a region covered with vegetation where there were many lakes (image).

www.reddit.com/r/MapPorn/comments/17j1hxc/map_of_north_africa_8000_years_ago_at_the_peak_of/

The variations are modulated by the moment at which the Earth passes at perihelion. In the Northern Hemisphere, the intensity of solar radiation in summer is greater when the Earth passes at perihelion in summer, and it is lower when the Earth passes at perihelion in winter (as now). These changes in intensity in summer are not very large, but they are big enough to change the monsoon regime (seasonal rains) in North Africa. Thus, 8000 years ago, rain could reach regions of the Sahara that are no longer reached today.

Effects of These Perturbations: Glaciations

Some of these small perturbations, such as the change in eccentricity, may slightly modify the average flux received on Earth from the Sun (Q), but not enough to cause significant temperature changes. Others, such as changing the tilt axis, don't even change this flux. However, these small disturbances are at the origin of the glaciation cycle. How can such a small change in average flux have such an impact?

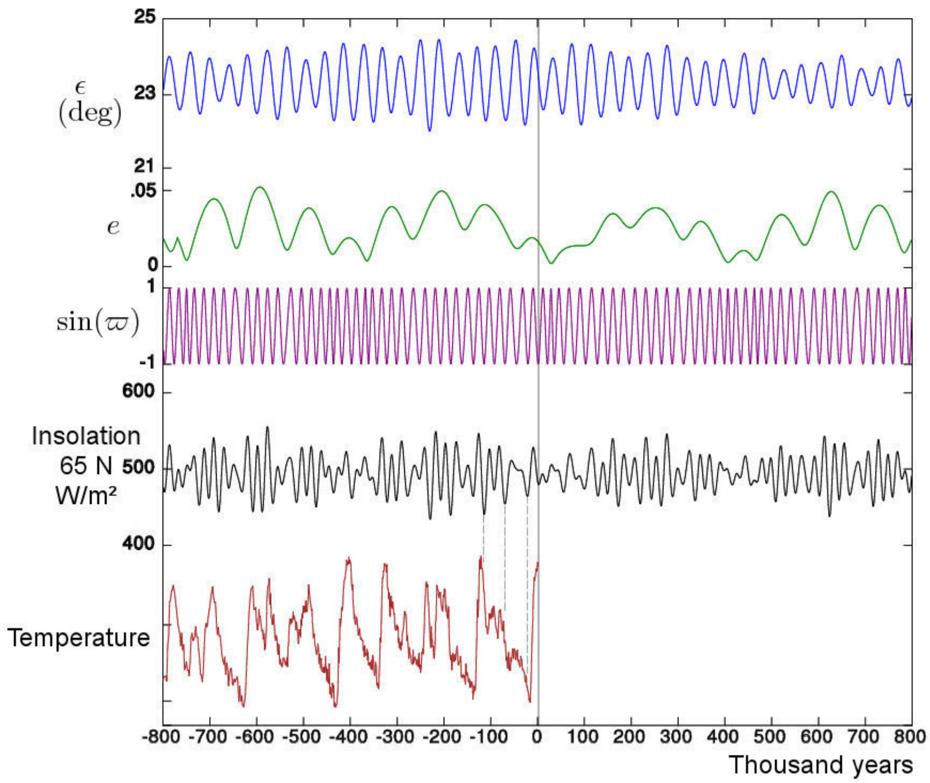
The important element is not the average flux received, but the average flux received at 65° north latitude (which is called *insolation*)! It is important to check if the ice has time to melt during the summer at this latitude. This is where the shape of the Earth's orbit and the inclination have an impact. If the orbit is very elongated so that summer occurs when the Earth is farthest from the Sun and if the Earth's tilt is not very great (which reduces the differences between the seasons), summer may be a little cold at 65° north latitude. If the summer is cold enough that the snow accumulated during the winter does not have time to melt completely during the summer, new snow will accumulate on top of this residual snow the following winter, and there will be even more snow when summer returns. This snow will not melt completely, and even more snow will be added on top of this residual snow the following winter and so on. Slowly, the snow accumulates and an ice cap forms.

