Chapter 12 Solutions

1. a) The number of protons is 18

The number of neutrons is 39 – 18 = 21

b) The number of protons is 76

The number of neutrons is 180 – 76 = 104

2. a) The radius is

$$\begin{aligned} r_{nucleus} &= 1.2 fm \sqrt[3]{A} \\ &= 1.2 fm \cdot \sqrt[3]{18} \\ &= 3.14 fm \end{aligned}$$

b) The radius is

$$\begin{aligned} r_{nucleus} &= 1.2 fm \sqrt[3]{A} \\ &= 1.2 fm \cdot \sqrt[3]{235} \\ &= 7.41 fm \end{aligned}$$

3. Let's find the density of the nucleus from its size and mass.

The nucleus of carbon has a radius of

$$\begin{aligned} r_{nucleus} &= 1.2 fm \sqrt[3]{A} \\ &= 1.2 fm \cdot \sqrt[3]{12} \\ &= 2.75 fm \end{aligned}$$

The mass of the nucleus is the mass of the atom minus the mass of the electrons.

$$\begin{aligned} m_{noyau} &= 12.000\,000u - 6 \cdot 0.000\,549u \\ &= 11.996\,706u \end{aligned}$$

Its density is, therefore,

$$\begin{aligned}\rho &= \frac{m}{Vol}\\ &= \frac{m}{\frac{4}{3}\pi r^3}\\ &= \frac{11.996\,706 \cdot 1.660539\times 10^{-27}kg}{\frac{4}{3}\pi\left(2.75\times 10^{-15}m\right)^3}\\ &= 2.287\times 10^{17}\,\tfrac{kg}{m^3}\end{aligned}$$

If the Earth had this density, then its radius would be

$$\begin{aligned}\rho &= \frac{m}{Vol}\\ 2.287\times 10^{17}\,\tfrac{kg}{m^3} &= \frac{6\times 10^{24}kg}{\frac{4}{3}\pi r^3}\\ r &= 184m\end{aligned}$$

The Earth would have a radius of only 184 m!

4. The reaction is

$$^{26}_{12}Mg \to {}^{25}_{12}Mg + {}^{1}_{0}n$$

Using the masses in the table, Q is

$$\begin{aligned}Q &= \left(m_{before} - m_{after}\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(25.982\ 593u - \left(24.985\ 837u + 1.008\ 665u\right)\right)\cdot 931.5\tfrac{MeV}{u}\\ &= -0.01191u\cdot 931.5\tfrac{MeV}{u}\\ &= -11.09MeV\end{aligned}$$

11.09 MeV must be provided to remove the neutron.

5. The reaction is

$$^{26}_{12}Mg \to {}^{25}_{11}Na + {}^{1}_{1}H$$

Using the masses in the table, Q is

$$\begin{aligned} Q &= \left(m_{before} - m_{after}\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(25.982\ 593u - \left(24.989\ 954u + 1.007\ 8255u\right)\right)\cdot 931.5\tfrac{MeV}{u} \\ &= -0.015186u \cdot 931.5\tfrac{MeV}{u} \\ &= -14.15MeV \end{aligned}$$

14.15 MeV must be provided to remove the proton.

6. a) The binding energy is

$$\begin{aligned} E_{binding} &= \left(Zm_{H1} + Nm_n - m_X\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(13\cdot 1.007\ 83u + 23\cdot 1.008\ 66u - 36.006\ 21u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(0.29476u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= 274.6MeV \end{aligned}$$

b) The binding energy is

$$\begin{aligned} E_{binding} &= \left(Zm_{H1} + Nm_n - m_X\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(84\cdot 1.007\ 825u + 120\cdot 1.008\ 665u - 203.980\ 318u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(1.716782u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= 1599MeV \end{aligned}$$

7. The binding energy of copper-60 is

$$\begin{aligned} E_{binding} &= \left(Zm_{H1} + Nm_n - m_X\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(29\cdot 1.007\ 825u + 31\cdot 1.008\ 665u - 59.937\ 365u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= \left(0.558175u\right)\cdot 931.5\tfrac{MeV}{u} \\ &= 519.9MeV \end{aligned}$$

The binding energy of cobalt-60 is

$$\begin{aligned}E_{binding} &= \left(Zm_{H1} + Nm_n - m_X\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(27\cdot 1.007\ 825u + 33\cdot 1.008\ 665u - 59.933\ 817u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(0.563403u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= 524.8MeV\end{aligned}$$

