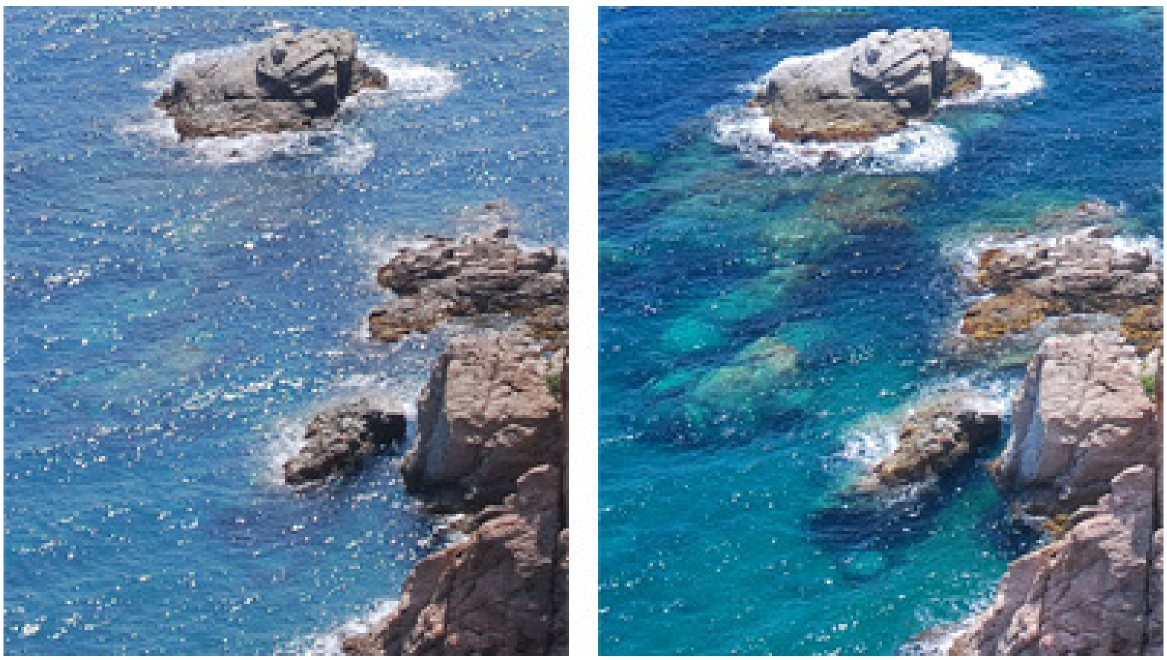
4 REFLECTION AND TRANSMISSION

How can light reflections on the surface of water be blocked to see what is on the bottom of the sea?



www.digital-photography-tips.net/Stay_Focussed-Newsletter-March-2013.html

Discover the answer to this question in this chapter.

It may happen that a wave passes from one medium to another. For example, a wave could move from a rope with a certain linear density to another string with a different linear density. In this chapter, we will examine how the wave changes when it passes from one medium to another.

4.1 FREQUENCY AND WAVELENGTH

When a wave passes from one medium to another, its frequency cannot change. If 50 oscillations per second arrive at the place where the medium changes, 50 oscillations per second must also leave. Oscillations cannot accumulate or vanish at the place where the media meet.

Frequency in a Change of Medium

The frequency does not change when a wave passes from one medium to another

$$f_1 = f_2$$

On the other hand, the speed may be different in the media because the speed depends on the characteristics of the medium. As

$$f = \frac{v}{\lambda}$$

and

$$f_1 = f_2$$

We obtain, by substituting v/λ for f,

Wavelength in a Change of Medium

$$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

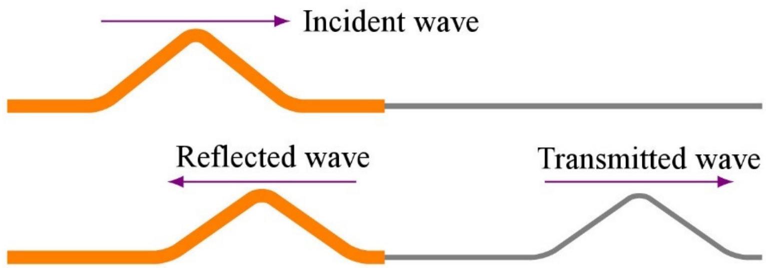
Thus, if the speed is greater in medium 2, the wavelength will be greater in this medium. This change can be seen in the following video.
http://www.youtube.com/watch?v=IEjdCAKubCc

The results obtained here are valid for all types of waves.

4.2 WAVES ON A ROPE

Amplitude of the Reflected and Transmitted Waves

When a wave on a rope passes from one rope to another, part of the wave will pass in the other medium while part of the wave will be reflected back in the same medium as the incident wave.



The amplitude of the transmitted and reflected waves depends on the impedance of the media. If the initial wave has an amplitude *A*, then the amplitude of the reflected and transmitted waves are as follows.

Amplitudes of the Reflected and Transmitted Waves

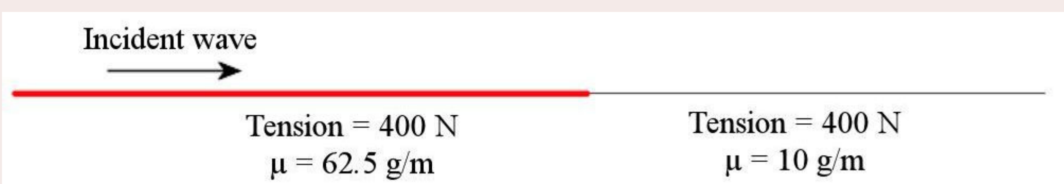
$$A_R = \frac{Z_1 - Z_2}{Z_1 + Z_2} A$$

$$A_T = \frac{2Z_1}{Z_1 + Z_2} A$$

If you wish, you can see proof of these formulas in this document.
http://physique.merici.ca/waves/proofArandAt.pdf

Example 4.2.1

A sine wave with a 1 cm amplitude and a 50 Hz frequency arrives at a junction between two strings. The following diagram gives the properties of the two strings.



What are the powers of the incident, reflected and transmitted waves?

The power of a wave is

$$P = \frac{1}{2} Z \omega^2 A^2$$

To calculate the powers, the impedances of the media are needed. These impedances are

$$Z_1 = \sqrt{F_T \mu_1} = \sqrt{400N \cdot 0.0625 \tfrac{kg}{m}} = 5 \tfrac{kg}{s}$$

$$Z_2 = \sqrt{F_T \mu_2} = \sqrt{400N \cdot 0.01 \tfrac{kg}{m}} = 2 \tfrac{kg}{s}$$

Thus, the power of the incident wave is

$$\begin{aligned} P_{incident} &= \frac{1}{2} Z_1 \omega^2 A^2 \\ &= \frac{1}{2} \cdot 5 \tfrac{kg}{s} \cdot (2\pi \cdot 50Hz)^2 \cdot (0.01m)^2 \\ &= 24.67W \end{aligned}$$

To find the powers of reflected and transmitted waves, the amplitudes of these waves are needed. These amplitudes are

$$\begin{aligned} A_R &= \frac{Z_1 - Z_2}{Z_1 + Z_2} A \\ &= \frac{5 \tfrac{kg}{s} - 2 \tfrac{kg}{s}}{5 \tfrac{kg}{s} + 2 \tfrac{kg}{s}} \cdot 1cm \\ &= 0.429cm \end{aligned}$$

$$\begin{aligned} A_T &= \frac{2Z_1}{Z_1 + Z_2} A \\ &= \frac{2 \cdot 5 \tfrac{kg}{s}}{5 \tfrac{kg}{s} + 2 \tfrac{kg}{s}} \cdot 1cm \\ &= 1.429cm \end{aligned}$$

It may seem odd that the transmitted wave has a greater amplitude than the incident wave, but it is possible. As the second medium has a smaller impedance, the wave can easily create a large disturbance in this medium. We will see that the power of this wave is less even if it has a greater amplitude.

The power of the reflected and transmitted wave are

$$\begin{aligned} P_R &= \frac{1}{2} Z_1 \omega^2 A_R^2 \\ &= \frac{1}{2} \cdot 5 \tfrac{kg}{s} \cdot (2\pi \cdot 50Hz)^2 \cdot (0.00429m)^2 \\ &= 4.53W \end{aligned}$$

$$\begin{aligned} P_T &= \frac{1}{2} Z_2 \omega^2 A_T^2 \\ &= \frac{1}{2} \cdot 2 \tfrac{kg}{s} \cdot (2\pi \cdot 50Hz)^2 \cdot (0.01429m)^2 \\ &= 20.14W \end{aligned}$$

It can be noted that the sum of the powers of the reflected and transmitted wave is equal to the power of the incident wave. This must always be true. In this example, 81.6% of the wave energy is transmitted and 18.4% of the wave energy is reflected.

Reflection and Transmission and Impedance Difference

Very Different Impedances (Impedance Mismatch)

Let's consider what happens if the impedance Z_1 is much larger than the impedance Z_2. In this case, the amplitude of the reflected wave is

$$A_R = \frac{Z_1 - Z_2}{Z_1 + Z_2} A \approx \frac{Z_1}{Z_1} A = A$$

The reflected wave has virtually the same amplitude as the initial wave. This means that almost all the energy of the initial wave is reflected back and that there is very little energy in the transmitted wave. The wave is, therefore, almost entirely reflected.

If the impedance Z_1 is much smaller than the impedance Z_2, then

$$A_R = \frac{Z_1 - Z_2}{Z_1 + Z_2} A \approx \frac{-Z_2}{Z_2} A = -A$$

The reflected wave has virtually the same amplitude as the incident wave. (We will deal with the meaning of the minus sign later.) Again, this means that there is a lot of reflection and that there is practically no energy in the transmitted wave.

Similar Impedances (Impedance Matching)

If the impedance Z_1 is very close to the impedance Z_2, then

$$A_R = \frac{Z_1 - Z_2}{Z_1 + Z_2} A \approx 0$$

In this case, there's no reflected wave and the wave is fully transmitted in the other medium.

This is called *impedance matching*. For example, this situation can be achieved with ropes having the same tension and made of different materials, but with identical linear densities. This is a very important concept for waves of currents (alternating currents) in electrical circuits. If the impedances are identical, there is no signal reflection. This is needed, for example when speakers are connected to a stereo system. Then, the amplifier, which has a certain impedance, sends signals to the speakers, who also have a certain impedance. If the impedance is not the same, there will be reflections of the signal between the receiver and speakers, which will result in an echo in the sound. Therefore, it is better to use speakers with the right impedance. The impedance values are often written on the back of the speakers and of the amplifier.

The following conclusion can then be drawn.

Rules for Reflection and Transmission.

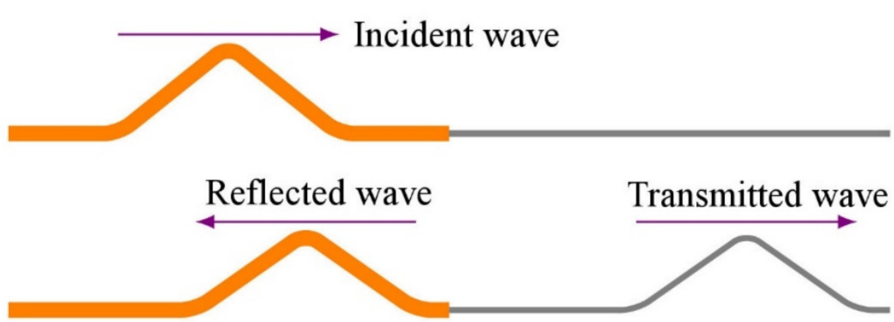
As the difference of impedance of the media gets larger, the reflection is larger and the transmission is smaller.

If the impedances of the media are identical, the wave is completely transmitted and there is no reflection.

Inversion of the Reflected Wave

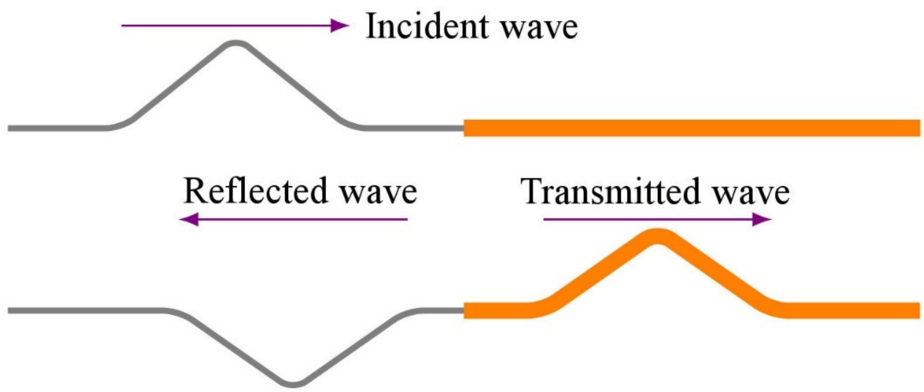
What is the meaning of the minus sign in front of the amplitude of the reflected wave when Z_2 is greater than Z_1? This sign means that the reflected wave is inverted compared to the incident wave. This is not so obvious to see with a sine wave, but it's pretty easy with a wave that consists of a single bump.

If the impedance Z_1 is larger than Z_2, the amplitude of the reflected wave is positive and we have the following situation.



The reflected wave is in the same direction as the incident wave.

If the impedance Z_1 is smaller than Z_2, the amplitude of the reflected wave is negative and we have the following situation.



The reflected wave is inverted compared to the incident wave.

The amplitude of the transmitted wave can never be negative and so it is never inverted compared to the incident wave.

Reflected and Transmitted Waves Inversion

The wave passes from medium 1 (impedance Z_1) to medium 2 (impedance Z_2).

If $Z_2 > Z_1$, the reflected wave is inverted.

If $Z_2 < Z_1$, the reflected wave is not inverted.

The transmitted wave is never inverted.

Let's see what this means for waves travelling on ropes. In this case, the impedance is

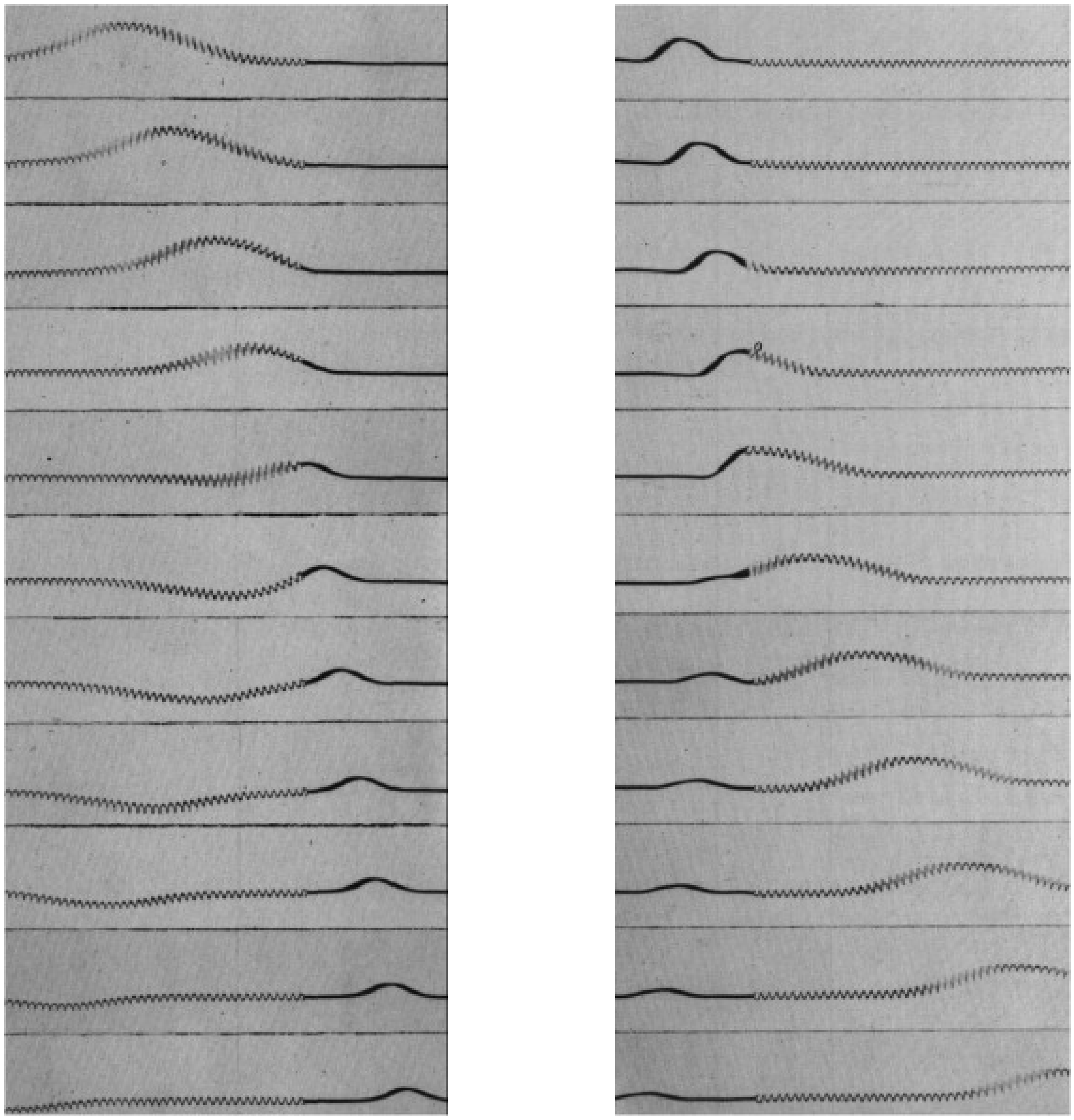
$$Z = \frac{F_T}{v} = \sqrt{F_T \mu}$$

Generally, the tensions of the strings tied end to end are the same and only the linear density is different. Note that the impedance is greater if the linear density is greater or if the speed of the wave is smaller. The rules for the inversion of the reflected wave then become

If $\mu_2 > \mu_1$ or if $v_2 < v_1$ the reflected wave is inverted.

