Chapter 11 Solutions

1. a) We'll start by calculating $h^2/8mL^2$.

$$\frac{h^2}{8mL^2}=\frac{\left(6.626\times10^{-34}Js\right)^2}{8\cdot1.6749\times10^{-27}kg\cdot\left(2\times10^{-14}m\right)^2}$$
$$=8.192\times10^{-14}J$$
$$=0.511MeV$$

The energy of the first level is

$$E_1=1^2\frac{h^2}{8mL^2}$$
$$=1\cdot0.511MeV$$
$$=0.511MeV$$

The energy of the second level is

$$E_2=2^2\frac{h^2}{8mL^2}$$
$$=4\cdot0.511MeV$$
$$=2.045MeV$$

The energy of the third level is

$$E_3=3^2\frac{h^2}{8mL^2}$$
$$=9\cdot0.511MeV$$
$$=4.602MeV$$

b) At level 1, the neutron's wavelength is

$$\lambda_n = \frac{2L}{n}$$
$$\lambda_1 = \frac{2L}{1}$$
$$= \frac{2 \cdot 2\times 10^{-14} m}{1}$$
$$= 4\times 10^{-14} m$$

2. This wave is the wave of the 6th level. The wavelength is

$$\lambda_n = \frac{2L}{n}$$
$$\lambda_6 = \frac{2L}{6}$$
$$= \frac{2 \cdot 4\times 10^{-9} m}{6}$$
$$= 1.333\times 10^{-9} m$$

Therefore, the momentum of the electron is

$$\lambda = \frac{h}{p}$$
$$1.333\times 10^{-9} m = \frac{6.626\times 10^{-34} Js}{p}$$
$$p = 4.9696\times 10^{-25} \tfrac{kgm}{s}$$

3. The smallest energy is at n = 1. The energy of this level is

$$E_1 = 1^2 \frac{h^2}{8mL^2}$$
$$= \frac{\left(6.626\times 10^{-34} Js\right)^2}{8 \cdot 9.1094\times 10^{-31} kg \cdot \left(1\times 10^{-9} m\right)^2}$$
$$= 6.025\times 10^{-20} J$$
$$= 0.376 eV$$

4. The width is found with the formula of the 4th energy level.

$$E_n = n^2 \frac{h^2}{8mL^2}$$
$$E_4 = 16\frac{h^2}{8mL^2}$$
$$10 \cdot 1.602\times 10^{-19} J = 16 \cdot \frac{\left(6.626\times 10^{-34} Js\right)^2}{8 \cdot 9.1094\times 10^{-31} kg \cdot L^2}$$
$$L = 7.757\times 10^{-10} m = 0.7757 nm$$

5. From the energy of the fourth level, the energy of the first level can be found

$$\begin{aligned} E_n &= n^2 \frac{h^2}{8mL^2} \\ E_4 &= 4^2 \cdot \frac{h^2}{8mL^2} \\ &= 16 \cdot \frac{h^2}{8mL^2} \\ &= 16 \cdot E_1 \end{aligned}$$

Therefore,

$$E_4 = 16 \cdot E_1$$
$$24eV = 16 \cdot E_1$$
$$E_1 = 1,5eV$$

Then the energy of the third level can be found from the energy of the first level.

$$\begin{aligned} E_n &= n^2 \frac{h^2}{8mL^2} \\ E_3 &= 3^2 \cdot \frac{h^2}{8mL^2} \\ &= 9 \cdot \frac{h^2}{8mL^2} \\ &= 9 \cdot E_1 \\ &= 9 \cdot 1,5eV \\ &= 13,5eV \end{aligned}$$

6. The energy of the level is

$$\begin{aligned} E_n &= \left(n_x^2 + n_y^2 + n_z^2\right)\frac{h^2}{8mL^2} \\ &= \left(3^2 + 4^2 + 2^2\right)\cdot\frac{\left(6.626\times10^{-34}Js\right)^2}{8\cdot 9.1094\times10^{-31}kg\cdot\left(1\times10^{-9}m\right)^2} \\ &= 1.747\times10^{-18}J \\ &= 10.90eV \end{aligned}$$