8. a) The reaction is

$${}^{112}_{54}Xe \to {}^{108}_{52}Te + {}^{4}_{2}He$$

b) The energy released is

$$\begin{aligned}Q &= \left(m_{Xe} - m_{Te} - m_{He4}\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(111.935\ 62u - 107.929\ 44u - 4.002\ 60u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(0.003\ 58u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= 3.33MeV\end{aligned}$$

c) The kinetic energy of the alpha particle is

$$\begin{aligned}E_{k\ He4} &= \frac{m_{Te}}{m_{He4} + m_{Te}}Q\\ &= \frac{107.929\ 44u}{4.002\ 60u + 107.929\ 44u}\cdot 3.33MeV\\ &= 3.21MeV\end{aligned}$$

9. The reaction is

$${}^{85}_{36}Kr \to {}^{85}_{37}Rb + e^- + \overline{\nu}$$

The energy released is

$$\begin{aligned}Q &= \left(m_{Kr} - m_{Rb}\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(84.912\ 527\ 3u - 84.911\ 789\ 7u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(0.000\ 737\ 6u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= 0.687MeV\end{aligned}$$

10. The reaction is

$$ {}^{40}_{19}K \rightarrow {}^{40}_{18}Ar + e^{+} + \nu $$

The energy released is

$$
\begin{aligned}
Q &= \left(m_K - m_{Ar} - 2m_e\right)\cdot 931.5\tfrac{MeV}{u} \\
&= \left(39.963\ 998\ 48u - 39.962\ 383\ 12u - 2\cdot 0.000\ 548\ 58\right)\cdot 931.5\tfrac{MeV}{u} \\
&= \left(0.000\,518\,2u\right)\cdot 931.5\tfrac{MeV}{u} \\
&= 0.483MeV
\end{aligned}
$$

11. The reaction is

$$ {}^{29}_{15}P \rightarrow {}^{29}_{14}Si + \nu $$

The energy released is

$$
\begin{aligned}
Q &= \left(m_P - m_{Si}\right)\cdot 931.5\tfrac{MeV}{u} \\
&= \left(28.981\ 800\ 6u - 28.976\ 494\ 7u\right)\cdot 931.5\tfrac{MeV}{u} \\
&= \left(0.005\,305\,9u\right)\cdot 931.5\tfrac{MeV}{u} \\
&= 4.94MeV
\end{aligned}
$$

12. The reaction is

$$ {}^{49}_{24}Cr^{*} \rightarrow {}^{49}_{24}Cr + \gamma $$

13. The decay is possible if Q is positive.

a) The reaction would be

$$ {}^{25}_{10}Ne \rightarrow {}^{25}_{11}Na + e^{-} + \overline{\nu} $$

The energy released would be

$$\begin{aligned}Q &= \left(m_{Ne} - m_{Na}\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(24.997\ 737u - 24.989\ 954u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(0.007\ 783u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= 7.25MeV\end{aligned}$$

As the value of Q is positive, this decay is possible.

b) The reaction would be

$${}^{28}_{13}Al \to {}^{28}_{12}Mg + e^+ + \nu$$

The energy released would be

$$\begin{aligned}Q &= \left(m_{Al} - m_{Mg} - 2m_e\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(27.981\ 910\ 3u - 27.983\ 876\ 8u - 2\cdot 0.000\ 548\ 6u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= \left(-0.003\ 063\ 7u\right)\cdot 931.5\tfrac{MeV}{u}\\ &= -2.85MeV\end{aligned}$$

As the value of Q is negative, this decay is impossible.

14. a) The initial number of nuclei is

$$\begin{aligned}N_0 &= \frac{mass}{atomic\ mass}N_A\\ &= \frac{0.002g}{83\frac{g}{mol}}\cdot 6.022\times 10^{23}\\ &= 1.45\times 10^{19}\end{aligned}$$

b) The number of nuclei is

$$\begin{aligned}N &= N_0 e^{-\lambda t}\\ &= N_0 e^{-\frac{\ln 2}{T_{1/2}}t}\\ &= 1.45\times 10^{19}\cdot e^{-\frac{\ln 2}{32.41h}\cdot 72h}\\ &= 3.11\times 10^{18}\end{aligned}$$