However, this ice cap that is forming is white and reflects light well. Its presence then increases the Earth's albedo, which cools the Earth. As the Earth cools, there is less evaporation. Since water is a greenhouse gas, the temperature decreases even more if the greenhouse effect is reduced. This kind of feedback greatly amplifies the small variation in radiation that the perturbations have brought. Thus began an ice age.

Why only the northern hemisphere? This is simply because almost all of the land mass is in the northern hemisphere, and there is a lot of it at 65° north latitude. In the Southern Hemisphere, there is almost no land at 65° latitude and ice caps cannot form in water. This also means that the onset of an ice age is partly linked to the position of the continents. Because plate tectonics cause continents to shift, glacial cycles are not a permanent phenomenon in Earth's history. This cycle has existed for about 2 million years.

When the perturbations make the average radiation increase again, the ice will melt a little, which will decrease the albedo a little, which will warm the Earth, which will melt the ice even more and decrease the albedo a little more, which will warm the Earth, and so on. The Earth is then slowly coming out of the ice age.

The following graph shows the effect of perturbations on the insolation at 65° north latitude. (The 0 on the timescale is today.) The first graph is the graph of the inclination of the Earth's axis, the second is the graph of eccentricity, and the third is the graph of the sine of the angle of the perihelion longitude, the position of which is related to the rotation of the axis of rotation. The fourth graph gives the average insolation at 65° north latitude at the summer solstice.



wikipedia.org/wiki/Cycles_de_Milankovitch

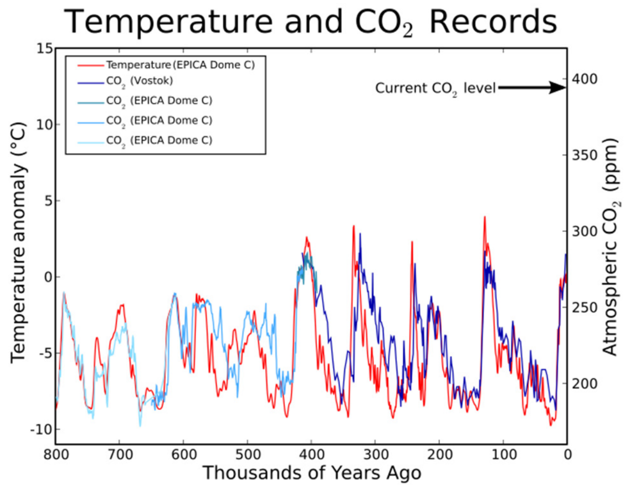
It can be seen that the last 3 minimums of the insolation curve correspond to drops in temperature.

Note that variations in insolation over the next thousands of years would have meant that there would not have been a new glaciation for 50 000 years, even if the greenhouse effect had not been increased.

Basically, for the last 2 million years, small perturbations in the orbit have caused the Earth to alternate between warm and cold modes. It is estimated that there were about thirty glaciations during this period.

CO_2 plays a role in amplifying the cycle. Variations in CO_2 concentrations are not the cause of glaciations, but they contribute to amplifying the cycle. When the glacial cycle begins, the ice increases the albedo and the temperature decreases. This allows the oceans to absorb more CO_2 and reduces the greenhouse effect. The cooling is therefore amplified by this reduction in the greenhouse effect.

The following graph, which shows the temperature variation and the CO_2 level, highlights this effect.



en.m.wikipedia.org/wiki/File:Co2-temperature-records.svg

We see that the CO_2 level is high when it is hot, and the CO_2 level is low when it is cold. There is clearly a link between CO_2 levels and temperature. However, the graph shows that, often, the CO_2 level curve lags behind the temperature changes with a certain delay. This is normal in this case, since the temperature changes first during glaciations, subsequently leading to variations in CO_2 levels which then amplify the temperature variations. Opponents of climate change often use this graph to say that CO_2 is not the cause of change because it changes after the temperature. This is indeed true in the case of glaciations, but just because it is true for glaciations does not mean that it is true for all climate change.

