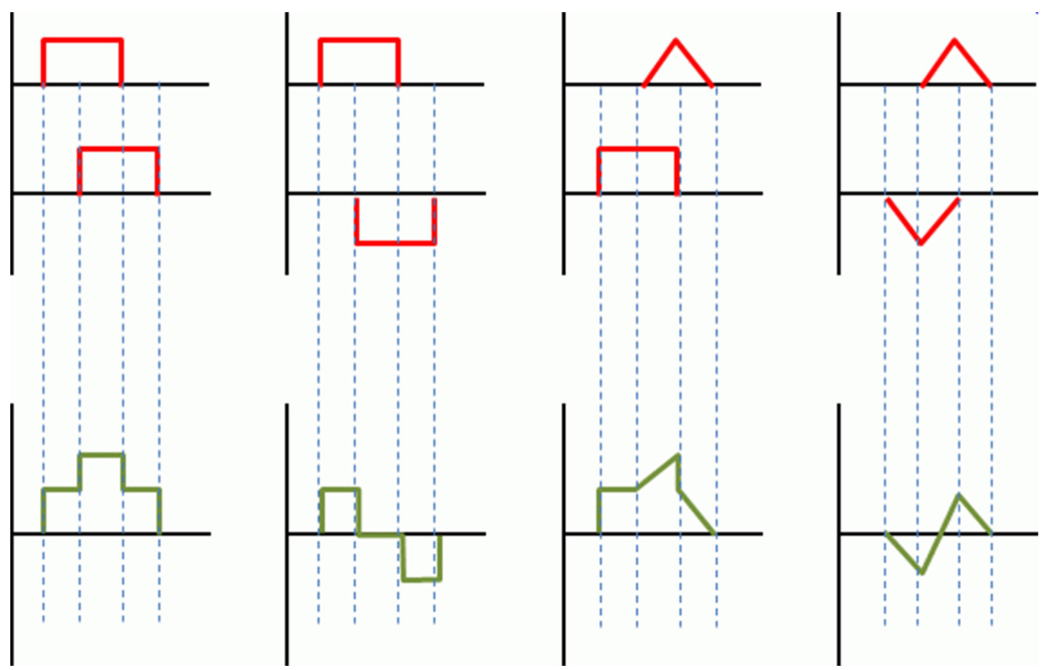
Chapter 5 Solutions

1.



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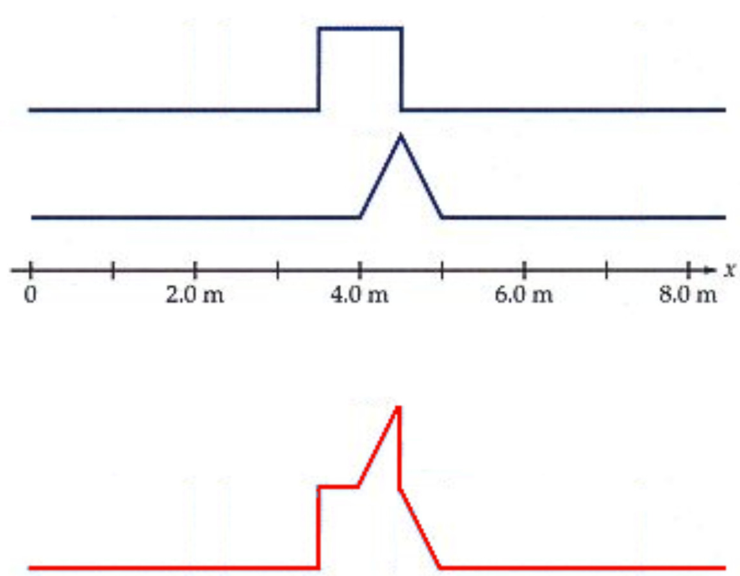

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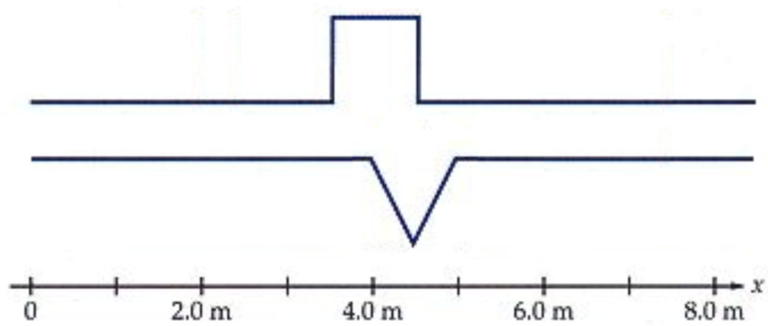
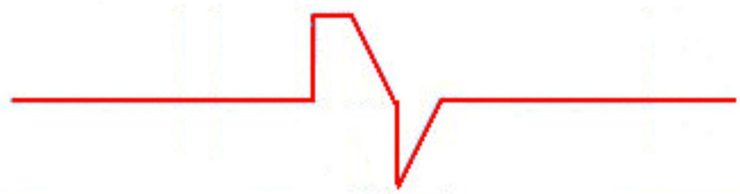




4. When the amplitudes of both waves are the same, the resulting amplitude is calculated with

$$A_{tot} = \left|2A\cos\left(\frac{\Delta\phi}{2}\right)\right|$$

We thus need the phase difference.

The phase difference between the oscillation is

$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 \\ &= -1rad - 4rad \\ &= -5rad\end{aligned}$$

Thus, the amplitude is

$$\begin{aligned}A_{tot} &= \left|2A\cos\left(\frac{\Delta\phi}{2}\right)\right| \\ &= \left|2\cdot 0.2m\cdot\cos\left(\frac{-5rad}{2}\right)\right| \\ &= 0.3205m\end{aligned}$$

5. When the amplitudes of the waves are different, the resulting amplitude is calculated with

$$A_{tot} = \sqrt{A_1^2 + 2A_1A_2\cos(\Delta\phi) + A_2^2}$$

We thus need the phase difference.