15. The initial number of atoms is

$$\begin{aligned}N_0 &= \frac{mass}{atomic\ mass} N_A \\ &= \frac{1g}{208\frac{g}{mol}} \cdot 6.022\times 10^{23} \\ &= 2.8952\times 10^{21}\end{aligned}$$

After 10 years, the number of remaining polonium nuclei is

$$\begin{aligned}N &= N_0 e^{-\lambda t} \\ &= N_0 e^{-\frac{\ln 2}{T_{1/2}}t} \\ &= 2.8952\times 10^{21} \cdot e^{-\frac{\ln 2}{2.898y}\cdot 10y} \\ &= 2.648\times 10^{20}\end{aligned}$$

The number of lead nuclei is equal to the number of radium atoms that have disintegrated. This number of atoms is equal to the initial number of polonium atoms minus the number of remaining polonium atoms.

$$\begin{aligned}N_{Pb} &= 2.8952\times 10^{21} - 2.648\times 10^{20} \\ &= 2.6304\times 10^{21}\end{aligned}$$

The mass of these lead atoms is

$$N = \frac{mass}{atomic\ mass} N_A$$

$$2.6304\times 10^{21} = \frac{m}{204\frac{g}{mol}} \cdot 6.022\times 10^{23}$$

$$m = 0.891g$$

16. a) The initial activity is

$$R_0 = \lambda N_0$$

The number of nuclei is

$$\begin{aligned} N_0 &= \frac{mass}{atomic\ mass} N_A \\ &= \frac{5\times 10^{-6} g}{201 \frac{g}{mol}} \cdot 6.022\times 10^{23} \\ &= 1.498\times 10^{16} \end{aligned}$$

The decay constant is

$$\begin{aligned} \lambda &= \frac{\ln 2}{T_{1/2}} \\ &= \frac{\ln 2}{15.3\cdot 60s} \\ &= 7.55\times 10^{-4} s^{-1} \end{aligned}$$

Therefore, the initial activity is

$$\begin{aligned} R_0 &= \lambda N_0 \\ &= 7.55\times 10^{-4} s \cdot 1.498\times 10^{16} \\ &= 1.131\times 10^{13} Bq \\ &= 305.6 Ci \end{aligned}$$

b) In one hour, the activity will be

$$\begin{aligned} R &= R_0 e^{-\lambda t} \\ &= R_0 e^{-\frac{\ln 2}{T_{1/2}} t} \\ &= 305.6 Ci \cdot e^{-\frac{\ln 2}{15.3\min}\cdot 60\min} \\ &= 20.17 Ci \end{aligned}$$

17. The half-life is

$$\begin{aligned} R &= R_0 e^{-\lambda t} \\ R &= R_0 e^{-\frac{\ln 2}{T_{1/2}}} \\ 16,9\mu Ci &= 20\mu Ci \cdot e^{-\frac{\ln 2}{T_{1/2}}\cdot 48h} \\ T_{1/2} &= 197,5h \end{aligned}$$

18. The initial number of nuclei is

$$\begin{aligned} N_0 &= \frac{mass}{atomic\ \ mass} N_A \\ &= \frac{10g}{227\frac{g}{mol}} \cdot 6.022\times 10^{23} \\ &= 2.653\times 10^{22} \end{aligned}$$

The number of nuclei remaining after 2 hours will be

$$\begin{aligned} N &= N_0 e^{-\lambda t} \\ &= N_0 e^{-\frac{\ln 2}{T_{1/2}}t} \\ &= 2.653\times 10^{22} \cdot e^{-\frac{\ln 2}{42.2\,\text{min}}\cdot 120\,\text{min}} \\ &= 3.696\times 10^{21} \end{aligned}$$

The number of nuclei which will decayed is equal to the difference between the initial number and the number of nuclei remaining after 2 hours

$$\begin{aligned} N_{\text{decayed}} &= N_0 - N \\ &= 2.653\times 10^{22} - 3.696\times 10^{21} \\ &= 2.283\times 10^{22} \end{aligned}$$

19. With 20 g of carbon, the initial activity was

$$\begin{aligned} R_0 &= 0,25\tfrac{Bq}{g} \cdot 20g \\ &= 5Bq \end{aligned}$$

Thus

$$\begin{aligned} R &= R_0 e^{-\lambda t} \\ R &= R_0 e^{-\frac{\ln 2}{T_{1/2}}} \\ 4.4Bq &= 5Bq \cdot e^{-\frac{\ln 2}{5730y}\cdot t} \\ T &= 1057y \end{aligned}$$

This piece of wood dates from the year 2021 – 1057 = 964. So it dates from the 10th century.

