Chapter 7 Solutions

1. The position of the maximum is given by

$$\tan\theta = \frac{y}{L}$$

For a and b, we have *L* but the angle must be found.

a) The angle of the first minimum is

$$a\sin\theta = \lambda$$
$$0.01\times10^{-3}m\cdot\sin\theta = 500\times10^{-9}m$$
$$\theta = 2.866°$$

Therefore, the position on the screen is

$$\tan\theta = \frac{y}{L}$$
$$\tan(2.866°) = \frac{y}{200cm}$$
$$y = 10.0cm$$

b) The angle of the second minimum is

$$a\sin\theta = 2\lambda$$
$$0.01\times10^{-3}m\cdot\sin\theta = 2\cdot500\times10^{-9}m$$
$$\theta = 5.74°$$

Therefore, the position on the screen is

$$\tan\theta = \frac{y}{L}$$
$$\tan(5.74°) = \frac{y}{200cm}$$
$$y = 20.1cm$$

2. The width of the slit will be found with the position of the 1st minimum.

$$a\sin\theta = \lambda$$

If the central maximum is 4 cm wide, then the distance between the first minimum and the centre of the central maximum is 2 cm. So, we have

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{2cm}{500cm}$$
$$\theta = 0.2292°$$

Therefore,

$$a\sin\theta = \lambda$$
$$a\cdot\sin(0.2292°) = 560\times 10^{-9}m$$
$$a = 0.14mm$$

3. The wavelength will be found with the position of the 1st minimum.

$$a\sin\theta = \lambda$$

If the central maximum is 50 cm wide, then the distance between the first minimum and the centre of the central maximum is 25 cm. So, we have

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{25cm}{160cm}$$
$$\theta = 8.88°$$

Therefore,

$$a\sin\theta = \lambda$$
$$0.01m\cdot\sin(8.88°) = \lambda$$
$$\lambda = 1.544mm$$

4. The light intensity is calculated with

$$I = I_0\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2$$

To calculate the intensity, α is needed. α is

$$\alpha = \frac{a\sin\theta}{\lambda}2\pi$$

With have λ but we need the angle.

0.5 cm from the centre of the central maximum, the angle is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{0.5cm}{200cm}$$
$$\theta = 0.1432°$$

Therefore, the value of α is

$$\begin{aligned}\alpha &= \frac{a\sin\theta}{\lambda}2\pi\\ &= \frac{0.1\times10^{-3}m\cdot\sin(0.1432°)}{600\times10^{-9}m}\cdot 2\pi\\ &= 2.618rad\end{aligned}$$

Thus, the intensity is

$$\begin{aligned}I &= I_0\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2\\ &= I_0\left(\frac{\sin(1.309)}{(1.309)}\right)^2\\ &= 0.5445I_0\end{aligned}$$

5. The half-length of the central maximum is

$$\sin\theta = \frac{\lambda}{a}$$

The 2 wavelength gives us these 2 equations.

At 450 nm, we have

$$\sin\theta_1 = \frac{450nm}{a}$$

At 650 nm, we have

$$\sin\theta_2 = \frac{650nm}{a}$$

Dividing these 2 equations gives

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{\frac{650nm}{a}}{\frac{450nm}{a}}$$
$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{13}{9}$$

Angle 1 (when the wavelength is 450 nm) can be found.

If the central maximum is 4 cm wide, then the distance between the first minimum and the centre of the central maximum is 2 cm. So, we have

$$\tan\theta_1 = \frac{y}{L}$$
$$\tan\theta_1 = \frac{2cm}{300cm}$$
$$\theta_1 = 0.382°$$

Therefore,

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{13}{9}$$
$$\frac{\sin\theta_2}{\sin(0,382°)} = \frac{13}{9}$$
$$\theta_2 = 0,5517°$$

So, the position of the first minimum on the screen is

$$\tan\theta_2 = \frac{y}{L}$$

$$\tan(0.5517°) = \frac{y}{300cm}$$

$$y = 2.889cm$$

The width of the central maximum is twice this value, so it is 5.778 cm.