7. a) The energy of the first level is

$$E_1 = 1^2\frac{h^2}{8mL^2} = \frac{h^2}{8mL^2}$$

We know that a level (n) has an 80 eV energy

$$\begin{aligned} E_n &= n^2\frac{h^2}{8mL^2} \\ 80eV &= n^2 E_1 \end{aligned}$$

And that the following level (n + 1) has a 96.8 eV energy.

$$\begin{aligned} E_{n+1} &= (n+1)^2\frac{h^2}{8mL^2} \\ 96.8eV &= (n+1)^2 E_1 \end{aligned}$$

We then have 2 equations and 2 unknowns. Dividing one equation by the other, the result is

$$\begin{aligned} \frac{96.8eV}{80eV} &= \frac{(n+1)^2 E_1}{n^2 E_1} \\ 1.21 &= \frac{(n+1)^2}{n^2} \end{aligned}$$

This gives

$$1.21n^2 = (n+1)^2$$
$$1.21n^2 = n^2 + 2n + 1$$
$$0.21n^2 - 2n - 1 = 0$$

The solutions of this equation are n =10 and n = -10/21. As n can only be a positive integer, the only acceptable value for n is n = 10.

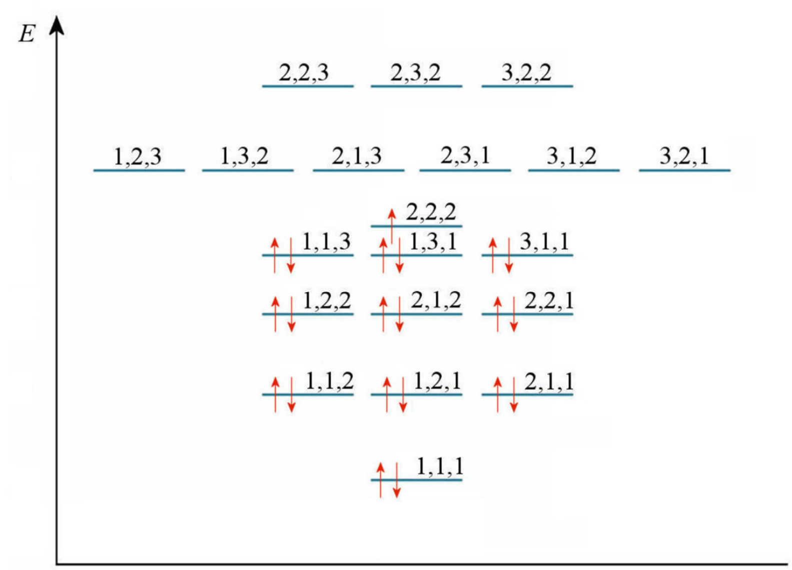
We can now find the energy of the first level.

$$80eV = n^2 E_1$$
$$80eV = 10^2 \cdot E_1$$
$$E_1 = 0.8eV$$

b) The width of the box is

$$E_1 = \frac{h^2}{8mL^2}$$
$$0.8eV \cdot 1.602 \times 10^{-19} \tfrac{J}{eV} = \frac{\left(6.626 \times 10^{-34} Js\right)^2}{8 \cdot 9.1094 \times 10^{-31} kg \cdot L^2}$$
$$L = 6.856 \times 10^{-10} m$$
$$L = 0.6856nm$$

8. Let's place the 21 electrons on the levels.



The last electron is at the level 2, 2, 2, thus at $n_x = 2$, $n_y = 2$ and $n_z = 2$

9. Let's find the energy levels. We will start by calculating $h^2/8mL^2$.

$$\begin{aligned}\frac{h^2}{8mL^2} &= \frac{\left(6.626\times10^{-34}Js\right)^2}{8\cdot1.6726\times10^{-27}kg\cdot\left(10^{-14}m\right)^2}\\ &= 3.281\times10^{-13}J\\ &= 2.048MeV\end{aligned}$$

The energy of the first level is

$$\begin{aligned}E_1 &= 1^2\frac{h^2}{8mL^2}\\ &= 1\cdot2.048MeV\\ &= 2.048MeV\end{aligned}$$

The energy of the second level is

$$\begin{aligned}E_2 &= 2^2\frac{h^2}{8mL^2}\\ &= 4\cdot2.048MeV\\ &= 8.192MeV\end{aligned}$$