The phase difference between the oscillation is

$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 \\ &= -1.5rad - 2rad \\ &= -3.5rad\end{aligned}$$

Thus, the amplitude is

$$\begin{aligned}A_{tot} &= \sqrt{A_1^2 + 2A_1A_2\cos(\Delta\phi) + A_2^2} \\ &= \sqrt{(0.5m)^2 + 2\cdot 0.5m\cdot 0.4m\cdot\cos(-3.5) + (0.4m)^2} \\ &= 0.1882m\end{aligned}$$

6. a) To have constructive interference, we must have $\Delta\phi = 2m\pi$. Then

$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 \\ 2m\pi &= \phi_2 - 1 \\ \phi_2 &= 2m\pi + 1\end{aligned}$$

To obtain a value between 0 and 2π, m = 0 must be chosen. Then ϕ_2 = 1. The oscillation to add is thus

$$y_2 = 0.1m\cdot\sin\left(30\tfrac{rad}{s}\cdot x + 100\tfrac{rad}{s}\cdot t + 1rad\right)$$

b) To have destructive interference, we must have $\Delta\phi = (2m + 1)\pi$. Then

$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 \\ (2m+1)\pi &= \phi_2 - 1 \\ \phi_2 &= (2m+1)\pi + 1\end{aligned}$$

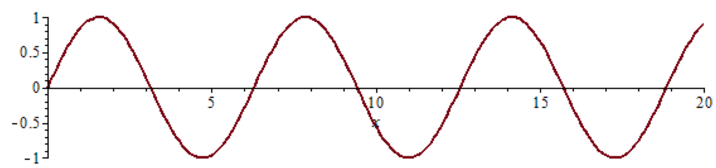
To obtain a value between 0 and 2π, $m = 0$ must be chosen. Then $\phi_2 = \pi + 1$. The oscillation to add is thus

$$y_2 = 0{,}1m \cdot \sin\left(30\tfrac{rad}{s} \cdot x + 100\tfrac{rad}{s} \cdot t + 4{,}142 rad\right)$$

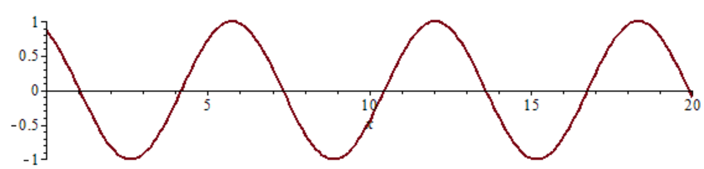
7. Here, the only phase difference is the one caused by the difference in distance $\Delta\phi_T$. Therefore, the phase shift is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{r_B - r_A}{\lambda}2\pi \\ &= -\frac{3.6m - 5.2m}{0.5m}\cdot 2\pi \\ &= \tfrac{32}{5}\pi\end{aligned}$$

8. Here, we have only the phase difference due to the sources $\Delta\phi_S$. Let's assume that source A has a vanishing constant phase



If source B is ahead by a third of a cycle on source A, then its graph must be



In this case, the phase constant is

$$\phi_{\text{source 2}} = \frac{2\pi}{3}$$

Thus, $\Delta\phi_S$ is

$$\begin{aligned}\Delta\phi_S &= \phi_{source\ 2} - \phi_{source\ 1} \\ &= \frac{2\pi}{3} rad - 0 \\ &= \frac{2\pi}{3} rad\end{aligned}$$

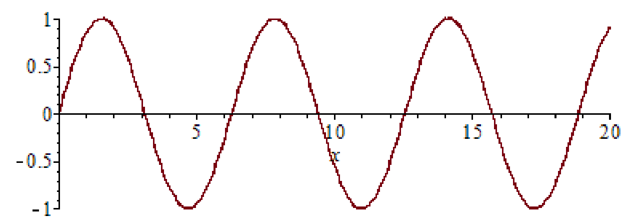
9. The total phase difference is

$$\Delta\phi = \Delta\phi_T + \Delta\phi_S + \Delta\phi_R$$

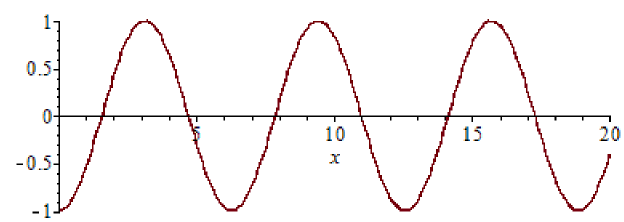
$\Delta\phi_T$ is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda} 2\pi \\ &= -\frac{r_B - r_A}{\lambda} 2\pi \\ &= -\frac{3m - 5m}{0.2m} \cdot 2\pi \\ &= 20\pi\end{aligned}$$

Let's assume that source A has a zero phase constant.



If source B lags by a quarter cycle on source A, then the graph must be



In this case, the phase constant is

$$\phi_{\text{source }2} = -\frac{\pi}{2}$$

Thus, $\Delta\phi_S$ is

$$\begin{aligned}\Delta\phi_S &= \phi_{source\ 2} - \phi_{source\ 1} \\ &= -\frac{\pi}{2} - 0 \\ &= -\frac{\pi}{2}\end{aligned}$$

The total phase difference is then

$$\begin{aligned}\Delta\phi &= \Delta\phi_T + \Delta\phi_S + \Delta\phi_R \\ &= 20\pi + -\frac{\pi}{2} + 0 \\ &= \frac{39\pi}{2} \\ &= 61.26 rad\end{aligned}$$

10. To have destructive interference, the phase difference must be equal to an odd number of π.

$$\Delta\phi = (2m+1)\pi$$

Thus, the phase difference is needed.

