Chapter 2 Solutions

1. a) The period is

$$T = \frac{1}{f} = \frac{1}{400Hz} = 0.0025s$$

b) We have

$$v = \lambda f$$
$$350\tfrac{m}{s} = \lambda \cdot 400Hz$$
$$\lambda = 0.875m$$

2. The wavelength will be found with

$$v = \lambda f$$

To calculate to wavelength, the speed and the frequency must be known.

The speed of the wave is

$$\begin{aligned} v &= \frac{\Delta x}{\Delta t} \\ &= \frac{30m}{4s} \\ &= 7.5\tfrac{m}{s} \end{aligned}$$

The wave period is

$$T = \frac{4s}{20 \text{ oscillations}} = 0.2s$$

The frequency is therefore

$$f = \frac{1}{T} = \frac{1}{0.2s} = 5Hz$$

Thus, the wavelength is

$$v = \lambda f$$
$$7.5\tfrac{m}{s} = \lambda \cdot 5Hz$$
$$\lambda = 1.5m$$

3. a)

According to the figure, the wavelength is 20 cm. The frequency is then found with

$$v = \lambda f$$
$$40\tfrac{m}{s} = 0.2m \cdot f$$
$$f = 200Hz$$

b) As the rope makes a harmonic oscillation, its maximum speed is

$$v_{\max} = \omega A$$

With the figure, we note that the amplitude is 6 cm. So the maximum speed is

$$\begin{aligned} v_{\max} &= \left(2\pi f\right) A \\ &= \left(2\pi \cdot 200Hz\right) \cdot 0.06m \\ &= 75.4\tfrac{m}{s} \end{aligned}$$

4. a) Since there a + in front of ωt, the wave is travelling towards the negative x-axis.

b) In the equation, we have k = 10 rad/m. the wavelength is thus

$$k = \frac{2\pi}{\lambda}$$
$$10m^{-1} = \frac{2\pi}{\lambda}$$
$$\lambda = 0.6283m$$

c) The speed is

$$\begin{aligned} v &= \frac{\omega}{k} \\ &= \frac{200s^{-1}}{10m^{-1}} \\ &= 20\tfrac{m}{s} \end{aligned}$$

d) The formula for the velocity of the rope is

$$\begin{aligned} v_y &= A\omega\cos\left(kx+\omega t+\phi\right) \\ &= 0.2m\cdot 200\tfrac{rad}{s}\cdot\cos\left(10\tfrac{rad}{m}\cdot x+200\tfrac{rad}{s}\cdot t+\frac{\pi}{4}rad\right) \\ &= 40\tfrac{m}{s}\cdot\cos\left(10\tfrac{rad}{m}\cdot x+200\tfrac{rad}{s}\cdot t+\frac{\pi}{4}rad\right) \end{aligned}$$

We could also have done the derivative of the position

$$\begin{aligned} v_y &= \frac{\partial y}{\partial t} \\ &= \frac{\partial}{\partial t}\left(0.2m\cdot\sin\left(10\tfrac{rad}{m}\cdot x+200\tfrac{rad}{s}\cdot t+\frac{\pi}{4}rad\right)\right) \\ &= 0.2m\cdot 200\tfrac{rad}{s}\cdot\cos\left(10\tfrac{rad}{m}\cdot x+200\tfrac{rad}{s}\cdot t+\frac{\pi}{4}rad\right) \\ &= 40\tfrac{m}{s}\cdot\cos\left(10\tfrac{rad}{m}\cdot x+200\tfrac{rad}{s}\cdot t+\frac{\pi}{4}rad\right) \end{aligned}$$

At x = 1 m and t = 1 s, the velocity of the rope is then

$$\begin{aligned} v_y &= 40\tfrac{m}{s}\cdot\cos\left(10\tfrac{rad}{m}\cdot 1m+200\tfrac{rad}{s}\cdot 1s+\frac{\pi}{4}rad\right) \\ &= 40\tfrac{m}{s}\cdot\cos\left(210rad+\frac{\pi}{4}rad\right) \\ &= -38.23\tfrac{m}{s} \end{aligned}$$