20. The mass can be found from the number of nucleus. The number of nucleus can be found with

$$R = \lambda N$$

The decay constant is

$$\begin{aligned}\lambda &= \frac{\ln 2}{T_{1/2}} \\ &= \frac{\ln 2}{1\,287\,360 s} \\ &= 5.384\times10^{-7} s^{-1}\end{aligned}$$

The number of nuclei is found with

$$\begin{aligned}R &= \lambda N \\ 25\cdot 3.7\times10^{10} Bq &= 5.384\times10^{-7} s^{-1}\cdot N \\ N &= 1.718\times10^{18}\end{aligned}$$

Finally, the mass can be found.

$$\begin{aligned}N &= \frac{mass}{atomic\ mass} N_A \\ 1.718\times10^{18} &= \frac{mass}{225 \frac{g}{mol}} 6.022\times10^{23} \\ mass &= 6.42\times10^{-4} g\end{aligned}$$

21. Only an alpha decay can change the number of nucleons. As this number changed from 235 nucleons to 207 nucleons and 4 nucleons are lost per decay, the number of alpha decay must be

$$\begin{aligned}N_\alpha &= \frac{235-207}{4} \\ &= 7\end{aligned}$$

With only alpha decays, the number of protons would have decreased by

$$7\cdot 2 = 14$$

Since 2 protons are lost in each decay. However, uranium has 92 protons and lead has 82. Only 10 protons were therefore lost. 4 beta decays must then have occurred to change 4 neutrons into protons so that only 10 protons are lost instead of 14.

Thus, there were 7 alpha decays and 4 beta decays.

22. The number of remaining potassium atoms is

$$N_K = N_0 e^{-\lambda t}$$

The number of argon atoms is equal to the number of potassium atoms which has transformed into argon. This number is equal to the initial number of potassium atoms minus the number of remaining potassium atoms. It is thus equal to

$$\begin{aligned} N_{Ar} &= N_0 - N_K \\ &= N_0 - N_0 e^{-\lambda t} \end{aligned}$$

Therefore, the ratio of the number of argon atoms and the number of potassium atoms is

$$\begin{aligned} \frac{N_{Ar}}{N_K} &= \frac{N_0 - N_0 e^{-\lambda t}}{N_0 e^{-\lambda t}} \\ &= \frac{1 - e^{-\lambda t}}{e^{-\lambda t}} \\ &= \frac{1}{e^{-\lambda t}} - \frac{e^{-\lambda t}}{e^{-\lambda t}} \\ &= e^{\lambda t} - 1 \end{aligned}$$

Thus

$$\begin{aligned} \frac{N_{Ar}}{N_K} &= e^{\lambda t} - 1 \\ 0.15 &= e^{\frac{\ln(2)}{1.248 Gy} t} - 1 \\ 1.15 &= e^{\frac{\ln(2)}{1.248 Gy} t} \\ \ln(1.15) &= \frac{\ln(2)}{1.248 Gy} t \\ t &= 0.252 Gy = 252 My \end{aligned}$$

23. For each reaction, the number of protons and the number of nucleons must be the same on each side of the equation. Therefore,

1) ${}^{15}_{7}N + {}^{1}_{1}H \to {}^{12}_{6}C + {}^{4}_{2}He$
2) ${}^{18}_{8}O + {}^{1}_{1}H \to {}^{18}_{9}F + {}^{1}_{0}n$
3) ${}^{7}_{3}Li + {}^{1}_{1}H \to {}^{7}_{4}Be + {}^{1}_{0}n$
4) ${}^{10}_{5}B + {}^{1}_{0}n \to {}^{7}_{3}Li + {}^{4}_{2}He$

24. The energy released is found from the masses with

$$Q = \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u}$$

1) The energy is

$$\begin{aligned} Q &= \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(\left(m_N + m_{He}\right) - \left(m_O + m_H\right)\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \begin{pmatrix} \left(14.003\ 074u + 4.002\ 603u\right) - \\ \left(16.999\ 131u + 1.007\ 825u\right) \end{pmatrix} \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(-0.001\,279u\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= -1.19 MeV \end{aligned}$$

Therefore, 1.19 MeV must be provided for this reaction to occur.