If $\mu_2 < \mu_1$ or if $v_2 > v_1$, the reflected wave is not inverted.

In the images on the following page, a wave is travelling on two stretched springs (which act as ropes) tied end-to-end. The largest spring has a smaller linear density than the smallest spring. (Yes, you read correctly. The smallest spring is perhaps made of metal while the largest is perhaps made of plastic.) The impedance of the big spring is, therefore, smaller than the impedance of the small spring.



Haber-Schaim, Cross, Dodge, Walter, Tougas, Physique PSSC, éditions CEC, 1974

In the series of images to the left, the wave arrives in the medium having the smaller impedance and attempts to go into the medium with a higher impedance. There's a transmitted wave, which is in the same direction as the initial wave. There is also a reflected wave that is inverted because the impedance of medium 2 is greater than the impedance of medium 1. It can be seen that the speed of the wave is smaller in medium 2, which is an indication that the impedance of medium 2 is greater.

In the series of images to the right, the wave arrives in the medium having the greater impedance and attempts to go into the medium with a smaller impedance. The transmitted wave is once again in the same direction as the initial wave. The reflected wave is also in the same direction since the impedance of medium 2 is smaller than the impedance of medium 1. It can be seen here that the speed in medium 2 is greater than in medium 1, confirming that the impedance of medium 2 is smaller than the impedance of medium 1.

A Wave Arriving at the End of a Rope

The formulas for the amplitude of the reflected wave will help to determine what happens when a wave arrives at the end of a rope.

Rope Attached at the End

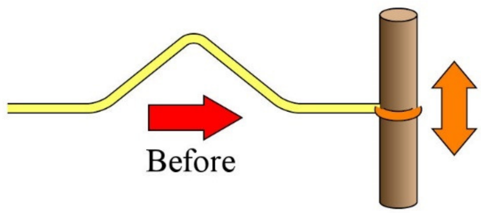
A wave is travelling along a rope whose end is securely attached to something. When the wave arrives at the end of the rope, the same thing happens as when the wave passes from one medium to another. Here the two media are the rope and the object on which the rope is attached. As this object is fixed, it will not be disturbed at all by the motion of the rope, which means that this object behaves as a medium with a huge impedance. This situation is, therefore, similar to the situation where Z_2 is much greater than Z_1. It was then found that the amplitude of the reflected wave is -A, which means that the wave is completely reflected, but inverted.

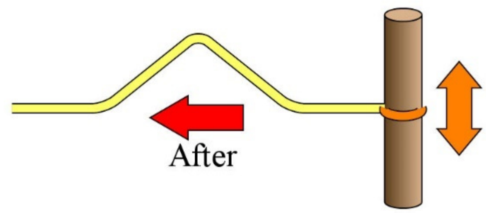
Here is a demonstration.
http://www.youtube.com/watch?v=LTWHxZ6Jvjs

This wave is inverted since the rope pulls on the point of attachment when the wave arrives at the end of the rope. According to Newton's third law, the attachment point then pulls down on the rope, which creates a downwards wave that moves towards the left.

Rope With a Free End

Now imagine that the rope is not attached at its end or that it is fixed to a rod by a massless ring that can slide without any friction along the rod. This is similar to a situation where the media are a rope and a massless rope. As the linear density of the second medium is zero, its impedance is zero. This corresponds to the situation where Z_2 is much smaller than Z_1, and this means that the amplitude of the reflected wave is A. This means that the wave is completely reflected and is not inverted.



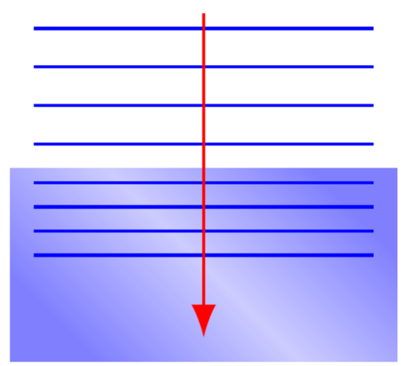


Here is a demonstration.
http://www.youtube.com/watch?v=aVCqq5AkePI

In this case, the end of the rope rises much higher than expected when the wave reaches the end because there is less tension force acting on the last piece of rope. Usually, each piece of rope is forced back to its equilibrium position by tension forces acting on each side of the piece. However, the tension force of only one side is exerted on the last piece of rope. This lack of tension force makes this last piece rise higher than expected. This upwards motion will exert an upwards force on the rest of the rope, which then creates a new upwards wave moving towards the left.

4.3 WAVES ARRIVING PERPENDICULARLY TO A SURFACE

Now, waves arriving on the surface at an angle of 90°, as in the diagram on the right, will be considered. Those waves could be, for example, sound or light.



Sound Waves

The amplitudes of the reflected and transmitted sound waves are given by the same formulas as for waves in a rope.

$$A_R = \frac{Z_1 - Z_2}{Z_1 + Z_2} A$$

$$A_T = \frac{2Z_1}{Z_1 + Z_2} A$$

If you wish, you can see proof of these formulas in this document.
http://physique.merici.ca/waves/proofArandAt2.pdf

(There is a small change compared to the proof of the previous section since it is done with the intensity of the wave rather than the power of the wave.)

Example 4.3.1

A 1000 Hz sound having an intensity of 100 dB in air enters into water. What is the intensity of the sound in water? Use the following data.

Air:	wave velocity = 340 m/s	density = 1.2 kg/m³
Water:	wave velocity = 1450 m/s	density = 1000 kg/m³

The intensity of the transmitted wave in water can be calculated from the amplitude of the wave with

$$I = \frac{1}{2} Z \omega^2 A^2$$

To find the intensity, the impedance of water and air must be found. These impedances are

$$\begin{aligned} Z_1 &= \rho_1 v_1 \\ &= 1.2 \tfrac{kg}{m^3} \cdot 340 \tfrac{m}{s} \\ &= 408 \tfrac{kg}{m^2 s} \end{aligned} \qquad \begin{aligned} Z_2 &= \rho_2 v_2 \\ &= 1000 \tfrac{kg}{m^3} \cdot 1450 \tfrac{m}{s} \\ &= 1\,450\,000 \tfrac{kg}{m^2 s} \end{aligned}$$

Also, the amplitude of the incident wave in water must be known. It can be found from the amplitude of the initial wave and impedances of the media.

<u>*Amplitude of the Initial Wave*</u>

With an intensity of 100 dB, the intensity of the wave is

$$\beta = 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$

$$100dB = 10dB \cdot \log \frac{I}{10^{-12} \frac{W}{m^2}}$$

$$I = 0.01 \tfrac{W}{m^2}$$

Therefore, the amplitude of the wave is

$$I = \frac{1}{2} Z_1 \omega^2 A^2$$

$$0.01 \tfrac{W}{m^2} = \frac{1}{2} \cdot 408 \tfrac{kg}{m^2 s} \cdot \left(2\pi \cdot 1000Hz\right)^2 \cdot A^2$$

$$A = 1.114 \times 10^{-6} m$$

Amplitude of the Transmitted Wave

Therefore, the amplitude of the transmitted wave is

$$\begin{aligned} A_T &= \frac{2Z_1}{Z_1+Z_2}A \\ &= \frac{2\cdot 408\frac{kg}{m^2s}}{408\frac{kg}{m^2s}+1\,450\,000\frac{kg}{m^2s}}\cdot 1.114\times 10^{-6}m \\ &= 6.267\times 10^{-10}m \end{aligned}$$

Intensity of the Transmitted Wave

The intensity of this wave in water is

$$\begin{aligned} I &= \frac{1}{2}Z_2\omega^2A^2 \\ &= \frac{1}{2}\cdot 1\,450\,000\frac{kg}{m^2s}\cdot\left(2\pi\cdot 1000Hz\right)^2\cdot\left(6.267\times 10^{-10}m\right)^2 \\ &= 1.125\times 10^{-5}\frac{W}{m^2} \end{aligned}$$

In decibels, this intensity is

$$\begin{aligned} \beta &= 10dB\cdot\log\frac{I}{10^{-12}\frac{W}{m^2}} \\ &= 10dB\cdot\log\frac{1.125\times 10^{-5}\frac{W}{m^2}}{10^{-12}\frac{W}{m^2}} \\ &= 70.5dB \end{aligned}$$

At 20 °C, the impedance of air is 413 Ns/m³, and the impedance of water is 1 439 000 Ns/m³. This huge difference of impedance has big repercussions when a sound wave tries to move from air to water or from water to air. With a big impedance mismatch, most of the energy of the wave is reflected, and there is almost no transmission.

During an ultrasound, some gel is put on the belly of the patient. This gel is not there simply to lubricate; it also helps prevent wave reflections. With no gel, the wave would travel in the machine, then in air and then in the belly. Then, there would be significant changes of impedance at each change of medium and, therefore, a lot of reflection. Thus, a significantly reduced sound wave would enter the person. With gel, the impedance variations are much smaller, and a large enough sound wave can penetrate into the person.

The impedance mismatch is even greater between air and steel (4 000 000 Ns/m²), which means that a sound propagating in a piece of steel has a hard time getting out of the metal to travel in air. It is for this reason that it is possible, it seems, to hear a train arriving well before it can be seen by listening to the sound in the rails. The sound made by the shock of the wheels of the train on the rails spreads in the rails and almost no sound dissipates in the air because of the great difference of impedance between the mediums. The sound can then travel very far in the rails before dissipating.

A nice experiment shows the effects of the impedance of the medium, but it is usually used, incorrectly, to demonstrate something else. In this experiment, a bell is ringing inside a bell jar. When the bell is ringing in the air, it can be heard. The air is then removed from the jar and the bell cannot be heard anymore, even if it's still ringing.
http://www.youtube.com/watch?v=ce7AMJdq0Gw
Usually, this experiment is used to show that sound cannot propagate in a vacuum. It's true that the sound is a mechanical wave and a medium is needed for this wave to propagate. This means that in a vacuum, there is no sound (unlike what is seen in almost every science fiction movies, with the exception of *2001, A Space Odyssey*). However, this is not what this experiment shows. Even if the pump removes 99% of the air, enough air is left for the sound to propagate. Here's a better explanation of what is happening: as air density decreases, the impedance of the air in the bell decreases so that it becomes increasingly different from the impedance of the air outside the bell jar and from the glass of the bell jar. As the pressure drops, it is more and more difficult for the sound to get out of the bell jar and is increasingly reflected within the bell jar. This produces a sound whose intensity decreases gradually, as can be seen in the video.

Light

Wavelength

With light, it is possible to go a little further. Since the frequency does not change from one medium to the other, the change of wavelength is

$$f_{\text{vacuum}} = f_{\text{substance}}$$
$$\frac{v_{\text{vacuum}}}{\lambda_{\text{vacuum}}} = \frac{v_{\text{substance}}}{\lambda_{\text{substance}}}$$
$$\frac{c}{\lambda_{\text{vacuum}}} = \frac{c/n}{\lambda_{\text{substance}}}$$

($v = c/n$ was used in the last line.)

Therefore, the change of wavelength is

Wavelength Change in a Transparent Substance

$$\lambda_{\text{substance}} = \frac{\lambda_{\text{vacuum}}}{n}$$

Amplitudes

For light, the formulas of the amplitudes of reflected and transmitted waves are a little similar to those for sound, except that the impedance is replaced by the index of refraction, which plays the role of the impedance with light.

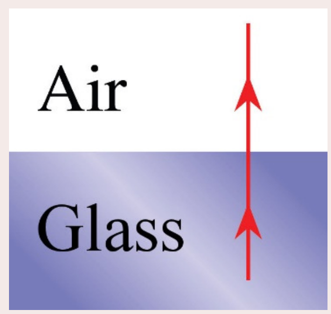
Amplitudes of the Reflected and Transmitted Light Waves

$$E_{0R} = \frac{n_1 - n_2}{n_1 + n_2} E_0$$

$$E_{0T} = \frac{2n_1}{n_1 + n_2} E_0$$

If you wish, you can see proof of these formulas in this document.
http://physique.merici.ca/waves/proofAtandAt3.pdf

Example 4.3.2



The wavelength of a light beam passing from glass to air changes from 430 nm to 623.5 nm. The light intensity in glass is 0.1 W/m².

a) What is the index of refraction of the glass?

Since 623.5 nm is the wavelength in air, we have

$$\lambda_{\text{substance}} = \frac{\lambda_{\text{vacuum}}}{n}$$

$$430nm = \frac{623.5nm}{n}$$

$$n = 1.45$$

b) What is the speed of light in glass?

The speed is

$$v = \frac{c}{n} = \frac{3\times10^8 \frac{m}{s}}{1.45} = 2.069\times10^8 \frac{m}{s}$$

c) What are the intensities of the reflected and transmitted wave?

The amplitude of the incident wave is found with the intensity formula.

$$I = \frac{cn\varepsilon_0 E_0^2}{2}$$

$$0.1\tfrac{W}{m^2} = \frac{3\times10^8 \tfrac{m}{s} \cdot 1.45 \cdot 8.854\times10^{-12} \tfrac{C^2}{Nm^2} \cdot E_0^2}{2}$$

$$E_0 = 7.206\tfrac{N}{C}$$

Therefore, the amplitudes of the reflected and transmitted waves are

$$\begin{aligned} E_{0R} &= \frac{n_1 - n_2}{n_1 + n_2} E_0 \\ &= \frac{1.45-1}{1.45+1} \cdot 7.206\tfrac{N}{C} \\ &= 1.324\tfrac{N}{C} \end{aligned}$$

$$\begin{aligned} E_{0T} &= \frac{2n_1}{n_1 + n_2} E_0 \\ &= \frac{2 \cdot 1.45}{1.45+1} \cdot 7.206\tfrac{N}{C} \\ &= 8.530\tfrac{N}{C} \end{aligned}$$

This means that the intensities of the reflected and transmitted waves are

$$\begin{aligned} I_R &= \frac{cn\varepsilon_0 E_{0R}^2}{2} \\ &= \frac{3\times10^8 \tfrac{m}{s} \cdot 1.45 \cdot 8.854\times10^{-12} \tfrac{C^2}{Nm^2} \cdot \left(1.324\tfrac{N}{C}\right)^2}{2} \\ &= 0.00337\tfrac{W}{m^2} \end{aligned}$$

$$\begin{aligned} I_T &= \frac{cn\varepsilon_0 E_{0T}^2}{2} \\ &= \frac{3\times10^8 \tfrac{m}{s} \cdot 1 \cdot 8.854\times10^{-12} \tfrac{C^2}{Nm^2} \cdot \left(8.530\tfrac{N}{C}\right)^2}{2} \\ &= 0.0966\tfrac{W}{m^2} \end{aligned}$$

Therefore, 96.6% of the energy is transmitted.

Note that since the refractive index plays the role of impedance with light, the rules for the inversion of the wave become the following rules for light.

Rules for Reflection and Transmission for Light

The wave passes from medium 1 (index n_1) to medium 2 (index n_2).

