Chapter 10 Solutions

1. The energy is

$$\begin{aligned} E_\gamma &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{550nm} \\ &= 2.25eV \end{aligned}$$

2. The number of photons is given by

$$N = \frac{\text{Total energy emitted}}{\text{Energy of one photon}}$$

The energy of a photon is

$$\begin{aligned} E_\gamma &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{632nm} \\ &= 1.962eV \\ &= 3.143\times10^{-19} J \end{aligned}$$

The energy emitted per second is

$$\begin{aligned} E &= Pt \\ &= 0.001W \cdot 1s \\ &= 0.001J \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned} N &= \frac{\text{Total energy emitted}}{\text{Energy of one photon}} \\ &= \frac{0.001J}{3.143\times10^{-19} \frac{J}{photons}} \\ &= 3.182\times10^{15}\ photons \end{aligned}$$

3. The number of photons is given by

$$N = \frac{\text{Total energy received}}{\text{Energy of one photon}}$$

The energy of a photon is

$$\begin{aligned} E_\gamma &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{585nm} \\ &= 2.12eV \\ &= 3.396\times10^{-19}J \end{aligned}$$

The energy received in 20 seconds is

$$\begin{aligned} E &= IA_{receiver}t \\ &= 50\tfrac{W}{m^2}\cdot 3m^2\cdot 20s \\ &= 3000J \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned} N &= \frac{\text{Total energy received}}{\text{Energy of one photon}} \\ &= \frac{3000J}{3.396\times10^{-19}\frac{J}{photons}} \\ &= 8.835\times10^{21}\ photons \end{aligned}$$

4. The number of photons is given by

$$N = \frac{\text{Total energy received}}{\text{Energy of one photon}}$$

The energy of a photon is

$$\begin{aligned} E_\gamma &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{470nm} \\ &= 2.638eV \\ &= 4.227\times10^{-19} J \end{aligned}$$

The energy received per second is

$$\begin{aligned} E &= IA_{receiver}t \\ &= 200\tfrac{W}{m^2}\cdot\pi(0.0025m)^2\cdot 1s \\ &= 0.003927J \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned} N &= \frac{\text{Total energy received}}{\text{Energy of one photon}} \\ &= \frac{0.003927J}{4.227\times10^{-19}\,\frac{J}{photons}} \\ &= 9.291\times10^{15}\ photons \end{aligned}$$

5. The maximal energy of the electrons is found with

$$E_{k\max} = hf - \phi$$

The photon energy is

$$\begin{aligned} E_\gamma &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{150nm} \\ &= 8.267eV \end{aligned}$$

The maximum energy of the ejected electrons is, therefore,

$$\begin{aligned} E_{k\max} &= hf - \phi \\ &= 8.267eV - 4.5eV \\ &= 3.767eV \end{aligned}$$

6. The maximal energy of the electrons is found with

$$E_{k\max} = hf - \phi$$

The work function of cesium is

$$\begin{aligned}\phi &= \frac{1240eVnm}{\lambda_0} \\ &= \frac{1240eVnm}{686nm} \\ &= 1.808eV\end{aligned}$$

a) With a wavelength of 690 nm, the energy of the photons is

$$\begin{aligned}E_\gamma &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{690nm} \\ &= 1.797eV\end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned}E_{k\max} &= hf - \phi \\ &= 1.797eV - 1.808eV \\ &= -0.011eV\end{aligned}$$

This means that there are no electrons ejected since a negative kinetic energy is impossible. Photons don't have enough energy to eject electrons.

b) With a wavelength of 450 nm, the energy of the photons is

$$\begin{aligned}E_\gamma &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{450nm} \\ &= 2.756eV\end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned} E_{k\max} &= hf - \phi \\ &= 2.756eV - 1.808eV \\ &= 0.948eV \end{aligned}$$

7. a) The threshold wavelength is

$$\phi = \frac{1240eVnm}{\lambda_0}$$

$$3.2eV = \frac{1240eVnm}{\lambda_0}$$

$$\lambda_0 = 387.5nm$$

b) The maximal speed is found with the maximum energy of the electrons, which is found with

$$E_{k\max} = hf - \phi$$

With a wavelength of 250 nm, the energy of the photons is

$$\begin{aligned} E_\gamma &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{250nm} \\ &= 4.96eV \end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned} E_{k\max} &= hf - \phi \\ &= 4.96eV - 3.2eV \\ &= 1.76eV \\ &= 2.82\times10^{-19}J \end{aligned}$$