The energy of the photon is

$$\begin{aligned}E_\gamma &= E_2 - E_1\\ &= 8.192MeV - 2.048MeV\\ &= 6.144MeV\end{aligned}$$

The wavelength of this photon is

$$\begin{aligned}E_\gamma &= \frac{1240eVnm}{\lambda}\\ 6.144\times10^6eV &= \frac{1240eVnm}{\lambda}\\ \lambda &= 2.018\times10^{-4}nm\end{aligned}$$

10. Let's find the energy levels. We will start by calculating $h^2/8mL^2$.

$$\frac{h^2}{8mL^2} = \frac{\left(6.626\times10^{-34}Js\right)^2}{8\cdot 9.1094\times10^{-31}kg\cdot\left(2\times10^{-9}m\right)^2}$$
$$= 1.506\times10^{-20}J$$
$$= 0.094eV$$

The energy of the first level is

$$E_1 = 1^2\frac{h^2}{8mL^2}$$
$$= 1\cdot 0,094eV$$
$$= 0,094eV$$

The energy of the fourth level is

$$E_4 = 4^2\frac{h^2}{8mL^2}$$
$$= 16\cdot 0,094eV$$
$$= 1,504eV$$

To move from level 1 to level 4, the photon's energy must be

$$E_\gamma = E_4 - E_1$$
$$= 1,504eV - 0,094eV$$
$$= 1,410eV$$

The wavelength of this photon is

$$E_\gamma = \frac{1240eVnm}{\lambda}$$
$$1,41eV = \frac{1240eVnm}{\lambda}$$
$$\lambda = 879nm$$

11. Let's find the energy levels. We will start by calculating $h^2/8mL^2$.

$$\frac{h^2}{8mL^2} = \frac{\left(6.626\times10^{-34}Js\right)^2}{8\cdot 9.1094\times10^{-31}kg\cdot\left(0.5\times10^{-9}m\right)^2}$$
$$= 2.410\times10^{-19}J$$
$$= 1.504eV$$

The energy of the starting level is

$$E_i = \left(2^2+2^2+2^2\right)\frac{h^2}{8mL^2}$$
$$= 12\cdot 1.504eV$$
$$= 18.049eV$$

The energy of the final level is

$$E_f = \left(1^2+1^2+1^2\right)\frac{h^2}{8mL^2}$$
$$= 3\cdot 1.504eV$$
$$= 4.512eV$$

The energy of the photon is

$$E_\gamma = E_i - E_f$$
$$= 18.049eV - 4.512eV$$
$$= 13.537eV$$

The wavelength of this photon is

$$E_\gamma = \frac{1240eVnm}{\lambda}$$
$$13.537eV = \frac{1240eVnm}{\lambda}$$
$$\lambda = 91.60nm$$

12. a) The energy $E_{1\infty}$ is

$$\begin{aligned}E_{1\infty} &= \frac{h^2}{8mL^2}\\&= \frac{\left(6.626\times10^{-34}Js\right)^2}{8\cdot 9.1094\times10^{-31}kg\cdot\left(0.6\times10^{-9}m\right)^2}\\&= 1.674\times10^{-19}J\\&= 1.0445eV\end{aligned}$$

La valeur de u est donc

$$\begin{aligned}U &= u^2E_{1\infty}\\20eV &= u^2\cdot 1.0445eV\\u &= 4.3758\end{aligned}$$

As $u + 1 = 5.3758$, there are 5 levels.

b) We have to solve the equations.

$$\begin{aligned}v\tan\left(\tfrac{\pi v}{2}\right) &= \sqrt{4,3758^2 - v^2}\\-v\cot\left(\tfrac{\pi v}{2}\right) &= \sqrt{4,3758^2 - v^2}\end{aligned}$$

According to the Wolfram website, the solutions to these equations are:

The first equation yields 4.1875, 2.5957, and 0.8722
The second equation gives 3.4272, and 1.7397

(We see that there are 5 levels and that there is a value of v between 0 and 1, a value of v between 1 and 2, a value of v between 2 and 3 and so on.)