Variations in CO_2 Levels

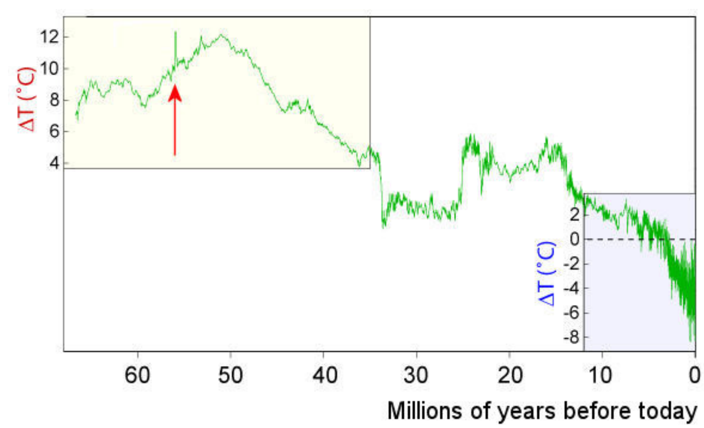
On some occasions, there have been large CO_2 emissions on Earth. This caused the CO_2 level to rise, and the temperature suddenly rose. In these cases, the increase in CO_2 preceded the increase in temperature (as is the case now).

There was such a large emission of CO_2 56 million years ago (at the transition of the Eocene and Paleocene). This was shortly after the extinction of the dinosaurs (65 million years ago) and there were, of course, no humans on Earth at that time. Essentially, there were small mammals and birds. One of the largest animals of the time was *Gastornis,* a large bird 1.75 m tall that could not fly.

Already, the climate of that time (before the CO_2 emission) was much warmer than that of today. The average temperature on Earth was on average 10 °C higher than today! The Earth was in fact in the midst of a long period of warming that lasted about ten million years and that caused the Earth to go from an average temperature of 8°C above the current temperature to 12°C above the current temperature. This warmer Earth, the warmest period in the last 200 million years, can perhaps give us an idea of what the Earth might look like in a few thousand years if the warming continues on. Essentially, it wasn't that much warmer at the equator than it is right now. On the other hand, the poles were much warmer (we can see that the poles are warming much faster than the rest of the Earth right now). There were no glaciers or polar ice caps on Earth, and the oceans were 100 m higher than they are now. A subtropical climate prevailed throughout the Earth, and tropical flora extended to a latitude of 50° on either side of the equator. There was very little change with the seasons. Antarctica was covered in forests, and its average temperature was 25 °C in summer. The average temperature of the Atlantic Ocean was 12°C higher than it is now. All in all, the climate was quite pleasant everywhere... Well, that doesn't mean that's exactly how the Earth would become with a 10°C warming because the conditions are a little different (Antarctica was still connected to South America while North America didn't yet connect to South America, so the ocean currents, which help to distribute heat, were very different), but it is not impossible. The CO_2 concentration was higher than it is now, at about 800 ppm. The greenhouse effect was certainly greater than today, but not enough to maintain such a temperature. Other factors were thought to contribute to the increase in temperature, but they have not yet been identified with certainty.

Then, suddenly, and we don't know exactly why, there was a vast release of CO_2 in the atmosphere. It seems to come from the waters near Greenland. Between 1500 and 2000 Gt of carbon have been added to the atmosphere in about 2000 years, which has increased the concentration of CO_2 from 800 ppm to nearly 2000 ppm. This is the fastest injection of carbon into the atmosphere in the entire history of the Earth (excluding, of course, the current period in which 10 Gt/year is added compared to about 1 Gt/year at that time). The average temperature, already high, suddenly rose by 5 to 8 °C. The rise in CO_2 was accompanied by species extinctions (but this extinction is one of the 5 great extinctions) and a decrease in the average size of the mammal species that survived. The warming then faded quite quickly (on a geological scale). In about 100 000 years, the CO_2 level dropped back to 800 ppm.

