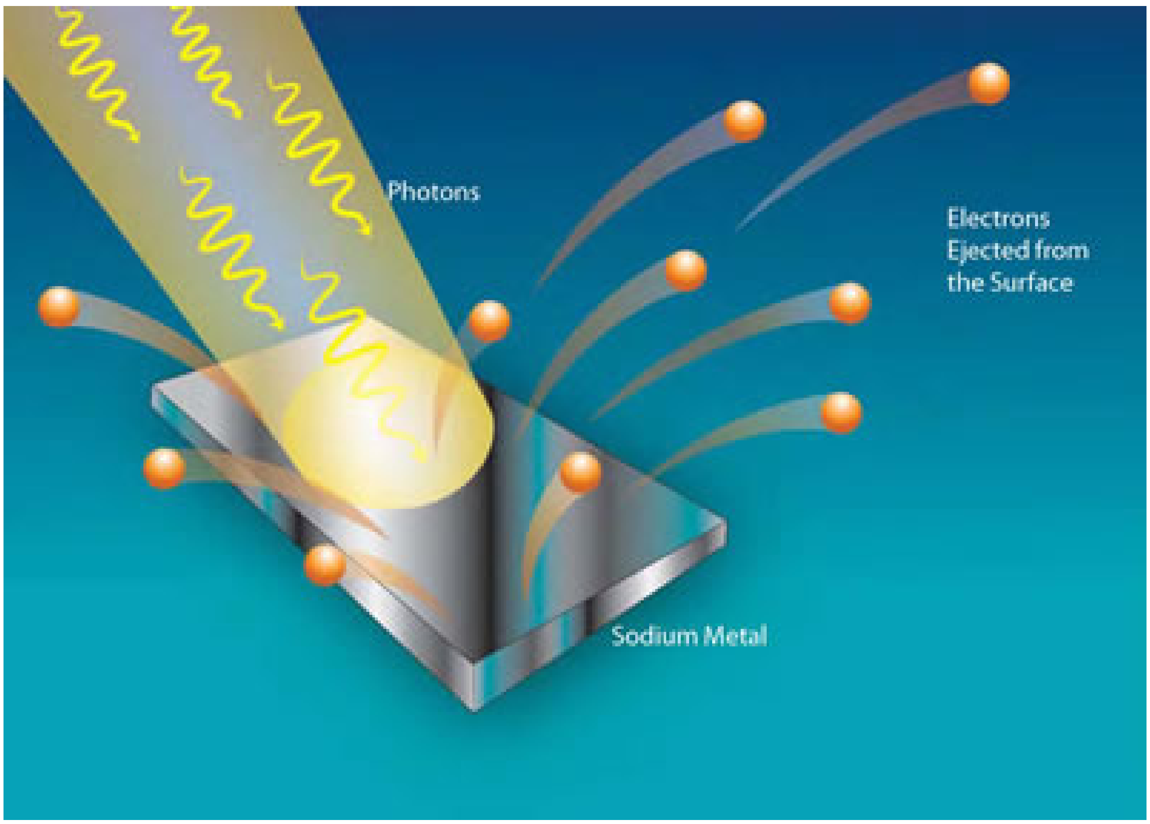
10 WAVES AND PARTICLES

Light of wavelength 310 nm is incident on a piece of sodium (work function = 2.46 eV). What is the maximum speed of the ejected electrons?



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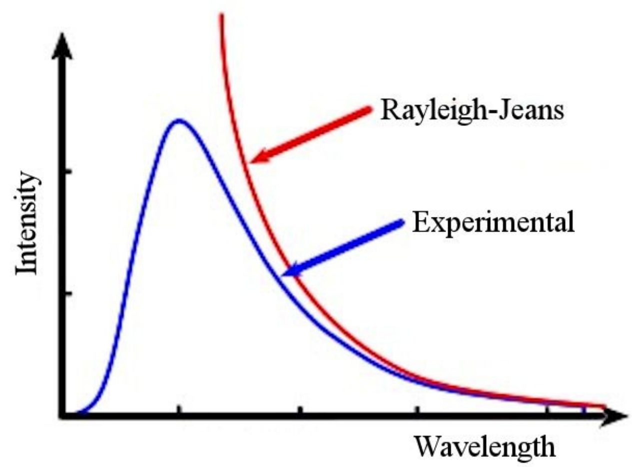
Discover how to solve this problem in this chapter.

10.1 PHOTONS

Rayleigh-Jeans Law

We have seen in the previous chapter that a hot object emits radiation. However, a serious problem occurred when physicists tried to explain this radiation. Basically, the principle is simple. The atoms of the warm object are oscillating due to temperature. Because of this oscillation, there are accelerating charged particles which generates electromagnetic waves. If the object is hotter the oscillations are larger, and more radiation is emitted. For the theoretical calculations, it was assumed that the object is in equilibrium with its environment and that it absorbs all the incoming radiation (hence the name *black body radiation* often given to this phenomenon).

But the calculations, made from 1860 (but more thoroughly after 1890), were unable to reproduce the experimental results. The theoretical formula, the Rayleigh-Jeans law, predicted instead the radiation curve shown in the graph. Even more annoying, the area under the theoretical curve (which represents the emitted power) is infinite! This means that the objects would cool instantly with a huge burst of radiation. This obviously wrong result was called the *ultraviolet catastrophe* since the theoretical formula was seriously different for small wavelengths (ultraviolet and smaller) while the agreement was better for the longer wavelengths.



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A way to reconcile theory and experiment had to be found.

Planck's Hypothesis

In 1900, Max Planck discovered that the right formula can be obtained if it is assumed that atoms emit energy in a very peculiar way. While atoms can emit any amount of energy in classical physics, Planck assumed that they must emit discrete packets of energy, which are called *quanta of energy* (*quantum* is the singular). The quantum of energy of an atom undergoing a harmonic oscillation at frequency f is

Quantum of Energy of an Atom in Harmonic Oscillation

$$E = hf$$

where h is a constant called *Planck constant* whose value is

Planck Constant

$$h = 6.626\ 070\ 15 \times 10^{-34}\ J \cdot s$$

(This value is exact. All decimals after the 5 are 0. It is so because the kilogram was defined, in 2019, by giving this value to h. For calculations, the value 6.626 x 10^{-34} Js will be used.)

According to Planck, the energy of the radiation emitted by an atom must be an integer multiple of the quantum of energy.

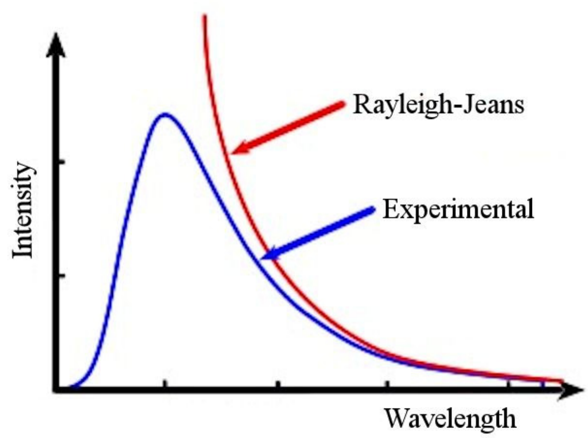
Energy Emitted by an Oscillating Atom

$$E = nhf$$

where n is an integer. This is called the quantification of energy.