Therefore, the speed of the electrons is

$$E_{k\max} = \frac{1}{2}mv_{\max}^2$$

$$2.82\times10^{-19}J = \frac{1}{2}\cdot 9.1094\times10^{-31}kg\cdot v_{\max}^2$$

$$v_{\max} = 7.868\times10^5\,\tfrac{m}{s}$$

8. The threshold wavelength id found with the work function, and this work function is found with

$$E_{k\,max} = hf - \phi$$

The maximum kinetic energy of the electrons is

$$\begin{aligned} E_{k\,max} &= \frac{1}{2} m v_{max}^2 \\ &= \frac{1}{2} \cdot 9.1094 \times 10^{-31} kg \cdot \left(5 \times 10^5 \tfrac{m}{s}\right)^2 \\ &= 1.139 \times 10^{-19} J \\ &= 0.711 eV \end{aligned}$$

The energy of the photons is

$$\begin{aligned} E_\gamma &= \frac{1240 eVnm}{\lambda} \\ &= \frac{1240 eVnm}{400 nm} \\ &= 3.1 eV \end{aligned}$$

The work function is then found with

$$\begin{aligned} E_{k\,max} &= hf - \phi \\ 0.711 eV &= 3.1 eV - \phi \\ \phi &= 2.389 eV \end{aligned}$$

Therefore, the threshold wavelength is

$$\begin{aligned} \phi &= \frac{1240 eVnm}{\lambda_0} \\ 2,389 eV &= \frac{1240 eVnm}{\lambda_0} \\ \lambda_0 &= 519 nm \end{aligned}$$

9. Since 3 % of the photons eject electrons, the number of ejected electrons is

$$N_{electrons} = 0.03 \cdot N_{photons}$$

The energy of a photon received is given by

$$N_{photons} = \frac{\text{Total energy}}{\text{Energy of one photon}}$$

$$\begin{aligned} E &= \frac{1240eVnm}{\lambda} \\ &= \frac{1240eVnm}{450nm} \\ &= 2.756eV \\ &= 4.414\times10^{-19} J \end{aligned}$$

The energy received per second per square centimetre is

$$\begin{aligned} E &= IA_{receiver}t \\ &= 40\tfrac{W}{m^2}\cdot 0.0001m^2 \cdot 1s \\ &= 0.004J \end{aligned}$$

Therefore, the number of photons received is

$$\begin{aligned} N &= \frac{\text{Total energy}}{\text{Energy of one photon}} \\ &= \frac{0.004J}{4.414\times10^{-19}\,\frac{J}{photons}} \\ &= 9.091\times10^{15}\ photons \end{aligned}$$

If only 3% of the photons eject an electron, then the number of ejected electrons is

$$\begin{aligned} N_{electrons} &= 0.03\cdot N_{photons} \\ &= 0.03\cdot 9.091\times10^{15} \\ &= 2.718\times10^{15} \end{aligned}$$

10. a) The wavelength shift is

$$\begin{aligned}\Delta\lambda &= 2.4263\times10^{-3}nm\cdot(1-\cos\theta)\\ &= 2.4263\times10^{-3}nm\cdot(1-\cos 45^\circ)\\ &= 0.0007106nm\end{aligned}$$

b) The wavelength of the incident photon is

$$\begin{aligned}E_\gamma &= \frac{1240eVnm}{\lambda}\\ 62,000eV &= \frac{1240eVnm}{\lambda}\\ \lambda &= 0.02nm\end{aligned}$$

The new wavelength is thus

$$\begin{aligned}\lambda' &= \lambda+\Delta\lambda\\ &= 0.02nm+0.0007106nm\\ &= 0.0207106nm\end{aligned}$$

c) The new energy of the photon is

$$\begin{aligned}E'_\gamma &= \frac{1240eVnm}{\lambda'}\\ &= \frac{1240eVnm}{0.0207106nm}\\ &= 59,873eV\end{aligned}$$

d) The kinetic energy of the electron is

$$\begin{aligned}E_\gamma &= E'_\gamma + E_{k\,e}\\ 62,000eV &= 59,873eV + E_{k\,e}\\ E_{k\,e} &= 2127eV\end{aligned}$$

e) The angle with the conservation of *y*-component of the momentum.