5. We have two equations:

$$\begin{aligned} v_{\max} &= A\omega \\ 2\tfrac{m}{s} &= A\omega \end{aligned}$$

and

$$a_{max} = A\omega^2$$
$$200\tfrac{m}{s^2} = A\omega^2$$

Dividing the second equation by the first, we get

$$\frac{200\tfrac{m}{s^2}}{2\tfrac{m}{s}} = \frac{A\omega^2}{A\omega}$$
$$100s^{-1} = \omega$$

Therefore,

$$2\tfrac{m}{s} = A\omega$$
$$2\tfrac{m}{s} = A\cdot 100s^{-1}$$
$$A = 0,02m$$

6. a) The maximum speed is

$$\begin{aligned} v_{max} &= A\omega \\ &= 0.1m\cdot 50s^{-1} \\ &= 5\tfrac{m}{s} \end{aligned}$$

b) The maximum acceleration is

$$\begin{aligned} a_{max} &= A\omega^2 \\ &= 0.1m\cdot\left(50s^{-1}\right)^2 \\ &= 250\tfrac{m}{s^2} \end{aligned}$$

c) When the crest is at the point x = 1 m, then the displacement at this point is equal to the amplitude. So we have

$$0.1m = 0.1m\cdot\sin\left(5\tfrac{rad}{m}\cdot 1m - 50\tfrac{rad}{s}\cdot t\right)$$

Solving for t, the result is

$$0.1m = 0.1m \cdot \sin\left(5\tfrac{rad}{m} \cdot 1m - 50\tfrac{rad}{s} \cdot t\right)$$
$$1 = \sin\left(5\tfrac{rad}{m} \cdot 1m - 50\tfrac{rad}{s} \cdot t\right)$$
$$\tfrac{\pi}{2} = 5 - 50s^{-1} \cdot t$$

(Normally, there would be a second solution for the arcsin, which is π minus this answer, but the 2nd solution is identical to the first.) Let's continue

$$\tfrac{\pi}{2} = 5 - 50s^{-1} \cdot t$$
$$t = 0.06858s$$

To this answer, the period can be added or subtracted to find all the moments at which the crest passes. Here, the period is

$$\begin{aligned} T &= \frac{2\pi}{\omega} \\ &= \frac{2\pi}{50s^{-1}} \\ &= 0.12566s \end{aligned}$$

It is clear that negative values will be obtained if the period is subtracted to our solution. As the first three positive value are sought, the period cannot be subtracted. It remains to add the period to find the other 2 times. Thus, the 3 moments are

$$\begin{aligned} t_1 &= 0.06858s \\ t_2 &= 0.06858s + 0.12566s = 0.19425s \\ t_3 &= 0.06858s + 0.12566s + 0.12566s = 0.31991s \end{aligned}$$

7. To write the equation, we need ω, k, A and ϕ.

ω is

$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{0.1s} = 20\pi\tfrac{rad}{s}$$

k is

$$v = \frac{\omega}{k}$$

$$50\tfrac{m}{s} = \frac{20\pi\tfrac{rad}{s}}{k}$$

$$k = \frac{2\pi}{5}\tfrac{rad}{m}$$

The amplitude is

$$\begin{aligned} A^2 &= y^2 + \left(\frac{v_y}{\omega}\right)^2 \\ &= (0.02m)^2 + \left(\frac{-1\tfrac{m}{s}}{20\pi s^{-1}}\right)^2 \\ &= 6.533\times 10^{-4} m^2 \\ A &= 0.02556m \end{aligned}$$

and the phase constant is

$$\tan(kx - \omega t + \phi) = \frac{\omega y}{-v_y}$$

$$\tan(0 + 0 + \phi) = \frac{20\pi s^{-1}\cdot 0.02m}{1\tfrac{m}{s}}$$

$$\phi = 0.8986$$

The equation is therefore

$$y = 0.02556m\cdot\sin\left(\tfrac{2\pi}{5}\tfrac{rad}{m}\cdot x - 20\pi\tfrac{rad}{s}\cdot t + 0.8986rad\right)$$