2) The energy is

$$\begin{aligned} Q &= \left(m_{before} - m_{after}\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(\left(m_{Be} + m_{He}\right) - \left(m_C + m_n\right)\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= \begin{pmatrix} \left(9.012\ 182u + 4.002\ 603u\right) - \\ \left(12.000\ 000u + 1.008\ 665u\right) \end{pmatrix} \cdot 931.5 \tfrac{MeV}{u} \\ &= \left(0.006\,120u\right) \cdot 931.5 \tfrac{MeV}{u} \\ &= 5.70 MeV \end{aligned}$$

So, this reaction releases 5.70 MeV.

3) The energy is

$$\begin{aligned}
Q &= \left(m_{before} - m_{after}\right)\cdot 931.5\tfrac{MeV}{u} \\
&= \left(\left(m_{Al} + m_{He}\right) - \left(m_P + m_n\right)\right)\cdot 931.5\tfrac{MeV}{u} \\
&= \begin{pmatrix} \left(26.981\ 539u + 4.002\ 603u\right) - \\ \left(29.978\ 314u + 1.008\ 665u\right) \end{pmatrix}\cdot 931.5\tfrac{MeV}{u} \\
&= \left(-0.002\,837u\right)\cdot 931.5\tfrac{MeV}{u} \\
&= -2.64MeV
\end{aligned}$$

Therefore, 2.64 MeV must be provided for this reaction to occur

4) The energy is

$$\begin{aligned}
Q &= \left(m_{before} - m_{after}\right)\cdot 931.5\tfrac{MeV}{u} \\
&= \left(\left(m_N + m_n\right) - \left(m_C + m_H\right)\right)\cdot 931.5\tfrac{MeV}{u} \\
&= \begin{pmatrix} \left(14.003\ 074u + 1.008\ 665u\right) - \\ \left(14.003\ 242u + 1.007\ 825u\right) \end{pmatrix}\cdot 931.5\tfrac{MeV}{u} \\
&= \left(0.000\,672u\right)\cdot 931.5\tfrac{MeV}{u} \\
&= 0.626MeV
\end{aligned}$$

So, this reaction releases 0.626 MeV.

25. In a reaction, the number of protons and the number of nucleons must be the same on each side of the equation. Therefore,

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{144}_{55}Cs + {}^{90}_{37}Rb + 2\,{}^{1}_{0}n$$

26. The energy released is found from the masses with

$$Q = \left(m_{before} - m_{after}\right)\cdot 931.5\tfrac{MeV}{u}$$

Therefore

$$\begin{aligned}
Q &= \left(m_{before} - m_{after}\right)\cdot 931.5\tfrac{MeV}{u}\\
&= \left(\left(m_n + m_U\right) - \left(m_{Xe} + m_{Sr} + 3\cdot m_n\right)\right)\cdot 931.5\tfrac{MeV}{u}\\
&= \begin{pmatrix}\left(1.008\ 66u + 235.043\ 93u\right) - \\ \qquad\left(142.935\ 11u + 89.907\ 74u + 3\times 1.008\ 66u\right)\end{pmatrix}\cdot 931.5\tfrac{MeV}{u}\\
&= \left(0.183\ 76u\right)\cdot 931.5\tfrac{MeV}{u}\\
&= 171.2MeV
\end{aligned}$$

27. The number of atoms in 100 g of pure uranium-235 is

$$\begin{aligned}
N &= \frac{100g}{235\frac{g}{mol}}\cdot 6.022\times 10^{23}\\
&= 2.562\times 10^{23}
\end{aligned}$$

This number is the number of fissions that can occur. With 200 MeV each, the total energy is

$$\begin{aligned}
E &= 2.562\times 10^{23}\cdot\left(200\times 10^{6}\cdot 1.602\times 10^{-19}J\right)\\
&= 8.208\times 10^{12}J
\end{aligned}$$

By consuming 250 MJ per day, this energy can last for

$$\frac{8.208\times 10^{12}J}{250\times 10^{6}\frac{J}{day}} = 32{,}830 days = 89.9y$$