If $n_2 > n_1$, the reflected wave is inverted.

If $n_2 < n_1$, the reflected wave is not inverted.

The transmitted wave is never inverted.

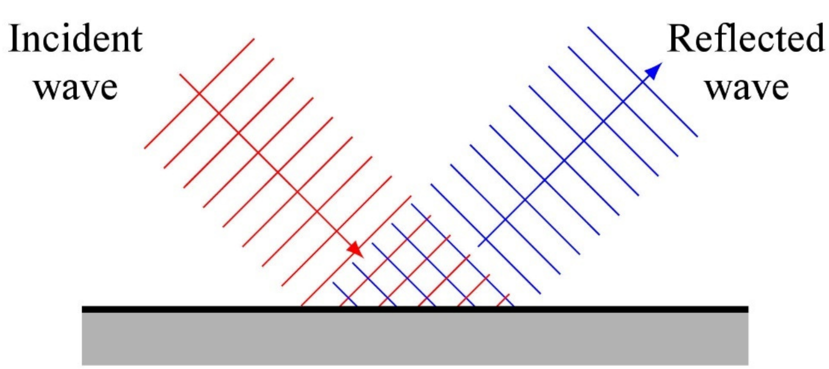
Metals behave like a substance with an infinite refractive index. Thus, if a wave arrives on a piece of metal, it is completely reflected and is inverted.

4.4 LAW OF REFLECTION

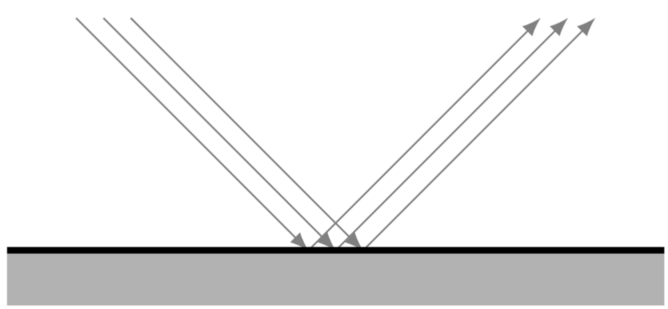
Now, a wave arriving with a certain angle with the surface will be considered.

The formulas of the amplitudes of the reflected and transmitted waves if the wave comes with a certain angle to an interface will not be given. These formulas are a little more complex than the formulas when the wave arrives perpendicularly to the surface, especially in the case of the light.

First, the reflected wave will be considered.

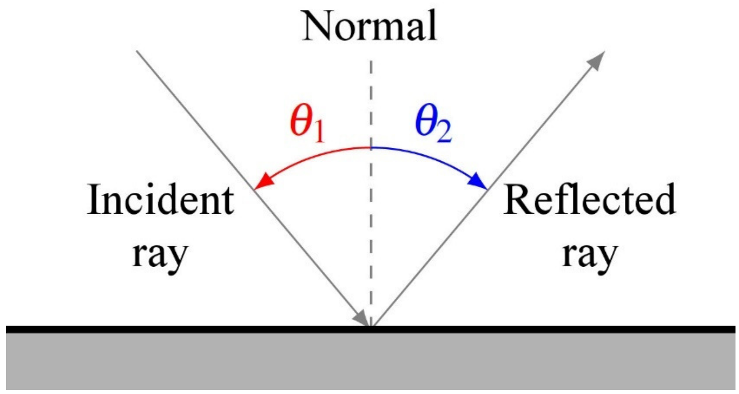


Here, the wave will not be followed with the wavefronts. Its displacement of the wave will be followed with lines showing the direction of propagation of the wave, i.e. the rays. With rays, a reflection is represented as in the following diagram.



Note that these rays are perpendicular to the wavefronts. So, the wavefronts can easily be added to the representation of the wave if they are needed.

When a wave is reflected, there is an incident ray and a reflected ray. The direction of these rays is given by measuring the angle between the ray and the normal to the surface.



It has been known for a long time (at least since Heron of Alexandria who lived in the 1st century AD) that the link between the angles of the incident and reflected rays is

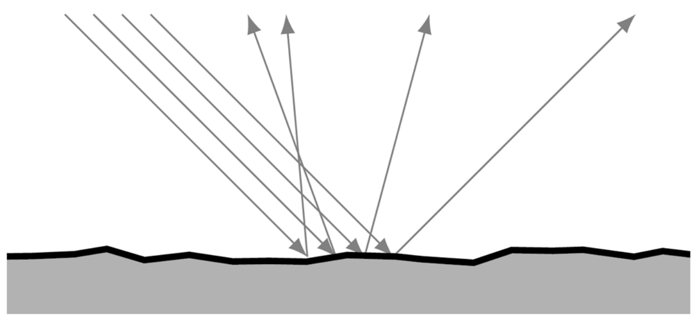
Law of Reflection

$$\theta_1 = \theta_2$$

An experimental demonstration of this law can be seen in this video.
http://www.youtube.com/watch?v=5wrQchqecjA

This law is true only if the surface is very smooth (having asperities much smaller than the wavelength of the light). Then, the reflection is a *specular reflection* (which comes from *speculum*, mirror in Latin).

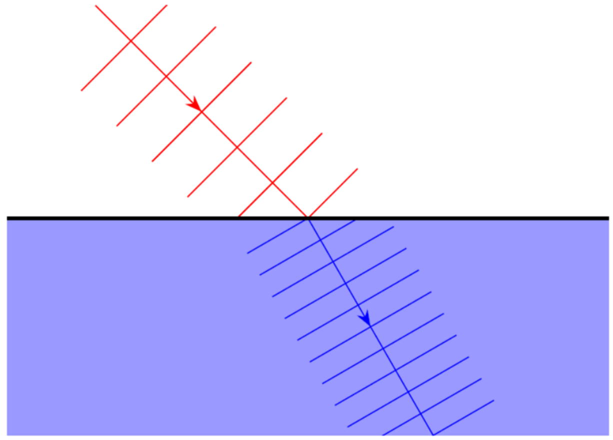
If the surface is not smooth (having asperities greater than the wavelength of the light), the light is reflected in every direction. This is called a *diffuse reflection*.



4.5 LAW OF REFRACTION

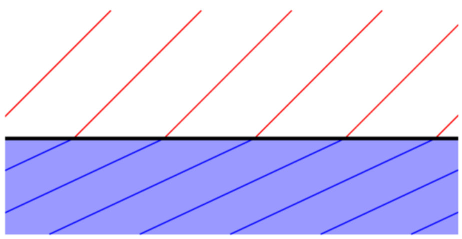
General Formula

Now, the transmitted wave will be considered. The diagram shows what happens when the speed of the wave is smaller in the second medium.

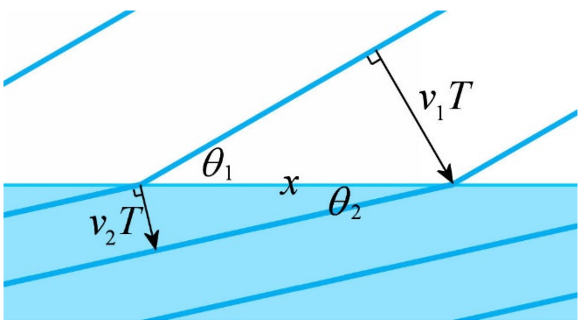


The passage of the wave in the second medium led a change in the direction of propagation of the wave. Let's try to understand why.

Since the speed is smaller in the second medium, the wave travels a smaller distance in the second medium for a specific time duration. The side of the wavefront side which is in the 2nd medium travels forward less quickly than the side which is in the first medium. Therefore, this side which travels at a lower speed lags compared to the other side, and this changes the orientation of the wavefront. Since the direction of propagation is perpendicular to the wavefront, this changes the direction of propagation of the wave.



This can also be seen in this clip.
https://www.youtube.com/watch?v=Bf1k9-4bb4w



Now, the change in orientation of the wave will be calculated. During a period, the wave travels a distance *vT*, which is also equal to the wavelength. Since the speed is smaller in the second medium, the wave travels a smaller distance in the second medium, which means that the wavelength is smaller in the 2nd medium.

According to the diagram,

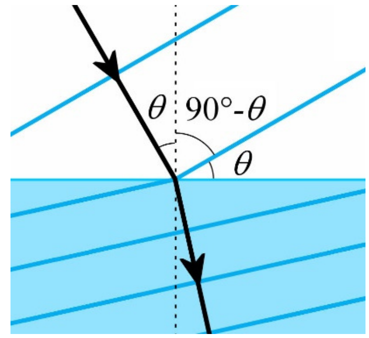
$$\sin\theta_1 = \frac{v_1T}{x} \qquad \text{and} \qquad \sin\theta_2 = \frac{v_2T}{x}$$

Index 1 refers to the medium where the light originates and index 2 refers to the medium where the light passes (or attempts to pass).

By dividing the first equation by the second, the following result is obtained.

$$\frac{\sin\theta_1}{\sin\theta_2}=\frac{v_1}{v_2}$$

Remember now that the direction of propagation of the wave is perpendicular to the wavefront. This direction of propagation also corresponds to the direction of propagation of light rays.



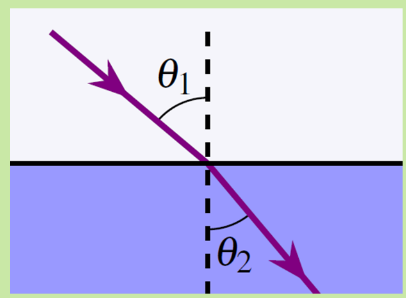
The angle between the wavefront and the surface is equal to the angle between the ray and the normal. In the diagram to the left, it is easy to see that these angles are the same.

The change in the direction of propagation of rays is, therefore, the same as the change of the angle of orientation of the wavefronts.

The following law for the change of the angle between the rays and the normal is thus obtained.

Law of Refraction

$$\frac{\sin\theta_1}{\sin\theta_2}=\frac{v_1}{v_2}$$



Law of Refraction for Light

Formula for refraction

For light, the speed is given by

$$v=\frac{c}{n}$$

Therefore,

$$\frac{\sin\theta_1}{\sin\theta_2}=\frac{c/n_1}{c/n_2}$$

$$\frac{\sin\theta_1}{\sin\theta_2}=\frac{n_2}{n_1}$$

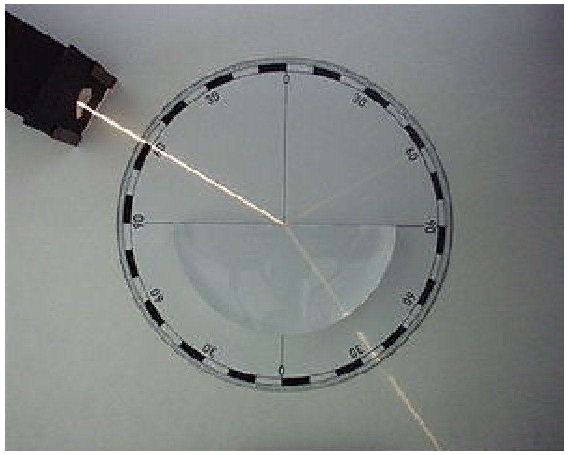
Thus. the law is

Law of Refraction for Light

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Now, it is easy to understand why n is called the index of refraction. It has this name because it has made his first appearance in this law.

This change in the direction of propagation when light passes from one medium to another can easily be seen in this diagram.



en.wikipedia.org/wiki/Refraction

Here is an experimental demonstration.
http://www.youtube.com/watch?v=yfawFJCRDSE
Look carefully at the photo showing the refraction and at the video and you will see that part of the light is reflected at the interface. When light arrives at a change of medium, there may be some reflection and some refraction.

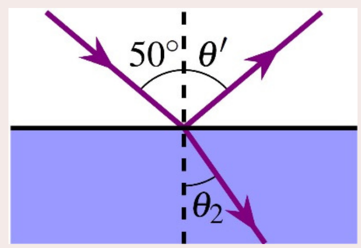
Corpuscular and Wave Theories

This phenomenon has been known for a long time since tables giving the values of the angles before and after the entry of the light beam into water were made by Ptolemy (1st century), although the values were not always right. The law connecting the angles θ_1 and θ_2 was discovered many centuries later. A manuscript dating from 984 by Ibn Sahl, a Muslim scholar, suggests that he knew the law. It was then rediscovered by Thomas Harriot in 1602 and by Willebrord Snell in 1621. However, René Descartes was the first to publish it in 1637. (So, there is some controversy regarding the name of this law. For everyone, except the French, the law is called Snell's law, while for the French, it is called Snell-Descartes' law or simply Descartes' law...)

One of the first proofs of the law of refraction was made by Descartes in 1637 but the proof was based on the corpuscular theory of light. (Descartes gave this proof even if he did not think that light was made of particles. He thought that light was an instantaneous transmission of pressure in ether). To get to the correct law with the corpuscular theory, Descartes had to assume that light travels faster if the refractive index is larger (while it's the opposite with the wave theory). Christiaan Huygens was the first to demonstrate the law of refraction in 1678 using the wave theory of light (which is essentially the same proof as the one given here).

You might think that measuring the speed of light in water would have therefore been an easy test to determine which theory was correct. It would have sufficed to measure the speed of light in water and in air. If light travels faster in water, the corpuscular theory is correct and if light travels slower in water, the wave theory is correct. However, the technology to measure the speed of light in water arrived only in 1850 when the wave theory was already the accepted theory of light for about 20 years. Even if this measure had been possible at an earlier date, it would not really have been a good way to decide between the corpuscular or the wave theory since it is possible to develop a corpuscular theory in which light travels at a slower speed in water.

Example 4.5.1



A ray of light with a 500 nm wavelength in air enters into water (n = 1.33) with a 50° incidence angle.

a) What is the angle of the reflected ray?

According to the law of reflection, the angle is

$$\theta' = 50°$$

b) What is the angle of the refracted ray?

According to the law of refraction, the angle is

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1 \cdot \sin 50° = 1.33 \cdot \sin\theta_2$$
$$\theta_2 = 35.16°$$

c) What is the wavelength of the light in water?

The wavelength is

$$\begin{aligned}\lambda_{water} &= \frac{\lambda_{vacuum}}{n} \\ &= \frac{500nm}{1.33} \\ &= 376nm\end{aligned}$$

(This result does not mean that the colour of the light changes. The colours as a function of the wavelength are given for wavelengths in vacuum. Actually, our eye measures the frequency of the light. As the frequency does not change when a wave passes from one medium to another, the colour remains the same.)

4.6 FERMAT'S PRINCIPLE

In 1657, Pierre de Fermat stated the following principle.

Fermat's Principle

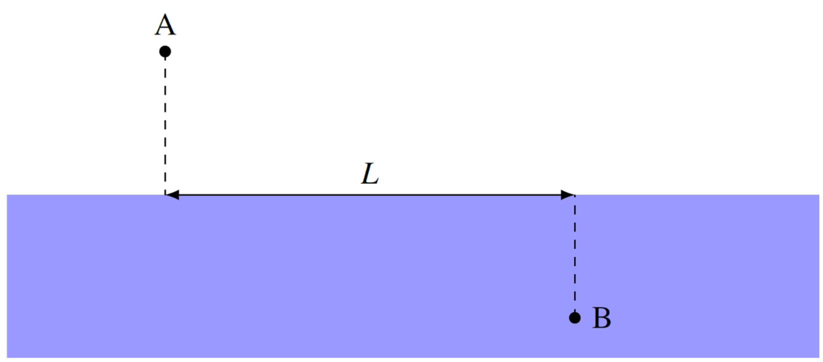
Passing from one point to another, light always follows the path that requires the least time to travel.

(Fermat stated this principle for light, but it is valid for all types of waves.)

This principle can sometimes be very useful, and it leads directly to the laws of reflection and refraction.

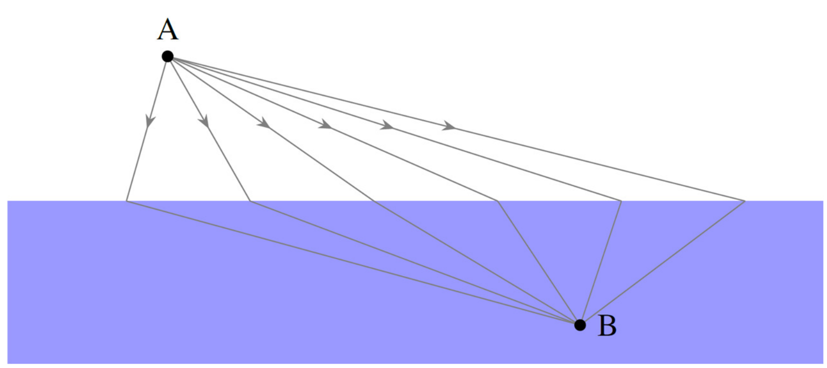
Law of Refraction From Fermat's Principle

Imagine that light goes from point A to point B in the situation shown in the diagram. The horizontal distance between points A and B is *L*.