8. a) The period will be found with

$$T = \frac{2\pi}{\omega}$$

ω can be found with

$$a = -\omega^2 x$$
$$-100\tfrac{m}{s^2} = -\omega^2 \cdot 0,01m$$
$$\omega^2 = 10000\tfrac{rad^2}{s^2}$$
$$\omega = 100\tfrac{rad}{s}$$

Then, the period is

$$\begin{aligned} T &= \frac{2\pi}{\omega} \\ &= \frac{2\pi}{100s^{-1}} \\ &= 0.06283s \end{aligned}$$

b) The amplitude is

$$\begin{aligned} A^2 &= y^2 + \left(\frac{v_y}{\omega}\right)^2 \\ &= (0.01m)^2 + \left(\frac{1.2\frac{m}{s}}{100s^{-1}}\right)^2 \\ &= 0.000244m^2 \\ A &= 0.01562m \end{aligned}$$

9. To write the wave equation, ω, k, A and ϕ must be found

The amplitude is 5 cm.

As the wavelength is 2 cm, k is

$$\begin{aligned} k &= \frac{2\pi}{\lambda} \\ &= \frac{2\pi}{2cm} \\ &= \pi\tfrac{rad}{cm} \end{aligned}$$

ω is

$$v = \frac{\omega}{k}$$

$$2\tfrac{cm}{s} = \frac{\omega}{\pi \tfrac{rad}{cm}}$$

$$\omega = 2\pi \tfrac{rad}{s}$$

For the phase constant, we're going to take a point on the graph where a crest is located. Let's take x = 1 cm. We then have (taking the positive signs in the formula since the wave goes towards the negative x's.)

$$\tan\left(kx + \omega t + \phi\right) = \frac{\omega y}{v_y}$$

$$\tan\left(\pi \tfrac{rad}{cm} \cdot 1cm + 2\pi \tfrac{rad}{s} \cdot 5s + \phi\right) = \frac{2\pi \tfrac{rad}{s} \cdot 0.05m}{0}$$

$$\tan\left(\pi rad + 10\pi rad + \phi\right) = \infty$$

$$\tan\left(11\pi rad + \phi\right) = \infty$$

$$11\pi rad + \phi = \frac{\pi}{2}$$

$$\phi = -10.5\pi$$

Adding 12π to bring the constant between 0 and 2π, we have

$$\phi = \tfrac{3\pi}{2}$$

Therefore, the equation is

$$y = 5cm \cdot \sin\left(\pi \tfrac{rad}{cm} \cdot x + 2\pi \tfrac{rad}{s} \cdot t + \tfrac{3\pi}{2} rad\right)$$

10. To write the wave equation, ω, k, A and ϕ must be found

The amplitude is 2 cm.

As the period is 2 s, ω is

$$\omega = \frac{2\pi}{T}$$

$$= \frac{2\pi}{2s}$$

$$= \pi \tfrac{rad}{s}$$

Therefore, *k* is

$$v = \frac{\omega}{k}$$
$$3\tfrac{cm}{s} = \frac{\pi \tfrac{rad}{s}}{k}$$
$$k = \tfrac{\pi}{3}\tfrac{rad}{cm}$$

For the phase constant, we're going to take an instant where there's a crest. Let's take t = 1.5 s. We then have (taking the negative signs in the formula since the wave goes towards the positive x's.)

$$\tan\left(kx - \omega t + \phi\right) = \frac{\omega y}{-v_y}$$
$$\tan\left(\tfrac{\pi}{3}\tfrac{rad}{cm}\cdot 3cm - \pi\tfrac{rad}{s}\cdot 1.5s + \phi\right) = \frac{\pi\tfrac{rad}{s}\cdot 0.02m}{0}$$
$$\tan\left(\pi rad - 1.5\pi rad + \phi\right) = \infty$$
$$\tan\left(-0.5\pi rad + \phi\right) = \infty$$
$$-0.5\pi rad + \phi = \frac{\pi}{2}$$
$$\phi = \pi$$