Let's see why this hypothesis eliminates the ultraviolet catastrophe.

For low-frequency radiation (so long wavelength radiation), the value of the quantum of energy hf is small. Let's use numbers to illustrate. Assume that energy of oscillation of the atom is 100 eV and that the quantum of energy at that frequency is 1 eV. The value of the quantum of energy is then much smaller than the energy of the atom, and the atom can easily emit radiation. It's easy to emit 1 eV (or 2 eV or 3 eV… because an integer number of quanta can be emitted) when you have an energy of 100 eV. The atom can emit a lot of quanta but as each quantum is relatively small, the overall intensity is not so great. This is why the curve is low in the right part of the graph.



Now, let's consider higher frequencies so that the quantum of energy is now worth 5 eV. An atom that has an energy of 100 eV can easily emit such quanta of energy. As each quantum has more energy, the emitted radiation is more intense. That's why the intensity curve (blue curve) rises when the wavelength decreases on the right part of the graph.

Let's consider still higher frequencies. Assume that hf is now 30 eV. An atom with an energy of 100 eV can still emit a quantum or quanta of energy, but it is more difficult. The emission of a large quantum is more unlikely than the emission of a small quantum. The amount of predicted radiation thus starts to decrease so that the radiation curve (in blue) increases less quickly than what was predicted by classical physics (in red) when the wavelength decreases.

For high frequencies (so for small wavelengths), the quantum of energy is even greater. Assume that *hf* is now worth 200 eV. It then becomes impossible for an atom with an energy of 100 eV to emit radiation. How can an atom with an energy of 100 eV emit a 200 eV quantum of energy? In this case, the radiation is completely eliminated. This inability to emit radiation at small wavelengths explains the sharp fall of the curve in the left part of the graph. (The fall is not instantaneous at 100 eV because this value would represent the average energy of the atoms. Some atoms have more energy than the average and can emit a quantum with an energy of 200 eV but there is not a lot of such atoms.)

Planck's hypothesis, therefore, leaves the radiation with large wavelengths identical to what was predicted while it decreases the radiation at medium frequencies and eliminates the radiation at low frequencies. This is exactly what was needed to eliminate the ultraviolet catastrophe. With this quanta hypothesis and the right choice for the value of Planck constant, the agreement is perfect between theory and experiment.

(It must be said that Planck did the inverse reasoning: he found a function that was in agreement with the theory and then he looked for what he had to assume to arrive at such a result. He then reached the conclusion that light has to be emitted by quanta.)

Still, no one knew why the emitted energy had to be an integer number of the quantum *hf*. Planck tried for several years to find a justification, but he never succeeded. Without plausible explanations, many simply regarded Planck's hypothesis as a mathematical trick.

Einstein's Photon Hypothesis

In 1905 (the same year he discovered relativity!), Einstein obtained a very interesting result concerning light. The properties of light trapped in a box (obviously, the sides are mirrors), are identical to those of a gas of particles. The results even show that these particles have an energy equal to *hf*! For Einstein, this was more than a coincidence. He, therefore, proposed the photon hypothesis.

Einstein's Photon Hypothesis

Light is composed of particles (photons) whose energy is

$$E_\gamma = hf$$

(Einstein instead uses the term *light quanta.* The name *photon* became common only after chemist C.S. Lewis used it in 1926, although it was for something different. The term had previously been used to refer to light quanta by René Wurmser in 1924 and Frithiof Wolfers in 1926. The term was also used for completely different concepts by Leonard Troland in 1916 and John Joly in 1921.)

This went much further than what Planck had proposed. For Planck, the energy emitted by a hot object has to be an integer number of *hf* but this is not necessarily implying that light remains in a packet of energy *hf* after the emission. Light is a wave and can have any energy. Only the emission process is quantified according to Planck.

For Einstein, there is quantification because hot objects emit photons. Since the energy of the photons is *hf*, the energy of the atoms has to decreases by *hf*. Once the light is emitted, it remained in the form of photons.

It is fair to say that the success was not immediate, and this is quite understandable. Since 1830, everything seemed to indicate that light is a wave. Interference, diffraction, and polarization experiments had convinced everyone that light is a wave. In addition, Maxwell's equations clearly indicate that light is a wave. And now, Einstein comes and proposes that light is made of particles. For nearly 20 years, Einstein was almost alone in believing his hypothesis, and he himself often doubted it during this period. There was, however, one element that seemed to support this idea: the photoelectric effect (that will be explored in the next section).

Example 10.1.1

A 10 W source emits light having a 600 nm wavelength. How many photons are emitted every second?

To find the number of photons emitted in 1 second, the energy emitted in 1 second and the energy of a photon at this frequency must be known. With these data, the number of photons can be found with

$$N = \frac{\text{Energy emitted in 1 second}}{\text{Energy of one photon}}$$

The energy emitted in 1 second is

$$\begin{aligned} E &= Pt \\ &= 10W \cdot 1s \\ &= 10J \end{aligned}$$

The energy of a single photon is

$$\begin{aligned} E_\gamma &= hf \\ &= \frac{hc}{\lambda} \\ &= \frac{6.626\times 10^{-34} Js \cdot 3\times 10^{8} \frac{m}{s}}{600\times 10^{-9} m} \\ &= 3.313\times 10^{-19} J \end{aligned}$$

Therefore, the number of emitted photons is

$$N = \frac{\text{Energy emitted in 1 second}}{\text{Energy of one photon}}$$
$$= \frac{10J}{3.313\times10^{-19}\,\frac{J}{photons}}$$
$$= 3.018\times10^{19}\,photons$$

Example 10.1.2

Red light with a 15 W/m² intensity and a 600 nm wavelength arrives on a sensor whose area is 2 square metres. How many photons are received in 5 seconds by this sensor?