The abrupt warming actually appears as a simple small temporary deviation on a graph of temperatures versus time.



en.wikipedia.org/wiki/Paleocene%E2%80%93Eocene_Thermal_Maximum

How can the Earth get out of such a warming caused by a significant emission of CO_2? How can you get rid of newly injected carbon into the carbon cycle? This carbon must be eliminated for the situation to return to normal. The oceans absorb some of it when the concentration increases because the partial pressure of the gas increases, but it cannot lower the concentration. Vegetation can absorb carbon, but it does not absorb anything permanently if the volume of biomass does not change permanently. In fact, as we have seen, rain is removing carbon from the cycle by depositing carbonates on the bottom of the oceans. The process is quite slow, but rain can bring the carbon cycle back to equilibrium in about 100 000 years. There is even a positive feedback loop that allows the rain to be more effective. The increase in heat increases evaporation and, therefore, the amount of rain. This heavier rain brings more carbonate into the oceans. In addition, this more abundant carbonate allows shelled organisms to proliferate. More of these organisms die and therefore more of the shell is deposited on the bottom of the oceans, thereby removing more carbon from the cycle.

Volcanoes add carbon to the atmosphere at a rate of just under 0.1 Gt of carbon per year. Even a very large eruption doesn't add much carbon; the explosion of Mount St. Helens in 1980 added 0.0065 Gt of carbon in the atmosphere. The eruption of Pinatubo in 1991 (the largest eruption of the 20th century) added 0.013 Gt. This is a far cry from the 10 Gt emitted by humans annually.

However, a really huge eruption can add significant amounts of carbon to the atmosphere. This is what happened 252 million years ago. At that time, there was a major episode of volcanism in Siberia. Siberia literally split in two to let out a lot of lava (between 3 000 000 and 7 000 000 km³ of lava, enough to cover Canada with a layer 500 m thick) and a lot of CO_2, making it the largest volcanic eruption known.

www.newscientist.com/article/2298056-worlds-largest-mass-extinction-may-have-begun-with-volcanic-winter/

The amount of CO_2 emitted was so great that the concentration rose from about 400 ppm to 2500 ppm (and possibly even up to 6000 ppm) in 75 000 years. The consequences were disastrous. It seems that the temperature of the equatorial regions has reached 50 to 60 °C on the continents and 40 °C above the oceans. The oceans have rapidly acidified, and a large part of marine species have become extinct. In fact, it is the largest extinction in the history of the Earth (70% of terrestrial species and 95% of marine species). Of course, such an eruption also released many other gases, some of which are toxic, so that CO_2 is probably not the only gas responsible for the extinction.

There have been no episodes of rapid decline in CO_2 in the history of the Earth since there is no mechanism that can rapidly eliminate this gas from the atmosphere. However, there was a significant, but slow, decline several billion years ago. When the Earth was formed, its atmosphere was mostly made of CO_2. The atmosphere would probably have been composed of 95% CO_2 (like the atmospheres of Venus and Mars), and the surface pressure was probably nearly 100 times greater than today. Obviously, there was a lot of the greenhouse effect, but, at the same time, the Sun had about 70% of its current brightness. (The brightness of a star increases gradually over the course of its life. For the Sun, the luminosity increases by about 1% per 100 million years.) Slowly, the rain removed this CO_2 by forming carbonic acid, which then reacted with the silicates as it reached the surface to form carbonates. This carbonate was carried into the oceans by rivers and deposited at the bottom of the oceans. Slowly, therefore, almost all of the carbon in the atmosphere ended up in the sediments, mostly limestone, at the bottom of the ocean. It is estimated that there are more than 30 000 000 Gt of carbon trapped in these sediments. Some even say that they contain about 80 000 000 Gt of carbon. This is really much more than the carbon of the atmosphere, plants and oceans (about 40 000 Gt in total).