Thus, we have

$$y = 2cm\cdot\sin\left(\tfrac{\pi}{3}\tfrac{rad}{cm}\cdot x - \pi\tfrac{rad}{s}\cdot t + \pi rad\right)$$

11. Since the waves are all going at the same speed on a rope, the speed is also 30 m/s.

12. The mass will be found with the linear density and this linear density will be found with the equation of the speed of the wave.

$$v = \sqrt{\frac{F_T}{\mu}}$$

If the wave goes from one end to the other in 0.05 s, its speed is

$$v = \frac{\Delta x}{\Delta t} = \frac{2m}{0.05s} = 40\tfrac{m}{s}$$

Then

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ 40\tfrac{m}{s} &= \sqrt{\frac{200N}{\mu}} \\ \mu &= 0.125\tfrac{kg}{m} \end{aligned}$$

The mass is therefore

$$\begin{aligned} \mu &= \frac{mass}{length} \\ 0.125\tfrac{kg}{m} &= \frac{mass}{2m} \\ mass &= 0.25kg \end{aligned}$$

13. The speed will be found with

$$v = \sqrt{\frac{F_T}{\mu}}$$

The tension is known but not the linear density.

The linear density of the rope is

$$\begin{aligned} \mu &= \frac{m}{L} \\ &= \frac{0.05kg}{2m} \\ &= 0.025\tfrac{kg}{m} \end{aligned}$$

Therefore,

$$v = \sqrt{\frac{F_T}{\mu}}$$
$$50\tfrac{m}{s} = \sqrt{\frac{F_T}{0.025\tfrac{kg}{m}}}$$
$$F_T = 62.5N$$

14. The speed will be found with

$$v = \sqrt{\frac{F_T}{\mu}}$$

The tension is known but not the linear density.

The linear density is found with the speed when the tension is 50 N.

$$v = \sqrt{\frac{F_T}{\mu}}$$
$$40\tfrac{m}{s} = \sqrt{\frac{50N}{\mu}}$$
$$\mu = 0,03125\tfrac{kg}{m}$$

If the tension is 80 N, then the speed is

$$v = \sqrt{\frac{F_T}{\mu}}$$
$$= \sqrt{\frac{80N}{0.03125\tfrac{kg}{m}}}$$
$$= 50.6\tfrac{m}{s}$$

15. a) The linear density will be found with the equation of the speed of the wave.

$$v = \sqrt{\frac{F_T}{\mu}}$$

The velocity of the wave is

$$\begin{aligned} v &= \frac{\omega}{k} \\ &= \frac{200s^{-1}}{10m^{-1}} \\ &= 20\tfrac{m}{s} \end{aligned}$$

Therefore

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ 20\tfrac{m}{s} &= \sqrt{\frac{50N}{\mu}} \\ \mu &= 0.125\tfrac{kg}{m} \end{aligned}$$

b) The maximum speed of the rope is

$$\begin{aligned} v_{\max} &= \omega A \\ &= 200s^{-1} \cdot 0.2m \\ &= 40\tfrac{m}{s} \end{aligned}$$

16. To find the speed with

$$v = \sqrt{\frac{F_T}{\mu}}$$

the linear density of the rope is needed. This linear density is

$$\mu = \frac{m}{L}$$

Therefore, the mass of a nylon rope with a length *L* and a radius of 2 mm must be found. The mass is found with the with density

$$m = \rho \cdot volume$$

Since the rope is a long cylinder of length *L*, its volume is $\pi r^2 L$. Thus,

$$m = \rho \cdot \pi r^2 L$$

Therefore, the linear density is

$$\begin{aligned}\mu &= \frac{m}{L} \\ &= \frac{\rho \pi r^2 L}{L} \\ &= \rho \pi r^2 \\ &= 1150 \tfrac{kg}{m^3} \cdot \pi \cdot (0.002m)^2 \\ &= 0.01445 \tfrac{kg}{m}\end{aligned}$$