6. We need to find the position of the two minimums. As the central maximum is 10 cm wide, we already know that the first minimum is at y = 5 cm.

It remains to find the position of the 2nd minimum.

The position of the minimums is given by

$$a\sin\theta = M\lambda$$

Thus, we have the following two equations.

First minimum

$$a\sin\theta_1 = \lambda$$

Second minimum

$$a\sin\theta_2 = 2\lambda$$

Dividing these equations gives

$$\frac{a\sin\theta_2}{a\sin\theta_1} = \frac{2\lambda}{\lambda}$$

$$\frac{\sin\theta_2}{\sin\theta_1} = 2$$

We need the angle of the first minimum. As the first minimum is at y = 5 cm, the angle is

$$\tan\theta_1 = \frac{y}{L}$$
$$\tan\theta_1 = \frac{5cm}{400cm}$$
$$\theta_1 = 0.716°$$

Therefore,

$$\frac{\sin\theta_2}{\sin\theta_1} = 2$$
$$\frac{\sin\theta_2}{\sin(0.716°)} = 2$$
$$\theta_2 = 1.432°$$

So, the position of the second minimum on the screen is

$$\tan\theta_2 = \frac{y}{L}$$
$$\tan(1.432°) = \frac{y}{400cm}$$
$$y = 10.002cm$$

The distance between the second minimum and the first minimum is therefore

$$\Delta y = 10.002cm - 5cm = 5.002cm$$

7. The angles of the minima are found with

$$a\sin\theta = M\lambda$$

The first minimum at 20° indicates that

$$a\sin 20° = \lambda$$
$$\sin 20° = \frac{\lambda}{a}$$

Therefore, the angle of the second minimum is

$$a\sin\theta = 2\lambda$$
$$\sin\theta = 2\frac{\lambda}{a}$$
$$\sin\theta = 2\cdot\sin 20°$$
$$\sin\theta = 0,68404$$
$$\theta = 43,16°$$

For the 3rd minimum, we have

$$a\sin\theta = 3\lambda$$
$$\sin\theta = 3\frac{\lambda}{a}$$
$$\sin\theta = 3\cdot\sin 20°$$
$$\sin\theta = 1,026$$

As there is no solution, there is no third minimum.

8. a) We have

$$\frac{d}{a} = \frac{0.2mm}{0.04mm} = 5$$

This means that $m_d = 4$. The number of maxima is therefore $2 \cdot 4 + 1 = 9$.

b) The intensity is given by

$$I_{tot} = 4I_{10}\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2 \cos^2\frac{\Delta\phi}{2}$$

To calculate this intensity, $\Delta\phi$ and α are needed. They are

$$\Delta\phi = \frac{d\sin\theta}{\lambda}2\pi \qquad \alpha = \frac{a\sin\theta}{\lambda}2\pi$$

To calculate them, θ is needed.

3 cm from the centre of the central maximum, the angle is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{3cm}{240cm}$$
$$\theta = 0.7162°$$

The value of $\Delta\phi$ is thus

$$\begin{aligned}\Delta\phi &= \frac{d\sin\theta}{\lambda}2\pi \\ &= \frac{0.2\times10^{-3}m\cdot\sin(0.7162°)}{600\times10^{-9}m}\cdot 2\pi \\ &= 26.178rad\end{aligned}$$

The value of α is

$$\begin{aligned}\alpha &= \frac{a\sin\theta}{\lambda}2\pi \\ &= \frac{0.04\times10^{-3}m\cdot\sin(0.7162°)}{600\times10^{-9}m}\cdot 2\pi \\ &= 5.236rad\end{aligned}$$