Since the path length difference is *d*, $\Delta\phi_T$ is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda} 2\pi \\ &= -\frac{d}{0.25m} \cdot 2\pi\end{aligned}$$

There is no other phase difference since there is no reflection, and the sources are in phase.

The total phase difference is thus

$$\Delta\phi = -\frac{d}{0.25m} \cdot 2\pi$$

Therefore

$$\Delta\phi = (2m+1)\pi$$
$$-\frac{d}{0.25m}\cdot 2\pi = (2m+1)\pi$$

If this equation is solved for Δr, we obtain

$$-\frac{d}{0.25m}\cdot 2\pi = (2m+1)\pi$$
$$d = -\frac{(2m+1)}{2}\cdot 0.25m$$

m is an integer (be careful not to confuse it with the unit m (meters)). Using some values for *m*, the results are

if $m = 2$ $d = -0.625$ m
if $m = 1$ $d = -0.375$ m
if $m = 0$ $d = -0.125$ m
if $m = -1$ $d = 0.125$ m
if $m = -2$ $d = 0.375$ m

Thus, the smallest value for the distance between the speakers is 0.125 m (in fact, we were looking for the smallest distance in absolute value because the negative sign depends on our choice of which source is source 1. Here, there are two responses equal to 12.5 cm. Since the two signs are good here, source 2 can be put 12.5 cm in front or behind source 1.)

11. To have destructive interference, the phase difference must be equal to an odd number of π.

$$\Delta\phi = (2m+1)\pi$$

Thus, the phase difference is needed.

Assuming that the speaker B is source 2, $\Delta\phi_T$ is

$$\Delta\phi_T = -\frac{\Delta r}{\lambda}2\pi$$
$$= -\frac{\Delta r}{0.32m}\cdot 2\pi$$

$\Delta\phi_S$ is

$$\Delta\phi_S = \frac{\pi}{4}$$

The value is positive since source 2 is ahead of source 1.

As there is no reflection, the total phase difference is

$$\Delta\phi = -\frac{\Delta r}{\lambda}2\pi + \frac{\pi}{4}$$

Therefore

$$\Delta\phi = (2m+1)\pi$$
$$-\frac{\Delta r}{\lambda}2\pi + \frac{\pi}{4} = (2m+1)\pi$$

Solving for the path length difference, we have

$$-\frac{\Delta r}{\lambda}2 + \frac{1}{4} = 2m+1$$
$$-\frac{\Delta r}{\lambda}2 = 2m + \frac{3}{4}$$
$$-\frac{\Delta r}{\lambda} = m + \frac{3}{8}$$
$$\Delta r = -\left(m + \frac{3}{8}\right)\lambda$$

If $m = 0$, then $\Delta r = -3\lambda/8$.
If $m = 1$, then $\Delta r = -11\lambda/8$.
As m increases from $m = 1$, the path length difference increases (in absolute value).
If $m = -1$, then $\Delta r = 5\lambda/8$.
As m decreases from $m = -1$, the path length difference increases.

The smallest path length difference (in absolute value) is thus

$$\Delta r = -\frac{3}{8}\lambda$$
$$= -\frac{3}{8} \cdot 32cm$$
$$= -12cm$$

Therefore

$$r_2 - r_1 = -12cm$$

Source 2 must be 12 cm closer than source 1.

12. To have destructive interference, the phase difference must be equal to an odd number of π.

$$\Delta\phi = (2m+1)\pi$$

Thus, the phase difference is needed.

The phase difference is

$$\Delta\phi = \frac{4\pi e n_f}{\lambda}$$
$$= \frac{4\pi \cdot 450nm \cdot 1.3}{\lambda}$$
$$= \frac{2340nm \cdot \pi}{\lambda}$$

Therefore, we must have

$$\Delta\phi = (2m+1)\pi$$
$$\frac{2340nm \cdot \pi}{\lambda} = (2m+1)\pi$$
$$\lambda = \frac{2340nm}{2m+1}$$

This equation gives the following values.

$m = 0$	$\lambda = 2340$ nm
$m = 1$	$\lambda = 780$ nm
$m = 2$	$\lambda = 468$ nm

$m = 3$ $\lambda = 334.3$ nm

In the visible spectrum, only the 468 nm wavelength is absent.

13. To have constructive interference, the phase difference must be equal to an even number of π.

$$\Delta\phi = 2m\pi$$

Thus, the phase difference is needed.

The phase difference is

$$\begin{aligned}\Delta\phi &= \frac{4\pi e n_f}{\lambda} \\ &= \frac{4\pi \cdot 450nm \cdot 1,3}{\lambda} \\ &= \frac{2340nm \cdot \pi}{\lambda}\end{aligned}$$

Then, we must have

$$\begin{aligned}\Delta\phi &= 2m\pi \\ \frac{2340nm \cdot \pi}{\lambda} &= 2m\pi \\ \lambda &= \frac{1170nm}{m}\end{aligned}$$

This equation gives the following values

$m = 1$	$\lambda = 1170$ nm
$m = 2$	$\lambda = 585$ nm
$m = 3$	$\lambda = 390$ nm

In the visible spectrum, only the 585 nm wavelength is strongly reflected.

14. a) The phase difference is

$$\begin{aligned}\Delta\phi &= \frac{4\pi e n_f}{\lambda} + \pi \\ &= \frac{4\pi \cdot 250nm \cdot 1.6}{450nm} + \pi \\ &= 14.31 rad\end{aligned}$$

b) The amplitude is

$$\begin{aligned}A_{tot} &= \left| 2A\cos\left(\frac{\Delta\phi}{2}\right) \right| \\ &= \left| 2A \cdot \cos\left(\frac{14.31 rad}{2}\right) \right| \\ &= 1.286 \cdot A\end{aligned}$$

Thus, the amplitude is 1.286 times greater than it would be without a thin film.

15. To have constructive interference, the phase difference must be equal to an even number of π.

$$\Delta\phi = 2m\pi$$

Thus, the phase difference is needed.