Therefore, the speed is

$$\begin{aligned}v &= \sqrt{\frac{F_T}{\mu}} \\ &= \sqrt{\frac{300N}{0.01445 \frac{kg}{m}}} \\ &= 144.1 \tfrac{m}{s}\end{aligned}$$

17. The distance between the nodes is equal to half of the wavelength. Therefore, the wavelength must be found. It is found with

$$v = \lambda f$$

The speed is found with

$$\begin{aligned}v &= \sqrt{\frac{F_T}{\mu}} \\ &= \sqrt{\frac{500N}{0.05 \frac{kg}{m}}} \\ &= 100 \tfrac{m}{s}\end{aligned}$$

The frequency is found with the formula for the maximum speed.

$$v_{\max} = A\omega$$
$$1\tfrac{m}{s} = 0.02m \cdot \omega$$
$$\omega = 50\tfrac{rad}{s}$$
$$f = \frac{\omega}{2\pi}$$
$$f = \frac{50}{2\pi}Hz$$

Thus, the wavelength is

$$v = \lambda f$$
$$100\tfrac{m}{s} = \lambda \cdot \frac{50}{2\pi}Hz$$
$$\lambda = 4\pi m$$

Therefore, the distance between the nodes is

$$\frac{\lambda}{2} = 2\pi m = 6.283m$$

18. a) To calculate the energy with the formula

$$E = \frac{1}{2}\mu D\omega^2 A^2$$

The frequency must be known.

The wave frequency is

$$v = \lambda f$$
$$50\tfrac{m}{s} = 0.4m \cdot f$$
$$f = 125Hz$$

The energy is therefore

$$\begin{aligned} E &= \frac{1}{2}\mu D\omega^2 A^2 \\ &= \frac{1}{2} \cdot 0.025\tfrac{kg}{m} \cdot 10m \cdot (2\pi \cdot 125Hz)^2 \cdot (0.002m)^2 \\ &= 0.3084J \end{aligned}$$

b) The power is

$$\begin{aligned} P &= \frac{1}{2}\mu v \omega^2 A^2 \\ &= \frac{1}{2}\cdot 0.025\tfrac{kg}{m}\cdot 50\tfrac{m}{s}\cdot\left(2\pi\cdot 125Hz\right)^2\cdot\left(0.002m\right)^2 \\ &= 1.542W \end{aligned}$$

19. The amplitude can be found with

$$P = \frac{1}{2}\mu v \omega^2 A^2$$

but the speed and the linear density must be known.

The speed of the wave is

$$\begin{aligned} v &= \lambda f \\ &= 0.125m\cdot 200Hz \\ &= 25\tfrac{m}{s} \end{aligned}$$

The linear density of the rope is

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ 25\tfrac{m}{s} &= \sqrt{\frac{80N}{\mu}} \\ \mu &= 0.128\tfrac{kg}{m} \end{aligned}$$

The amplitude is then found with the power formula.

$$\begin{aligned} P &= \frac{1}{2}\mu v \omega^2 A^2 \\ 20W &= \frac{1}{2}\cdot 0.128\tfrac{kg}{m}\cdot 25\tfrac{m}{s}\cdot\left(2\pi\cdot 200Hz\right)^2\cdot A^2 \\ A &= 0.002813m \end{aligned}$$

20. The power will be found with

$$P = \frac{1}{2}\mu v \omega^2 A^2$$

However, the speed must be found first.

The wave speed is

$$\begin{aligned} v &= \frac{\omega}{k} \\ &= \frac{50\pi s^{-1}}{10\pi m^{-1}} \\ &= 5\tfrac{m}{s} \end{aligned}$$

The power is thus

$$\begin{aligned} P &= \frac{1}{2}\mu v \omega^2 A^2 \\ &= \frac{1}{2} \cdot 0.05\tfrac{kg}{m} \cdot 5\tfrac{m}{s} \cdot \left(50\pi s^{-1}\right)^2 \cdot \left(0.02m\right)^2 \\ &= 1.234W \end{aligned}$$

