Chapter 6 Solutions

1. To have destructive interference, the phase difference between the waves must be an odd number of π.

$$\Delta\phi = (2m+1)\pi$$

Therefore, the phase difference between the waves must be found.

$\Delta\phi_T$ is

$$\Delta\phi_T = -\frac{\Delta r}{\lambda}2\pi$$

The distances of the sources are needed. The distances are (assuming that wave 2 is the one coming from speaker B)

$$\begin{aligned} r_1 &= \sqrt{(1m)^2 + (2.4m)^2} \\ r_2 &= 2.4m \end{aligned}$$

The path length difference is

$$\begin{aligned} \Delta r &= 2.4m - \sqrt{(1m)^2 + (2.4m)^2} \\ &= -0.2m \end{aligned}$$

Thus, $\Delta\phi_T$ is

$$\begin{aligned} \Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{-0.2m}{\lambda}\cdot 2\pi \\ &= \frac{0.2m}{\lambda}\cdot 2\pi \end{aligned}$$

As there is no phase difference between the sources and no reflection, this phase difference is the total phase difference.

$$\Delta\phi = \frac{0.2m}{\lambda}\cdot 2\pi$$

The condition for destructive interference thus becomes

$$\Delta\phi = (2m+1)\pi$$
$$\frac{0.2m}{\lambda}\cdot 2\pi = (2m+1)\pi$$

If this equation is solved for λ, we arrive at

$$\frac{0.2m}{\lambda}2 = (2m+1)$$
$$\lambda = \frac{0.4m}{2m+1}$$

To get the minimum frequency, the longest wavelength is needed. Thus, we need the smallest numerator, which we obtain with $m = 0$. Then, the maximum wavelength is

$$\lambda_{\max} = \frac{0.4m}{2\cdot 0+1} = 0.4m$$

The minimum frequency is thus

$$\begin{aligned} f_{\min} &= \frac{v}{\lambda_{\max}} \\ &= \frac{340\frac{m}{s}}{0.4m} \\ &= 850Hz \end{aligned}$$

2. To have constructive interference, the phase difference between the waves must be an even number of π.

$$\Delta\phi = 2m\pi$$

Therefore, the phase difference between the waves must be found.

$\Delta\phi_T$ is

$$\Delta\phi_T = -\frac{\Delta r}{\lambda}2\pi$$

The distances of the sources are needed. The distances are (assuming that wave 1 is the wave reflected by the wall)

$$r_1 = 2\sqrt{(3m)^2 + d^2}$$
$$r_2 = 6m$$

The path length difference is thus

$$\Delta r = 6m - 2\sqrt{(3m)^2 + d^2}$$

We also need the wavelength. This wavelength is

$$\begin{aligned}\lambda &= \frac{v}{f} \\ &= \frac{343 \frac{m}{s}}{490Hz} \\ &= 0.7m\end{aligned}$$

Then, $\Delta\phi_T$ is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda} 2\pi \\ &= -\frac{6m - 2\sqrt{(3m)^2 + d^2}}{0.7m} \cdot 2\pi \\ &= \frac{2\sqrt{(3m)^2 + d^2} - 6m}{0.7m} \cdot 2\pi\end{aligned}$$

As the sound reflected on a wall is inverted, $\Delta\phi_R$ is

$$\begin{aligned}\Delta\phi_R &= \phi_{2R} - \phi_{1R} \\ &= 0 - \pi \\ &= -\pi\end{aligned}$$

As $\Delta\phi_S = 0$, the total phase difference is

$$\Delta\phi = \frac{2\sqrt{(3m)^2 + d^2} - 6m}{0.7m} \cdot 2\pi - \pi$$

The condition for constructive interference thus becomes

$$\Delta\phi = 2m\pi$$
$$\frac{2\sqrt{(3\text{m})^2 + d^2} - 6\text{m}}{0.7\text{m}} \cdot 2\pi - \pi = 2m\pi$$

Note here that since the two terms on the left are positive, the term on the right must also be positive, thereby eliminating all the negative values of *m*.