28. a) The energy released is found from the masses with

$$Q = \left(m_{before} - m_{after}\right)\cdot 931.5\tfrac{MeV}{u}$$

Therefore,

$$\begin{aligned}
Q &= \left(m_{before} - m_{after}\right)\cdot 931.5\tfrac{MeV}{u}\\
&= \left(\left(m_{H2} + m_{H2}\right) - \left(m_{He3} + m_n\right)\right)\cdot 931.5\tfrac{MeV}{u}\\
&= \left(\left(2.014\ 101u + 2.014\ 101u\right) - \left(3.016\ 029u + 1.008\ 665u\right)\right)\cdot 931.5\tfrac{MeV}{u}\\
&= \left(0.003\ 508u\right)\cdot 931.5\tfrac{MeV}{u}\\
&= 3.27MeV
\end{aligned}$$

b) A molecule of water has a molar mass of 18 g/mol. Therefore, the number of water molecules in a ton of water is

$$\begin{aligned} N &= \frac{1,000,000g}{18\frac{g}{mol}} \cdot 6.022\times 10^{23} \\ &= 3.34\times 10^{28} \end{aligned}$$

The number of hydrogen atoms is twice as large since there are 2 atoms of hydrogen per water molecule. There are, therefore, 6.68×10^{28} hydrogen atoms.

Since one hydrogen atom in 6500 is a deuterium atom, the number of deuterium atoms is

$$\frac{6.68\times 10^{28}}{6500} = 1.029\times 10^{25}$$

As two atoms of deuterium are needed for each nuclear reaction, the number of reactions that can be done is

$$\frac{1.029\times 10^{25}}{2} = 5.145\times 10^{24}$$

With 3.27 MeV per reaction, the energy that can be obtained is

$$5.145\times 10^{24} \cdot \left(3.27\times 10^{6} \cdot 1.602\times 10^{-19} J\right) = 2.7\times 10^{12} J$$

N.B. This is equivalent to the energy that can be obtained from 57 tons of gasoline, or from about 135 tons of coal…

29. The decay of radium is as usual and obeys the following law.

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

However, the equation for actinium is a little different.

$$\frac{dN_2}{dt} = \left(\text{atoms coming from radium}\right) - \left(\text{disintegration}\right)$$

The number of atoms of actinium coming from radium is equal to the number of radium atoms which disintegrates. Thus

$$\left(\text{atoms coming from radium}\right)=\lambda_1 N_1$$

(This is positive because the number of atoms of actinium increases.)

Then, there is the actinium decay, which is proportional to the decay constant and the number of atoms, as for any decay.

$$\left(\text{decay}\right)=-\lambda_2 N_2$$

(This is negative because the number of atoms of actinium decreases.)

Therefore, we have the following two equations.

$$\frac{dN_1}{dt}=-\lambda_1 N_1$$
$$\frac{dN_2}{dt}=\lambda_1 N_1-\lambda_2 N_2$$

The first one is identical to the equation of a normal decay. So, the solution is

$$N_1=N_{01}e^{-\lambda_1 t}$$

Thus, the second equation becomes

$$\frac{dN_2}{dt}=\lambda_1 N_{01}e^{-\lambda_1 t}-\lambda_2 N_2$$

Knowing that the form of the solution is

$$N_2=Ae^{-\lambda_1 t}+Be^{-\lambda_2 t}$$

Initially, there are no actinium atoms. At $t = 0$, the equation gives

$$0=Ae^{-\lambda_1\cdot 0}+Be^{-\lambda_2\cdot 0}$$
$$0=A+B$$
$$B=-A$$

Therefore

$$N_2=A\left(e^{-\lambda_1 t}-e^{-\lambda_2 t}\right)$$

In addition, the differential equation for actinium gives

$$\begin{aligned}
\frac{dN_2}{dt} &= \lambda_1 N_{01} e^{-\lambda_1 t} - \lambda_2 N_2 \\
\frac{dA\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)}{dt} &= \lambda_1 N_{01} e^{-\lambda_1 t} - \lambda_2 A\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \\
-\lambda_1 A e^{-\lambda_1 t} + \lambda_2 A e^{-\lambda_2 t} &= \lambda_1 N_{01} e^{-\lambda_1 t} - \lambda_2 A e^{-\lambda_1 t} + \lambda_2 A e^{-\lambda_2 t} \\
-\lambda_1 A e^{-\lambda_1 t} + \lambda_2 A e^{-\lambda_2 t} &= \left(\lambda_1 N_{01} - \lambda_2 A\right) e^{-\lambda_1 t} + \lambda_2 A e^{-\lambda_2 t}
\end{aligned}$$