21. We have two equations. They are

$$\begin{aligned} Z &= \sqrt{F_T \mu} \\ 4\tfrac{kg}{s} &= \sqrt{F_T \mu} \\ 16\tfrac{kg^2}{s^2} &= F_T \mu \end{aligned}$$

and

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ 80\tfrac{m}{s^2} &= \sqrt{\frac{F_T}{\mu}} \\ 6400\tfrac{m^2}{s^2} &= \frac{F_T}{\mu} \end{aligned}$$

By multiplying these 2 equations, we have

$$16\tfrac{kg^2}{s^2}\cdot 6400\tfrac{m^2}{s^2} = F_T\mu\cdot\frac{F_T}{\mu}$$
$$102,400\tfrac{kg^2m^2}{s^4} = F_T^2$$
$$F_T = 320N$$

From that, the linear density is found.

$$16\tfrac{kg^2}{s^2} = F_T\mu$$
$$16\tfrac{kg^2}{s^2} = 320N\cdot\mu$$
$$\mu = 0.05\tfrac{kg}{m}$$

22. For the first wave, we have

$$P_1 = \frac{1}{2}Z_1\omega^2 A_1^2$$

When the rope is changed, the impedance is changed since the linear density of the rope is changed. This linear density is

$$\mu = \frac{m}{L}$$

The mass is found with the density

$$m = \rho\cdot volume$$

Since the rope is a long cylinder of length L, its volume is $\pi r^2 L$. Thus,

$$m = \rho\cdot\pi r^2 L$$

Therefore, the lineic density is

$$\begin{aligned}\mu &= \frac{m}{L}\\ &= \frac{\rho\cdot\pi r^2 L}{L}\\ &= \rho\cdot\pi r^2\end{aligned}$$

If the radius is doubled, then

$$\begin{aligned}\frac{\mu_2}{\mu_1} &= \frac{\rho \cdot \pi r_2^2}{\rho \cdot \pi r_1^2} \\ &= \frac{r_2^2}{r_1^2} \\ &= \frac{(2r_1)^2}{r_1^2} \\ &= 4\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{Z_2}{Z_1} &= \frac{\sqrt{F_T \mu_2}}{\sqrt{F_T \mu_1}} \\ &= \sqrt{\frac{F_T \mu_2}{F_T \mu_1}} \\ &= \sqrt{\frac{\mu_2}{\mu_1}} \\ &= \sqrt{4} \\ &= 2\end{aligned}$$

Thus, the ratio of power is

$$\frac{P_2}{P_1} = \frac{\frac{1}{2} Z_2 \omega^2 A_2^2}{\frac{1}{2} Z_1 \omega^2 A_1^2}$$

$$\frac{P_2}{P_1} = \frac{Z_2 A_2^2}{Z_1 A_1^2}$$

Since the powers are the same and the ratio of impedances is 2, we have

$$1 = 2\frac{A_2^2}{A_1^2}$$

$$1 = 2\frac{A_2^2}{(0.02m)^2}$$

$$A_2 = 1.414cm$$

23. The distance traveled by the front of the wave during time Δt is

$$l = v\Delta t$$

The mass of the rope over this distance (which is the mass of the rope set in motion) is

$$\begin{aligned} m &= \mu l \\ &= \mu v \Delta t \end{aligned}$$

The mass of rope set in motion per unit of time is

$$\frac{m}{\Delta t} = \mu v$$

We then have

$$\begin{aligned} \frac{m}{\Delta t} &= \mu \sqrt{\frac{F_T}{\mu}} \\ &= \sqrt{F_T \mu} \\ &= Z \end{aligned}$$

And there you have it, the mass of the string set in motion per unit of time is equal to the impedance. This is consistent with the fact that impedance is measured in kg/s.