Solving for *d,* we arrive at

$$\frac{2\sqrt{(3\text{m})^2 + d^2} - 6\text{m}}{0.7\text{m}} \cdot 2 - 1 = 2m$$
$$\frac{2\sqrt{(3\text{m})^2 + d^2} - 6\text{m}}{0.7\text{m}} \cdot 2 = 2m + 1$$
$$2 \cdot \sqrt{(3\text{m})^2 + d^2} - 6\text{m} = \frac{2m+1}{2} \cdot 0.7\text{m}$$
$$2 \cdot \sqrt{(3\text{m})^2 + d^2} = \frac{2m+1}{2} \cdot 0.7\text{m} + 6\text{m}$$
$$\sqrt{(3\text{m})^2 + d^2} = \frac{2m+1}{4} \cdot 0.7\text{m} + 3\text{m}$$
$$(3\text{m})^2 + d^2 = \left(\frac{2m+1}{4} \cdot 0.7\text{m} + 3\text{m}\right)^2$$
$$d^2 = \left(\frac{2m+1}{4} \cdot 0.7\text{m} + 3\text{m}\right)^2 - (3\text{m})^2$$

Here's what you get for different values of *m*.

$m = 0$	$d^2 = 1.080625\ \text{m}^2$
$m = 1$ and more	d^2 is greater than $1.080625\ \text{m}^2$

The minimum value is, therefore, found with $m = 0$. The distance is then

$$d = \sqrt{1.080625m^2}$$
$$= 1.0395m$$

3. The intensity will be found from the resulting amplitude. This amplitude is

$$A = \sqrt{A_1^2 + 2A_1A_2\cos(\Delta\phi) + A_2^2}$$

As the amplitude for light is denoted E_0 instead of A, this equation becomes

$$E_{0tot} = \sqrt{E_{01}^2 + 2E_{01}E_{02}\cos(\Delta\phi) + E_{02}^2}$$

Therefore, the phase difference between the waves must be found.

$\Delta\phi_T$ is

$$\Delta\phi_T = -\frac{\Delta r}{\lambda}2\pi$$

The distances of the sources are needed. The distances are (assuming that wave 2 is the wave reflected by the wall)

$$r_1 = 6m$$
$$r_2 = 2\sqrt{(3m)^2 + (2m)^2}$$

The path length difference is thus

$$\begin{aligned}\Delta r &= 2\sqrt{(3m)^2 + (2m)^2} - 6m \\ &= 1.211m\end{aligned}$$

Therefore, $\Delta\phi_T$ is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{1.211m}{1.4m}\cdot 2\pi \\ &= -5.435rad\end{aligned}$$

As the wave reflected on a wall is inverted, $\Delta\phi_R$ is

$$\Delta\phi_R = \pi$$

As $\Delta\phi_S = 0$, the total phase difference is

$$\Delta\phi = -5.435 + \pi = -2.294rad$$

Since $E_{02} = 0,7E_{01}$, the amplitude is

$$\begin{aligned}
E_{0tot} &= \sqrt{E_{01}^2 + 2E_{01}E_{02}\cos(\Delta\phi) + E_{02}^2} \\
&= \sqrt{E_{01}^2 + 2\cdot E_{01}\cdot 0.7\cdot E_{01}\cdot\cos(-2.294) + (0.7\cdot E_{01})^2} \\
&= \sqrt{E_{01}^2\cdot\left(1 + 2\cdot 0.7\cdot\cos(-2.294) + (0.7)^2\right)} \\
&= \sqrt{E_{01}^2\cdot 0.5637} \\
&= E_{01}\cdot 0.7508
\end{aligned}$$

Thus, the intensity is

$$\begin{aligned}
I_{tot} &= \frac{1}{2}cn\varepsilon_0 E_{0tot}^2 \\
&= \frac{1}{2}cn\varepsilon_0\left(0.7508\cdot E_{01}\right)^2
\end{aligned}$$

Without reflection, the amplitude of the wave would have been simply E_{01} and the intensity would have been

$$I_1 = \frac{1}{2}cn\varepsilon_0 E_{01}^2$$

By dividing one intensity by the other intensity, the result is

$$\begin{aligned}
\frac{I_{tot}}{I_1} &= \frac{\frac{1}{2}cn\varepsilon_0\left(0.7508\cdot E_{01}\right)^2}{\frac{1}{2}cn\varepsilon_0 E_{01}^2} \\
&= (0.7508)^2 \\
&= 0.5637
\end{aligned}$$

Therefore, the intensity is 56.37% of the intensity we would have if there was only the wave arriving directly from the source.