For the equation to be true at any time, the coefficients in front of the different exponential must be equal. Therefore,

$$\begin{aligned}
&\text{for } e^{-\lambda_1 t} \quad \rightarrow \quad -\lambda_1 A = \lambda_1 N_{01} - \lambda_2 A \\
&\text{for } e^{-\lambda_1 t} \quad \rightarrow \quad \lambda_2 A = \lambda_2 A
\end{aligned}$$

The second equation is always true. The first equation can be used to find the value of A.

$$\begin{aligned}
-\lambda_1 A &= \lambda_1 N_{01} - \lambda_2 A \\
-\lambda_1 A + \lambda_2 A &= \lambda_1 N_{01} \\
\left(-\lambda_1 + \lambda_2\right) A &= \lambda_1 N_{01} \\
A &= \frac{\lambda_1 N_{01}}{\lambda_2 - \lambda_1}
\end{aligned}$$

Therefore, the number of actinium atom as a function of time is

$$\begin{aligned}
N_1 &= N_{01} e^{-\lambda_1 t} \\
N_2 &= \frac{\lambda_1 N_{01}}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)
\end{aligned}$$

At the end of 300 s, the number of radium atoms is

$$\begin{aligned}
N_1 &= N_{01} e^{-\lambda_1 t} \\
&= N_{01} e^{-\frac{\ln 2}{250 s} 300 s} \\
&= 0.43528 N_{01}
\end{aligned}$$

and the number of actinium atom is

$$
\begin{aligned}
N_2 &= \frac{\lambda_1 N_{01}}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \\
&= \frac{\frac{\ln 2}{250s} N_{01}}{\frac{\ln 2}{250s} - \frac{\ln 2}{250s}}\left(e^{-\frac{\ln 2}{250s}300s} - e^{-\frac{\ln 2}{119s}300s}\right) \\
&= \frac{\frac{1}{250s} N_{01}}{\frac{1}{250s} - \frac{1}{250s}}\left(e^{-\frac{\ln 2}{250s}300s} - e^{-\frac{\ln 2}{119s}300s}\right) \\
&= 0.23714 N_{01}
\end{aligned}
$$

The remaining atoms are thorium atoms

$$
\begin{aligned}
N_3 &= N_{01} - N_1 - N_2 \\
&= N_{01} - 0.53428 N_{01} - 0.23714 N_{01} \\
&= 0.32758 N_{01}
\end{aligned}
$$

Thus, 43.53% of atoms are radium atoms, 23.71% are actinium atoms and 32.76% are thorium atoms.

30. The energy released is

$$
\begin{aligned}
Q &= \left(m_{before} - m_{after}\right)\cdot 931.5 \tfrac{MeV}{u} \\
&= \left(3\cdot 4.002\,603\,254u - 12.000\,000\,000u\right)\cdot 931.5 \tfrac{MeV}{u} \\
&= \left(0.007\,809\,762u\right)\cdot 931.5 \tfrac{MeV}{u} \\
&= 7.275 MeV
\end{aligned}
$$

With a 30% mass ratio, the mass of helium in the star is

$$
\begin{aligned}
m_{He} &= 0.30 \cdot 5\times 10^{30} kg \\
&= 1.5\times 10^{30} kg
\end{aligned}
$$

The number of helium atoms is

$$
\begin{aligned}
N_{atom} &= \frac{M_{helium}}{m_{atom}} \\
&= \frac{1.5\times 10^{-30} kg}{4.002\,603\,254 \cdot 1.660\,539\times 10^{-27} kg} \\
&= 2.2568\times 10^{56}
\end{aligned}
$$

As 3 atoms are needed to make one reaction, the number of reactions is

$$\begin{aligned} N_{reactions} &= \frac{2.2568\times10^{56}}{3} \\ &= 7.5227\times10^{55} \end{aligned}$$

The total energy that can be obtained is, therefore,

$$\begin{aligned} E &= N_{reactions}\cdot E_{1\,reaction} \\ &= 7.5227\times10^{55}\cdot 7.275MeV\cdot 1.602\times10^{-13}\,\tfrac{J}{MeV} \\ &= 8.767\times10^{43}J \end{aligned}$$

Finally, the lifetime is

$$\begin{gathered} E = Pt \\ 8.767\times10^{43}J = 10^{27}W\cdot t \\ t = 8.767\times10^{16}s \\ t = 2.78\times10^{9}\ years \end{gathered}$$

This is 2.78 billion years.