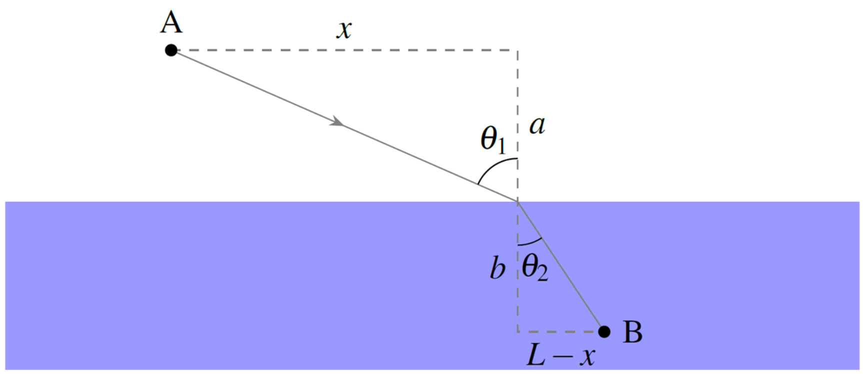


It will now be shown that the path followed by a ray of light according to the law of refraction is the one that requires the least time to travel from A to B.

Assuming that light travels in a straight line when the refractive index is constant, a ray of light only changes direction when it encounters a change of media. Thus, the trajectories shown in this diagram are all possible trajectories.



To find the fastest path, let first assume that the ray of light enters the second medium at a horizontal distance *x* from point A.



The time it takes to travel from point A to the interface is

$$t_1 = \frac{\sqrt{a^2 + x^2}}{v_1}$$

The time it takes to travel from the interface to point B is

$$t_2 = \frac{\sqrt{b^2 + (L-x)^2}}{v_2}$$

The total time it takes to travel from A to B is thus

$$t = \frac{\sqrt{a^2 + x^2}}{v_1} + \frac{\sqrt{b^2 + (L-x)^2}}{v_2}$$

The value of *x* that makes the time to travel from A to B minimal is found with the derivative of *t*. At a minimum, the derivative of *t* must vanish.

$$\frac{dt}{dx} = 0$$

This means that

$$\frac{1}{2}\frac{2x}{v_1\sqrt{a^2 + x^2}} + \frac{1}{2}\frac{2(L-x)(-1)}{v_2\sqrt{b^2 + (L-x)^2}} = 0$$

$$\frac{x}{v_1\sqrt{a^2 + x^2}} + \frac{-(L-x)}{v_2\sqrt{b^2 + (L-x)^2}} = 0$$

$$\frac{x}{v_1\sqrt{a^2 + x^2}} = \frac{(L-x)}{v_2\sqrt{b^2 + (L-x)^2}}$$

However, it can be noted (from the last diagram) that

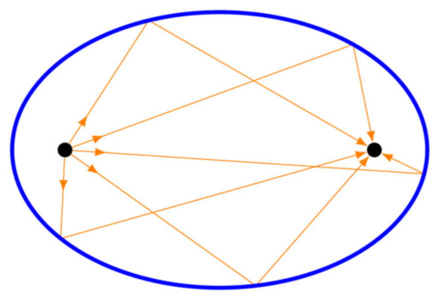
$$\sin\theta_1 = \frac{x}{\sqrt{a^2 + x^2}} \qquad \sin\theta_2 = \frac{(L-x)}{\sqrt{b^2 + (L-x)^2}}$$

The equation then becomes

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

This is the formula for the law of refraction. This means that among all possible paths that light could take between A and B, it follows the path that takes the least time.

What If the Time Is the Same for Many Trajectories?

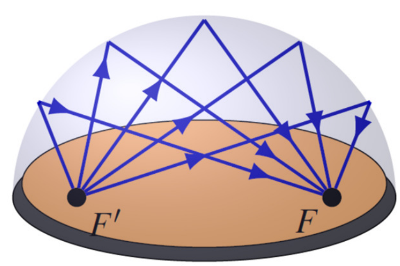


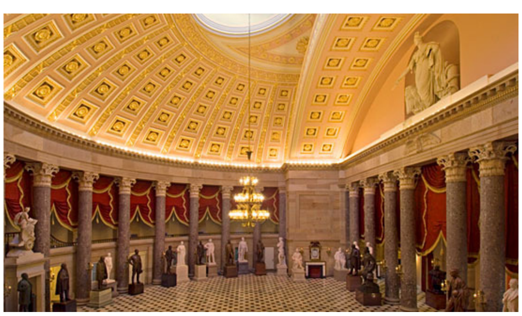
It is possible to have a situation in which the time is the same for multiple paths. For example, the path taking the leat time to travel from one focus to the other focus of an ellipse is such a situation.

The principle says that the wave is going to follow the path that takes the least time. However, the distance travelled from one focus to the other after a reflection is the same for every path (this is the very definition of the ellipse). Since the time is the same for all paths, all paths lead to the other focus. This means that all the waves emitted at one focus of an ellipse will travel along all these paths and meet at the other focus.

The wave travelling from one focus to the other can be seen in this video.
https://www.youtube.com/watch?v=3leJFxzxPIc

Some architects even use this idea to design elliptical rooms where, if you are located at one of the focus, you can easily hear a person whispering at the other focus.





The Statuary Hall in the Capitol building in Washington is such a room. (Although there is only half of an ellipse, it still works.)

www.winstonchurchill.org/news-and-events/churchill-centre-news/2527-churchill-centre-to-donate-bust-of-winston-churchill-to-us-capitol

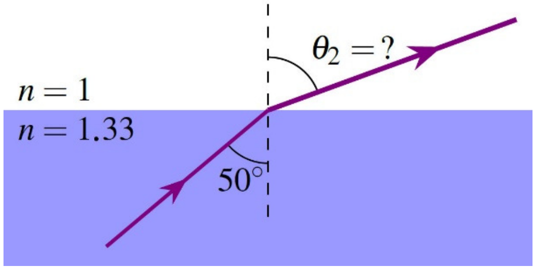
Similarly, the Paris metro was built on the same principle: passengers on a platform can hear their fellow passengers perfectly on the opposite platform. They are both located in the focii of the ellipse, which is the shape of the tunnel vault.

de.wikipedia.org/wiki/Notre-Dame-des-Champs_(Métro_Paris)

4.7 TOTAL REFLECTION

Critical Angle

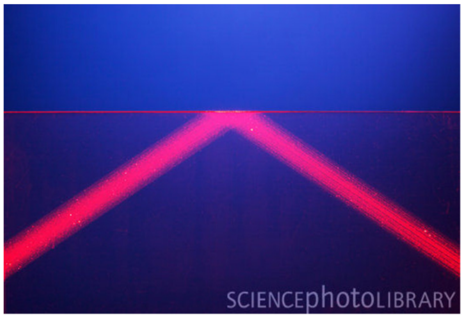
Sometimes, refraction is impossible. This will be illustrated by an example. Let's calculate the refraction angle in the situation shown in the diagram.



The angle is found with the law of refraction.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1.33 \cdot \sin 50° = 1 \cdot \sin\theta_2$$
$$\sin\theta_2 = 1.019$$

Then, there is a little problem: there is no solution to this equation! This simply means that there is no refraction in this case. The light is, therefore, fully reflected and no light comes out of the water. This phenomenon is called *total reflection* (also known as *total internal reflection*) and was discovered by Kepler in 1604. The following image shows you a red beam of light making a total reflection on a surface.



www.sciencephoto.com/media/97795/view

To have a total reflection, the sine of the refracted angle must be larger than 1.

$$\sin\theta_2 > 1$$

Using the law of refraction, this means that

$$\sin\theta_2 > 1$$
$$\frac{v_2 \sin\theta_1}{v_1} > 1$$

$$\sin\theta_1 > \frac{v_1}{v_2}$$

This condition can be rewritten in the following form.

Critical Angle for Total Reflection

$$\sin\theta_c = \frac{v_1}{v_2}$$

If θ_1 is larger than θ_c, then the reflection is total.

With light, the condition can be written in another form. Since

$$\sin\theta_c = \frac{c/n_1}{c/n_2}$$

the formula becomes

Critical Angle for Total Reflection With Light

$$\sin\theta_c = \frac{n_2}{n_1}$$

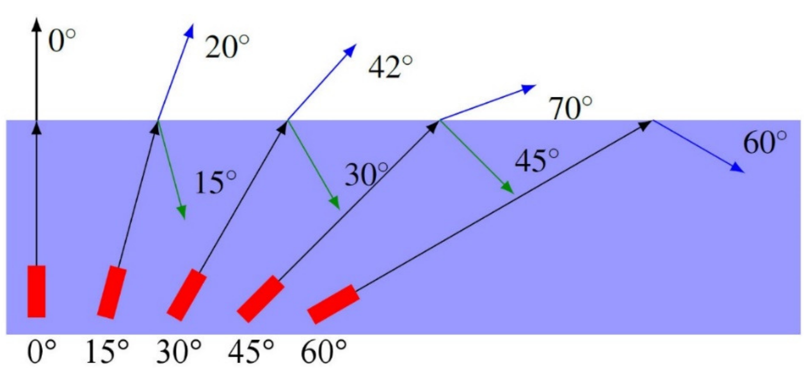
If θ_1 is larger than θ_c, then the reflection is total.

For a light ray in water trying to pass into air, the critical angle is

$$\sin\theta_c = \frac{1}{1.33}$$
$$\theta_c = 48.75°$$

It is then obvious that the 50° angle of the previous example was over the critical angle, and that is why there was a total reflection.

The following diagram shows what happens when light in water is trying to pass into air depending on the angle of incidence.



For angles smaller than the critical angle, there are some reflection and some refraction. When the angle of incidence exceeds the critical angle (last ray in the diagram), there is no refraction, and all the light is reflected.

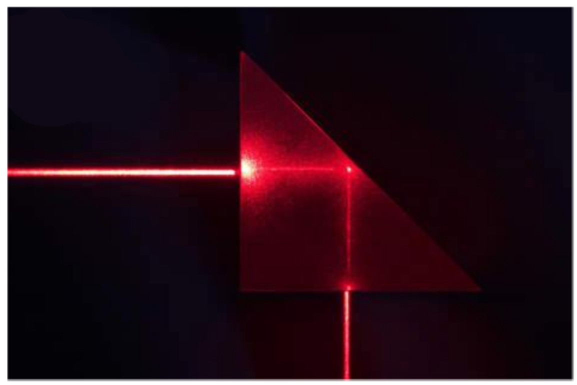
This phenomenon can be seen in this unnecessarily long video.
http://www.youtube.com/watch?v=2kBOqfS0nmE

For light in air trying to enter into water, the critical angle is

$$\sin\theta_c = \frac{1.33}{1}$$

As there has no solution, it is impossible for light in air to make a total reflection on a water surface. In order to have a total reflection, the index n_1 must be larger than n_2.

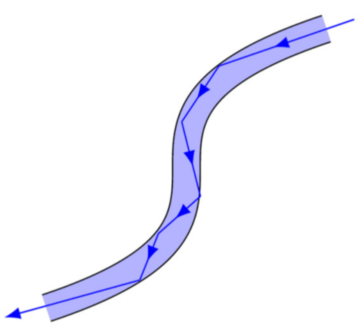
Prisms are used to reflect light in some optical instruments such as binoculars. The critical angle of glass in air being around 42°, rays arriving at 45° on a surface are totally reflected, as seen in this image to the right.

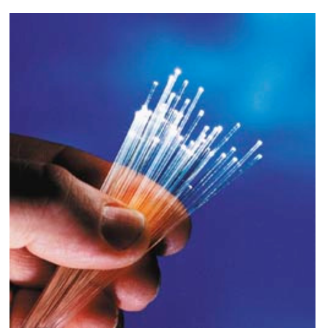


www.visualphotos.com/image/1x7467350/total_internal_reflection_at_the_hypotenuse_of_a

Why use a prism rather than a mirror? A mirror is never 100% efficient, and part of the light is not reflected back and passes through the mirror. Some of the light is then lost. With total reflection, all the light is reflected and nothing is lost.

Total reflection also explains how optical fibres work. A laser light sent in the fibre always makes a total reflection on the sides of the fibre. Therefore, the light remains trapped inside the fibre until it arrives at the other end. Light signals can, therefore, be transmitted over long distances without any loss.



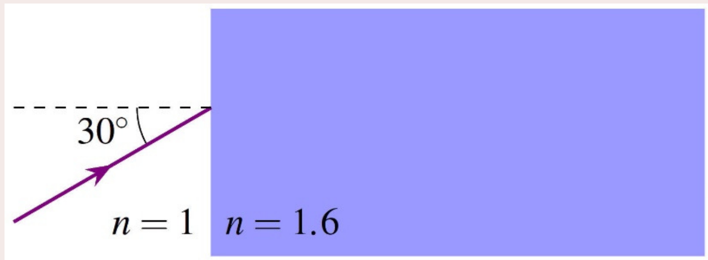


www.thimphutech.com/2009/04/optical-fiber-link-fixed.html

This effect can also be observed in this video.
http://www.youtube.com/watch?v=rlo2XeB2qt4

Example 4.7.1

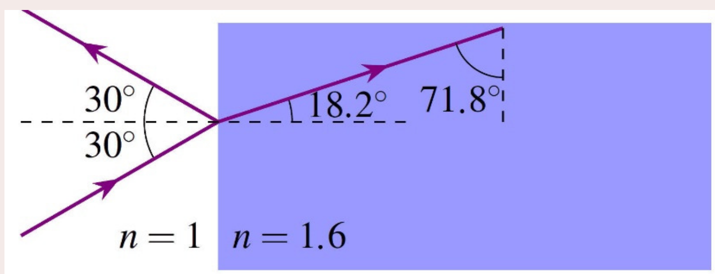
What will the path of this light ray be after its entry into this piece of glass?



When the ray arrives at the surface of the glass, there is some reflection (at 30° according to the law of reflection) and (possibly) some refraction. The refraction angle is

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1 \cdot \sin 30° = 1.6 \cdot \sin\theta_2$$
$$\theta_2 = 18.2°$$

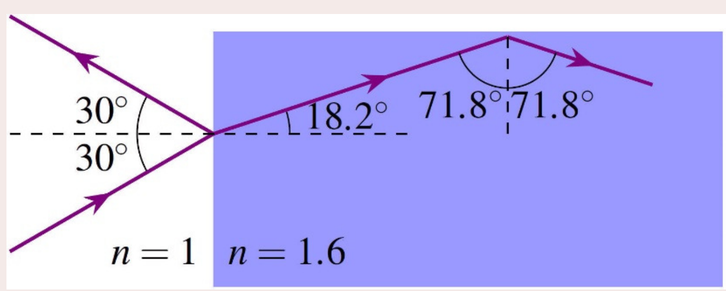
(Since there is a solution, there is some refraction.) This ray will continue and will arrive at another interface, this time with a 71.8° incidence angle.



Once again, there will be some reflection (at 71.8°) and (possibly) some refraction. The refraction angle is found with

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1.6 \cdot \sin 71,8° = 1 \cdot \sin\theta_2$$
$$\sin\theta_2 = 1.52$$

As there is no solution, there is no refraction and the reflection is total here. Therefore, the final answer is as follows.



4.8 DISPERSION

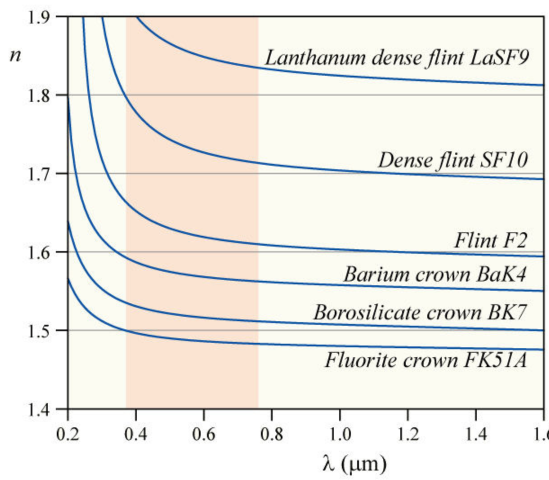
In the previous chapter, it was said that all waves, regardless of their frequency or wavelength, travel at the same speed if the environment is non-dispersive.