The phase difference is

$$\begin{aligned}\Delta\phi &= \frac{4\pi e n_f}{\lambda} + \pi \\ &= \frac{4\pi \cdot e \cdot 1.8}{550nm} + \pi \\ &= \frac{7.2 \cdot \pi \cdot e}{550nm} + \pi\end{aligned}$$

Then, we must have

$$\Delta\phi = 2m\pi$$
$$\frac{7.2\cdot\pi\cdot e}{550nm} + \pi = 2m\pi$$
$$\frac{7.2\cdot e}{550nm} + 1 = 2m$$
$$e = \frac{550nm\cdot(2m-1)}{7.2}$$

The minimum thickness is found with $m = 1$. The minimum thickness is

$$e = \frac{550nm}{7.2} = 76.39nm$$

16. The phase difference is

$$\Delta\phi = \frac{4\pi e n_f}{\lambda} + \pi$$

For the wavelength making constructive interference (λ_1), we have

$$\Delta\phi = 2m\pi$$

Therefore

$$\frac{4\pi e n_f}{\lambda_1} + \pi = 2m_1\pi$$

Solving for the thickness, we found

$$\begin{aligned} e &= \frac{(2m_1 - 1)\lambda_1}{4n_f} \\ &= \frac{(2m_1 - 1)\cdot 638.4nm}{4\cdot 1.33} \\ &= (2m_1 - 1)120nm \end{aligned}$$

Giving the values 1, 2, 3, 4, 5…. to m_1, the following thicknesses are obtained: 120 nm, 360 nm, 600 nm, 840 nm, 1080 nm.

For the wavelength making destructive interference (λ_2), we have

$$\Delta\phi = (2m+1)\pi$$

Therefore

$$\frac{4\pi e n_f}{\lambda_2} + \pi = (2m_2 + 1)\pi$$

Simplified, this equation is

$$\frac{4\pi e n_f}{\lambda_2} = 2m_2\pi$$

Solving for the thickness, we obtain

$$\begin{aligned} e &= \frac{2m_2\lambda_1}{4n_f} \\ &= \frac{2m_2 \cdot 478.8nm}{4 \cdot 1.33} \\ &= m_2 \cdot 180nm \end{aligned}$$

Giving the values 1, 2, 3, 4, 5…. to m_2 , the following thicknesses are obtained: 180 nm, 360 nm, 540 nm, 720 nm, 900 nm.

The smallest common value for the thickness is, therefore, 360 nm.

17. a) The sound frequency is

$$\begin{aligned} f_{sound} &= \frac{f_1 + f_2}{2} \\ &= \frac{500Hz + 508Hz}{2} \\ &= 504Hz \end{aligned}$$

b) The beat frequency is

$$\begin{aligned} f_{beats} &= |f_1 - f_2| \\ &= 508Hz - 500Hz \\ &= 8Hz \end{aligned}$$

18. We have the equations

$$f_{sound} = \frac{f_1 + f_2}{2}$$
$$350Hz = \frac{f_1 + f_2}{2}$$
$$700Hz = f_1 + f_2$$

and

$$f_{beats} = |f_1 - f_2|$$
$$6Hz = f_1 - f_2$$

(The absolute value was removed since we will assume that f_1 is greater than f_2.)

Adding these two equations, we obtain

$$700Hz + 6Hz = (f_1 + f_2) + (f_1 - f_2)$$
$$706Hz = 2f_1$$
$$f_1 = 353Hz$$

The other frequency is then

$$6Hz = f_1 - f_2$$
$$6Hz = 353Hz - f_2$$
$$f_2 = 347Hz$$

19. If there are 4.2 Hz beats, then there is a 4.2 Hz gap between the frequency of the rope and the frequency of the machine. As the machine has a 329.6 Hz frequency, this means that the rope has a frequency of 333.8 Hz or 325.4 Hz. Which of these frequencies is good?

To find out, we'll use the fact that the beat frequency decreases if the tension of the rope is increased. As the frequency of the string is given by

$$f_1 = \frac{1}{2L}\sqrt{\frac{F_T}{\mu}}$$

We can see that frequency of the rope increases if tension is increased.

Now assume that the frequency of the rope is 333.8 Hz. If the tension is increased, the frequency increases, and the gap between the frequency of the string and the 329.6 Hz sound will become larger, and the beat frequency will increase.

Now assume that the frequency of the rope is 325.4 Hz. If the tension is increased, the frequency increases, and the gap between the frequency of the string and the 329.6 Hz sound will become smaller, and the beat frequency will decrease.

As it is said that the beat frequency decreases if the tension is increased, then the frequency of the rope must be 325.4 Hz.

With a 1300 N tension, we have

$$325.4Hz = \frac{1}{2L}\sqrt{\frac{1300N}{\mu}}$$

If a 329,6 Hz frequency is needed, then

$$329.6Hz = \frac{1}{2L}\sqrt{\frac{F_T'}{\mu}}$$

By dividing this last equation by the previous one, we obtain

$$\frac{329.6Hz}{325.4Hz} = \frac{\left(\frac{1}{2L}\sqrt{\frac{F_T'}{\mu}}\right)}{\left(\frac{1}{2L}\sqrt{\frac{1300N}{\mu}}\right)}$$

$$\frac{329.6}{325.4} = \sqrt{\frac{F_T'}{1300N}}$$

$$F_T' = 1333.8N$$

20. Since the amplitude varies with a frequency of 6000 Hz, then the difference in frequency is

$$\Delta f = 6000Hz$$

Now let's calculate this frequency shift with the Doppler effect.

First, we will calculate the frequency of the waves received by the car. According to the Doppler effect formula, this frequency is

$$f' = f\frac{c+v_{car}}{c}$$

(Since the car is heading towards the radar gun, the speed of the car is negative in the Doppler Effect formula, which gives us the positive sign in the formula.)