Therefore, the energy levels, given by $E = v^2E_{1\infty}$, are

$$\begin{aligned}E_1 &= (0.8722)^2\cdot 1.0445eV\\&= 0.795eV\end{aligned}$$

$$\begin{aligned}E_2 &= (1,7397)^2\cdot 1.0445eV\\&= 3.161eV\end{aligned}$$

$$\begin{aligned}E_3 &= (2.5957)^2\cdot 1.0445eV\\&= 7.038eV\end{aligned}$$

$$\begin{aligned} E_4 &= (3.4272)^2 \cdot 1.0445 eV \\ &= 12.27 eV \end{aligned}$$

$$\begin{aligned} E_5 &= (4.1875)^2 \cdot 1.0445 eV \\ &= 18.32 eV \end{aligned}$$

c) To get out of the box, the particle's energy must be greater than 20 eV. Therefore

$$E_f > 20 eV$$

The smallest energy is 20 eV.

Therefore, the minimum energy of the photon is

$$\begin{aligned} E_{\gamma\, min} &= E_f - E_i \\ &= 20 eV - 3.161 eV \\ &= 16.839 eV \end{aligned}$$

Therefore, the maximum wavelength is

$$\begin{aligned} E_{\gamma\, min} &= \frac{1240 eVnm}{\lambda_{max}} \\ 16.839 eV &= \frac{1240 eVnm}{\lambda_{max}} \\ &= 73.64 nm \end{aligned}$$

13. The probability is

$$T = \frac{16E(U-E)}{U^2} e^{-2\alpha L}$$

where

$$\begin{aligned} \alpha &= \frac{\pi\sqrt{8m(U-E)}}{h} \\ &= \frac{\pi\sqrt{8 \cdot 9.1094 kg \cdot (10eV - 3eV) \cdot 1.602\times 10^{-19} \frac{J}{eV}}}{6.626\times 10^{-24} Js} \\ &= 1.3555\times 10^{10} m^{-1} \end{aligned}$$

Therefore

$$\begin{aligned} T &= \frac{16E(U-E)}{U^2} e^{-2\alpha L} \\ &= \frac{16 \cdot 3eV(10eV - 3eV)}{(10eV)^2} e^{-2 \cdot 1.3555\times 10^{10} m^{-1} \cdot 0.3\times 10^{-9} m} \\ &= \frac{16 \cdot 3eV(10eV - 3eV)}{(10eV)^2} \cdot 0.002938 \\ &= 0.000987 \\ &= 0.0987\% \end{aligned}$$

14. The probability is

$$T = \frac{16E(U-E)}{U^2} e^{-2\alpha L}$$

where

$$\begin{aligned} \alpha &= \frac{\pi\sqrt{8m(U-E)}}{h} \\ &= \frac{\pi\sqrt{8 \cdot 9.1094kg \cdot (12eV - 4eV) \cdot 1.602\times 10^{-19} \frac{J}{eV}}}{6.626\times 10^{-24} Js} \\ &= 1.4491\times 10^{10} m^{-1} \end{aligned}$$

Therefore

$$\begin{aligned} 0.05 &= \frac{16 \cdot 4eV \cdot (12eV - 4eV)}{(12eV)^2} e^{-2 \cdot 1.4491\times 10^{10} m^{-1} \cdot L} \\ 0.05 &= \tfrac{32}{9} \cdot e^{-2.8981\times 10^{10} m^{-1} \cdot L} \\ \tfrac{9}{640} &= e^{-2.8981\times 10^{10} m^{-1} \cdot L} \\ \ln\left(\tfrac{9}{640}\right) &= -2.8981\times 10^{10} m^{-1} \cdot L \\ L &= 1.471\times 10^{-10} m \\ L &= 0.1471 nm \end{aligned}$$

15. The probability is

$$T = \frac{16E(U-E)}{U^2} e^{-2\alpha L}$$

Initially, we have

$$0.1 = \frac{16 \cdot 4eV \cdot (8eV - 4eV)}{(8eV)^2} e^{-2\alpha L}$$
$$0.1 = 4 \cdot e^{-2\alpha L}$$
$$e^{-2\alpha L} = 0.025$$