For several types of wave, this is often true. For example, air is a non-dispersive medium for sound waves. So, low-pitched sounds and high-pitched sounds travel at the same speed in air. If it were not the case, a sound could be dramatically distorted if, for example, high-pitched sounds were to travel faster than low-pitched sounds. We would then hear high-pitched sounds before the low-pitched sounds. This would drastically alter the beauty of a musical piece if the sound of the bass was heard 1 second after the sound of the guitar.

For light, vacuum is a non-dispersive medium. On the other hand, many substances are dispersive for light. Let's see what happens in this case.

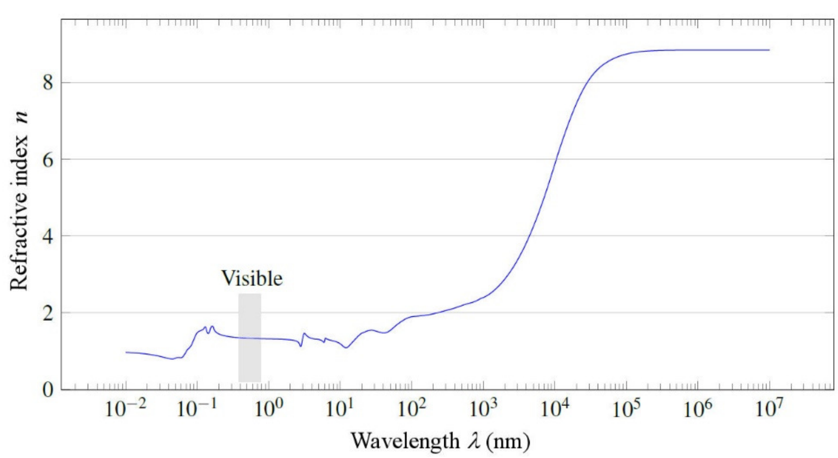
Refractive Index Variations

There is sometimes a slight variation of speed depending on the wavelength of the light in transparent media. This means that the refractive index varies with the wavelength of the light. This graph on the right shows the variation of the index for some transparent media.



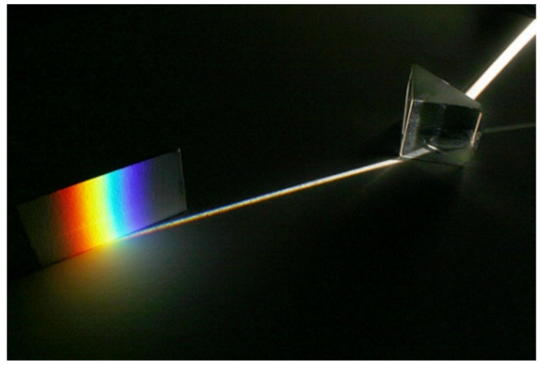
mathscinotes.wordpress.com/2010/10/06/dispersion-power-penalty-modeling-part-1/

The graph seems to suggest that the refractive index always decreases as the wavelength increases, but this is not the case. The following graph shows the value of the refractive index of water for a greater range of wavelengths. Sometimes it increases, sometimes it decreases.



Colours Separation

The change of the index as a function of the wavelength implies a slightly different refraction for each colour. White light, which is a superposition of all the colours, is thus separated into its components by refraction.

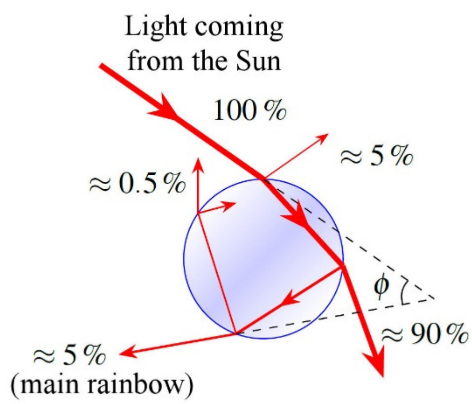


www.e-education.psu.edu/astro801/content/l3_p3.html

For glass, the refractive index for small wavelengths of visible light is often larger than for large wavelengths of visible light. This means that small-wavelength light (such as violet) is more deflected by refraction than large-wavelength light (such as red). This is exactly what can be seen in the diagram. This separation of colours is called *dispersion*.

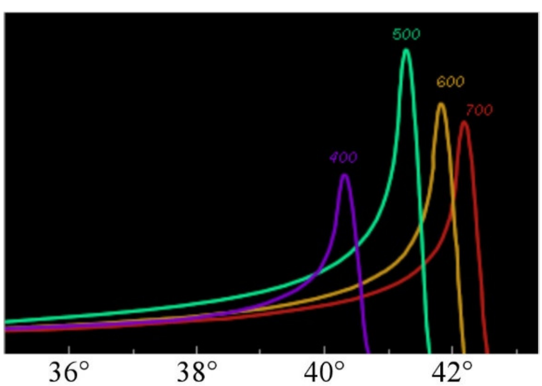
Rainbows

Dispersion is at the origin of a quite spectacular optical phenomenon: the rainbow. A rainbow is produced when the Sun's light is dispersed as it passes through raindrops. Two elements are, therefore, essential to the formation of a rainbow: sunlight and raindrops.



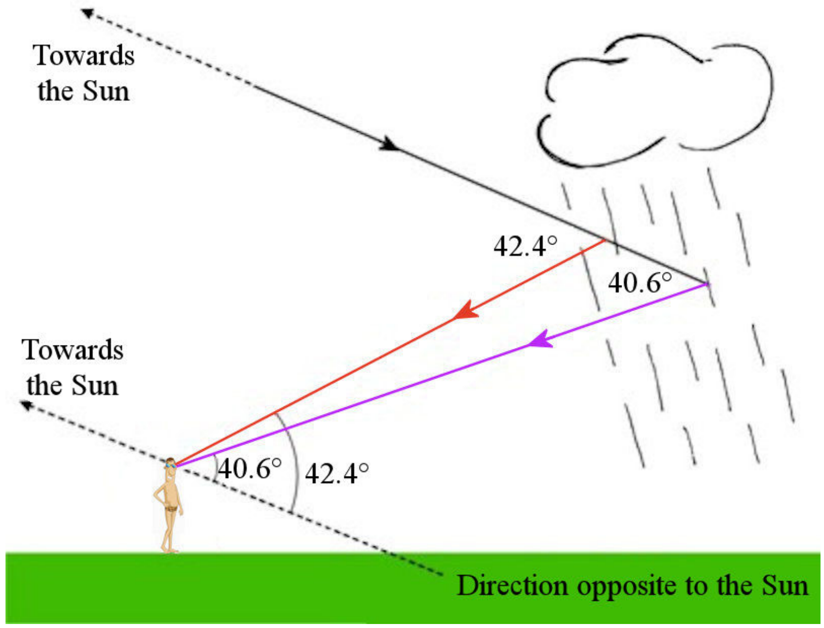
In fact, the light making up a rainbow has a somewhat unusual path: it enters the drop with a refraction, then is reflected in the drop, and then gets out with another refraction. Approximately 5% of the light makes this peculiar journey.

The rays coming from the Sun entering the drop with any angles will emerge with different angles. However, with this path, there is something special happening: there is a specific direction for which the intensity of the light is very large. In addition, the direction of this intense ray varies with the colour because of dispersion. For red light, the intensity is maximum when there are 42.4° (the refractive index is 1.3311) between the initial direction of the rays and the emerging ray (angle ϕ in the diagram). For blue light, the angle is 40.6° (the refractive index is 1.3435). The following graph of light intensity as a function of the angle of deflection shows the difference according to the wavelength.



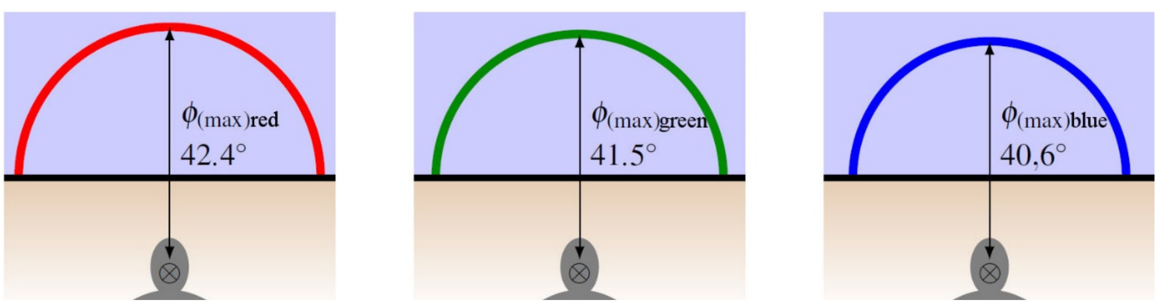
www.atoptics.co.uk/rainbows/primcol.htm

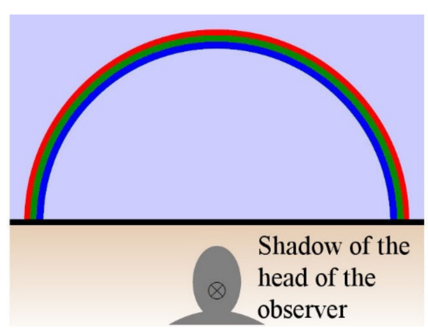
As the angle of the intense ray changes with colour, the following situation is obtained.



Red light can then be seen at 42.4° from the direction opposite to the Sun while blue light can be seen at 40.6° from the direction opposite to the Sun.

However, there are several directions for which the angle is 42.4°. All the points at 42.4° from the direction opposite to the Sun form a circle. There is a circle like this for each colour, but the angle is different. Note that the direction opposite to the Sun is the centre of the shadow of your head.





In these diagrams, there is only a part of a full circle since the arc ends at the ground. (There are no raindrops below the horizon, so no light is coming from this direction.) By combining all these circular arcs of different sizes, a rainbow with violet light on the inside and red light on the outside is obtained.

The arc is higher above the ground if the Sun is low on the horizon. This is so because there are always 42° between the rainbow and the centre of the shadow of your head. If the Sun is low, the shadow is higher and the arc is higher. If the Sun is too high above the horizon (more than

42 °), there is no rainbow, as the highest point of the arc is still below the horizon. The following picture shows the best rainbow it is possible to see.

www.rainbowsymphony.com/blogs/blog/the-science-behind-fully-double-rainbows

Here, the largest possible arc (half of a circle) can be seen. Sometimes, only a portion of the arc can be seen when it is not raining everywhere along the arc. Also, this arc is very high in the sky. This arc was formed shortly before sunset, which means that the Sun was very low above the horizon. If the Sun is low, the rainbow is high since the Sun and the rainbow are always opposed to each other.

Actually, a whole rainbow circle can be seen on a plane. Then, there may be raindrops in every direction and this can be observed.

apod.nasa.gov/apod/ap140930.html

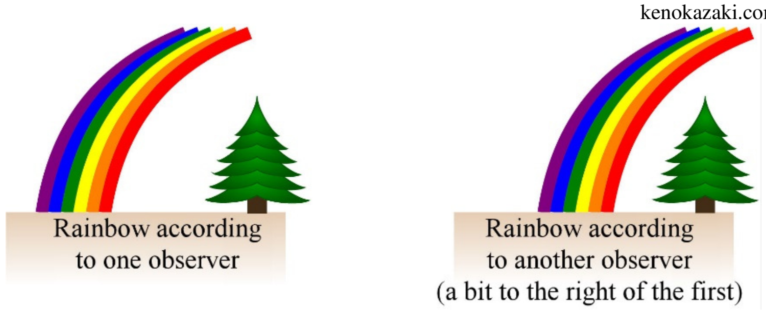
Perhaps you have noticed in the two previous pictures that a second arc may also be present. It is created by light rays making two reflections inside a raindrop (ray with a 0.5% intensity on the first diagram in this section). The angle between the centre of the shadow of your head and this secondary arc is about 72°. This angle changes slightly depending on the colour, but this time, red light is more strongly deflected than violet light. On the secondary arc, red is, therefore, on the inside of the arc and violet on the outside.


wiki.pingry.org/u/physics/index.php/Rainbows,_Reflection,_and_Refraction

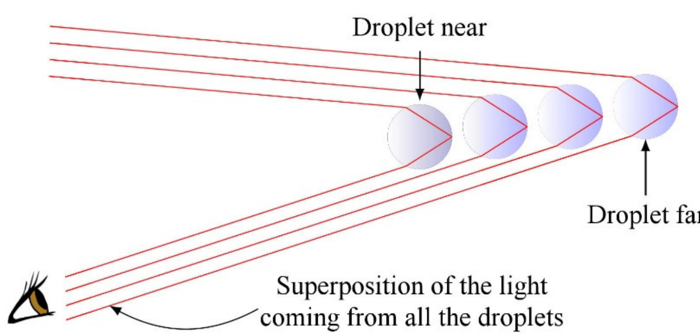
It is sometimes said there is gold at the foot of rainbows. Besides being ridiculous, this is not possible for two reasons.


kenokazaki.com/tag/rainbow/

1) A rainbow is not at the same place for every observer. As it is centred on the shadow of the head of the observer, the centre of the arc is at a different place for each person.



2) A rainbow is not at a precise distance. Every raindrop sends light to form the rainbow and there are drops which are close to the observer and drops which are further away. If there is rain from 1 km to 2 km from the observer, the light he sees comes from every drop between these distances.



If it's not raining, a rainbow can be created with a sprinkler.
http://www.youtube.com/watch?v=_c6HsiixFS8

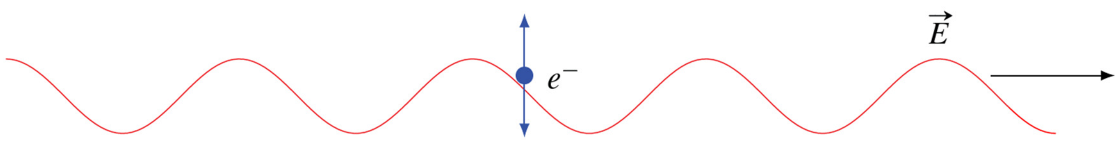
And the colours do not come from contaminants in the water, as stated in the clip.

Here is a guy who likes rainbows a little too much.
http://www.youtube.com/watch?v=OQSNhk5ICTI

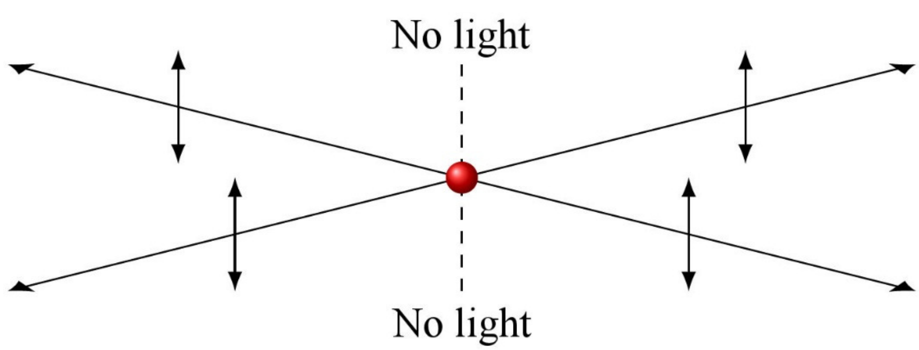
4.9 POLARIZATION BY REFLECTION

A beam of light reflected on a surface can become polarized. To understand why let's consider how light is reflected from a surface.

When light interacts with charged particles, two things happen. First, the oscillating electric field of the wave exerts an oscillating force on the charged particles. This oscillating force makes the charged particles oscillate in the direction of the electric field, so in the direction of the polarization of the wave, with the same frequency as the frequency of the wave.