$$0 = p'_\gamma \sin\theta - p'_e \sin\phi$$

The momentum of the photon is found with

$$E'_\gamma = p'_\gamma c$$
$$59{,}873 \cdot 1.602\times10^{-19} J = p'_\gamma \cdot 3\times10^{8} \tfrac{m}{s}$$
$$p'_\gamma = 3.197\times10^{-23} \tfrac{kgm}{s}$$

The momentum of the electron is found with

$$E_e = \frac{p^2}{2m}$$
$$2127 \cdot 1.602\times10^{-19} J = \frac{p'^2_e}{2\cdot 9.1094\times10^{-31} kg}$$
$$p'_e = 2.491\times10^{-23} \tfrac{kgm}{s}$$

The conservation equation then becomes

$$0 = p'_\gamma \sin\theta - p'_e \sin\phi$$
$$0 = 3.197\times10^{-23} \tfrac{kgm}{s} \cdot \sin 45° - 2.491\times10^{-23} \tfrac{kgm}{s} \cdot \sin\phi$$
$$0 = 3.197 \cdot \sin 45° - 2.491 \cdot \sin\phi$$
$$\phi = 65.1°$$

11. The diffusion angle is found with

$$\Delta\lambda = 2.4263\times10^{-3} nm \cdot (1-\cos\theta)$$

To find the angle, we need the wavelength shift. This shift is found with the wavelengths before and after the collision.

The initial wavelength is

$$E_\gamma = \frac{1240eVnm}{\lambda}$$
$$50{,}000eV = \frac{1240eVnm}{\lambda}$$
$$\lambda = 0.0248nm$$

The wavelength after the scattering is

$$E'_\gamma = \frac{1240eVnm}{\lambda'}$$
$$49,500eV = \frac{1240eVnm}{\lambda'}$$
$$\lambda' = 0.02505nm$$

So, the wavelength shift is

$$\begin{aligned}\Delta\lambda &= \lambda' - \lambda \\ &= 0.02505nm - 0.0248nm \\ &= 0.00025nm\end{aligned}$$

Therefore, the angle is

$$\Delta\lambda = 2.4263\times10^{-3}\,nm\cdot(1-\cos\theta)$$
$$0.00025nm = 2.4263\times10^{-3}\,nm\cdot(1-\cos\theta)$$
$$\theta = 26.3°$$

12. The wavelength is

$$\begin{aligned}\lambda &= \frac{h}{p} \\ &= \frac{h}{mv} \\ &= \frac{6.626\times10^{-34}\,Js}{1.6726\times10^{-27}\,kg\cdot10^4\,\frac{m}{s}} \\ &= 3.96\times10^{-11}m = 0.0396nm\end{aligned}$$

13. As the speed is close to the speed of light, the relativistic momentum formula must be used. The wavelength is, therefore,

$$\begin{aligned}\lambda &= \frac{h}{p}\\ &= \frac{h}{\gamma m v}\\ &= \frac{6.626\times10^{-34}Js}{\frac{1}{\sqrt{1-\left(\frac{2\times10^{8}\frac{m}{s}}{3\times10^{8}\frac{m}{s}}\right)^{2}}}\cdot 1.6726\times10^{-27}kg\cdot 2\times10^{8}\frac{m}{s}}\\ &= 1.476\times10^{-15}m\end{aligned}$$

14. The wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

The speed will be found from the kinetic energy.