If the width is doubled, we have

$$\begin{aligned} T &= \frac{16 \cdot 4eV \cdot (8eV - 4eV)}{(8eV)^2} e^{-2\alpha(2L)} \\ &= 4 \cdot \left(e^{-2\alpha L}\right)^2 \\ &= 4 \cdot (0.025)^2 \\ &= 0.0025 \\ &= 0.25\% \end{aligned}$$

16. With a period of 4 x 10^{-15} s, the frequency is

$$f = \frac{1}{T} = \frac{1}{4\times10^{-15}s} = 2.5\times10^{14}Hz$$

a) The smallest energy is at the level $n = 0$.

$$\begin{aligned} E_n &= \left(n+\tfrac{1}{2}\right)hf \\ E_0 &= \left(0+\tfrac{1}{2}\right)hf \\ &= \frac{1}{2} \cdot 6.626\times10^{-34}Js \cdot 2.5\times10^{14}Hz \\ &= 8.283\times10^{-20}J \\ &= 0.517eV \end{aligned}$$

b) The wavelength of the photon will be found from the energy of the photons. This energy is given by

$$E_\gamma = E_i - E_f$$

E_i is the energy of the level n =3.

$$\begin{aligned} E_n &= \left(n+\tfrac{1}{2}\right)hf \\ E_3 &= \left(3+\tfrac{1}{2}\right)hf \\ &= \frac{7}{2}hf \\ &= 7\cdot\frac{1}{2}hf \\ &= 7\cdot E_0 \\ &= 7\cdot 0.517eV \\ &= 3.619eV \end{aligned}$$

E_f is the energy of the level n = 1

$$\begin{aligned} E_n &= \left(n+\tfrac{1}{2}\right)hf \\ E_1 &= \left(1+\tfrac{1}{2}\right)hf \\ &= \frac{3}{2}hf \\ &= 3\cdot\frac{1}{2}hf \\ &= 3\cdot E_0 \\ &= 3\cdot 0.517eV \\ &= 1.551eV \end{aligned}$$

Therefore, the photon energy is

$$\begin{aligned} E_\gamma &= E_3 - E_1 \\ &= 3.619eV - 1.551eV \\ &= 2.068eV \end{aligned}$$

The wavelength of this photon is

$$\begin{aligned} E_\gamma &= \frac{1240eVnm}{\lambda} \\ 2.068eV &= \frac{1240eVnm}{\lambda} \\ \lambda &= 599.6nm \end{aligned}$$

17. The period will be found with the frequency. The frequency can be found with the energy formula

$$E_n = \left(n+\tfrac{1}{2}\right)hf$$

Since the energy of the photon is

$$E_\gamma = E_i - E_f$$

and the photon energy is

$$\begin{aligned} E_\gamma &= \frac{1240eV\cdot nm}{\lambda} \\ &= \frac{1240eV\cdot nm}{496nm} \\ &= 2.5eV \end{aligned}$$

we have

$$\begin{aligned} E_\gamma &= E_5 - E_2 \\ 2.5eV &= \left(5+\tfrac{1}{2}\right)hf - \left(2+\tfrac{1}{2}\right)hf \\ 2.5eV &= 5hf + \tfrac{1}{2}hf - 2hf - \tfrac{1}{2}hf \\ 2.5eV &= 3hf \end{aligned}$$

Therefore

$$\begin{aligned} 2.5eV &= 3hf \\ 2.5\cdot 1.602\times 10^{-19}J &= 3\cdot 6.626\times 10^{-34}Js\cdot f \\ f &= 2.015\times 10^{14}Hz \end{aligned}$$

The period of oscillation is therefore

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{2{,}015\times 10^{14}Hz} \\ &= 4.963\times 10^{-15}s \end{aligned}$$

18. The wavelength is found with the energy of the photon, which itself is given by

$$E_\gamma = E_i - E_f$$

The energy of the 5th level is

$$\begin{aligned} E_n &= \frac{-13.598eV}{n^2} \\ E_5 &= \frac{-13.598eV}{5^2} \\ &= -0.54392eV \end{aligned}$$

The energy of the 2nd level is

$$\begin{aligned} E_n &= \frac{-13.598eV}{n^2} \\ E_2 &= \frac{-13.598eV}{2^2} \\ &= -3.3995eV \end{aligned}$$

The energy of the photon is

$$\begin{aligned} E_\gamma &= E_i - E_f \\ &= -0.54392eV - -3.3995eV \\ &= 2.85558eV \end{aligned}$$

Therefore, the wavelength of light is

$$\begin{aligned} E_\gamma &= \frac{1240eV \cdot nm}{\lambda} \\ 2.85558eV &= \frac{1240eV \cdot nm}{\lambda} \\ \lambda &= 434.2nm \end{aligned}$$

19. The wavelength is found with the energy of the photon. For the photon to be absorbed, its energy must be exactly equal to the energy gap between levels 1 and 6.