4. We know that the amplitude is given by

$$E_{0tot} = \sqrt{E_{01}^2 + 2E_{01}E_{02}\cos(\Delta\phi) + E_{02}^2}$$

To know this amplitude, the amplitudes and phase difference must be known.

The amplitude of wave A is

$$I_1 = \frac{1}{2} c\varepsilon_0 E_{01}^2$$

$$0,001 \tfrac{W}{m^2} = \frac{1}{2} \cdot 3\times 10^8 \tfrac{m}{s} \cdot 8,854\times 10^{-12} \tfrac{m^2}{NC^2} \cdot E_{0A}^2$$

$$E_{0A} = 0,8677 \tfrac{N}{C}$$

To calculate the amplitude of wave 2, the fact that the intensity decreases with to the inverse of the square of the distance is used. This ratio of intensity gives

$$\frac{I_B}{I_A} = \frac{\frac{1}{2} cn\varepsilon_0 E_{0B}^2}{\frac{1}{2} cn\varepsilon_0 E_{0A}^2}$$

$$\frac{I_B}{I_A} = \frac{E_{0B}^2}{E_{0A}^2}$$

$$\frac{\frac{P}{4\pi r_B^2}}{\frac{P}{4\pi r_A^2}} = \frac{E_{0B}^2}{E_{0A}^2}$$

$$\frac{r_A^2}{r_B^2} = \frac{E_{0B}^2}{E_{0A}^2}$$

$$\frac{r_A}{r_B} = \frac{E_{0B}}{E_{0A}}$$

Thus

$$\begin{aligned} \frac{E_{0B}}{E_{0A}} &= \frac{r_A}{r_B} \\ &= \frac{300m}{\sqrt{(300m)^2 + (200m)^2}} \\ &= \frac{3}{\sqrt{13}} \end{aligned}$$

Therefore, the amplitude of the wave 2 is

$$\begin{aligned} E_{0B} &= \frac{3}{\sqrt{13}} E_{0A} \\ &= \frac{3}{\sqrt{13}} \cdot 0.8677 \tfrac{N}{C} \\ &= 0.7220 \tfrac{N}{C} \end{aligned}$$

It remains to find the phase difference. As we have only the phase difference caused by the difference in distance, the phase shift is

$$\begin{aligned}
\Delta\phi &= -\frac{r_B - r_A}{\lambda} 2\pi \\
&= \frac{\sqrt{(300m)^2 + (200m)^2} - 300m}{3\times 10^8 \frac{m}{s} / 100\,\text{MHz}} \cdot 2\pi \\
&= \frac{60.555m}{3m} \cdot 2\pi \\
&= 126.826 rad
\end{aligned}$$

Therefore, the amplitude is

$$\begin{aligned}
E_{0tot} &= \sqrt{E_{01}^2 + 2E_{01}E_{02}\cos(\Delta\phi) + E_{02}^2} \\
&= \sqrt{\left(0.8677\tfrac{N}{C}\right)^2 + 2\cdot 0.8677\tfrac{N}{C}\cdot 0.7220\tfrac{N}{C}\cdot\cos(126{,}826) + \left(0{,}7220\tfrac{N}{C}\right)^2} \\
&= 1.3311\tfrac{N}{C}
\end{aligned}$$

Finally, the intensity can be found

$$\begin{aligned}
I_{tot} &= \frac{1}{2} c\varepsilon_0 E_{01}^2 \\
&= \frac{1}{2}\cdot 3\times 10^8 \tfrac{m}{s}\cdot 8.854\times 10^{-12} \tfrac{m^2}{NC^2}\cdot\left(1.3311\tfrac{N}{C}\right)^2 \\
&= 2.353\times 10^{-3} \tfrac{W}{m^2}
\end{aligned}$$

5. The phase difference is

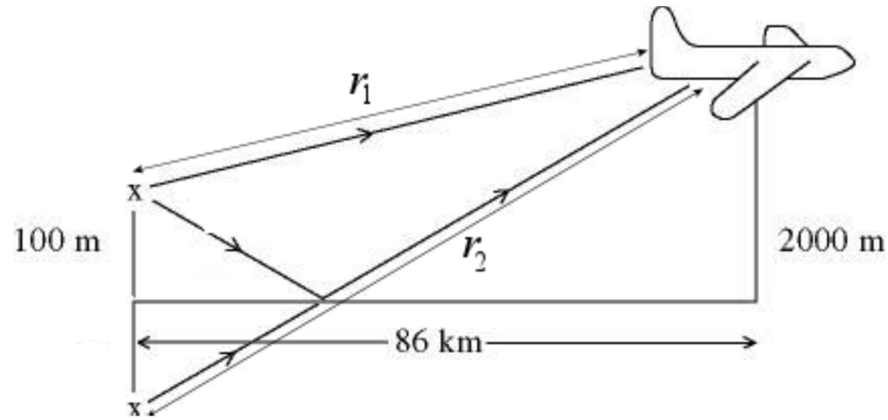
$$\Delta\phi = \Delta\phi_T + \Delta\phi_S + \Delta\phi_R$$