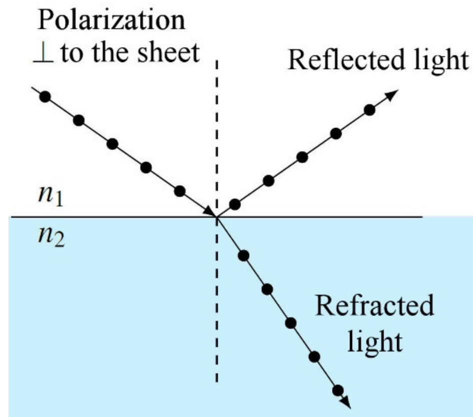


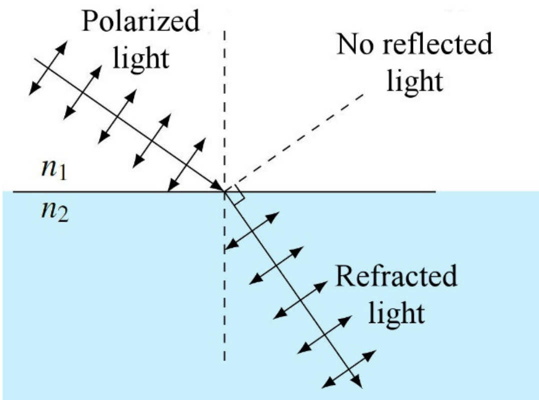
Then, the oscillating charged particle emits an electromagnetic wave with the same frequency as the frequency of the oscillation of the particle. The emitted wave is polarized in the direction of the oscillation of the particle. However, the emission is not isotropic. There is some radiation in the plane perpendicular to the oscillation of the particle, but there is none in the direction of the oscillation of the particle.



Now, let's look at what happens when light is reflected. Let's take a specific example to simplify the reasoning: a beam of light travelling in air reflects and refracts when entering into water. When the electromagnetic wave arrives in the water, charged particles in water started oscillating. In turn, these particles emit an electromagnetic wave. The reflected light comes from these waves emitted by charged particles while the refracted light is a combination of the original wave and the wave emitted by the particles.

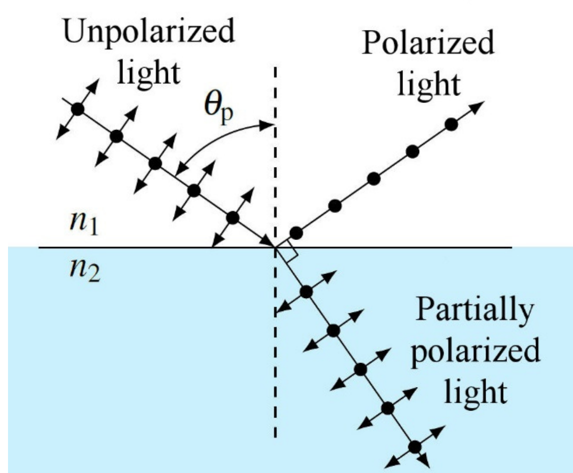
If the light that comes on the surface is polarized in a direction parallel to the surface (i.e. perpendicular to the sheet), the particles of the medium will also oscillate in that direction. As the direction of the reflected wave is perpendicular to the direction of the oscillation of the particles, there is some reflected light with this polarization.



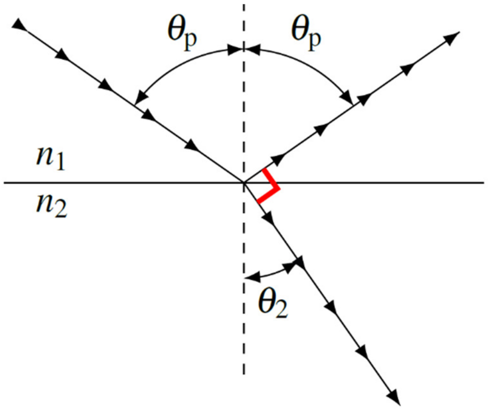


If the polarization of the light is in a direction not parallel to the surface (so along the sheet), then the situation is quite different. Light makes particles in water oscillate in the direction shown in the diagram. This oscillation causes the emission of light, but it is impossible for these oscillations to send light in the direction of the reflection if the reflected light is in the same direction as the oscillation of the particles. In this case, there is no reflected light because the particles which oscillate cannot send light in that direction. As this oscillation is perpendicular to the direction of the refracted ray, there is no light reflected with this polarization if there are 90° between the refracted ray and the reflected ray.

So, if unpolarized light is reflected on a surface, the two polarizations are present. To find out what happens then, the two diagrams obtained for each polarization must be added. The result is shown on the diagram to the right. The two components of the polarization of the light come on the surface. However, as only one of these polarizations can be reflected, the reflected light is polarized. The two components of the polarization can be refracted, and the refracted ray is not polarized. It is, however, partially polarized, because one of the polarization components is stronger than the other. The polarization that can be reflected has lost some of its intensity to the reflection and less intensity is left for the refracted ray compared to the polarization that is only refracted. This is how polarized light can be obtained from unpolarized light with a reflection.



In short, there must be 90° between the reflected ray and the refracted ray to have a completely polarized reflected light beam. This is the situation shown in the diagram. According to Snell's law, we have



$$n_1 \sin\theta_p = n_2 \sin\theta_2$$

Since there are 90° between the reflected ray and the refracted ray, the angle θ_2 is

$$\theta_p + 90° + \theta_2 = 180°$$
$$\theta_2 = 90° - \theta_p$$

Snell's law then becomes

$$n_1 \sin\theta_p = n_2 \sin\theta_2$$
$$n_1 \sin\theta_p = n_2 \sin\left(90° - \theta_p\right)$$
$$n_1 \sin\theta_p = n_2 \cos\theta_p$$

Since $\sin\theta / \cos\theta = \tan\theta$, the end result is

Brewster's Angle or Polarization Angle

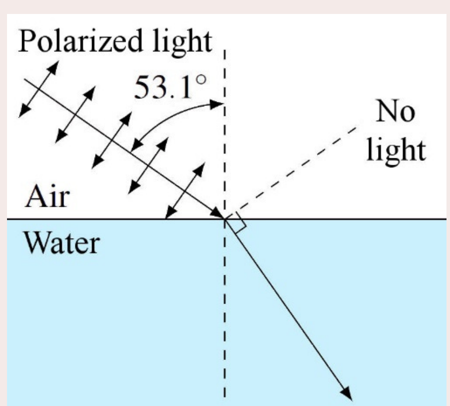
$$\tan\theta_p = \frac{n_2}{n_1}$$

Example 4.9.1

What is the polarization angle for light travelling in air and reflected from the surface of water?

The angle is

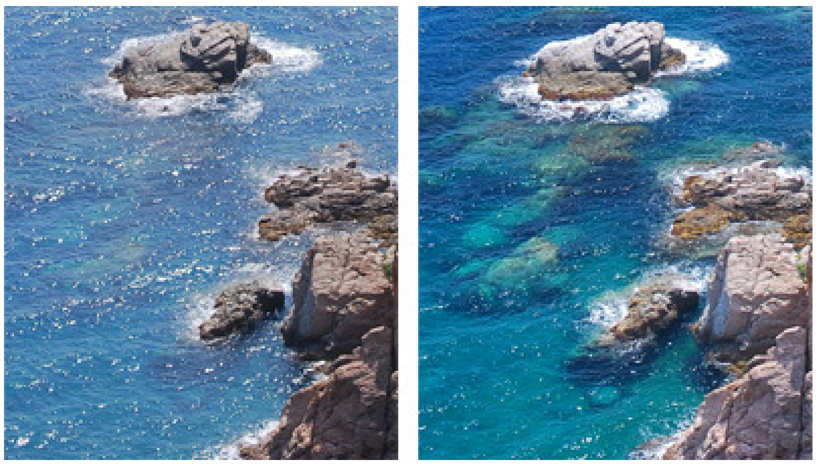
$$\tan\theta_p = \frac{n_2}{n_1}$$
$$\tan\theta_p = \frac{1.33}{1}$$
$$\theta_p = 53.1°$$



This means that the light polarized in the direction shown in the diagram cannot be reflected on water if the angle of incidence is 53.1°.

If the angle of incidence is not 53.1°, then there will be some reflected light. The farther away the angle of polarization is from the angle of incidence, the greater is the intensity of the reflected light.

This effect can be seen in the following images. In this first image, everything is as usual. The bottom of the sea is hard to see because the light reflected by the surface is brighter than the light coming from the bottom of the sea. In the picture to the right, a polarizing filter having a vertical axis is used. As the light reflected on the water is horizontally polarized, the filter blocks the reflected light. Now, the light coming from the bottom of the sea is more intense than the reflected light, and the bottom of the sea can be seen.



www.digital-photography-tips.net/Stay_Focussed-Newsletter-March-2013.html

The following left image shows reflected light off the car. Using a polarizing filter with a horizontal axis, the light reflected on the vertical surfaces (which is vertically polarized) is now blocked. The reflected light is now gone (image to the right).

fotografium.com/bw-55mm-polarize-filtre#.UxyRqvl5PTo

Reflected light is rarely completely polarized. For this to happen, the angle of incidence must be exactly equal to the polarization angle. But even if the angle is not exactly equal to the angle of polarization, the intensity of the polarization parallel to the surface is often stronger than the other component in the reflected light. There is a partial polarization. A filter can then block this strongest polarization and reflected light is less intense with the filter. This phenomenon can be seen in the following image. The light reflected off a lake is seen through a polarizing filter with a vertical axis.

paraselene.de/cgi/bin?_SID=7e65d76b84105709c35aeec86f67c20bdca7aabd00268925652735&_bereich=artikel&_aktion=detail&idartikel=116150&_sprache=paraselene_englisch

At the bottom of the picture, there is virtually no light reflected on the lake. This is because the light coming from this place comes on the lake with an angle of incidence close to the polarization angle. This strongly polarized reflected light is then almost all blocked by the polarizing filter and no reflected light can be seen. Elsewhere on the lake reflected light can be seen. The reflection seen in these places comes from light having an angle of incidence not that close to the polarization angle. In this case, the reflected light is only partially polarized. Although the filter blocks the horizontal polarization, the other polarization remains, and some reflected light can be seen.

Polarized glasses are simply polarizing filters with a vertical axis. The effect is not spectacular with unpolarized light: they simply absorb half the light. The light is polarized after its passage through the glasses, but the human eye is not sensitive to polarization, which means that there is no difference between light polarized in one direction or another or between unpolarized light and polarized light. There is, however, a difference with reflected light. Reflected light is polarized in a direction parallel to the surface so that light reflected on a lake or on the floor is horizontally polarized (totally or partially). With glasses having a vertical axis, this polarized reflected light is blocked. The reflected light is thus strongly attenuated with polarized glasses. This is shown in this video.
http://www.youtube.com/watch?v=MNbg4Go8NR0

4.10 BIREFRINGENCE

In 1669, the Danish scientist Rasmus Bartholin discovers a strange phenomenon: when a calcite crystal is placed over a text, two images of the text are seen! Note that the two images have exactly the same intensity.

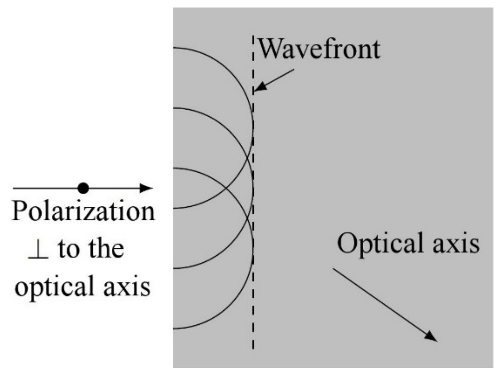
faculty.kutztown.edu/friehauf/beer/ (yes, it's correct)

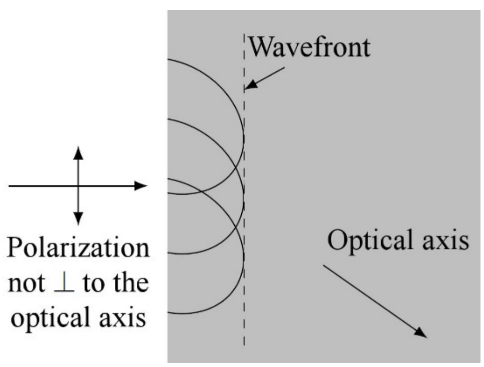
This phenomenon is called *double refraction* or *birefringence* because the image splitting comes from the fact that the refraction is different for each image when light passes through the crystal. This idea led Newton to mention that light seems to have two different aspects, like the two poles of a magnet, which brought the name *polarization*.

Explanation of birefringence

Some crystals are not isotropic (this happens if the molecules are all aligned in the same direction). This is often means that light can travel faster in one direction in the crystal for a certain polarization. This direction is indicated by the optic axis of the crystal.

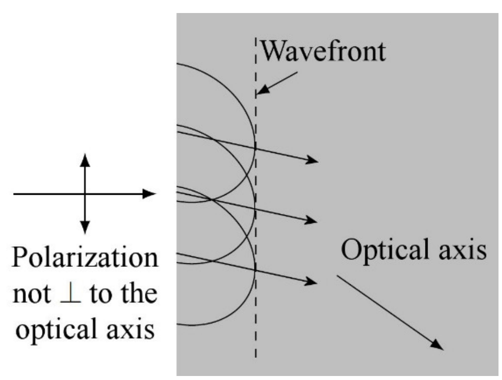
Let's see what this means for light polarized in a direction perpendicular to the optic axis of the crystal. This polarization creates waves that propagate at the same speed in every direction (circles in the diagram) and so it propagates normally in the substance (perpendicular to the wavefront). This polarization forms the *ordinary ray*.



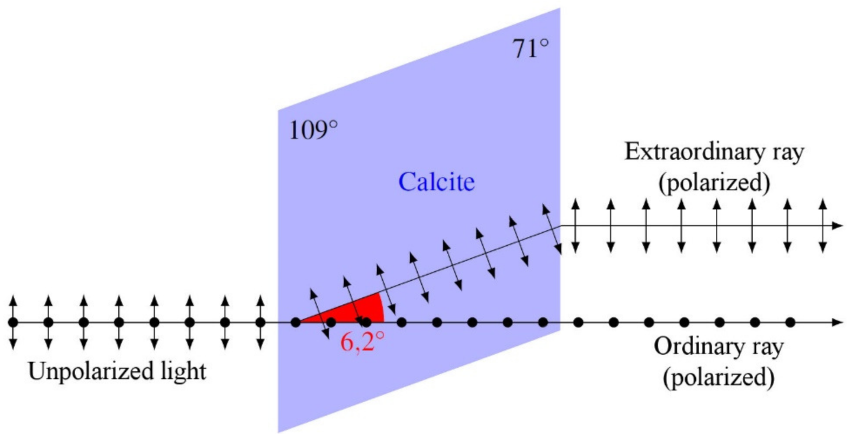


For the other polarization, the wave is propagating faster in the direction of the optic axis. The waves are not circles anymore but ellipses stretched in the direction of the optic axis.

In a previous chapter, it was said that the rays are always perpendicular to the wavefronts. This is true if the speed of light is the same in every direction but this is no longer true if the speed is different, as here. The direction is rather as follows.



This ray goes from the centre of the ellipse to the point of the ellipse tangent to the wavefront. This means that ray does not travels in the expected direction (which would have been directly towards the right here because the angle of incidence was zero). The ray travelling in this unexpected direction is called the *extraordinary ray*. For calcite, the angle between the ordinary ray and the extraordinary ray is 6.2°.



If unpolarized light passes through such a crystal, then the ordinary ray and the extraordinary ray are present at the same time. Unpolarized light is thus separated into two polarized rays with the same intensity.

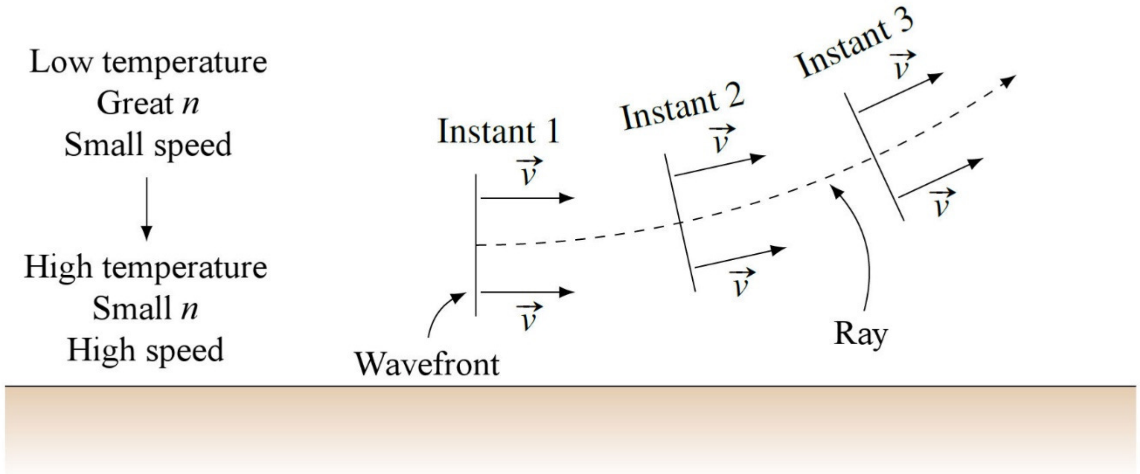
With a polarizing filter, it is quite easy to see that the two images obtained with a crystal of calcite are polarized. By rotating the filter, it is possible to switch from one image to the other.
http://www.youtube.com/watch?v=WdrYRJfiUv0

The study of the passage of light in anisotropic crystals is quite complex. The refractive index then becomes a 3 x 3 matrix, and it is possible to have a refraction with a certain angle even if the incidence angle is zero (as for the extraordinary ray in calcite). These complex cases will not be explored in these notes.