Therefore, the intensity is

$$\begin{aligned}I_{tot} &= 4I_{10}\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2\cos^2\frac{\Delta\phi}{2} \\ &= 4I_{10}\cdot\left(\frac{\sin\left(\frac{5.236}{2}\right)}{\left(\frac{5.236}{2}\right)}\right)^2\cdot\cos^2\frac{26.178}{2} \\ &= 0.1097I_{10}\end{aligned}$$

c) The intensity is given by

$$I_{tot} = 4I_{10}\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2\cos^2\frac{\Delta\phi}{2}$$

To calculate this intensity, $\Delta\phi$ and α are needed. They are

$$\Delta\phi = \frac{d\sin\theta}{\lambda}2\pi \qquad \alpha = \frac{a\sin\theta}{\lambda}2\pi$$

To calculate them, θ is needed.

The angle of the first interference maximum is

$$d\sin\theta = \lambda$$
$$\sin\theta = \frac{\lambda}{d}$$

The value of $\Delta\phi$ is therefore

$$\begin{aligned}\Delta\phi &= \frac{d\sin\theta}{\lambda}2\pi \\ &= \frac{d\,\frac{\lambda}{d}}{\lambda}2\pi \\ &= 2\pi\end{aligned}$$

The value of α is

$$\begin{aligned}\alpha &= \frac{a\sin\theta}{\lambda}2\pi \\ &= \frac{a\,\frac{\lambda}{d}}{\lambda}2\pi \\ &= \frac{a}{d}2\pi \\ &= \frac{0.04mm}{0.2mm}\cdot 2\pi \\ &= \frac{2\pi}{5}\end{aligned}$$

Thus, the intensity is

$$\begin{aligned}I_{tot} &= 4I_{10}\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2\cos^2\frac{\Delta\phi}{2} \\ &= 4I_{10}\cdot\left(\frac{\sin\left(\frac{\pi}{5}\right)}{\left(\frac{\pi}{5}\right)}\right)^2\cdot\cos^2\frac{2\pi}{2} \\ &= 3.5I_{10}\end{aligned}$$

As the intensity of the central interference maximum is 4 I_{10}, the ratio of intensity is

$$ratio = \frac{3.5I_{10}}{4I_{10}} = 0.875$$

The intensity is thus 87.5% of the intensity of the central interference maximum.

9. a)

The distance between the slits will be found with the position of the interference maxima

$$d\sin\theta = m\lambda$$

We notice that the 8th-order interference maximum is close to y = 5 cm. (Any maximum or minimum can be used). At this position, the angle is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{5cm}{200cm}$$
$$\theta = 1.432°$$

For the 8th-order maximum, we have

$$d\sin\theta = 8\lambda$$
$$d\cdot\sin(1.432°) = 8\cdot 650\times10^{-9}m$$
$$d = 2.0806\times10^{-4}m = 0.20806mm$$

As this is a little approximate, let's say 0.2 mm.

b) The distance between the slits will be found with the position of the minimum of diffraction.

$$a\sin\theta = M\lambda$$

We notice that the 1st-order diffraction minimum is close to y = 3.2 cm. (Any minimum can be used). At this position, the angle is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{3.2cm}{200cm}$$
$$\theta = 0.9167°$$

For the first-order minimum, we have

$$a\sin\theta = \lambda$$
$$a\cdot\sin(0.9167°) = 650\times10^{-9}m$$
$$a = 4.063\times10^{-5}m = 0.04063mm$$

As this is a little approximate, let's say 0.04 mm.

10. The angle of the first-order minimum is

$$\sin\theta = 1,22\frac{\lambda}{a}$$
$$\sin\theta = 1.22\cdot\frac{560\times10^{-9}m}{0,1\times10^{-3}m}$$
$$\theta = 0.39145°$$

Therefore, the distance between the centre of the diffraction pattern and the first minimum on the screen is

$$\tan\theta = \frac{y}{L}$$
$$\tan(0.3914°) = \frac{y}{200cm}$$
$$y = 1.366cm$$

11. The diameter of the hole will be found with

$$\sin\theta = 1.22\frac{\lambda}{a}$$

If the central maximum has a 6 mm diameter, then the distance between the first minimum and the centre of the central maximum is 3 mm. So, we have