$\Delta\phi_T$ is

$$\Delta\phi_T = -\frac{\Delta r}{\lambda} 2\pi$$

The distances of the sources and the wavelength are needed.

The distances are (assuming that wave 2 is the reflected wave)



$$r_1 = \sqrt{(86000m)^2 + (1900m)^2}$$
$$r_2 = \sqrt{(86000m)^2 + (2100m)^2}$$

The path length difference is thus

$$\begin{aligned}\Delta r &= \sqrt{(86000m)^2 + (2100m)^2} - \sqrt{(86000m)^2 + (1900m)^2} \\ &= 4.6499m\end{aligned}$$

The wavelength is

$$\begin{aligned}\lambda &= \frac{c}{f} \\ &= \frac{3\times10^8 \frac{m}{s}}{120\times10^6 Hz} \\ &= 2.5m\end{aligned}$$

Then, $\Delta\phi_T$ is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda} 2\pi \\ &= -\frac{-4.6499m}{2.5m}\cdot 2\pi \\ &= -11.686rad\end{aligned}$$

$\Delta\phi_S$ is 0 since both waves are coming from the same source

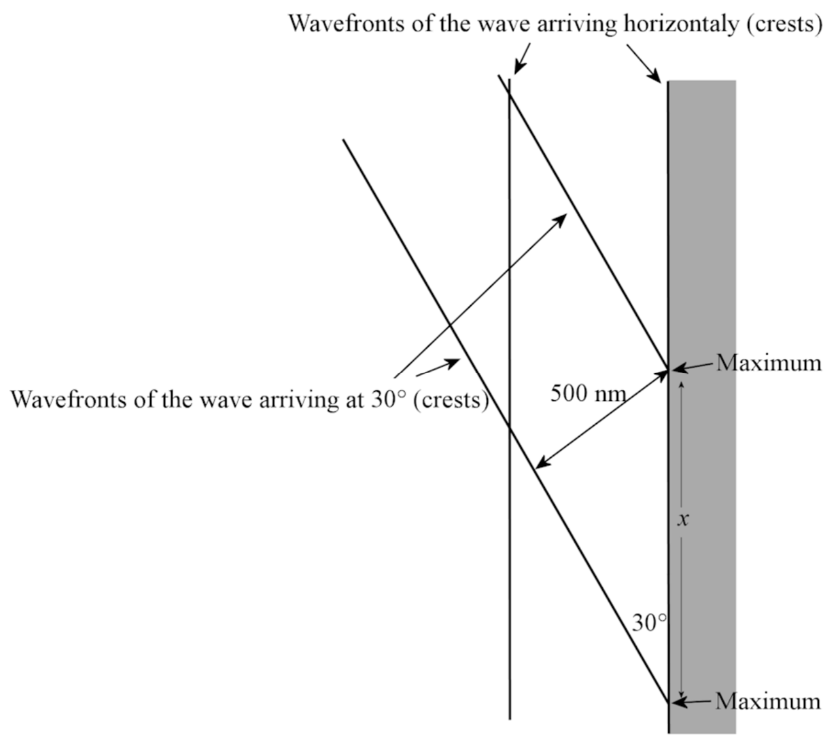
Since the reflected wave is inverted, $\Delta\phi_R$ is

$$\Delta\phi_R = \pi$$

The total phase difference is thus

$$\begin{aligned}\Delta\phi &= -11.686rad + \pi \\ &= -8.545rad\end{aligned}$$

6. There is constructive interference when the crests intersect. Considering that the wavefronts are the crests, the distance between the maximums on the screen is therefore equal to the distance between two places where the wavefronts intersect.