4.11 REFRACTION WHEN THE SPEED OF THE WAVE CHANGES CONTINUOUSLY

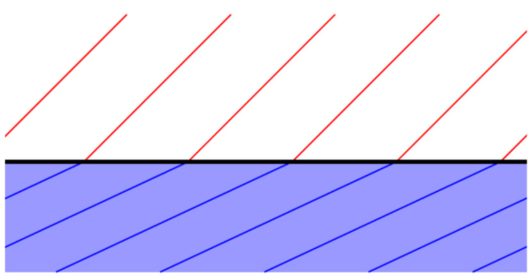
Now let's see what happens if the speed of the wave changes continuously from one place to another. This can happen, for example, with light if the refractive index of air changes gradually with height above the ground. This can easily happen since the refractive index of air depends on the air density and thus on its temperature. Most of the light from the Sun passes through the atmosphere and heats up the ground, which in turn heats up the air. Thus, the air near the ground is often warmer than the air at a higher altitude, thereby reducing its density and its refractive index. The speed of the wave is, therefore, greater close to the ground.

Let's consider a vertical wavefront (instant 1) to illustrate what happens then. Remember that the direction of propagation of a wave must be perpendicular to the wavefront.



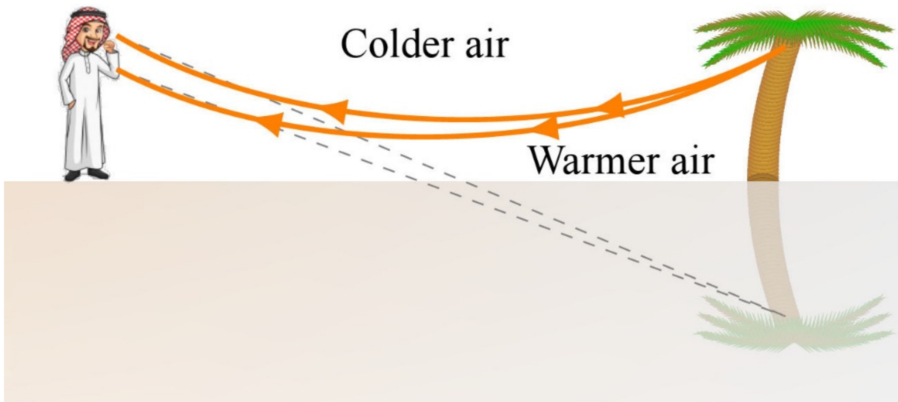
As the speed of the wave is not the same everywhere, different parts of the wavefront travel at different speeds. In our example, the speed of the wavefront is greater at the bottom of the wavefront than at the top. Therefore, the bottom of the wavefront will overtake the top of the wavefront. Then, the wavefront will not be vertical anymore but will be tilted (instant 2). This also implies that the direction of propagation of the wave has changed because it must be perpendicular to the wavefront. The wave is now travelling a little upwards. The more the bottom of the wave will overtake the top of the wave, the more the direction of propagation of the wave will be deflected upwards.

The shape of the path can even be calculated if the change of the refraction index with height is known. If you wish, you can see an example in this document.
http://physique.merici.ca/waves/trajectory-light.pdf

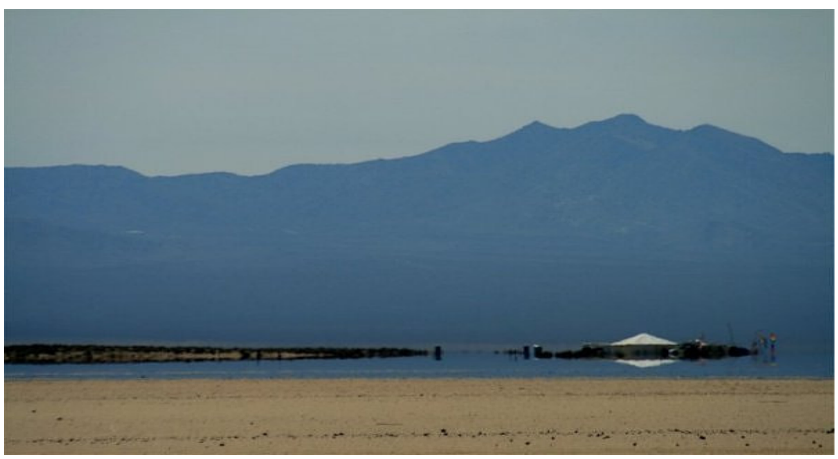


This deviation is also a refraction because it is similar to what happens when light passes from one medium to another. When the wavefront in air enters into water, its speed decreases. The part of the wavefront still in air then overtakes the part of the wave in water, and this changes the orientation of the wavefront and the direction of propagation of the wave.

Now, let's suppose that some light is coming from a palm tree and that this light is moving towards the ground. Since the wave travels a bit downwards, the top of the wavefront has the lead over the bottom of the wavefront at the start (look at the wavefront on the right part of the next diagram). But as the bottom of the wavefront travels faster, it will slowly overtake the top of the wavefront and take the lead. This implies that the direction of propagation of the wave will slowly change so that it will be directed a bit upwards after a certain time. (The path is way more curved in the diagram than it actually is.)



When the observer receives the light, he interprets the light as he always does. He thinks that the light rays travel in a straight line. The observer, therefore, thinks that the palm tree is at the point of intersection of the straight lines in the direction of the rays he sees (dotted lines). He, therefore, sees the palm tree while looking towards the ground! If there were no palm tree, he would see the sky in that direction. As the sky is blue, he would see something blue while looking towards the ground and he'll think that there is a lake there. You can see this effect on the following pictures.

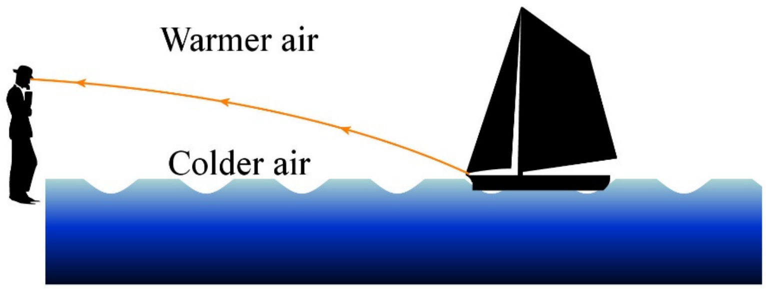


www.sflorg.com/nature_trail/atmospheric/atmospheric_10

www.crystalinks.com/mirage.html

The mirage seen in this video (at 1:10) is quite impressive. It really looks like there is a lake.
http://www.youtube.com/watch?v=HzIBmuLHMSE

This effect can also occur in the opposite direction above water if the air is warmer than the water. The cold water cools the air above the lake, and the refractive index is greater near the water. The light, therefore, deflects in the direction shown in this diagram.



You can see a demonstration of such a refraction in this video.
http://www.youtube.com/watch?v=BV3aRiL64Ak
Corn syrup has been added to the water in a tank. After a few days, the syrup is dissolved, but the water is denser near the bottom, resulting in a greater refractive index at the bottom of the tank.

Over a cold body of water, it is then possible to see deformed images such as the one shown in this picture.

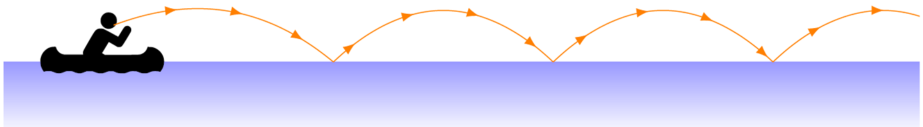
scribol.com/featured/a-world-of-mirages-10-dazzling-optical-phenomena-from-round-the-globe/7497

Finally, more complex temperature variation can deflect the light in a dramatic manner, as can be seen in this next image.

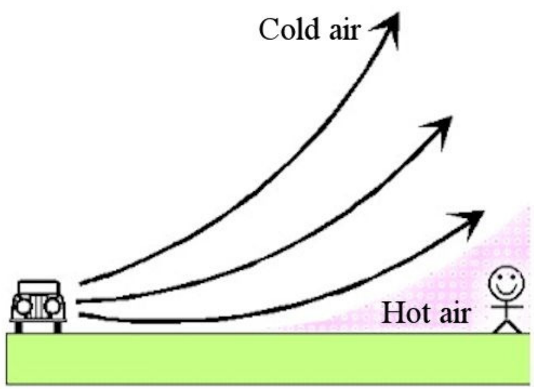
www.flickr.com/photos/dcartiersr/2283060804/

This phenomenon is not specific to light since several types of waves can be refracted. Sound can also be refracted since the speed of sound depends on the temperature and a large speed has exactly the same effect as a small refractive index.

For example, suppose two people are talking on a boat on the surface of a lake. During the summer, the air is generally warmer than the lake, and this makes the air near the surface of the lake colder than the air above. The sound near the lake, therefore, travels at a lower speed than at a higher altitude. This speed difference then deflects the trajectory of the sound towards the surface of the lake. When the sound comes back on the lake, it is reflected. Refraction again deflects the trajectory towards the surface of the lake where the sound is again reflected. This process repeats itself until the sound reaches the edge of the lake.

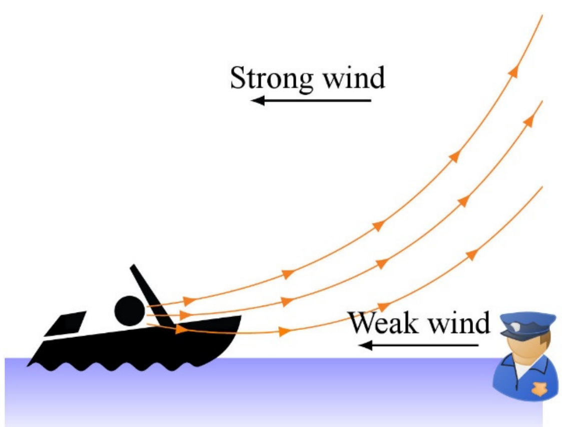


That’s why, on a hot summer day, every sound created on the lake, like motors, can be easily heard on the shore.



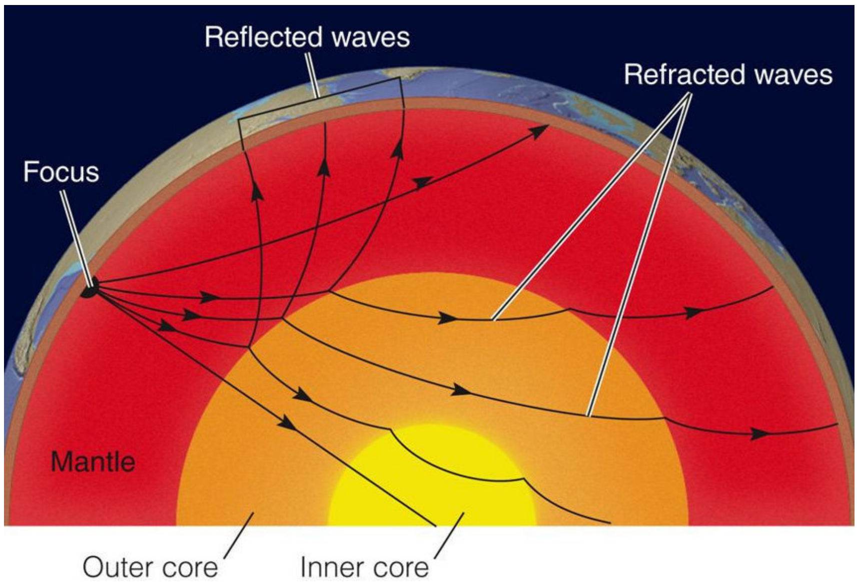
www.pa.op.dlr.de/acoustics/essay1/brechung_en.html

If the temperature of the air gets lower with altitude, the deviation of the sound is directed upwards instead. In this image, the person does not even hear the noise of the car since the sound is deflected and pass over his head. This is what is occurring at the surface of a lake at night when the lake is warmer than the air. You can have a big party on the lake, and nobody hears you around the lake…



Be aware that a large wind can also have a similar effect with sound. In the diagram to the left, the wind gets stronger with altitude (which is usually the case). This has the effect of slowing down the top of wavefront more than the bottom of the wavefront. Therefore, the orientation of the wavefront slowly changes as shown in the diagram, which has the effect of changing the direction of propagation of the wave. The sound is, therefore, deflected towards the sky in this situation, and the police officer does not hear the speedboat. Then, if you shout something at someone who is far away upwind, it is possible that the person will not hear you at all. This is not because the wind has stopped the sound (which is impossible since the speed of sound is much greater than the speed of the wind), but rather because the wind, whose speed changes with altitude, has deflected the sound upwards.

The waves generated by earthquakes also undergo refraction inside the Earth. The waves undergo a sudden refraction from the mantle to the outer core and from the outer core to the inner core. There is also a slow refraction inside of each of these structures because the speed of the wave changes with the pressure and the temperature of the rocks.



slideplayer.com/slide/7080633/

By measuring waves that arrive at the surface of the Earth, the way they have been refracted can be calculated and then the internal structure of the Earth can be inferred.

SUMMARY OF EQUATIONS

Frequency and Wavelength in a Change of Medium

$$f_1 = f_2$$
$$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

Amplitudes of the Reflected and Transmitted Waves

$$A_R = \frac{Z_1 - Z_2}{Z_1 + Z_2} A$$
$$A_T = \frac{2Z_1}{Z_1 + Z_2} A$$

Rules for Reflection and Transmission

As the difference of impedance of the media gets larger, the reflection is larger and the transmission is smaller.

If the impedances of the media are identical, the wave is completely transmitted and there is no reflection.

Reflected and Transmitted Waves Inversion

If $Z_2 > Z_1$, the reflected wave is inverted.

Si $Z_2 < Z_1$, the reflected wave is not inverted.

The transmitted wave is never inverted.

Wavelength Change in a Transparent Substance

$$\lambda_{\text{substance}} = \frac{\lambda_{\text{vacuum}}}{n}$$

Amplitudes of the Reflected and Transmitted Light Waves

$$E_{0R} = \frac{n_1 - n_2}{n_1 + n_2} E_0$$

$$E_{0T} = \frac{2n_1}{n_1 + n_2} E_0$$

Rules for Reflection and Transmission of Light

The wave passes from medium 1 (index n_1) to medium 2 (index n_2).
If $n_2 > n_1$, the reflected wave is inverted.
If $n_2 < n_1$, the reflected wave is not inverted.
The transmitted wave is never inverted.

Law of Reflection

$$\theta_1 = \theta_2$$

Law of Refraction

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

Law of Refraction for Light

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Fermat's Principle

Passing from one point to another, light always follows the path that requires the least time to travel.

Critical Angle for Total Reflection

$$\sin\theta_c = \frac{v_1}{v_2}$$

If θ_1 is larger than θ_c, then there is total reflection.

Critical Angle for Total Reflection With Light

$$\sin\theta_c = \frac{n_2}{n_1}$$

If θ_1 is larger than θ_c, then there is total reflection.