To find the number of photons in 5 seconds, the energy received in 5 seconds and the energy of a photon at this frequency must be known. With these data, the number of photons can be found with

$$N = \frac{\text{Energy received in 5 seconds}}{\text{Energy of one photon}}$$

The energy received by the sensor in 5 seconds is

$$E = IA_{receiver}t$$
$$= 15\frac{W}{m^2}\cdot 2m^2\cdot 5s$$
$$= 150J$$

The energy of a single photon is

$$E_\gamma = hf$$
$$= \frac{hc}{\lambda}$$
$$= \frac{6.626\times10^{-34}\,Js\cdot 3\times10^{8}\,\frac{m}{s}}{600\times10^{-9}\,m}$$
$$= 3.313\times10^{-19}\,J$$

Therefore, the number of photons received is

$$N = \frac{\text{Energy received in 5 seconds}}{\text{Energy of one photon}}$$
$$= \frac{150J}{3.313\times10^{-19}\,\frac{J}{photons}}$$
$$= 4.53\times10^{20}\,photons$$

A little shortcut to calculate the energy of a photon in eV from the wavelength in nm can be made. The energy is calculated with the following formula.

$$E_\gamma = hf = \frac{hc}{\lambda}$$

The energy is therefore calculated with the combination of constant *hc* which is

$$\begin{aligned} hc &= 6.626\ 070\ 15\times 10^{-34}\ Js \cdot 2.99792458\times 10^{8}\ \tfrac{m}{s} \\ &= 1.9864\times 10^{-25}\ Jm \end{aligned}$$

If the joules are changed into electronvolts and the meters are changed into nanometers, the value of *hc* is

$$\begin{aligned} hc &= 1.9864\times 10^{25}\ Jm \cdot \frac{10^{9}\ nm}{1m} \cdot \frac{1eV}{1.602\times 10^{-19}\ J} \\ &= 1239.84 eVnm \end{aligned}$$

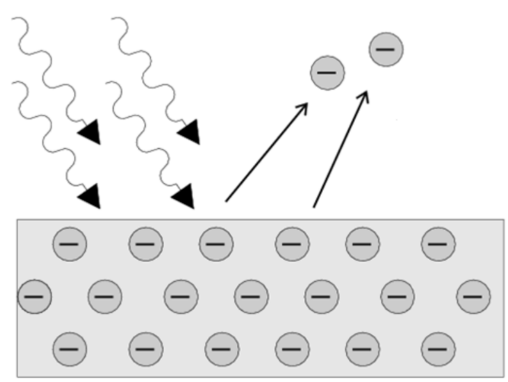
This result means that the energy can then be calculated from the wavelength with the following formula.

Photon Energy from Its Wavelength

$$E_\gamma = \frac{1240 eVnm}{\lambda}$$

10.2 PHOTOELECTRIC EFFECT

To support his photon hypothesis, Einstein use it to explain a phenomenon that seemed to defy the wave theory: the photoelectric effect.



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In this effect, light shining on a metal causes the ejection of electrons present in the metal. Hertz discovered the effect in 1887, but it was not realized before 1899 that electrons were ejected (J.J. Thomson discovered this). You can see in this video that light shining on the soda can eject electrons from the can, thereby changing the electric charge of the can so that the little bits of metal fixed to the soda can do not repel each other as much as before.
http://www.youtube.com/watch?v=WO38qVDGgqw
In theory, a light wave can eject electrons, but some disturbing facts were highlighted by the experiments made by Lenard in 1902.

He discovered that the maximum kinetic energy of the emitted electrons is independent of the intensity of the light, so of the amplitude of the wave. He also discovered that this maximum energy increases if the frequency of the light increases.

To understand why these results were troubling, consider the following analogy. Imagine that waves are pushing pebbles on a beach. The fact that the energy is independent of the amplitude of the wave would mean that the speed at which pebbles are pushed is independent of the amplitude of the wave. A 1 cm high wave would push the pebbles with the same speed as a 20 m high wave! (If both waves have the same λ.) The only thing influencing the speed of the pebbles would be the wavelength of the wave. The smaller the wavelength is, the more violently the pebbles are pushed. A 1 cm high wave having a 10 cm wavelength would push the pebbles with more force than a 20 m high wave with a 30 m wavelength. Obviously, these results were not making any sense.

Einstein proposed the following explanation in 1905. The electrons are ejected when they absorb a photon. By absorbing the photon, they gain the energy of the photon, which allows them to be ejected if the photon had enough energy.

The following result is thus obtained.

Electron energy = Photon energy - Work to get out of the metal

Once the electron is out of the metal, its energy is in the form of kinetic energy. The energy of the photon is hf and the energy needed to get out of the metal is a constant that depends only on the metal. This energy is called the *work function* and is noted ϕ. For example, it takes 4.08 eV to eject an electron out of a piece of aluminum.

The equation is thus

Photoelectric Effect

$$E_{k\,\max} = hf - \phi$$

This formula predicts that the maximum kinetic energy of the ejected electrons increases with the frequency. It also predicts that this energy is independent of the intensity of the wave since this intensity is nowhere to be found in this equation. According to Einstein, the number of ejected electrons increases if the light intensity increases (since there are more photons to eject electrons then) but the energy of each electron remains the same since one electron absorbs only one photon. This is in total agreement with Lenard's observations of 1902.

The equation also predicts the existence of a threshold frequency. For light having a frequency smaller than a specific frequency, the photons don't have enough energy to eject electrons. If it takes 4 eV to eject an electron from a metal and if a photon has an energy of only 3 eV, there is little doubt that nothing will happen. The ejection occurs only if the photon energy is greater than the work function. This means that there are ejected electrons only if

$$hf \geq \phi$$

Therefore, the threshold frequency, which is the minimum frequency, is

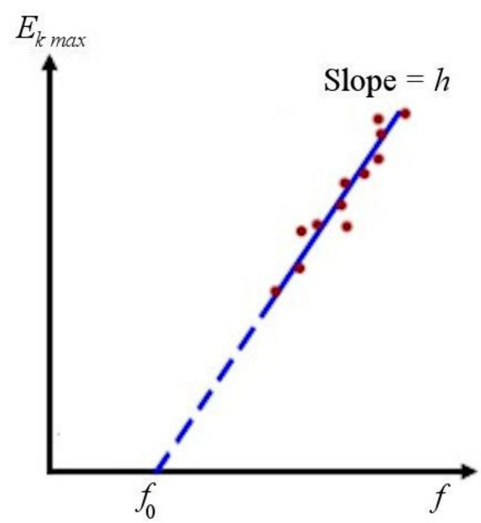
Threshold Frequency for the Photoelectric Effect

$$f_0 = \frac{\phi}{h}$$

Einstein's theory also helps to understand why the maximum kinetic energy of the electrons is obtained. The electron has the maximum kinetic energy if it does nothing more than absorb the photon and get out of the metal. It is possible, however, for the electron to lose some energy in a collision with another electron or an atomic nucleus.

In 1905, not many experimental details were known about the photoelectric effect. It was known that the energy of the electrons increases with the frequency, but it wasn't known how it was increasing. The exact relation between energy and frequency was discovered only in 1916 when Millikan did an experiment. Actually, Millikan wanted to show that Einstein was wrong and that his photon hypothesis was ridiculous. But when Millikan did the experiment, he obtained the following graph.