A rectangle triangle is then formed (bottom right). Thus

$$\sin 30° = \frac{500nm}{x}$$
$$x = 1000nm$$

7. The distance between the slits is found with

$$d\sin\theta = m\lambda$$

To obtain it, we need the angle of the 4$^{\text{th}}$ maximum.

The angle is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{1cm}{200cm}$$
$$\theta = 0.2865°$$

The distance is then found with

$$d\sin\theta = m\lambda$$
$$d \cdot \sin(0.2865°) = 4 \cdot 600nm$$
$$d = 4.8\times10^{-4}m = 0.48mm$$

8. The position will be calculated with the angle. The angle is given by

$$d\sin\theta = m\lambda$$
$$0.1\times10^{-3}m \cdot \sin\theta = 5 \cdot 500nm$$
$$\theta = 1.4325°$$

The position of the maximum is thus

$$\tan\theta = \frac{y}{L}$$
$$\tan(1.4325°) = \frac{y}{160cm}$$
$$y = 4.001cm$$

9. The wavelength is found with

$$d\sin\theta = m\lambda$$

We do not know d and the angle but the value of $d\sin\theta$ can be found can be found with the information we have concerning light with a 550 nm wavelength since the maxima are at the same position.

With a 550 nm wavelength, we have

$$d\sin\theta = m\lambda$$
$$d\sin\theta = 5\cdot 550nm$$
$$d\sin\theta = 2750nm$$

With the other wavelength, we have

$$d\sin\theta = m\lambda$$
$$2750nm = 4\lambda$$
$$\lambda = 687.5nm$$

10. The distance is found with

$$\tan\theta = \frac{y}{L}$$

The diagram shows that y = 11.5 mm for the 3rd minimum (m = 2). We need the angle.

The angle is of the 3rd minimum is

$$d\sin\theta = \left(m+\tfrac{1}{2}\right)\lambda$$
$$0.2\times10^{-3}m\cdot\sin\theta = \left(2+\tfrac{1}{2}\right)\cdot 632\times10^{-9}m$$
$$\theta = 0.4526°$$

Therefore

$$\tan\theta = \frac{y}{L}$$
$$\tan\left(0.4526°\right) = \frac{1.15cm}{L}$$
$$L = 145.6cm$$

11. In Young's experiment, the path length difference can be found from the phase difference with

$$\Delta\phi = -\frac{\Delta r}{\lambda} 2\pi$$

$$2 = \left| -\frac{\Delta r}{450 \times 10^{-9} m} \cdot 2\pi \right|$$

$$\Delta r = 143.24 nm$$

As the path length difference is

$$\Delta r = d \sin\theta$$

We have

$$143.24 \times 10^{-9} m = 0.2 \times 10^{-3} m \cdot \sin\theta$$

$$\theta = 0.041°$$

The position is then found with

$$\tan\theta = \frac{y}{L}$$

$$\tan(0.041°) = \frac{y}{240 cm}$$

$$y = 0.1719 cm$$

12. a)

There are 9 slits since there are 7 small maxima. (The number of slits is always equal to the number of small maxima + 2.)

b) The distance between the slits will be found from the position of one of the large maxima on the screen.

$$d \sin\theta = m\lambda$$

We need the angle.

Any maximum can be used. Here we will use the order-2 maximum which is at y = 5 cm. At this position, the angle is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{5cm}{30cm}$$
$$\theta = 9.46°$$

Thus, the distance is

$$d\sin\theta = m\lambda$$
$$d\cdot\sin(9.46°) = 2\cdot 500\times10^{-9}m$$
$$d = 6.083\times10^{-6}m = 6.083\mu m$$

c) The intensity being N^2I_1 with N slits, the intensity is $81I_1$ with 9 slits.

13. a) The maximum value of m is

$$m < \frac{d}{\lambda} = \frac{\frac{1}{300}\times10^{-3}m}{650\times10^{-9}m} = 5.13$$

The maximum value of m is thus 5. There are therefore 11 maxima (the m = 1 to m = 5 maxima on the right, the m = 1 to m = 5 maxima on the left and the central maximum).

b) The distance is found with

$$\tan\theta = \frac{y}{L}$$

We have L but we need the angle.

The angle of the first-order maximum is

$$d\sin\theta = m\lambda$$
$$\tfrac{1}{300}\times10^{-3}m\cdot\sin\theta = 1\cdot 650\times10^{-9}m$$
$$\theta = 11.24°$$

Therefore, the position is

$$\tan\theta = \frac{y}{L}$$
$$\tan(11.24°) = \frac{y}{240cm}$$
$$y = 47.7cm$$

14. a) The wavelength is found with

$$d\sin\theta = m\lambda$$

We have d but we need the angle.