Since the level of CO_2 has varied a lot over time, we can certainly find times when the level of CO_2 was equal to what we have now. As the level was very high several tens of millions of years ago and very low during the glaciations, there was certainly a time when the CO_2 level was equal to what we have today. A look at this period could give us an idea of what to expect once the Earth has regained its thermal equilibrium (at the moment, the earth is not at equilibrium since there is still a difference of 1 W/m² between the flux emitted and the flux received by the Earth). 3.3 million years ago (during the so-called *Mean Pliocene Warm Period*), the CO_2 concentration was 400 ± 50 ppm, which is pretty much what we have today. The continents were not exactly in the same place as they are today, but the difference was not huge. At that time, the average temperature was 2 to 3.5 °C higher than today, and the sea level was 10 to 20 m higher than today. That gives us an idea of what awaits us (if we stopped increasing CO_2 concentration now, which seems unlikely).

SUMMARY OF EQUATIONS

Conductive Heat Transfer

$$P = k\frac{A}{\ell}\Delta T$$

Wavelength of the Emission Peak (Wien's Law)

$$\lambda_{pic} = \frac{2,898\times 10^{-3} mK}{T}$$

Power Radiated by a Hot Object (Stephan-Boltzmann Law)

$$P = \varepsilon\sigma A T^4$$

Average Flux (Power per Unit Area) Arriving on a Planet

$$Q = \frac{I}{4} = \frac{P_{star}}{16\pi D^2}$$

Equilibrium Temperature on the Surface of a Planet

$$T_e = \sqrt[4]{\frac{Q(1-A)}{\sigma}}$$

Temperature at the Surface of a Planet (With the Greenhouse Effect)

$$T_s = T_e \sqrt[4]{1 - \ln\sqrt{1-\varepsilon}}$$

Relationship between the amount of carbon and the concentration of CO_2 in the atmosphere

$$2.214 Gt_C = 1 ppm_{CO_2}$$

Radiative Forcing Due to Increased CO_2 Concentration

$$\Delta F_{CO2} \approx 5.35 \tfrac{W}{m^2} \cdot \ln\left(\frac{C}{278\,ppm}\right)$$

Temperature Increase Related to CO_2 Radiative Forcing

$$\Delta T \approx 0.55 \tfrac{°Cm^2}{W} \cdot \Delta F_{CO2}$$

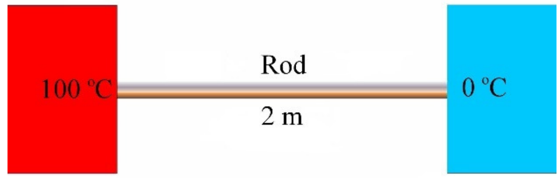
EXERCISES

Use the following values for some of these exercises.

Power of the Sun = 3,828 x 10^{26} W
Average distance between the Earth and the Sun = 149 600 000 km
Distance between the Earth and the Sun at aphelion = 152 100 000 km
Distance between the Earth and the Sun at perihelion = 147 100 000 km
Distance between Mars and the Sun = 227 340 000 km
Albedo of Earth = 0.30
Albedo of Mars = 0.25

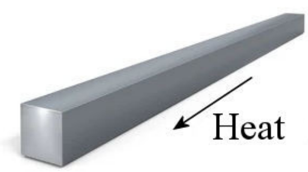
9.2 Conduction

1. An object at 100 °C is at a distance of 2 metres from another object at 0 °C. Both objects are huge and perfectly conductive. A lead rod with a diametre of 4 cm is then placed between the two objects.



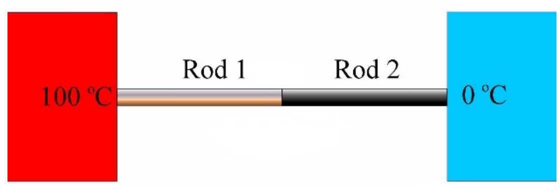
   a) How much energy passes through the rod in one hour knowing that the thermal conductivity of lead is 34.7 W/m°C?
   b) What is the temperature of the rod 10 cm from the end that touches the object at 100 °C?