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{0.3cm}{180cm}$$
$$\theta = 0.0955°$$

Therefore,

$$\sin\theta = 1.22\frac{\lambda}{a}$$
$$\sin(0.0955°) = 1.22\cdot\frac{620\times10^{-9}m}{a}$$
$$a = 4.538\times10^{-4}m = 0.4538mm$$

12. According to the Babinet's principle, the diffraction pattern obtained with a hair is identical to the pattern obtained with a slit. Thus, the width of the hair is the same as the width of the slit that corresponds to the same diffraction pattern. So, this problem will be treated as a slit problem.

Thus, the width of the hair can be found with the formula giving the with of a slit

$$a\sin\theta = \lambda$$

The angle of the first minimum is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{0.065m}{9.67m}$$
$$\theta = 0.3851°$$

The width of the hair is then found with

$$a\sin\theta = \lambda$$
$$a\cdot\sin(0.3851°) = 523\times10^{-9}m$$
$$a = 7.78\times10^{-5}m$$
$$a = 77.8\mu m$$

13. The distance will be found with

$$\theta_{c(rad)} = \frac{d}{L}$$

To obtain it, we need the critical angle. This angle is

$$\sin\theta_c = 1.22\frac{\lambda}{a}$$

$$\sin\theta_c = 1.22\cdot\frac{550\times10^{-9}m/1.33}{3\times10^{-3}m}$$

$$\theta_c = 0.009635°$$

Therefore, the distance is

$$\theta_{c(rad)} = \frac{d}{L}$$

$$1.6817\times10^{-4}rad = \frac{0.02m}{L}$$

$$L = 118.9m$$

14. The distance will be found with

$$\theta_{c(rad)} = \frac{d}{L}$$

To obtain it, we need the critical angle. This angle is

$$\sin\theta_c = 1.22\frac{\lambda}{a}$$

$$\sin\theta_c = 1.22\cdot\frac{550\times10^{-9}m}{0.25m}$$

$$\theta_c = 1.538\times10^{-4}°$$

Therefore,

$$\theta_{c(rad)} = \frac{d}{L}$$

$$2.684\times10^{-6}rad = \frac{d}{200{,}000m}$$

$$d = 0.5368m$$

15. The diameter will be found with

$$\sin\theta_c = 1.22\frac{\lambda}{a}$$

To obtain it, we need the angle between the objects. This angle is

$$\begin{aligned}\theta_{c(rad)} &= \frac{d}{L} \\ &= \frac{8\times 10^{7}\,km}{4.73\times 10^{13}\,km} \\ &= 1.691\times 10^{-6}\,rad\end{aligned}$$

Therefore,

$$\sin\theta_c = 1.22\frac{\lambda}{a}$$

$$\sin\left(1.691\times 10^{-4}\,rad\right) = 1.22\cdot\frac{550\times 10^{-9}\,m}{a}$$

$$a = 0.3967m$$

16. The intensity is

$$I = I_0\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}\right)^2$$

At a maximum (or a minimum), we must have $dI/d\alpha = 0$. Thus, we must have

$$\frac{dI}{d\alpha} = 0$$

$$\frac{d}{d\alpha}\left(I_0\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}\right)^2\right) = 0$$

$$I_0 2\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}\right)\left(\frac{\cos\left(\frac{\alpha}{2}\right)\frac{1}{2}}{\frac{\alpha}{2}} - \frac{\sin\left(\frac{\alpha}{2}\right)\cdot\frac{1}{2}}{\left(\frac{\alpha}{2}\right)^2}\right) = 0$$

There are two possibilities for this derivative to vanish. First possibility: the first term in parentheses vanishes.

$$\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} = 0$$

The solution of this equation is

$$\frac{\alpha}{2} = M\pi$$

where M = 1, 2, 3,,,. We recognize this solution: those are the minimum of intensity.