This is exactly what Einstein had predicted! The graph shows that the ejection of electrons starts at a threshold frequency f_0 and that the kinetic energy of the electrons increases linearly with the frequency. According to Einstein, the slope had to be equal to the Planck constant, and this is exactly what Millikan observed experimentally.



www.a-levelphysicstutor.com/quantphys-photo-elect.php

Example 10.2.1

Light of wavelength 310 nm is incident on a piece of sodium (work function = 2.46 eV).

a) What is the maximum kinetic energy of the ejected electrons?

The maximum kinetic energy is

$$\begin{aligned}E_{k\,max} &= hf - \phi \\ &= \frac{hc}{\lambda} - \phi \\ &= \frac{1240eVnm}{310nm} - 2.46eV \\ &= 1.54eV\end{aligned}$$

b) What is the maximum speed of the ejected electrons?

With the maximum kinetic energy, the maximum speed is found.

$$E_{k\,max} = \frac{1}{2} m v_{max}^2$$

$$2.467\times 10^{-19} J = \frac{1}{2} \cdot 9.11\times 10^{-31} kg \cdot v_{max}^2$$

$$v_{max} = 7.36\times 10^{5} \tfrac{m}{s}$$

Note that the units of energy were changed to joules before putting it into the equation. You probably also notice that $\frac{1}{2}mv^2$ was used for the kinetic energy because the speed of the electrons is not close to the speed of light.

c) What is the threshold wavelength of this metal?

The threshold wavelength is found with the threshold frequency

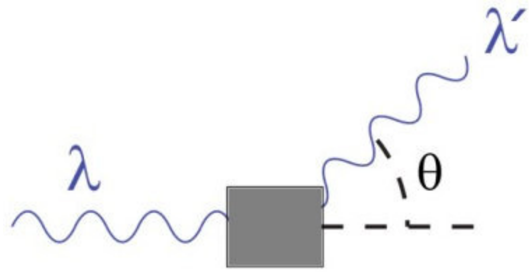
$$\begin{aligned}\lambda_0 &= \frac{c}{f_0} \\ &= \frac{hc}{\phi} \qquad (\text{since } f_0 = \frac{\phi}{h}) \\ &= \frac{1240eVnm}{2.46eV} \\ &= 504nm\end{aligned}$$

Has the corpuscular theory of light resurrected? The problem was that the photoelectric effect was the only phenomenon explained by this theory. There was no other experimental evidence of this theory whereas there were many for the wave theory. Not surprisingly, the photon hypothesis had only few supporters before 1923. Einstein himself had reservations until about 1917. Everything changed in 1923 when the Compton effect was explained with photons.

10.3 COMPTON EFFECT

Since the beginning of the 20th century, the passage of X-rays (discovered in 1895) through matter was studied. At the end of the 1910s, it was known that the scattered rays (rays deflected from their original path) had a wavelength slightly greater than the wavelength of the incident rays ($\lambda' > \lambda$).

(Actually, there are X-rays scattered in every direction simultaneously. Only one of these directions is shown in the diagram.)



en.wikipedia.org/wiki/Compton_scattering

Experiments had shown that the wavelength shift depended only on the scattering angle according to the formula

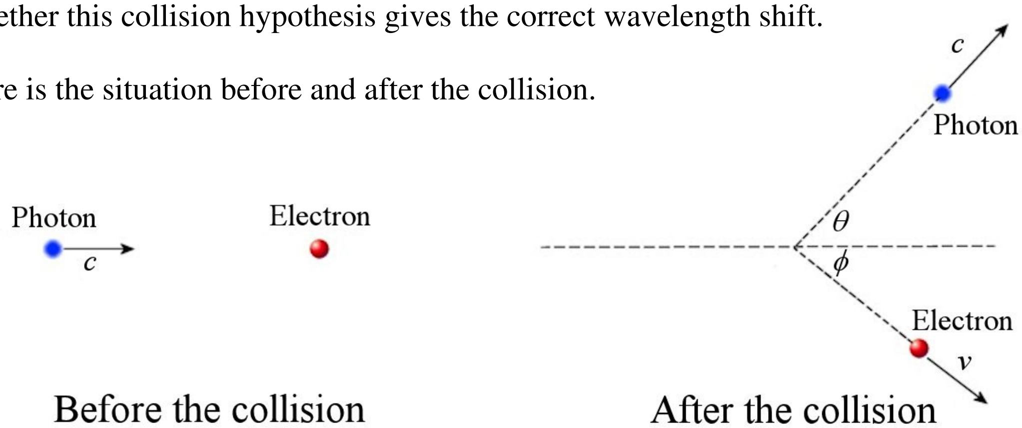
$$\Delta\lambda = 0.0024263nm \cdot (1-\cos\theta)$$

This value of 0.0024263 nm bears the name *Compton wavelength* and is denoted λ_c.

The scattering itself is not mysterious. The incident wave is an oscillating electric field that makes the electrons oscillate in the substance. The scattered radiation comes from these oscillating electrons since they emit electromagnetic radiation. But the wavelength shift was mysterious. According to the wave theory, the oscillation of the electrons should have the same frequency as the original wave and the wave emitted by the electrons should have the same frequency as the oscillation of electrons. The scattered wave should, therefore, have the same frequency, and thus the same wavelength, as the initial wave. This, however, is not what is observed.

In 1923, Arthur Compton, discover a solution to this problem by assuming that light is composed of photons and that the incident photons make a perfectly elastic collision with the electrons in the substance. In this collision, the photons lose a part of their energy to the electrons, which lowers their frequency and increases their wavelength. Let's examine whether this collision hypothesis gives the correct wavelength shift.

Here is the situation before and after the collision.



In such an elastic collision, the momentum and the kinetic energy is conserved. In these equations, γ will represent the photon and e the electron. The conservation equations are

Equations of Conservation for the Compton Effect

Energy conservation	$E_\gamma = E'_\gamma + E_{ke}$
x-component of momentum conservation	$p_\gamma = p'_\gamma \cos\theta + p'_e \cos\phi$
y-component of momentum conservation	$0 = p'_\gamma \sin\theta - p'_e \sin\phi$

First, ϕ will be eliminated. To do so, the momentum equations will be solved for $p'_e \cos\phi$ and $p'_e \sin\phi$.

$$p'_e \cos\phi = p_\gamma - p'_\gamma \cos\theta$$
$$p'_e \sin\phi = p'_\gamma \sin\theta$$

Then, $\sin^2\phi + \cos^2\phi = 1$ is used to obtain

$$\left(p'_e \cos\phi\right)^2 + \left(p'_e \sin\phi\right)^2 = \left(p_\gamma - p'_\gamma \cos\theta\right)^2 + \left(p'_\gamma \sin\theta\right)^2$$
$$p'^2_e \cos^2\phi + p'^2_e \sin^2\phi = p^2_\gamma - 2p_\gamma p'_\gamma \cos\theta + p'^2_\gamma \cos^2\theta + p'^2_\gamma \sin^2\theta$$
$$p'^2_e \left(\cos^2\phi + \sin^2\phi\right) = p^2_\gamma - 2p_\gamma p'_\gamma \cos\theta + p'^2_\gamma \left(\cos^2\theta + \sin^2\theta\right)$$
$$p'^2_e = p^2_\gamma - 2p_\gamma p'_\gamma \cos\theta + p'^2_\gamma$$

This is our first equation.