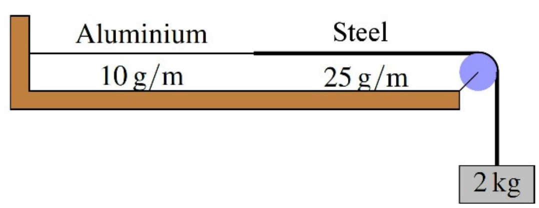
Brewster's Angle or Polarization Angle

$$\tan\theta_p = \frac{n_2}{n_1}$$

EXERCISES

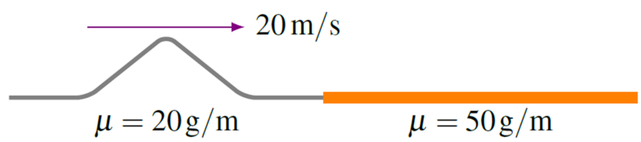
4.2 Waves on a Rope

1. In the situation shown in the diagram, a sine wave with a wavelength of 20 cm and an amplitude of 5 mm starts on the left side of the aluminum wire and moves towards the steel wire.



   a) What is the wave speed in the aluminum wire?
   b) What is the frequency of the wave when it is travelling in the aluminum wire?
   c) What is the wave speed in the steel wire?
   d) What is the frequency of the wave when it is travelling in the steel wire?
   e) What is the wavelength of the wave when it is travelling in steel wire?
   f) What is the impedance of the aluminum wire?
   g) What is the impedance of the steel wire?
   h) What is the amplitude of the reflected wave? (Specify if the reflected wave is inverted.)
   i) What is the amplitude of the transmitted wave?
   j) What percentage of the power of the initial wave is transmitted in the steel cable?

2. A wave on a string arrives at a place where the linear density of the string changes.



a) In what direction (upwards or downwards) will the transmitted wave be?
b) In what direction (upwards or downwards) will the reflected wave be?
c) What will the speed of the transmitted wave be?
d) What will the speed of the reflected wave be?

3. A wave is travelling on a string with a certain impedance. The wave then arrives at a junction between two strings. Passing from one string to the other, half the power of the wave is transmitted and the other half is reflected. Knowing that the impedance of the second string (Z_2) is greater than the impedance of the first string (Z_1), determine the ratio of the impedances of the two strings (Z_2/Z_1)?

4.3 Waves Arriving Perpendicularly to a Surface

4.
a) What is the impedance of air if its density is 1.3 kg/m³, and its temperature is 15 °C?
b) What is the impedance of water if its density is 1000 kg/m³ and the speed of sound waves in water is 1520 m/s?
c) Can we conclude that sound can easily travel from air to water?

5. A 200 Hz sound wave having an intensity of 60 dB in air enters into water.

a) What is the intensity of the transmitted sound in water and of the reflected sound in air?
b) Show that the intensity of the transmitted sound is always 29.3 dB lower than the initial intensity when a sound, regardless of its initial intensity and frequency, passes from air to water.

Use the following data

Air: speed of sound = 330 m/s density = 1.3 kg/m³
Water: speed of sound = 1450 m/s density = 1000 kg/m³

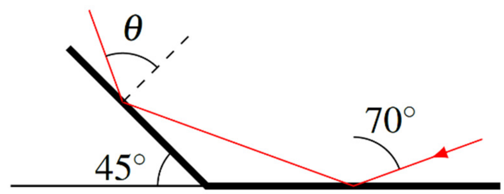
6. A light wave has a 500 nm wavelength in vacuum. What will be the wavelength of this wave if it enters into water (which has a refractive index of 1.33)?

7. A light wave has a 600 nm wavelength in vacuum and a 480 nm wavelength in a transparent substance. What is the speed of light in this substance?

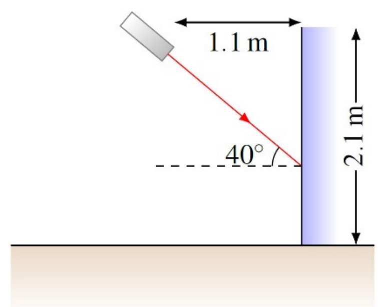
8. A light wave with an intensity of 5 W/m² passes from air to water (n = 1.33). What are the intensities of the transmitted and reflected light?

9. Light travelling in air enters into a transparent substance. It is then measured that the intensity of the transmitted light is 92% of the initial intensity. What is the index of refraction of this substance?

4.4 Law of Reflection

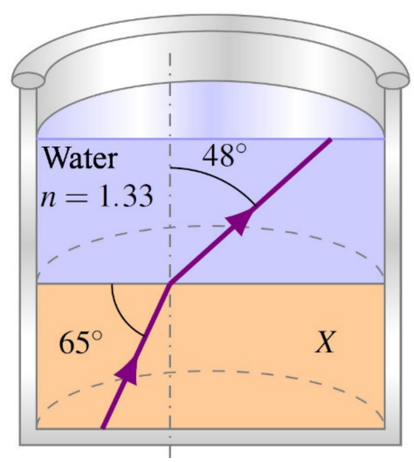


10. What is the angle θ in this diagram?



11. How far from the wall will the laser touch the ground in the configuration shown in the diagram?

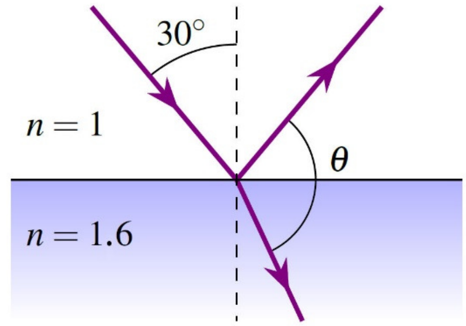
4.5 Law of Refraction



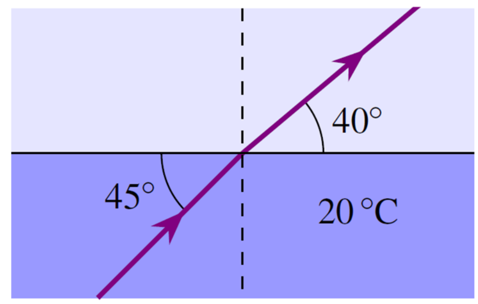
12. A ray of light passes from a transparent substance X to water as shown in the diagram.

   a) What is the refractive index of substance X?
   b) What is the speed of light in substance X?

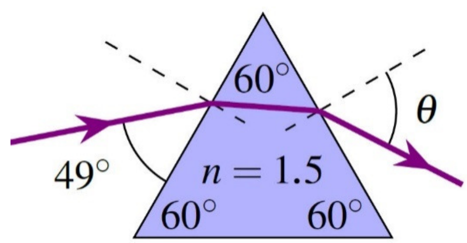
13. What is the angle θ on this diagram?



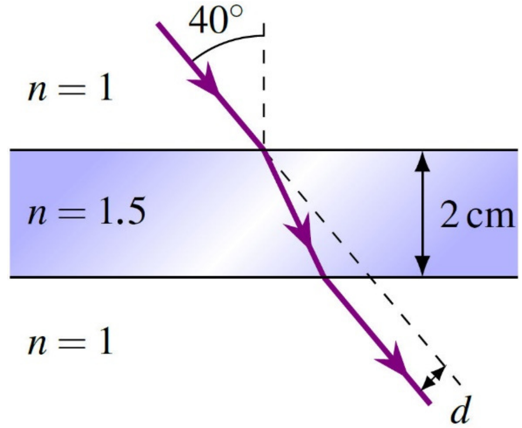
14. A sound wave refracts as shown in the diagram. It passes from an area where the air is at 20 °C to a region where air has a different temperature. What is the temperature of the air in this region?



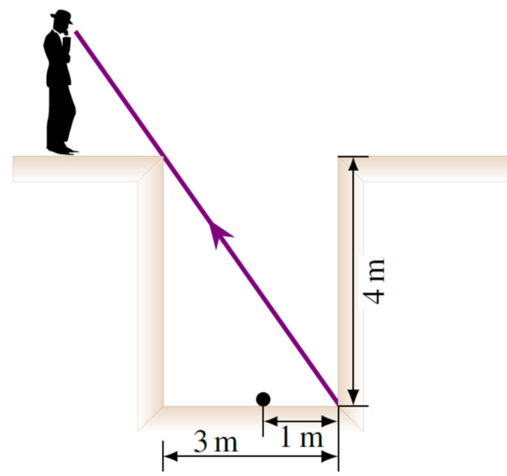
15. A beam of light passes through a prism. What is the angle θ in this diagram?



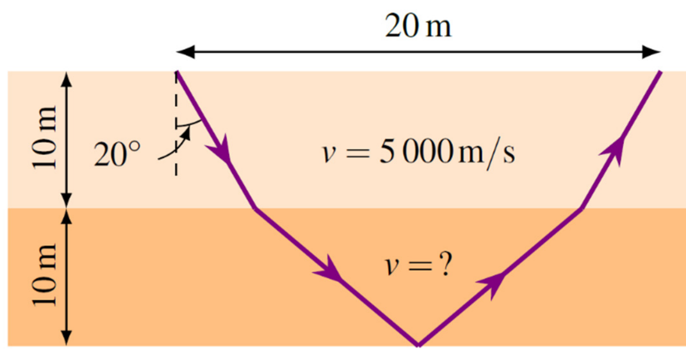
16. A ray of light passes through a plate of glass as illustrated in the diagram. After passing through the glass, the beam is shifted a distance d compared to its original trajectory. What is the value of d?



17. In the situation shown in the diagram, Sonia cannot see the black spot at the bottom of the hole. She can, however, see it if the hole is completely filled with a transparent substance. What should be the minimum refractive index of the substance so that Sonia could see the black spot at the bottom of the hole?



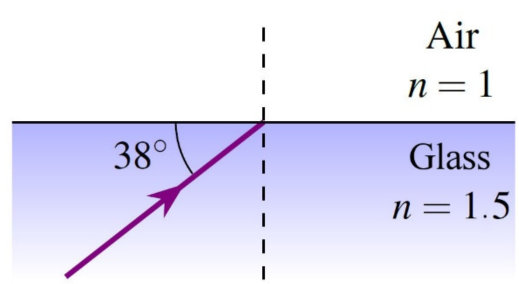
18. A seismic wave propagates into the ground. However, there is a change of rock type, as shown in the diagram. From the information given on the diagram, determine the speed of the wave in the rock at the highest depth.



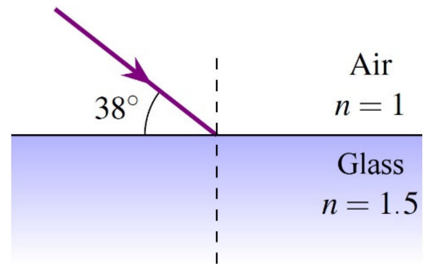
4.7 Total Reflection

19. The critical angle is 60° when light arrives at the interface between an unknown substance and water, coming from the water. What is the speed of light in the substance?

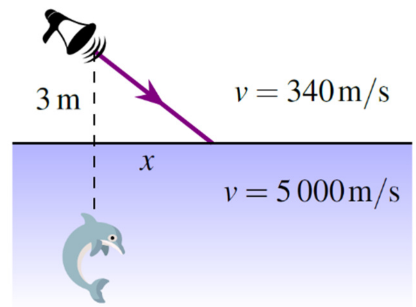
20. Light arrives at an interface between glass and air as shown in the diagram. Will the light make a total reflection or not?



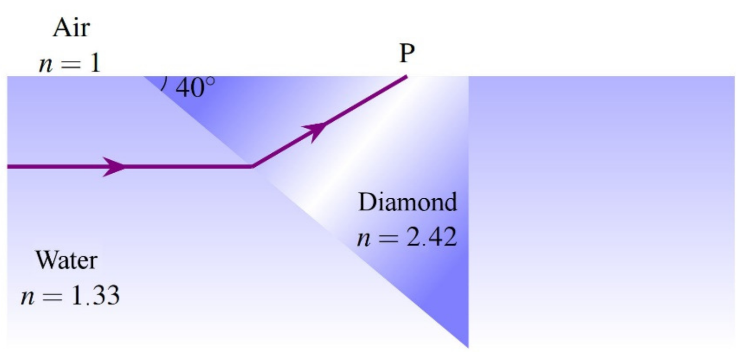
21. Light arrives at an interface between glass and air as shown in the diagram. Will the light make a total reflection or not?



22. We want to communicate with a dolphin with sound waves. Using the information given on the diagram, determine the maximum distance x for which sound can enter into the water.

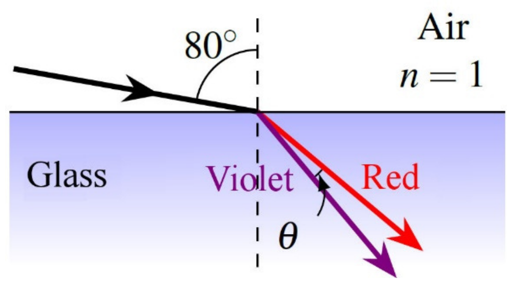


23. Light passes from water to diamond and then arrives at point P where there is an interface between diamond and air as shown in the diagram. Will the light make a total reflection at point P?



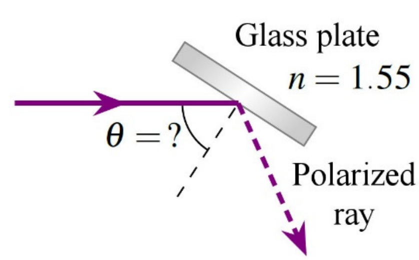
4.8 Dispersion

24. White light propagating in air comes with an incidence angle of 80° on the surface of a piece of glass. With the refraction in the glass, the colours are separated since the refractive index is not the same depending on the colours. The index goes from 1.66 for violet to 1.62 for red. What is the angle between the violet ray and the red ray in the glass?

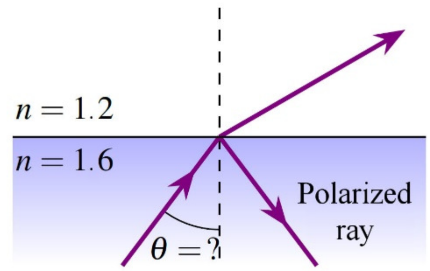


4.9 Polarization by Reflection

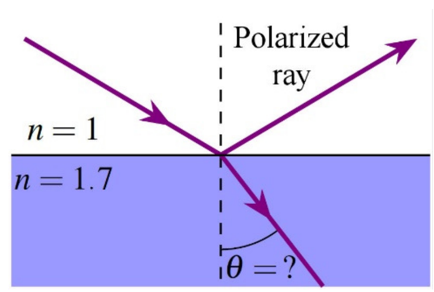
25. What should the angle in this diagram be in order to have a reflected ray of light totally polarized?



26. What should the angle in this diagram be in order to have a reflected ray of light totally polarized?



27. Light reflects off a glass surface. The refractive index of the glass is 1.7. What is the angle between the normal and the refracted ray (θ in the diagram) if the reflected ray is completely polarized?



28. Light arrives at an interface between two media (and one of the media is not necessarily air). The critical angle for internal reflection is 48°.

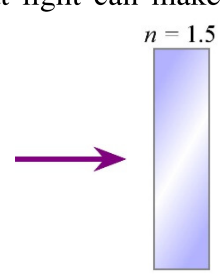
   a) What is the angle of polarization?
   b) Is it possible to have a totally polarized total reflection?

Challenges

(Questions more difficult than the exam questions.)

29. A beam of light passes through a layer of glass. Knowing that light can make multiple reflections between the two surfaces of the glass plate, calculate the percentages of the transmitted and reflected light.

   (For those of you who know a little bit more physics, let's assume that white light is used, which means that the effects of interference (see Chapter 7) can be ignored since there are several wavelengths.)



ANSWERS

4.2 Waves on a Rope

1. a) 44.27 m/s b) 221.4 Hz c) 28 m/s d) 221.4 Hz e) 12.65 cm f) 0.4427 kg/s g) 0.7 kg/s h) It's an inverted wave with a 1.126 mm amplitude i) 3.874 mm j) 94.9%
2. a) upwards b) downwards c) 12.65 m/s d) 20 m/s
3. $Z_2/Z_2 = 5.828$

4.3 Waves Arriving Perpendicularly to a Surface

4. a) 442.4 kg/m²s b) 1 520 000 kg/m²s c) No, because the impedance of the water is too different from that of air (3435 times greater)
5. a) Transmitted wave: 30.7 dB Reflected wave: 59.99 dB
6. 375.9 m
7. 2.4×10^8 m/s
8. Transmitted: 4.9 W/m² Reflected: 0.1 W/m²
9. 1.7888

4.4 Law of Reflection

10. 65°
11. 1.403 m

4.5 Law of Refraction

12. a) 2.34 b) 1.28×10^8 m/s
13. 131.8°
14. 70.9 °C
15. 57.16°
16. 5.59 mm
17. 1.342
18. 7846 m/s

4.7 Total Reflection

19. 2.60×10^8 m/s
20. There is a total reflection
21. There is no total reflection
22. 20.45 cm
23. There is a total reflection

4.8 Dispersion

24. 1.05°

4.9 Polarization by Reflection

25. 57.2°
26. 36.9°
27. 30.5°
28. b) 36.6° b) No

Challenges

29. Transmitted: 92.31% Reflected: 7.69%