2. The heat transfer rate in this silver rod is 20 W. The rod is 4 m long, and the end of the rod is in a square of 3 cm on each side. Knowing that the thermal conductivity of silver is 427 W/m°C, determine the temperature difference between the two ends of the rod.



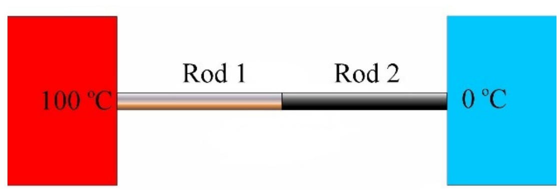
www.shutterstock.com/fi/image-illustration/one-single-square-steel-bar-isolated-2012616197

3. An object at 100 °C is at a distance of 2 metres from another object at 0 °C. Both objects are huge and perfectly conductive. The two objects are connected with two rods in contact end to end, both having the same length and diametre. One rod is made of lead (rod 1) and the other is made of iron (rod 2). The thermal conductivity of lead is 34.7 W/m°C, and the thermal conductivity of iron is 79.5 W/m°C.



   a) What is the temperature at the junction of the two rods?
   b) What is the rate of heat transfer in the rod?

4. An object at 100 °C is at a distance of 2 metres from another object at 0 °C. Both objects are huge and perfectly conductive. The two objects are connected with two rods in contact end to end, both of which are 1 m long and have a radius of 2 cm. The thermal conductivity of rod 1 is 100 W/m °C. What is the thermal conductivity of rod 2 if the temperature at the junction point is 40 °C?



5. The glass of a window is 125 cm high and 50 cm wide. The side of the window inside the house has a temperature of 15 °C, and the side of the window outside the house is at -20 °C. The thermal conductivity of glass is 0.837 W/m °C.

   a) What is the rate of heat transfer in the glass if it is 1.2 cm thick?

   b) Rather than using a single 1.2 cm thick pane of glass, we will now use two 0.6 cm thick panes of glass separated by a 0.6 cm thick layer of air. What is the rate of heat transfer in the window if the thermal conductivity of the air is 0.0234 W/m °C?

9.3 Radiation Emitted by Hot Objects

6. An object has a temperature of 3000 °C. What is the wavelength of the emission peak of this object?

7. The Sun's emission peak has a wavelength of 502 nm. What is the surface temperature of the Sun?

8. The North Star (Polaris) has a 32 000 000 km radius and a 6015 K surface temperature. What is the power of the radiation emitted by this star?

9. You are naked outside. Your body has an area of 1.8 m² and a surface temperature of 37 °C. What is the power emitted by your body if the air temperature is…

   a) 20 °C

   b) -30 °C

   Remember that in a medium at a certain temperature T_0 that gives you radiation, the net power emitted is $P = \varepsilon\sigma A\left(T^4 - T_0^4\right)$.

10. The power of the radiation emitted by a light bulb is 60 W. This light comes from a 10 cm long hot cylindrical filament whose diametre is 1 mm. What is the temperature of the filament (in °C) when the lamp is on? (The temperature of the environment is 20 °C.)

9.4 The Surface Temperature of a Planet

11. As the distance between the Earth and the Sun changes during the year, the average temperature of the Earth must also change. Calculate the difference in equilibrium temperature between the time when the Earth is at perihelion and the time when the Earth is at aphelion (if the greenhouse effect is neglected).

12. In 1 billion years, the Sun will be 10% more powerful than it is now. By how much will the equilibrium temperature increase compared to its value today?

13. What should be the Earth's albedo for the Earth's equilibrium temperature to be 0 °C, knowing that Q = 340.275 W/m²?

14. How far from the Sun would the Earth have to be for its average equilibrium temperature to be 80 °C if its albedo remains at 0.3?