The energy of the 1st level is

$$E_n = \frac{-13.598eV}{n^2}$$
$$E_1 = \frac{-13.598eV}{1^2}$$
$$= -13.598eV$$

The energy of the 6th level is

$$E_n = \frac{-13.598eV}{n^2}$$
$$E_2 = \frac{-13.598eV}{6^2}$$
$$= -0.37772eV$$

The energy that the photon must have is

$$E_\gamma = E_f - E_i$$
$$= -0.37773eV - -13.598eV$$
$$= 13.2203eV$$

Therefore, the wavelength of light must be

$$E_\gamma = \frac{1240eV \cdot nm}{\lambda}$$
$$13.2203eV = \frac{1240eV \cdot nm}{\lambda}$$
$$\lambda = 93.80nm$$

20. a) The energy of the 1st level is

$$E_n = \frac{-13.598eV}{n^2}$$
$$E_1 = \frac{-13.598eV}{1^2}$$
$$= -13.598eV$$

The energy of the final level is

$$E_\gamma = E_f - E_i$$
$$13eV = E_f - -13.598eV$$
$$E_f = -0.598eV$$

Let's see to which level this energy corresponds to

$$E_n = \frac{-13.598eV}{n^2}$$
$$-0.598 = \frac{-13.598eV}{n^2}$$
$$n = 4.769$$

As n can only be an integer value, there is no level having this energy. Therefore, the photons cannot be absorbed and the electrons are always on the 1st level.

b) If bombarded with electrons, any energy between 0 and 13 eV can be transferred during the collision. In a), we found that an addition of 13 eV brings us somewhere between the 4th and 5th level. So, the electrons can go to level 2, level 3, and level 4.

21. At a level n, the values of l the can take the integer values between 0 and n – 1.

For each value of l, m can take integer values between -l and l. Therefore

1 value for l = 0 (m = 0)
3 values for l = 1 (m = -1, 0, and 1)
5 values for l = 2 (m = -2, -1, 0, 1, and 2)
7 values for l = 3 (m = -3, -2, -1, 0, 1, 2, and 3)
…and so on.

Here we find the list of the odd numbers. The last odd number will be $2l_{\max} + 1$.

As $l_{\max}$ is equal to n – 1, the last odd number will be

$$2l_{\max} + 1 = 2(n-1) + 1$$
$$= 2n - 1$$

The total number of levels is therefore obtained by adding up these odd numbers.

$$N = 1 + 3 + 5 + 7 + \ldots + (2n - 1)$$

If $n = 1$, $N = 1$. If $n = 2$, $N = 4$. If $n = 3$, $N = 9$. If $n = 4$, $N = 16$. The number of levels seems to be given by n^2.

Can we prove that this sum of odd numbers is equal to n^2? Of course.

Let's start by adding these two series of numbers

$$N = 1 + 3 + 5 + 7 + \ldots + (2n - 5) + (2n - 3) + (2n - 1)$$

and

$$N = (2n - 1) + (2n - 3) + (2n - 5) + \ldots + 7 + 5 + 3 + 1$$

If the first terms of each series are summed, the result is $2n$.
If the second terms of each series are summed, the result is $2n$.
If the third terms of each series are summed, the result is $2n$.
…
The addition of terms 2 by 2 gives us a series of $2n$.

$$2N = 2n + 2n + 2n + \ldots + 2n$$

As there are n terms in the series, the result is

$$2N = n \cdot 2n$$
$$2N = 2n^2$$
$$N = n^2$$

22. a) The energy is

$$E_n = -\frac{Z^2 \cdot 13,60eV}{n^2}$$
$$E_1 = -\frac{3^2 \cdot 13,60eV}{1^2} = -122,4eV$$

b) To ionize the atom, the energy of the electron must be positive. Therefore, at least 122.4 eV must be given to ionize it.

c) The wavelength is found with the energy of the photon, which itself is found with

$$E_\gamma = E_i - E_f$$

The energy of the 2nd level is

$$\begin{aligned} E_n &= -\frac{Z^2 \cdot 13.60eV}{n^2} \\ E_2 &= -\frac{3^2 \cdot 13.60eV}{2^2} \\ &= -30.6eV \end{aligned}$$

We already know that the energy of the 1st level is -122.4 eV.