The angle of the first-order maximum is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{43.6cm}{100cm}$$
$$\theta = 23.56°$$

Therefore, the wavelength is

$$d\sin\theta = m\lambda$$
$$\tfrac{1}{800}\times 10^{-3} m\cdot\sin(23.56°) = 1\cdot\lambda$$
$$\lambda = 499.6nm$$

b) The position of the 2nd maximum is found with

$$\tan\theta = \frac{y}{L}$$

We have L but we need the angle.

The angle of the second-order maximum is

$$d\sin\theta = m\lambda$$
$$\tfrac{1}{800}\times 10^{-3} m\cdot\sin\theta = 2\cdot 499.6nm$$
$$\theta = 53.07°$$

Thus, the position is

$$\tan\theta = \frac{y}{L}$$
$$\tan(53.07°) = \frac{y}{100cm}$$
$$y = 133.02cm$$

The distance between the second-order maximum and the first-order maximum is, therefore,

$$x = 133.02cm - 43.60cm = 89.42cm$$

c) The maximum value of *m* is

$$m < \frac{d}{\lambda} = \frac{\frac{1}{800}\times 10^{-3}m}{499.6\times 10^{-9}m} = 2.50$$

The maximum value of *m* is thus 2. There are therefore 5 maxima (the $m = 1$ and $m = 2$ maxima on the right, the $m = 1$ and $m = 2$ maxima on the left and the central maximum).

15. The positions of the maxima are found with

$$\tan\theta = \frac{y}{L}$$

We have *L* but we need the angle.

For the first wavelength, the angle of the first-order maximum is

$$d\sin\theta = m\lambda$$
$$\tfrac{1}{300}\times 10^{-3}m \cdot \sin\theta = 1\cdot 589.0nm$$
$$\theta = 10.1776°$$

The position of this maximum is

$$\tan\theta = \frac{y}{L}$$
$$\tan(10{,}1776°) = \frac{y}{200cm}$$
$$y = 35{,}9050cm$$

For the second wavelength, the angle of the first-order maximum is

$$d\sin\theta = m\lambda$$
$$\tfrac{1}{300}\times 10^{-3}m\cdot\sin\theta = 1\cdot 589.6nm$$
$$\theta = 10.1881°$$

The position of this maximum is

$$\tan\theta = \frac{y}{L}$$
$$\tan(10.1881°) = \frac{y}{200cm}$$
$$y = 35.9427cm$$

Therefore, the distance between these two maxima is

$$x = 35.9427cm - 35.9050cm = 0.0377cm$$

16. The position of the order-2 maximum is found with

$$\tan\theta_2 = \frac{y}{L}$$

We have L but we need the angle (called θ_2 here)

The angle of the 2nd maximum is found with

$$d\sin\theta = m\lambda$$
$$d\sin\theta_2 = 2\lambda$$

However, d and λ are not known.

The angle can be found with what is known about of the first maximum. For the first maximum, we

$$d\sin\theta = m\lambda$$
$$d\sin\theta_1 = \lambda$$

So, we have the following two equations.

$$d\sin\theta_2 = 2\lambda$$
$$d\sin\theta_1 = \lambda$$

By dividing the equations, we have

$$\frac{d\sin\theta_2}{d\sin\theta_1} = \frac{2\lambda}{\lambda}$$
$$\frac{\sin\theta_2}{\sin\theta_1} = 2$$

To find the angle, you need the angle of the 1st maximum. This angle is

$$\tan\theta_1 = \frac{y}{L}$$
$$\tan\theta_1 = \frac{35cm}{100cm}$$
$$\theta_1 = 19.29°$$

Therefore

$$\frac{\sin\theta_2}{\sin\theta_1} = 2$$
$$\frac{\sin\theta_2}{\sin(19.29°)} = 2$$
$$\theta_2 = 41.35°$$

Thus, the position is

$$\tan\theta_2 = \frac{y}{L}$$
$$\tan(41.35°) = \frac{y}{100cm}$$
$$y = 88.02cm$$