Now, the formula connecting relativistic energy and momentum of the electron is used.

$$E^2 = p'^2_e c^2 + m^2c^4$$
$$\left(E_k + mc^2\right)^2 = p'^2_e c^2 + m^2c^4$$

With the energy conservation during the collision ($E_k = E_\gamma - E'_\gamma$), this last equation becomes

$$\left(E_\gamma - E'_\gamma + mc^2\right)^2 = p'^2_e c^2 + m^2c^4$$

For a photon, the energy is $E = pc$. Therefore,

$$\left(E_\gamma - E'_\gamma + mc^2\right)^2 = p'^2_e c^2 + m^2c^4$$
$$\left(p_\gamma c - p'_\gamma c + mc^2\right)^2 = p'^2_e c^2 + m^2c^4$$
$$\left(p_\gamma - p'_\gamma\right)^2 c^2 + 2mc^3\left(p_\gamma - p'_\gamma\right) + \cancel{m^2c^4} = p'^2_e c^2 + \cancel{m^2c^4}$$

$$\left(p_\gamma - p'_\gamma\right)^2 + 2mc\left(p_\gamma - p'_\gamma\right) = p_e'^2$$

This is our second equation.

We then have 2 equations giving the momentum of the electron after the collision.

$$p_e'^2 = p_\gamma^2 - 2p_\gamma p'_\gamma \cos\theta + p_\gamma'^2$$
$$p_e'^2 = \left(p_\gamma - p'_\gamma\right)^2 + 2mc\left(p_\gamma - p'_\gamma\right)$$

The right side of these two equations must then be equal.

$$\left(p_\gamma - p'_\gamma\right)^2 + 2mc\left(p_\gamma - p'_\gamma\right) = p_\gamma^2 - 2p_\gamma p'_\gamma \cos\theta + p_\gamma'^2$$

This gives

$$\left(p_\gamma - p'_\gamma\right)^2 + 2mc\left(p_\gamma - p'_\gamma\right) = p_\gamma^2 - 2p_\gamma p'_\gamma \cos\theta + p_\gamma'^2$$
$$p_\gamma^2 - 2p_\gamma p'_\gamma + p'^2_\gamma + 2mc\left(p_\gamma - p'_\gamma\right) = p_\gamma^2 - 2p_\gamma p'_\gamma \cos\theta + p_\gamma'^2$$
$$-2p_\gamma p'_\gamma + 2mc\left(p_\gamma - p'_\gamma\right) = -2p_\gamma p'_\gamma \cos\theta$$
$$mc\left(p_\gamma - p'_\gamma\right) = p_\gamma p'_\gamma\left(1 - \cos\theta\right)$$

The momentum of the photon is related to the wavelength of light by

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

The equation thus becomes

$$mc\left(p_\gamma - p'_\gamma\right) = p_\gamma p'_\gamma\left(1 - \cos\theta\right)$$
$$mc\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right) = \frac{h}{\lambda}\frac{h}{\lambda'}\left(1 - \cos\theta\right)$$
$$mc\left(\lambda' - \lambda\right) = h\left(1 - \cos\theta\right)$$
$$\Delta\lambda = \frac{h}{mc}\left(1 - \cos\theta\right)$$

As

$$\frac{h}{mc} = \frac{6.62607\times10^{-34}\,Js}{9.1094\times10^{-31}\,kg\cdot 2.99792\times10^{8}\,\frac{m}{s}} = 0.0024263nm$$

the end result is

Compton Effect

$$\Delta\lambda = 0.0024263nm \cdot (1 - \cos\theta)$$

The equation is exactly identical to the experimental equation! A collision between a photon and an electron thus explains the Compton effect.

Example 10.3.1

X-rays having a 0.01 nm wavelength collide with electrons.

a) What is the wavelength of the X-rays scattered at 30°?

The wavelength shift is

$$\begin{aligned}\Delta\lambda &= 2.43\times10^{-3}nm \cdot (1-\cos 30°)\\ &= 3.2\times10^{-4}nm\end{aligned}$$

(Notice that all electromagnetic waves undergo this wavelength shift. Obviously, such a small shift would not be easily observable for visible light.)

The wavelength of the X-rays scattered at 30° is, therefore,

$$\lambda' = \lambda + \Delta\lambda = 0.01nm + 0.00032nm = 0.01032nm$$

b) What are the energies of the photons before and after the collision for X-rays scattered at 30°?

The photon energy before the collision is

$$\begin{aligned}E_\gamma &= \frac{hc}{\lambda}\\ &= \frac{1240eVnm}{0.01nm}\\ &= 124keV\end{aligned}$$

The photon energy after the collision is

$$\begin{aligned}E'_\gamma &= \frac{hc}{\lambda'}\\ &= \frac{1240eVnm}{0.01032nm}\\ &= 120.155keV\end{aligned}$$

It is obvious that the photon has lost some energy in the collision.

c) What is the kinetic energy of the electron after the collision for X-rays scattered at 30°?

The kinetic energy of the electron is found with the formula of energy conservation in the collision.

$$E_\gamma = E'_\gamma + E_{ke}$$
$$124keV = 120.155keV + E_{ke}$$
$$E_{ke} = 3.845keV$$

(This means that the speed of the electron is 3.68 x 10^7 m/s.)

Actually, there are two frequencies received at some specific angle θ because the photons can be scattered either by an electron or by an atomic nucleus. As the frequency shift depends on the mass of the object struck by the photon (according to *h*/*mc* in the formula), the frequency shift is not the same. The nucleus, being so heavy, receives virtually no energy in the collision and the frequency of the photon practically does not shift when it scatters on a nucleus.

In summary, Compton's calculations clearly showed that light is made of photons. This was new evidence to support Einstein's photon hypothesis. From this time on, photons were accepted by the scientific community.

The interaction between a photon and an electron is completely different in the photoelectric and Compton effects. The photon is absorbed by the electron in the photoelectric effect whereas it simply makes an elastic collision with the electron in the Compton effect.

10.4 IS LIGHT A WAVE OR A PARTICLE?

The beginning of quantum physics showed that light sometimes acts like a particle with an energy *hf*. Can this idea be reconciled with all the experiments and the theories made in the 19th century which showed that light is a wave? Here is a summary of the situation in 1923.

The following phenomena show that light is a wave:

Interference Diffraction Polarization
Maxwell's Equations

The following phenomena show that light is a particle:

Photoelectric effect Compton effect

There is a serious problem here since it is absolutely impossible to explain interference, diffraction, and polarization with a corpuscular theory and it is absolutely impossible to explain the photoelectric effect and the Compton effect with a wave theory.