The energy of the photon is

$$\begin{aligned} E_\gamma &= E_i - E_f \\ &= -30.6eV - -122.4eV \\ &= 91.8eV \end{aligned}$$

Therefore, the wavelength of light is

$$\begin{aligned} E_\gamma &= \frac{1240eV \cdot nm}{\lambda} \\ 91.8eV &= \frac{1240eV \cdot nm}{\lambda} \\ \lambda &= 13.51nm \end{aligned}$$

23. The energy of the absorbed photon is

$$\begin{aligned} E_\gamma &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{250nm} \\ &= 4.96eV \end{aligned}$$

The energy of the electron has increased by 4.96 eV.

By emitting the first photon, the electron loses the energy of this photon. This energy is

$$\begin{aligned} E_\gamma &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{800nm} \\ &= 1.55eV \end{aligned}$$

As the electron had gained 4.96 eV and had just lost 1.55 eV, he still has 3.41 eV to lose to return to its initial energy level. It had to emit a 3.41 eV photon, which has the wavelength given by this formula.

$$E_\gamma = \frac{1240eVnm}{\lambda}$$
$$3.41eV = \frac{1240eVnm}{\lambda}$$
$$\lambda = 363.6nm$$

24. In the box, there is a standing wave which is identical to a standing wave on a string. Thus, the amplitude of the wave will be identical to the amplitude standing wave on a string.

$$\psi = 2A\sin kx$$

The wavelength of the first level is

$$\begin{aligned}\lambda_1 &= \frac{2L}{1}\\ &= 2L\\ &= 2\cdot 10nm\\ &= 20nm\end{aligned}$$

Thus, the amplitude of the wave is

$$\psi = 2A\sin kx$$
$$\psi = 2A\sin\left(\frac{2\pi}{\lambda}x\right)$$
$$\psi = 2A\sin\left(\frac{2\pi}{20nm}x\right)$$

First, this wave function must comply with

$$\int_{\substack{\text{all possible}\\ \text{locations}}} \psi^2 dx = 1$$

As the only possible locations are between 0 nm and 10 nm, this equation becomes

$$\int_{0nm}^{10nm} \left(2A\sin\left(\frac{2\pi}{20nm}x\right)\right)^2 dx = 1$$

The value of *A* can then be found with this equation.

$$\int_{0nm}^{10nm} 4A^2 \sin^2\left(\frac{2\pi}{20nm}x\right) dx = 1$$

$$4A^2\left[\frac{x}{2} - \frac{20nm}{8\pi}\sin\left(\frac{4\pi}{20nm}x\right)\right]_{0nm}^{10nm} = 1$$

$$4A^2\left[\frac{10nm}{2} - \frac{20nm}{8\pi}\sin\left(\frac{4\pi}{20nm}10nm\right)\right] - A^2\left[\frac{0nm}{2} - \frac{10nm}{8\pi}\sin\left(\frac{4\pi}{20nm}0nm\right)\right] = 1$$

$$4A^2\left[\frac{10nm}{2} - \frac{20nm}{8\pi}\sin(2\pi)\right] - A^2\left[0 - \frac{10nm}{8\pi}\sin(0)\right] = 1$$

$$4A^2 \cdot 5nm = 1$$

$$A = \frac{1}{2\sqrt{5nm}}$$

Thus, the amplitude of the wave is

$$\psi = 2A\sin\left(\frac{2\pi}{20nm}x\right)$$

$$\psi = \frac{1}{\sqrt{5nm}}\sin\left(\frac{2\pi}{20nm}x\right)$$

Therefore, the probability of finding the particle between $x = 0$ nm and $x = 3$ nm is