After the reflection, the car becomes a moving source emitting the frequency f', and the radar becomes the observer (at rest). The frequency received by the radar is once again shifted by this second Doppler Effect. The frequency becomes

$$f'' = f'\frac{c}{c-v_{car}}$$

The frequency received is thus

$$f'' = f\frac{c+v_{car}}{c}\frac{c}{c-v_{car}} = f\frac{c+v_{car}}{c-v_{car}}$$

Thus, the frequency shift is

$$\Delta f = f\frac{c+v_{car}}{c-v_{car}} - f$$

This equation must be solved for the speed of the car. Here is two ways to do this.

Version 1

$$\begin{aligned}
\Delta f &= f\frac{c+v_{car}}{c-v_{car}} - f \\
&= f\frac{c-v_{car}+2v_{car}}{c-v_{auto}} - f \\
&= f\left(1+\frac{2v_{car}}{c-v_{car}}\right) - f \\
&= \frac{2fv_{car}}{c-v_{car}}
\end{aligned}$$

As the speed of the car is negligible compared to the speed of light, the shift becomes

$$\Delta f = \frac{2fv_{car}}{c}$$

Since we know that the frequency difference is 6000 Hz, we have

$$6000Hz = \frac{2 \cdot 25\times10^9\,Hz \cdot v_{car}}{3\times10^8\,\frac{m}{s}}$$
$$v_{car} = 36\,\tfrac{m}{s} = 129.6\,\tfrac{km}{h}$$

Version 2

Let's solve the equation for the speed of the car

$$\Delta f = f\,\frac{c+v_{car}}{c-v_{car}} - f$$
$$\tfrac{\Delta f}{f} = \frac{c+v_{car}}{c-v_{car}} - 1$$
$$\tfrac{\Delta f}{f} + 1 = \frac{c+v_{car}}{c-v_{car}}$$
$$\left(\tfrac{\Delta f}{f}+1\right)\left(c-v_{car}\right) = c+v_{car}$$
$$\left(\tfrac{\Delta f}{f}+1\right)c - \left(\tfrac{\Delta f}{f}+1\right)v_{car} = c+v_{car}$$
$$\left(\tfrac{\Delta f}{f}+1\right)c - c = v_{car} + \left(\tfrac{\Delta f}{f}+1\right)v_{car}$$
$$\tfrac{\Delta f}{f}c + c - c = v_{car} + \tfrac{\Delta f}{f}v_{car} + v_{car}$$
$$\tfrac{\Delta f}{f}c = 2v_{car} + \tfrac{\Delta f}{f}v_{car}$$
$$\tfrac{\Delta f}{f}c = \left(2+\tfrac{\Delta f}{f}\right)v_{car}$$
$$v_{car} = \frac{\tfrac{\Delta f}{f}c}{2+\tfrac{\Delta f}{f}}$$
$$v_{car} = \frac{\Delta f c}{2f+\Delta f}$$

Thus

$$v_{car} = \frac{6000Hz \cdot 3\times10^8\,\frac{m}{s}}{2\cdot 25\times10^9\,Hz + 6000Hz}$$
$$= 36\,\tfrac{m}{s} = 129.6\,\tfrac{km}{h}$$

21. a) Since $\omega = 200\pi$ rad/s, the frequency is

$$\begin{aligned} f &= \frac{\omega}{2\pi} \\ &= \frac{200\pi s^{-1}}{2\pi} \\ &= 100Hz \end{aligned}$$

b) Since $k = 40\pi$ rad/m, the wavelength is

$$\begin{aligned} k &= \frac{2\pi}{\lambda} \\ 40\pi \tfrac{rad}{m} &= \frac{2\pi}{\lambda} \\ \lambda &= 0.05m \end{aligned}$$

c) The speed is

$$\begin{aligned} v &= \lambda f \\ &= 0.05m \cdot 100Hz \\ &= 5\tfrac{m}{s} \end{aligned}$$

d) The formula for the velocity of the rope is obtained by deriving the formula for the position with respect to time

$$\begin{aligned} v_y &= \frac{\partial y}{\partial t} \\ &= \frac{\partial}{\partial t}\left(0.06m \cdot \sin\left(40\pi \tfrac{rad}{m} \cdot x\right) \cdot \cos\left(200\pi \tfrac{rad}{s} \cdot t\right)\right) \\ &= -0.06m \cdot 200\pi s^{-1} \cdot \sin\left(40\pi \tfrac{rad}{m} \cdot x\right) \cdot \sin\left(200\pi \tfrac{rad}{s} \cdot t\right) \\ &= -12\pi \tfrac{m}{s} \cdot \sin\left(40\pi \tfrac{rad}{m} \cdot x\right) \cdot \sin\left(200\pi \tfrac{rad}{s} \cdot t\right) \end{aligned}$$

At $x = 0.02$ m and $t = 0.022$ s the velocity is

$$\begin{aligned} v_y &= -12\pi \tfrac{m}{s} \cdot \sin\left(40\pi \tfrac{rad}{m} \cdot 0,02m\right) \cdot \sin\left(200\pi \tfrac{rad}{s} \cdot 0,022s\right) \\ &= -12\pi \tfrac{m}{s} \cdot \sin\left(\tfrac{4\pi}{5}\right) \cdot \sin\left(\tfrac{22\pi}{5}\right) \\ &= -21.07 \tfrac{m}{s} \end{aligned}$$

e) The amplitude is

$$A_{tot} = \left|2A\sin kx\right|$$
$$= \left|6cm \cdot \sin\left(40\pi \tfrac{rad}{m} \cdot x\right)\right|$$

At x = 0.5 cm, we have

$$A_{tot} = \left|6cm \cdot \sin\left(40\pi \tfrac{rad}{m} \cdot 0.005m\right)\right|$$
$$= 3.527cm$$

22. The equation is

$$y_{tot} = 4cm \cdot \sin\left(10\pi \tfrac{rad}{m} \cdot x\right)\cos\left(50\pi \tfrac{rad}{s} \cdot t\right)$$

23. a) To write the standing wave equation

$$x = 2A\sin(kx)\cos(\omega t)$$

We need, A, k and ω.