In 1909, Einstein, then the only supporter of photons, justified his ideas, saying that the next phase will be to find a new more complex model. With this new model, the light would sometimes act as a wave in certain situations and would sometimes act as a particle in certain other situations. This new theory combining these two aspects was never found.

In fact, the situation was about to get even more complicated as this problem is not limited to light…

10.5 DE BROGLIE WAVES

In 1923, Louis de Broglie came to a surprising conclusion: matter can also act as a wave! He even found the formula giving the wavelength of these matter waves.

De Broglie Wavelength

$$\lambda = \frac{h}{p}$$

where p is the momentum of the particle.

Example 10.5.1

What is the wavelength of an electron travelling at 3 x 10^6 m/s (1% of the speed of light)?

In this case, the momentum can be calculated with the non-relativistic formula. Therefore,

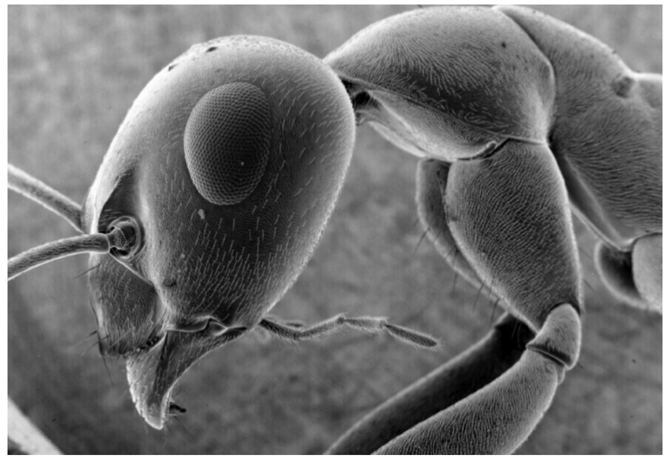
$$\begin{aligned}\lambda &= \frac{h}{mv} \\ &= \frac{6.626\times 10^{-34} Js}{9.11\times 10^{-31} kg \cdot 3\times 10^{6} \frac{m}{s}} \\ &= 0.243 nm\end{aligned}$$

De Broglie arguments were not based on any experiment. Instead, it consisted of a series of theoretical arguments on the coherence of physics. To illustrate the idea, here one of the proofs presented by de Broglie.

> In a more formal version, vectors with three components have no place in relativity. Often, vectorial quantities are found in something called a four-vectors which has 4 components. For example, the relativistic energy and the momentum are the four components of the energy-momentum four-vector.
>
> $$\left(\frac{E}{c}, p_x, p_y, p_z\right)$$
>
> This was never mentioned, but k is also a vector (we would have seen it with a more advanced study of three-dimensional waves). It is a vector of magnitude $2\pi/\lambda$ directed towards the direction of propagation of the wave. In relativity, this vector is part of a four-vector with the following 4 components.
>
> $$\left(\frac{\omega}{c}, k_x, k_y, k_z\right)$$
>
> The formula $E = hf$ is a relationship between one of the components of these two four-vectors.
>
> $$E = hf$$
>
> $$E = \frac{h}{2\pi} 2\pi f$$
>
> $$E = \hbar\omega$$
>
> $$\left(\frac{E}{c}\right) = \hbar\left(\frac{\omega}{c}\right)$$
>
> However, if there is a relationship between one of the components of a four-vector, the same relationship must hold for the other components. Thus, the following relationship must also be true.
>
> $$p = \hbar k$$
>
> This leads to
>
> $$p = \frac{h}{2\pi}\frac{2\pi}{\lambda}$$
>
> $$\lambda = \frac{h}{p}$$

De Broglie presented his theory in a paper in September 1923. Many scientists were initially skeptical until Einstein came to the same conclusion as de Broglie in January 1925 starting from different premises. Erwin Schrödinger also showed that the trajectory of a projectile could be explained by a refraction if the projectile is a wave whose wavelength is given by de Broglie formula. The ideas of de Broglie then seemed to be confirmed.

Nowadays, the wave aspect of matter is used in electron microscopes. In these devices, electrons rather than light are used to form an image. Otherwise, the principle is the same as for a conventional microscope. Electric fields are used to play the role of lenses in these microscopes. Very accurate images can be obtained because electrons with a wavelength smaller than 1 nm are used. The resolution of a microscope being roughly equal to the wavelength, very small details can be seen. As the lenses for electrons are not perfect, the maximum resolution is of the order of 5 to 10 nm, which is much better than a microscope operating in visible light whose maximum resolution is approximately 200 nm. Here is an image produced by an electron microscope.



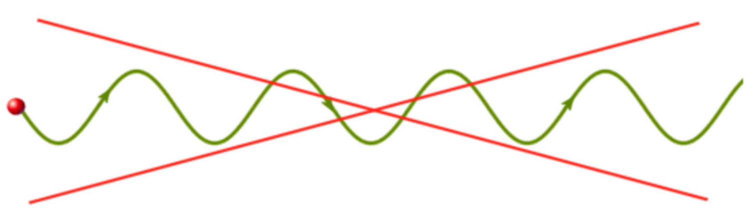
www.boston.com/bigpicture/2008/11/peering_into_the_micro_world.html

De Broglie paper was purely theoretical, and the proofs in favor of the theory were mostly aesthetic and coherence proofs. The idea had to be proven experimentally. But to see how the wave nature of matter can be proven experimentally, we have to understand the meaning of *matter can act as a wave*.

10.6 INTERPRETATION OF THE WAVE

The idea that matter can act as wave is not a simple restatement of what was learned about mechanical waves, which are waves propagating in matter. For these waves, matter is not a wave, it only composes the medium that supports the waves.

Also, it does not mean that particles travels following an undulating path.

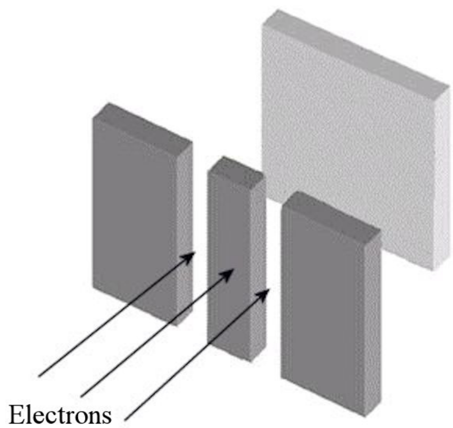


Common Mistake: Thinking That the Electron Follows an Oscillating Path

Matter waves do not represent the path of the particles. The electrons do not move along a wavy path whose wavelength is *h/p*.

The idea that the matter can act as a wave rather means that particles like electrons can, for example, undergo diffraction when passing through a small hole. But what happens then? With diffraction, the wave spreads out after its passage through a hole. Does that mean that an electron spreads out after its passage through a small hole? How can a particle spread?