$$P=\int_{0nm}^{3nm}\frac{1}{5nm}\sin^2\left(\frac{2\pi}{20nm}x\right)dx$$
$$=\frac{1}{5nm}\left[\frac{x}{2}-\frac{20nm}{8\pi}\sin\left(\frac{4\pi}{20nm}x\right)\right]_{0nm}^{3nm}$$
$$=\frac{1}{5nm}\left[\frac{3nm}{2}-\frac{20nm}{8\pi}\sin\left(\frac{4\pi}{20nm}3nm\right)\right]-\frac{1}{5nm}\left[\frac{0nm}{2}-\frac{20nm}{8\pi}\sin\left(\frac{4\pi}{20nm}0nm\right)\right]$$
$$=\frac{1}{5nm}\left[\frac{3nm}{2}-\frac{20nm}{8\pi}\sin(0.6\pi)\right]-\frac{1}{5nm}\left[0-\frac{20nm}{8\pi}\sin(0)\right]$$
$$=\frac{1}{5nm}\left[\frac{3nm}{2}-\frac{20nm}{8\pi}\sin(1.2\pi)\right]$$
$$=\frac{3}{10}-\frac{1}{2\pi}\sin(0.6\pi)$$
$$=0.1486$$

Therefore, the probability is 14.86%

25. The Schrödinger equation with the potential $U=½kx^2$ is

$$\frac{d^2\psi}{dx^2}+\frac{2m}{\hbar^2}\left(E-\frac{1}{2}kx^2\right)\psi=0$$

Let's check whether

$$\psi=Ae^{-Bx^2}$$

is a solution. To check this, the second derivative of this function is needed.

$$\frac{d\psi}{dx}=Ae^{-Bx^2}(-2Bx)$$
$$=2ABxe^{-Bx^2}$$

$$\frac{d^2\psi}{dx^2}=\frac{d}{dx}\left(-2ABxe^{-Bx^2}\right)$$
$$=-2ABxe^{-Bx^2}(-2Bx)+-2ABe^{-Bx^2}$$
$$=4AB^2x^2e^{-Bx^2}-2ABe^{-Bx^2}$$

Therefore

$$\frac{d^2\psi}{dx^2}+\frac{2m}{\hbar^2}\left(E-\frac{1}{2}kx^2\right)\psi=0$$

$$4AB^2x^2e^{-Bx^2}-2ABe^{-Bx^2}+\frac{2m}{\hbar^2}\left(E-\frac{1}{2}kx^2\right)Ae^{-Bx^2}=0$$

$$4B^2x^2-2B+\frac{2m}{\hbar^2}\left(E-\frac{1}{2}kx^2\right)=0$$

$$\frac{2m}{\hbar^2}\left(E-\frac{1}{2}kx^2\right)=2B-4B^2x^2$$

$$\frac{2m}{\hbar^2}E-\frac{mk}{\hbar^2}x^2=2B-4B^2x^2$$

The function is a solution if both sides are equal. To be equal, the constant terms must be equal and that the x^2 terms must also be equal. Thus

$$\frac{2m}{\hbar^2}E=2B \qquad \text{and} \qquad \frac{mk}{\hbar^2}x^2=4B^2x^2$$

The second equation gives

$$\frac{mk}{\hbar^2}x^2=4B^2x^2$$

$$\frac{mk}{4\hbar^2}=B^2$$

Using this value in the first equation, the result is

$$\frac{2m}{\hbar^2}E=2B$$

$$\frac{2m}{\hbar^2}E=2\sqrt{\frac{mk}{4\hbar^2}}$$

$$\frac{m}{\hbar^2}E=\frac{\sqrt{mk}}{2\hbar}$$

$$E=\frac{\sqrt{mk}}{2m}\hbar$$

$$E=\frac{1}{2}\sqrt{\frac{k}{m}}\hbar$$

For a harmonic oscillation, we have

$$\omega = \sqrt{\frac{k}{m}}$$

Thus

$$\begin{aligned} E &= \frac{1}{2}\hbar\omega \\ &= \frac{1}{2}\frac{h}{2\pi}2\pi f \\ &= \frac{1}{2}hf \end{aligned}$$