It is written that A = 0,05 m.

With a 20 cm wavelength, k is

$$k = \frac{2\pi}{\lambda}$$
$$= \frac{2\pi}{0.2m}$$
$$= 10\pi \tfrac{rad}{m}$$

With a 0.05 s period, ω is

$$\omega = \frac{2\pi}{T}$$
$$= \frac{2\pi}{0.05s}$$
$$= 40\pi \tfrac{rad}{s}$$

Therefore, the equation is

$$y_{tot} = 0.1m \cdot \sin\left(10\pi \tfrac{rad}{m} \cdot x\right) \cdot \cos\left(40\pi \tfrac{rad}{s} \cdot t\right)$$

b) The distance between the nodes is equal to half of the wavelength. It is thus 10 cm.

c) Since there is a node at $x = 0$, the amplitude 1 cm from the nodes is the amplitude at $x = 1$ cm. This amplitude is

$$\begin{aligned} A_{tot} &= \left|2A\sin kx\right| \\ &= \left|0.1m\cdot\sin\left(10\pi\tfrac{rad}{m}\cdot x\right)\right| \\ &= \left|0.1m\cdot\sin\left(10\pi\tfrac{rad}{m}\cdot 0.01m\right)\right| \\ &= 0.03090m \end{aligned}$$

24. To write the standing wave equation

$$x = 2A\sin(kx)\cos(\omega t)$$

We need, A, k and ω.

Since the amplitude at the centre of the antinodes is $2A$, we know that $2A = 5$ mm.

Since the distance between the nodes is 20 cm, the wavelength is 40 cm. With a 40 cm wavelength, k is

$$\begin{aligned} k &= \frac{2\pi}{\lambda} \\ &= \frac{2\pi}{0.4m} \\ &= 5\pi\tfrac{rad}{m} \end{aligned}$$

For ω, we will use

$$v = \frac{\omega}{k}$$

We have k but we need the speed. The wave speed is

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ &= \sqrt{\frac{12N}{0.03\frac{kg}{m}}} \\ &= 20\tfrac{m}{s} \end{aligned}$$

Therefore, ω is

$$\begin{aligned} v &= \frac{\omega}{k} \\ 20\tfrac{m}{s} &= \frac{\omega}{5\pi\frac{rad}{m}} \\ \omega &= 100\pi\tfrac{rad}{s} \end{aligned}$$

This means that

$$y_{tot} = 0.005m \cdot \sin\left(5\pi\tfrac{rad}{m} \cdot x\right) \cdot \cos\left(100\pi\tfrac{rad}{s} \cdot t\right)$$

25. We have

$$\begin{aligned} f_n &= \frac{n}{2L}\sqrt{\frac{F_T}{\mu}} \\ 400Hz &= \frac{4}{2 \cdot 0.6m} \cdot \sqrt{\frac{F_T}{0.02\frac{kg}{m}}} \\ F_T &= 288N \end{aligned}$$

26. We have

$$\begin{aligned} f_n &= \frac{nv}{2L} \\ 400Hz &= \frac{2v}{2 \cdot 2m} \\ v &= 800\tfrac{m}{s} \end{aligned}$$

27. At the 3rd harmonics, we have

$$\lambda = \frac{2L}{n}$$
$$\lambda = \frac{2L}{3}$$

The length of the rope can thus be found if we have the wavelength.

Since $k = 20\pi$ rad/m, the wavelength is

$$k = \frac{2\pi}{\lambda}$$
$$20\pi m^{-1} = \frac{2\pi}{\lambda}$$
$$\lambda = 0.1m$$

Therefore

$$\lambda = \frac{2L}{n}$$
$$0,1m = \frac{2L}{3}$$
$$L = 0,15m$$

28. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$50Hz = \frac{1}{2\cdot 0.5m}\cdot\sqrt{\frac{350N}{\mu}}$$
$$\mu = 0.14\tfrac{kg}{m}$$

29. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$200Hz = \frac{1}{2L}\sqrt{\frac{100N}{\mu}}$$

and

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$500Hz = \frac{5}{2L}\sqrt{\frac{F_T'}{\mu}}$$

By dividing the second equation by the first equation, we obtain

$$\frac{500Hz}{200Hz} = \frac{\frac{5}{2L}\sqrt{\frac{F_T'}{\mu}}}{\frac{1}{2L}\sqrt{\frac{100N}{\mu}}}$$
$$2.5 = \frac{5\sqrt{F_T'}}{\sqrt{100N}}$$
$$F_T' = 25N$$

30. We will obtain the mass from the tension of the rope. The tension is

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$160Hz = \frac{4}{2\cdot 1.2m}\cdot\sqrt{\frac{F_T}{0.036\frac{kg}{m}}}$$
$$F_T = 331.8N$$

The mass is then

$$F_T = mg$$
$$331.8N = m\cdot 9.8\tfrac{N}{kg}$$
$$m = 33.85kg$$

31. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$50Hz = \frac{1}{2\cdot 1m}\sqrt{\frac{F_T}{\mu}}$$

and

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$400Hz = \frac{2}{2L'}\sqrt{\frac{F_T}{\mu}}$$

By dividing the second equation by the first equation, we obtain

$$\frac{400Hz}{50Hz} = \frac{\frac{2}{2L'}\sqrt{\frac{F_T}{\mu}}}{\frac{1}{2\cdot 1m}\sqrt{\frac{F_T}{\mu}}}$$
$$8 = \frac{\frac{2}{L'}}{\frac{1}{1m}}$$
$$8 = \frac{2}{L'}\cdot\frac{1m}{1}$$
$$L' = 0.25m$$

32. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$f_{1A} = \frac{1}{2\cdot 1m}\sqrt{\frac{100N}{\mu}}$$

and

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$

$$f_{1B} = \frac{1}{2 \cdot 0.25m}\sqrt{\frac{25N}{\mu}}$$

The ratio of the frequencies is

$$\frac{f_{1A}}{f_{1B}} = \frac{\frac{1}{2 \cdot 1m}\sqrt{\frac{100N}{\mu}}}{\frac{1}{2 \cdot 0.25m}\sqrt{\frac{25N}{\mu}}}$$

$$\frac{f_{1A}}{f_{1B}} = \frac{\frac{1}{1m}\sqrt{100N}}{\frac{1}{0.25m}\sqrt{25N}}$$

$$\frac{f_{1A}}{f_{1B}} = \frac{0.25m \cdot \sqrt{100N}}{1m \cdot \sqrt{25N}}$$

$$\frac{f_{1A}}{f_{1B}} = 0.5$$

33. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$

$$360Hz = \frac{1}{2 \cdot L}\sqrt{\frac{F_T}{\mu}}$$

and

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$

$$f_{1B} = \frac{1}{2 \cdot (0.4L)}\sqrt{\frac{F_T}{\mu}}$$

By dividing the second equation by the first equation, we obtain

$$\frac{f_{1B}}{360Hz}=\frac{\frac{1}{2\cdot(0.4L)}\sqrt{\frac{F_T}{\mu}}}{\frac{1}{2\cdot L}\sqrt{\frac{F_T}{\mu}}}$$
$$\frac{f_{1B}}{360Hz}=\frac{1}{0.4}$$
$$f_{1B}=900Hz$$

34. We have

$$f_n=520Hz$$
$$nf_1=520Hz$$

At the following harmonic, the integer in front of f_1 is $n + 1$. Then

$$(n+1)f_1=650Hz$$

Using the first equation, we obtain

$$(n+1)f_1=650Hz$$
$$nf_1+f_1=650Hz$$
$$520Hz+f_1=650Hz$$
$$f_1=130Hz$$

35. The frequency of the first harmonic is

$$\begin{aligned}f_1&=\frac{1\cdot v}{2L}\\&=\frac{1\cdot 340\frac{m}{s}}{2\cdot 0.25m}\\&=680Hz\end{aligned}$$

The frequency of the second harmonic is

$$f_2 = \frac{3 \cdot v}{2L}$$
$$= \frac{2 \cdot 340 \frac{m}{s}}{2 \cdot 0.25m}$$
$$= 1360Hz$$

The frequency of the third harmonic is

$$f_3 = \frac{3 \cdot v}{2L}$$
$$= \frac{3 \cdot 340 \frac{m}{s}}{2 \cdot 0.25m}$$
$$= 2040Hz$$

36. The frequency of the first harmonic is

$$f_1 = \frac{1 \cdot v}{4L}$$
$$= \frac{1 \cdot 340 \frac{m}{s}}{4 \cdot 0.40m}$$
$$= 212.5Hz$$

The frequency of the third harmonic is

$$f_3 = \frac{3 \cdot v}{4L}$$
$$= \frac{3 \cdot 340 \frac{m}{s}}{4 \cdot 0.40m}$$
$$= 637.5Hz$$

The frequency of the fifth harmonic is

$$f_5 = \frac{5 \cdot v}{4L}$$
$$= \frac{5 \cdot 340 \frac{m}{s}}{4 \cdot 0.40m}$$
$$= 1062.5Hz$$

37. The frequency of the 3rd harmonic is

$$f_3 = \frac{3 \cdot v}{4L}$$
$$500Hz = \frac{3 \cdot v}{4L}$$

The length can be found, but we need the speed of the wave. This speed is

$$\begin{aligned} v &= 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}} \\ &= 331.3\tfrac{m}{s} \cdot \sqrt{\frac{303.15K}{273.15K}} \\ &= 349.0\tfrac{m}{s} \end{aligned}$$

Then, we have

$$\begin{aligned} f_3 &= \frac{3 \cdot v}{4L} \\ 500Hz &= \frac{3 \cdot 349.0\tfrac{m}{s}}{4 \cdot L} \\ L &= 0.5235m \end{aligned}$$

38. At this temperature, the wave speeds are

$$\begin{aligned} v &= 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}} \\ &= 331.3\tfrac{m}{s} \cdot \sqrt{\frac{298.15K}{273.15K}} \\ &= 346.1\tfrac{m}{s} \end{aligned}$$

$$\begin{aligned} v &= 331.3\tfrac{m}{s} \cdot \sqrt{\frac{T}{273.15K}} \\ &= 331.3\tfrac{m}{s} \cdot \sqrt{\frac{273.15K}{273.15K}} \\ &= 331.3\tfrac{m}{s} \end{aligned}$$

At 25 °C, we have

$$\begin{aligned} f_1 &= \frac{1 \cdot v}{x \cdot L} \\ 500Hz &= \frac{1 \cdot 346.1\tfrac{m}{s}}{x \cdot L} \end{aligned}$$

(x has been used in the denominator since we do not know whether the pipe is open or closed. If it is open, we have $x = 2$, and if it is closed, then $x = 4$.)