With a 10 eV kinetic energy, the speed of the electron is

$$\begin{aligned}E_k &= \frac{1}{2}mv^2\\ 10\cdot 1.602\times10^{-19}J &= \frac{1}{2}\cdot 9.1094\times10^{-31}kg\cdot v^2\\ v &= 1.875\times10^{6}\frac{m}{s}\end{aligned}$$

Thus, the wavelength is

$$\begin{aligned}\lambda &= \frac{h}{mv}\\ &= \frac{6.626\times10^{-34}Js}{9.1094\times10^{-31}kg\cdot 1.875\times10^{6}\frac{m}{s}}\\ &= 3.879\times10^{-10}m = 0.3879nm\end{aligned}$$

15. With a 6 eV kinetic energy, the speed of the electron is

$$E_k = \frac{1}{2} m v^2$$
$$6 \cdot 1.602 \times 10^{-19} J = \frac{1}{2} \cdot 9{,}1094 \times 10^{-31} kg \cdot v^2$$
$$v = 1.453 \times 10^6 \tfrac{m}{s}$$

Thus, the wavelength is

$$\begin{aligned} \lambda &= \frac{h}{p} \\ &= \frac{h}{mv} \\ &= \frac{6.626 \times 10^{-34} Js}{9.1094 \times 10^{-31} kg \cdot 1.453 \times 10^6 \tfrac{m}{s}} \\ &= 5.007 \times 10^{-10} m = 0.5007 nm \end{aligned}$$

When U increases to 2 eV, the kinetic energy decreases to 4 eV. The speed of the electron is then

$$E_k = \frac{1}{2} m v^2$$
$$4 \cdot 1.602 \times 10^{-19} J = \frac{1}{2} \cdot 9.1094 \times 10^{-31} kg \cdot v^2$$
$$v = 1.186 \times 10^6 \tfrac{m}{s}$$

And the wavelength is

$$\begin{aligned} \lambda &= \frac{h}{p} \\ &= \frac{h}{mv} \\ &= \frac{6.626 \times 10^{-34} Js}{9.1094 \times 10^{-31} kg \cdot 1.186 \times 10^6 \tfrac{m}{s}} \\ &= 6.132 \times 10^{-10} m = 0.6132 nm \end{aligned}$$

The change in wavelength is, therefore,

$$\begin{aligned} \Delta\lambda &= \lambda' - \lambda \\ &= 0.6132 nm - 0.5007 nm \\ &= 0.1125 nm \end{aligned}$$

16. The distance x is the distance between the order-2- maxima. The position of these maxima will be found with

$$d\sin\theta = m\lambda$$

We have d but not λ. We will find this wavelength with h/p.

With a 2 eV kinetic energy, the speed of the electron is

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ 2\cdot 1.602\times10^{-19}J &= \frac{1}{2}\cdot 9.1094\times10^{-31}kg\cdot v^2 \\ v &= 8.3877\times10^5 \tfrac{m}{s} \end{aligned}$$

The wavelength is

$$\begin{aligned} \lambda &= \frac{h}{p} \\ &= \frac{h}{mv} \\ &= \frac{6.626\times10^{-34}Js}{9.1094\times10^{-31}kg\cdot 8.3877\times10^5 \tfrac{m}{s}} \\ &= 8.672\times10^{-10}m = 0.8672nm \end{aligned}$$

Therefore, the angle of the order-2 maximum is

$$\begin{aligned} d\sin\theta &= m\lambda \\ 0.1\times10^{-6}m\cdot\sin\theta &= 2\cdot 8.672\times10^{-10}m \\ \theta &= 0.9938° \end{aligned}$$

The distance from the central maximum to the order 2 maximum is, therefore,

$$\begin{aligned} \tan\theta &= \frac{y}{L} \\ \tan(0.9938°) &= \frac{y}{300cm} \\ y &= 5.204cm \end{aligned}$$

The distance between the two order 2 maxima is twice as big. Therefore, it is 10.408 cm.

17. The uncertainty of the momentum is

$$\begin{aligned}\Delta p &= p_{\max} - p_{\min} \\ &= 2.05\times10^{-23} \tfrac{kgm}{s} - 2\times10^{-23} \tfrac{kgm}{s} \\ &= 5\times10^{-25} \tfrac{kgm}{s}\end{aligned}$$

Therefore, the uncertainty of the position is

$$\Delta x \Delta p = h$$
$$\Delta x \cdot 5\times10^{-25} \tfrac{kgm}{s} = 6.626\times10^{-34} Js$$
$$\Delta x = 1.325\times10^{-9} m = 1.325 nm$$

18. The uncertainty of the energy is

$$\Delta E \Delta t = h$$
$$\Delta E \cdot 10^{-8} s = 6.626\times10^{-34} Js$$
$$\Delta E = 6.626\times10^{-26} J = 4.136\times10^{-7} eV$$