9.5 The Greenhouse Effect

15. What would be the average temperature of the Earth (in °C) if the value of ε were 0.8?

16. What would have to be the value of ε for the average temperature of the Earth to be 50 °C?

17. There is virtually no greenhouse effect on Mars.

    a) What is the surface temperature on Mars without the greenhouse effect (in °C)?
    b) Now imagine that an atmosphere is added on Mars whose value of ε is 0.7. What would then be the average temperature on the surface of Mars (in °C)?

18. What should be the albedo for the average temperature of the Earth to be 5 °C taking into account the greenhouse effect (if ε and Q remain at ε = 0.71 and Q = 340.275 W/m²)?

19. What is the absorption coefficient of a single layer of the atmosphere if the atmosphere is divided into 50 layers and the total absorption coefficient is 0.71?

20. Knowing that the Earth's temperature has risen from 13.6 °C in 1850 to 14.8 °C in 2023 and that the Earth's equilibrium temperature is -18.57 °C, calculate by how much the coefficient ε has increased since 1850.

9.7 Global Warming

21. By how much would the temperature increase if the CO_2 concentration were 1000 ppm?

22. By how much would the temperature rise if we used the 6000 Gt of carbon from fossil fuels and 41% of this carbon remained in the atmosphere?

23. What would the concentration of CO_2 have to be for the average temperature on Earth to be 20 °C if the average temperature in 1850 were 13.6 °C?

24. Some propose to put aerosols in the upper atmosphere to increase the percentage of light reflected by the Earth and thus increase the Earth's albedo. By increasing the albedo, the increase in temperature could be offset by an increase in ε. We know, according to exercise 20, that the coefficient ε has increased from 0.7045 to 0.7201 since 1850. Knowing that the albedo is 0.3 now, to what value should the albedo be increased to cancel the warming?

25. All the computers in the world that run non-stop just to mine Bitcoins consume about 1.1×10^{11} kWh (1 kWh is equivalent to 3.6×10^6 J). This is almost 2 times more than the electricity consumption of all of the province of Quebec. It is estimated that the production of 1 kWh of energy generates 0.68 kg of CO_2. Knowing this, how much carbon is added to the atmosphere each year to mine Bitcoins (in millions of tons of carbon)?

26. Some estimate that 3.4 billion years ago, the atmosphere was about 90 times more massive and that it was composed of 95% CO_2. The rain then slowly removed the CO_2 from the atmosphere and deposited it on the ocean floor in the form of carbonate. Let's try to assess the amount of carbon in marine sediments.

   a) The atmosphere exerts an average pressure of 101.3 kPa on the Earth's surface. What is the force exerted on each square metre of the Earth's surface?

b) This force corresponds to the weight of all the air above this square metre. What is the mass of air above this square metre?

c) Knowing that the radius of the Earth is 6371 km, determine the total mass of the atmosphere.

d) If the atmosphere was 90 times more massive at the beginning, what was the mass of the atmosphere at that time?

e) What was the mass of carbon in the atmosphere at the beginning (in Gt)? This is the amount that should be found at the bottom of the oceans.

ANSWERS

9.2 Conduction

1. a) 7849 J b) 95 °C
2. 208.2 °C
3. a) 30.39 °C b) 3.036 W
4. 150 W/m°C
5. a) 1526 W b) 80.79 W

9.3 Radiation Emitted by Hot Objects

6. 885 nm
7. 5773 K
8. 9.551×10^{29} W
9. a) 186.6 W b) 575.0 W
10. 1752 °C

9.4 The Surface Temperature of a Planet

11. 4.26 °C
12. 6.14 °C
13. 0.0723
14. 77 750 000 km

9.5 The Greenhouse Effect

15. 21.92 °C
16. 0.9582
17. a) -63.04 °C b) -36.77 °C
18. 0.3839
19. 0.0244

20. The value of ε passed from 0.7045 to 0.7201.

9.7 Global Warming

21. 3.8 °C
22. 4.7 °C
23. 2447 ppm
24. 0.3116
25. 20 Mt
26. a) 101 300 N b) 10 337 kg c) 5.3×10^{18} kg d) 4.8×10^{20} kg
    e) Approximately 120 000 000 Gt