At 0 °C, we have

$$\begin{aligned} f_1' &= \frac{1\cdot v}{x\cdot L} \\ &= \frac{1\cdot 331.3\frac{m}{s}}{x\cdot L} \end{aligned}$$

If we divide this equation by the equation at 25 °C, we obtain

$$\frac{f_1'}{500Hz} = \frac{\left(\frac{1\cdot 331.3\frac{m}{s}}{x\cdot L}\right)}{\left(\frac{1\cdot 346.1\frac{m}{s}}{x\cdot L}\right)}$$

$$\frac{f_1'}{500Hz} = \frac{331.3\frac{m}{s}}{346.1\frac{m}{s}}$$

$$f_1' = 478.6Hz$$

39. a)

Let's assume that the pipe is open. Then we would have the following two equations.

$$f_n = \frac{n\cdot v}{2L} \qquad f_{n+1} = \frac{(n+1)\cdot v}{2L}$$

$$630Hz = \frac{n\cdot v}{2L} \qquad 840Hz = \frac{(n+1)\cdot v}{2L}$$

By dividing the equation to the right by the equation to the left, we obtain

$$\frac{840Hz}{630Hz} = \frac{\frac{(n+1)\cdot v}{2L}}{\frac{n\cdot v}{2L}}$$

$$\frac{840}{630} = \frac{n+1}{n}$$

$$840\cdot n = 630\cdot(n+1)$$

$$840\cdot n = 630\cdot n + 630$$

$$210\cdot n = 630$$

$$n = 3$$

This is an acceptable solution (because n must be an integer).

Let's now assume that the pipe is closed. Then we would have the following two equations.

$$f_n = \frac{n \cdot v}{4L} \qquad f_{n+1} = \frac{(n+2)\cdot v}{4L}$$

$$630Hz = \frac{n \cdot v}{4L} \qquad 840Hz = \frac{(n+2)\cdot v}{4L}$$

By dividing the equation to the right by the equation to the left, we obtain

$$\frac{840Hz}{630Hz} = \frac{\frac{(n+2)\cdot v}{4L}}{\frac{n \cdot v}{4L}}$$

$$\frac{840}{630} = \frac{n+2}{n}$$

$$840 \cdot n = 630 \cdot (n+2)$$

$$840 \cdot n = 630 \cdot n + 1260$$

$$210 \cdot n = 1260$$

$$n = 6$$

This is not an acceptable solution (because *n* must be an odd integer with a closed pipe).

Therefore, the tube is open.

b) We have

$$f_3 = \frac{3 \cdot v}{2L}$$

$$630Hz = \frac{3 \cdot 336Hz}{2L}$$

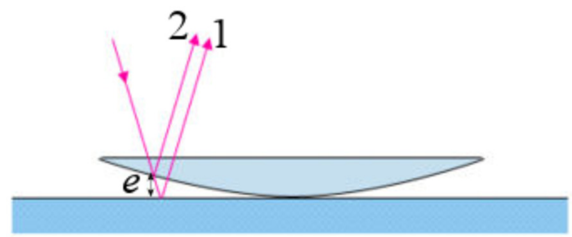
$$L = 0.8m$$

40. At a dark ring, there must be destructive interference. Therefore, the phase difference is

$$\Delta\phi = (2m+1)\pi$$

As this is a thin film, the phase difference is

$$\Delta\phi = \frac{4\pi e n_f}{\lambda} + \Delta\phi_R$$



$\Delta\phi_R$ must then be found. As wave 1 (the one with the longest path) is travelling in air and is reflected on a medium having a larger index (glass), the wave is inverted and undergoes a phase shift ($\phi_{R1} = \pi$). As wave 2 (the one with the shortest path) is travelling in glass and is reflected on a medium having a smaller index of refraction (air), the wave is not inverted and does not undergo a phase shift ($\phi_{R2} = 0$). The difference between these two phase shifts is

$$\begin{aligned}\Delta\phi_R &= \phi_{R2} - \phi_{R1} \\ &= 0 - \pi \\ &= -\pi\end{aligned}$$

Therefore, the total phase difference is

$$\Delta\phi = \frac{4\pi e n_f}{\lambda} - \pi$$

With destructive interference, this means that

$$\frac{4\pi e n_f}{\lambda} - \pi = (2m+1)\pi$$

This gives

$$\begin{aligned}\frac{4 e n_f}{\lambda} - 1 &= 2m+1 \\ \frac{4 e n_f}{\lambda} &= 2m+2 \\ e &= \frac{(2m+2)\lambda}{4n_f}\end{aligned}$$

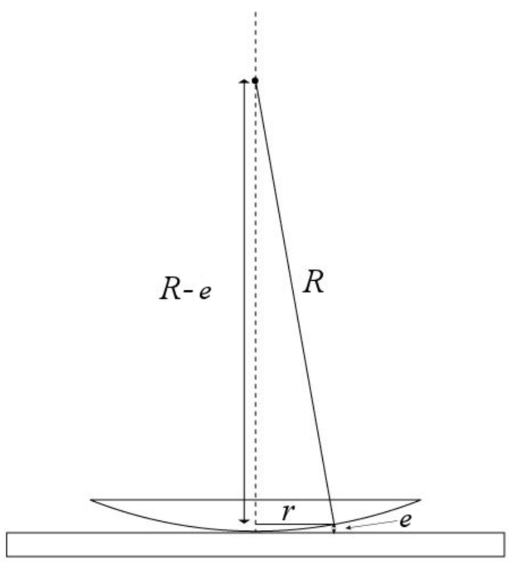
As the index of the film is 1, the equation becomes

$$e = \frac{(2m+2)\lambda}{4}$$
$$= \frac{(m+1)\lambda}{2}$$

Values of *m* smaller than -2 are not possible here. *m* = -1 corresponds to the central dark spot, *m* = 0 corresponds to the first ring, *m* = 1 to the second ring and *m* = 2 to the third ring. Therefore, *e* for the third ring is

$$e = \frac{(2+1)\lambda}{2}$$
$$= \frac{3\lambda}{2}$$
$$= \frac{3 \cdot 600nm}{2}$$
$$= 900nm$$

It only remains to find the radius of the ring. This radius is



$$(R-e)^2 + r^2 = R^2$$
$$(0.60m - 900\times 10^{-9} m)^2 + r^2 = (0.60m)^2$$
$$r^2 = (0.60m)^2 - (0.60m - 900\times 10^{-9} m)^2$$
$$r = 1.039mm$$

tr.wikipedia.org/wiki/Newton_halkalar%C4%B1