17. a) The light intensity with N slits is

$$I_N = I_1 \frac{\sin^2\left(\frac{N\Delta\phi}{2}\right)}{\sin^2\left(\frac{\Delta\phi}{2}\right)}$$

The maximum begins and ends when the intensity is zero. The intensity is zero when the sine function in the numerator is zero (but without the denominator being zero, because the fraction is then 0/0, which is not 0). Thus the intensity vanishes when

$$\sin\left(\tfrac{N\Delta\phi}{2}\right)=0$$
$$\frac{N\Delta\phi}{2}=M\pi$$
$$\frac{\Delta\phi}{2}=\frac{M}{N}\pi$$

where M is an integer (but it cannot be a whole number of N, because then the denominator is zero and this corresponds to the phase difference of the large maxima).

The angle of the large order 1 maximum is

$$\frac{\Delta\phi}{2}=\pi$$

This means that the first order maximum occurs when $M = N$. The minimum preceding it is thus at $M = N - 1$ and the minimum following it is at $M = N + 1$. Therefore, the change of phase difference between the two minima (which is the width of the central maximum) is given by

$$\frac{\Delta\phi_{\text{min after}}}{2}-\frac{\Delta\phi_{\text{min before}}}{2}=\frac{N+1}{N}\pi-\frac{N-1}{N}\pi$$
$$\frac{\Delta\phi_{\text{min after}}}{2}-\frac{\Delta\phi_{\text{min before}}}{2}=\frac{2\pi}{N}$$
$$\Delta\phi_{\text{min after}}-\Delta\phi_{\text{min before}}=\frac{4\pi}{N}$$

But the phase difference is

$$\Delta\phi=\frac{d\sin\theta}{\lambda}2\pi$$

so that

$$\frac{d\sin\theta_{\text{min after}}}{\lambda}2\pi - \frac{d\sin\theta_{\text{min before}}}{\lambda}2\pi = \frac{4\pi}{N}$$
$$\sin\theta_{\text{min after}} - \sin\theta_{\text{min before}} = \frac{2\lambda}{Nd}$$
$$\Delta(\sin\theta) = \frac{2\lambda}{Nd}$$

Since the angle are close to each other, the following relation holds.

$$\begin{aligned}\Delta(\sin\theta) &= \frac{d\sin\theta}{d\theta}\Delta\theta \\ &= \cos\theta\Delta\theta\end{aligned}$$

Thus, the angle is

$$\cos\theta\Delta\theta = \frac{2\lambda}{Nd}$$
$$\Delta\theta = \frac{2\lambda}{Nd\cos\theta}$$

It is then obvious that the width of the maxima gets smaller as N gets larger.

Note: This can be simplified further since

$$\sin\theta = \frac{\lambda}{d}$$

Thus, we could have written

$$\Delta\theta = \frac{2\lambda}{Nd\cos\theta}$$
$$\Delta\theta = \frac{2\sin\theta}{N\cos\theta}$$
$$\Delta\theta = \frac{2\tan\theta}{N}$$

b) Let's find by how much the angle of the first order maximum changes when the wavelength is changed a little. This means that we're looking for $\Delta\theta$ when $\Delta\lambda$ is small. It is found with

$$\Delta\theta = \frac{d\theta}{d\lambda}\Delta\lambda$$

Since the angle of the first order maximum is given by

$$d\sin\theta = \lambda$$

we have

$$\sin\theta = \frac{\lambda}{d}$$
$$\frac{d\sin\theta}{d\theta} = \frac{1}{d}\frac{d\lambda}{d\theta}$$
$$\cos\theta = \frac{1}{d}\frac{d\lambda}{d\theta}$$
$$\frac{d\theta}{d\lambda} = \frac{1}{d\cos\theta}$$

Thus

$$\Delta\theta = \frac{d\theta}{d\lambda}\Delta\lambda$$
$$\Delta\theta = \frac{1}{d\cos\theta}\Delta\lambda$$

However, this angle must be (approximately) greater than or equal to half the width of the central maximum. This means that

$$\frac{1}{d\cos\theta}\Delta\lambda \geq \frac{\lambda}{Nd\cos\theta}$$

Simplifying, the result is

$$\Delta\lambda \geq \frac{\lambda}{N}$$

Thus, the number of slits needed is

$$0.59nm \geq \frac{589.00nm}{N}$$
$$N \geq 998.3\text{slits}$$

Approximately, it takes 1000 slits to see the two maxima separately.