24. The speed of the wave depends on the tension. Let's find the tension as a function of the position on the string. To ensure that there is an equilibrium of force, this tension must be equal to the weight of the rope under the point considered. A y-axis pointing downwards with an origin $y = 0$ at the top of the rope will be used. At the position y, the length of the rope under this position is

$$l = L - y$$

where L is the total length of the rope. The mass of this part of rope is

$$m = \mu \left(L - y \right)$$

This means that the tension of the rope at y is

$$F_T = \mu g \left(L - y \right)$$

Then, the speed of the wave at y is

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{g\left(L - y\right)}$$

To travel the small distance *dy*, the time is

$$dt = \frac{dy}{v} = \frac{dy}{\sqrt{g\left(L - y\right)}}$$

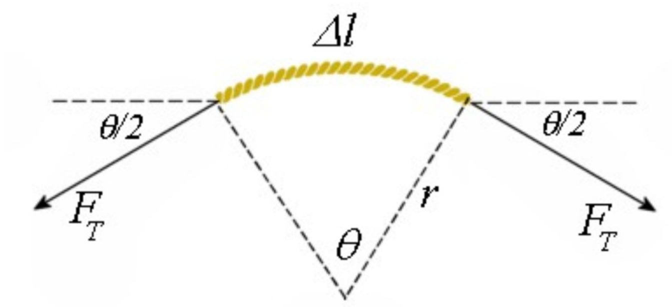
If all these times are added now, we obtain

$$\begin{aligned} t &= \int_0^L \frac{dy}{\sqrt{g\left(L - y\right)}} \\ &= \frac{1}{\sqrt{g}} \int_0^L \frac{dy}{\sqrt{\left(L - y\right)}} \end{aligned}$$

Setting $u = L - y$, the integral becomes

$$\begin{aligned} t &= \frac{1}{\sqrt{g}} \int_L^0 \frac{-du}{\sqrt{u}} \\ &= \frac{-1}{\sqrt{g}} \int_L^0 u^{-1/2} du \\ &= \frac{-1}{\sqrt{g}} \left[\frac{u^{1/2}}{1/2} \right]_L^0 \\ &= \frac{-2}{\sqrt{g}} \left[0^{1/2} - L^{1/2} \right] \\ &= 2\sqrt{\frac{L}{g}} \\ &= 2\sqrt{\frac{2m}{9.8\frac{m}{s^2}}} \\ &= 0.9035s \end{aligned}$$

25. The speed of the wave depends on the tension. Therefore, the tension in this ring must be found. Let's take a small piece of rope and examine the forces on this piece. (The angle θ is small on the figure.)



Since this piece makes a circular motion, the sum of *y*-components of the force acting on this piece is (using a *y*-axis directed upwards)

$$\sum F_y = ma_y$$
$$\to F_T \sin\left(-\frac{\theta}{2}\right) + F_T \sin\left(180° + \frac{\theta}{2}\right) = -m\omega^2 r$$

Since sin -*x* = -sin *x* and sin (180°+*x*) = -sin *x*, it becomes

$$-F_T \sin\left(\frac{\theta}{2}\right) - F_T \sin\left(\frac{\theta}{2}\right) = -m\omega^2 r$$
$$2F_T \sin\left(\frac{\theta}{2}\right) = m\omega^2 r$$

Since the angle is small, sin *x* = *x*. (This means that we are now working with angles in radians.)

$$2F_T \frac{\theta}{2} = m\omega^2 r$$
$$F_T \theta = m\omega^2 r$$

The mass of the piece depends on the angle. Since the density is μ, the mass of the piece is

$$m = \mu \Delta l$$
$$= \mu r \theta$$

The force equation then becomes

$$F_T \theta = m\omega^2 r$$
$$F_T \theta = \mu r \theta \omega^2 r$$
$$F_T = \mu r^2 \omega^2$$

Thus, the speed of the wave is

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ &= \sqrt{\frac{\mu r^2 \omega^2}{\mu}} \\ &= r\omega \end{aligned}$$

With the values, we arrive at

$$\begin{aligned} v &= 0.25m \cdot 2\tfrac{rad}{s} \\ &= 0.5\tfrac{m}{s} \end{aligned}$$

Note that this speed ($r\omega$) is also the speed of the rope. Thus, the wave, if it goes in the opposite direction to the rope, always remains at the same place!