Second possibility: the second term in parentheses vanishes.

$$\frac{\cos\left(\frac{\alpha}{2}\right)\frac{1}{2}}{\frac{\alpha}{2}} - \frac{\sin\left(\frac{\alpha}{2}\right)\cdot\frac{1}{2}}{\left(\frac{\alpha}{2}\right)^2} = 0$$

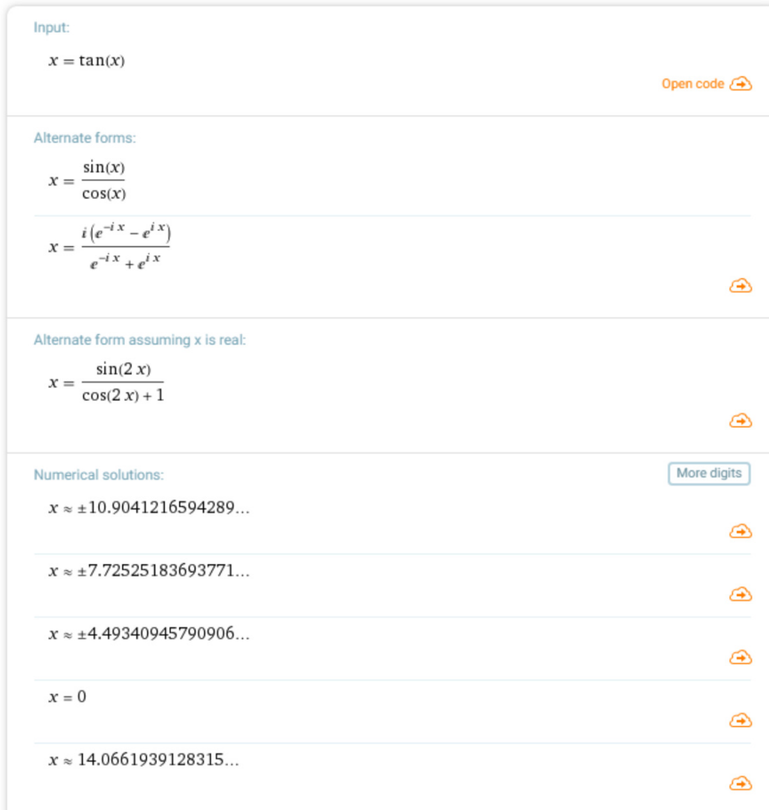
The solution leads to

$$\frac{\cos\left(\frac{\alpha}{2}\right)\frac{1}{2}}{\frac{\alpha}{2}} = \frac{\sin\left(\frac{\alpha}{2}\right)\cdot\frac{1}{2}}{\left(\frac{\alpha}{2}\right)^2}$$

$$\cos\left(\frac{\alpha}{2}\right) = \frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}$$

$$\frac{\alpha}{2} = \tan\left(\frac{\alpha}{2}\right)$$

This equation is not easy to solve. Among other things, it can be solved with a software like Maple or with the given internet site. Here is the solution according to Wolfram.



The first maximum is thus at x = 4.49341. (The approximation in which the maxima were assumed to be exactly between the minima gives 4.7124) Thus,

$$\frac{\alpha}{2} = 4.49341$$
$$\alpha = 8.98682$$

Since

$$\alpha = \frac{a \sin\theta}{\lambda} 2\pi$$

the angle is

$$\frac{a \sin\theta}{\lambda} 2\pi = 8.98682$$
$$\frac{0.1\times 10^{-3} m \cdot \sin\theta}{600\times 10^{-9} m} \cdot 2\pi = 8.98682$$
$$\sin\theta = 0.00858178$$
$$\theta = 0.4917°$$

Therefore, the distance is

$$\tan\theta = \frac{y}{L}$$

$$\tan(0.4917°) = \frac{y}{2m}$$

$$y = 0.01716m$$

$$y = 1.716cm$$