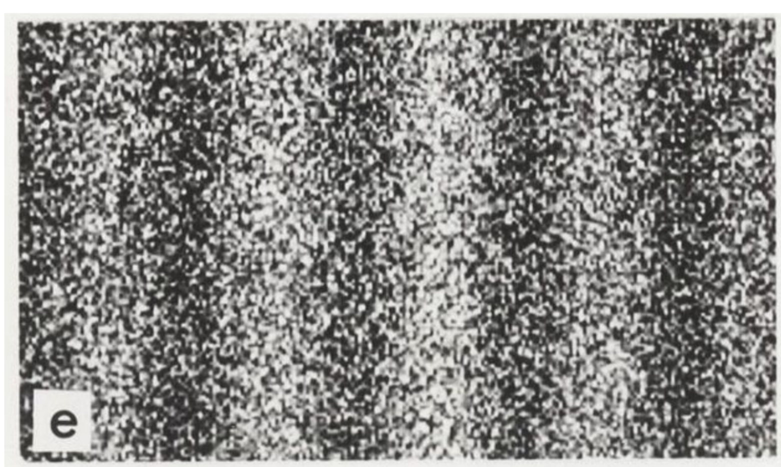
The Copenhagen Interpretation



www.blacklightpower.com/theory-2/theory/double-slit/

To understand what the wave represents, let's look at what happens if Young's experiment is done with electrons. If electrons act as a wave, there should be an interference pattern with maxima and minima on the screen, exactly as with light. (To allow the scientists to record the position where an electron hits the screen, a special screen is used. This screen becomes luminous at the spot where the electron hits it.)

This is the result that was obtained when this experiment was done (in 1989).



www.hitachi.com/rd/portal/research/em/doubleslit.html

The interference pattern bears a striking resemblance to the one obtained in Young's experiment performed with light. In addition, the spacing between the bright fringes fits with the wavelength of the wave given by the de Broglie formula.

The result of this experiment shows that many particles are hitting the screen at locations where there is a maximum of interference, so at the places where the amplitude of the wave is maximum. Few particles hit the screen at locations where there is a minimum of interference, so at the places where the amplitude of the wave is very small.

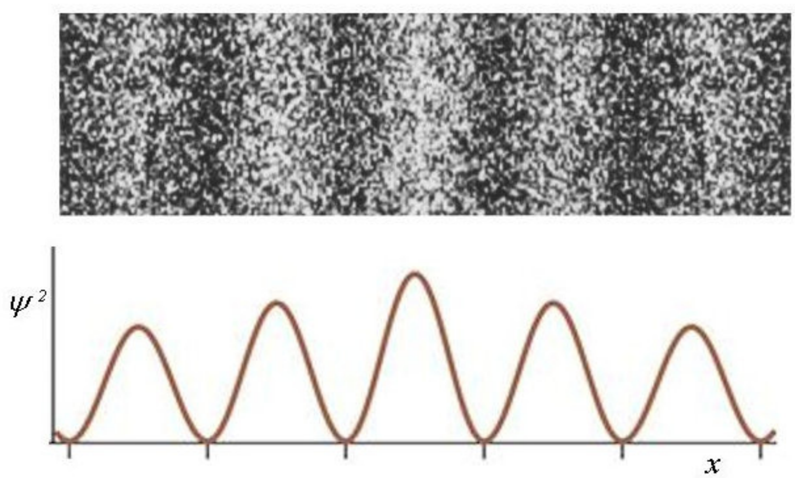
This suggests the following interpretation, made by Max Born in 1926, known as *the Copenhagen interpretation*.

Copenhagen Interpretation

The square of the amplitude at a location is proportional to the probability of finding the particle at this location.

Thus, more particles hit the screen at the places where there is a maximum of interference because the amplitude is greater at these places and there is more chance of finding the particle at these locations.

The symbol used for the amplitude of the wave is ψ. So, for the double-slit experiment with electrons, the following graph shows the link between ψ^2 and the number of electrons arriving on the screen.



www.chegg.com/homework-help/questions-and-answers/physics-archive-2011-november-20

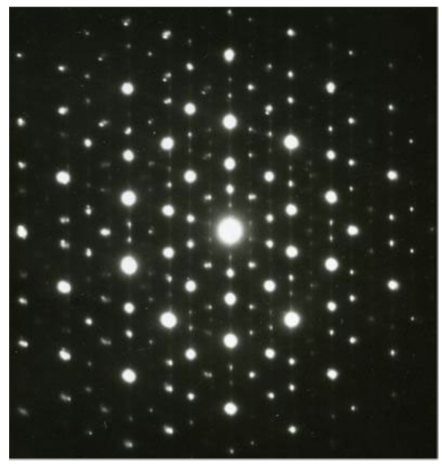
It is obvious that more electrons hit the screen at the positions where ψ^2 is great.

This interpretation is valid for every kind of particles, including photons. Note that Einstein had suggested something similar in 1905 when he suggested for the first time that light was composed of photons. He proposed that the intensity of light on a surface, which is proportional to the square of the wave amplitude, should be proportional the number of photons arriving on this surface.

Experimental Proofs

It was impossible to perform the double-slit experiment when de Broglie published its results, but they still managed to confirm the ideas of De Broglie with interference as soon as 1926.

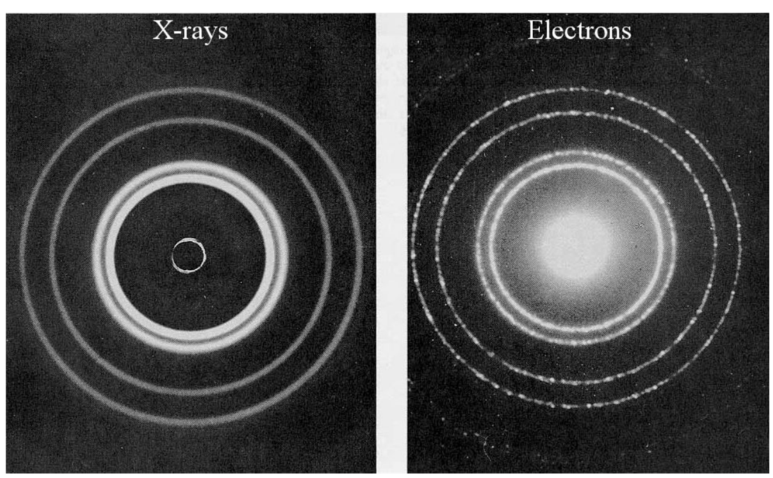
At that time, Clinton Davisson, and Lester Germer, and, independently, George Paget Thomson and Alexander Reid obtain interference pattern with electrons. In the Davisson and Germer experiment, electrons were passing through a nickel crystal. The regular spacing of the atoms inside the crystal is such that it acts as a grating. If the electrons act as a wave, an interference pattern is supposed to be seen. The wavelength of the wave can even be found from the spacing between the interference maxima with $d\sin\theta = m\lambda$. The image to the right shows the interference pattern obtained when electrons pass through a manganese and aluminum crystal.



scienceblogs.com/gregladen/2011/10/05/there-can-be-no-such-creature/

The bright spots are the positions where many electrons hit the screen, and these correspond to interference maxima. Thus, there is an interference pattern, and this confirms that electrons act like waves.

The image to the right shows the diffraction patterns obtained when X-rays and electrons with the same wavelength pass through a piece of aluminum foil. The patterns are circular because the foil is made up of several small randomly oriented crystals. The similarity of the two images is striking. This shows that electrons behave like a wave (X-rays in this case).



www.pems.adfa.edu.au/~s9471553/level1/Teaching/Physics1BWaves/Physics1BWaves.html

Davisson's and Thomson's experiments thus confirmed that electrons can act as a wave and that the wavelength is actually given by de Broglie formula. For this discovery, they all received a Nobel Prize (Broglie in 1929 and Davisson and Thomson in 1937) (quasi-interesting fact: Thomson received the Nobel Prize for showing that electrons are waves while his father, J.J. Thomson, had received it in 1906 for showing that electrons are particles in 1897!)

An experiment made in 1945 also showed that interference patterns can also be obtained with neutrons.

Example 10.6.1

The kinetic energy of an electron is 350 keV. What is its wavelength?

The wavelength is found with

$$\lambda = \frac{h}{p}$$

Therefore, the momentum of the electron is needed.

First possibility: calculate the speed

The mass energy of the electron is

$$\begin{aligned} mc^2 &= 9.11\times10^{-31}kg \cdot \left(3\times10^8 \tfrac{m}{s}\right)^2 \\ &= 8.199\times10^{-14} J \\ &= 511.8 keV \end{aligned}$$

The speed of the electron is found with

$$\begin{aligned} E_k &= (\gamma - 1) mc^2 \\ 350keV &= (\gamma - 1)\cdot 511.8keV \\ \gamma &= 1.6839 \\ \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} &= 1.6839 \\ u &= 0.8046c \end{aligned}$$

Therefore, the wavelength is

$$\begin{aligned} \lambda &= \frac{h}{\gamma mu} \\ &= \frac{6.626\times10^{-34} Js}{1.6839\cdot 9.11\times10^{-31}kg \cdot 0.8046 \cdot 3\times10^8 \frac{m}{s}} \end{aligned}$$

$$= 0.00179nm$$

Second possibility: calculate the momentum

The momentum can be found with

$$E^2 - p^2c^2 = m^2c^4$$
$$(511.8keV + 350keV)^2 - p^2c^2 = (511.8keV)^2$$
$$pc = 693.4keV$$
$$pc = 1.111\times10^{-13}J$$
$$p = 3.703\times10^{-22}\,\tfrac{kgm}{s}$$

Then, the wavelength is

$$\lambda = \frac{h}{p}$$
$$= \frac{6.626\times10^{-34}Js}{3.703\times10^{-22}\,\frac{kgm}{s}}$$
$$= 0.00179nm$$

To obtain a diffraction pattern with these electrons moving at this speed, they must pass through a really small hole.

In this last example, you could have been tempted to use $E = hf$ (where E is the relativistic energy) to find the frequency, and then use $v = \lambda f$ to find the wavelength. This method would have been wrong because, with what was learned here, $E = hf$ cannot be used for something else than a photon.

Common Mistake: using $E = hf$ or $v = \lambda f$ for particles other than a photon

The formula $E = hf$ is valid for every particle, but there are a few subtleties if it is applied to particles that do not travel at the speed of light. For reasons not given here, (but given here: http://physique.merici.ca/waves/proof-Ehf.pdf) $\lambda = h/p$ is valid for any particle (including photons), but not $E = hf$ if $v = \lambda f$ is used to find the frequency. As you have no formula to find f for matter in these notes, $E = hf$ cannot be used for something else than a photon.

Example 10.6.2

What is the wavelength of a baseball (m = 145 g) travelling at 15 m/s?

The wavelength is

$$\begin{aligned}\lambda &= \frac{h}{mv} \\ &= \frac{6.626\times 10^{-34} Js}{0.145kg \cdot 15 \frac{m}{s}} \\ &= 3\times 10^{-34} m\end{aligned}$$

In this case, there is no chance to obtain a diffraction pattern with baseballs. The balls would have to pass through a hole whose diameter is about 10^{-34} m, which is impossible (the atomic nucleus has a diameter of about 10^{-14} m).

Since matter can act as a wave, everything that was learned about waves can be applied to matter.

Example 10.6.3

Electrons travelling at 5000 m/s pass through a circular hole whose diameter is 0.1 mm. A diffraction pattern is then observed on a screen located 2 m from the hole. What is the diameter of the central diffraction maximum?

The central maximum ends at the first minimum. The angle of this first minimum is given by

$$\sin\theta = \frac{1.22\lambda}{a}$$

To find this angle, the wavelength of the electrons is needed. This wavelength is

$$\begin{aligned}\lambda &= \frac{h}{mv} \\ &= \frac{6.626\times 10^{-34} Js}{9.11\times 10^{-31} kg \cdot 5000 \frac{m}{s}} \\ &= 145.5nm\end{aligned}$$

Therefore, the angle of the first minimum is

$$\sin\theta = \frac{1,22\lambda}{a}$$

$$\sin\theta = \frac{1.22 \cdot 145.5\times10^{-9}m}{0.1\times10^{-3}m}$$
$$\sin\theta = 0.001775$$
$$\theta = 0.1017°$$

On the screen, the distance between the centre of the central maximum and the end of the central maximum is

$$\tan\theta = \frac{y}{L}$$
$$\tan(0.1017°) = \frac{y}{2m}$$
$$y = 3.55mm$$

Therefore, the diameter of the central maximum is 7.10 mm.

Wave-Particle Duality

In the previous chapter, we were wondering if light is a wave or a particle. Obviously, the problem is broader because matter also has the same properties.

So, the question is: *Are <u>matter and light</u> waves or particles*?

Unfortunately, there is no simple answer to this question. We can only observe that they sometimes act like waves, sometimes like particles. Generally, they act like particles when the wavelength is small (so when the energy is high) and like waves when the wavelength is great (so when the energy is low). For macroscopic objects such as a baseball, the wave aspect of matter can never be seen.

When matter acts like a particle, it only acts like a particle, and not at all like a wave. For example, a collision with another particle is described by the equations of a collision between particles, and this collision is impossible to describe with the wave theory, exactly as what is happening with light in the Compton effect. The result isn't just a simple approximation of the wave theory for long wavelengths because the result is simply impossible to explain if it is assumed that matter is a wave. However, when matter acts like a wave, it only acts like a wave, and not at all like a particle. It is impossible to explain the result of an electron diffraction experiment if it is assumed that matter is made of particles.

This is the *wave-particle duality*: both theories are needed to explain all the observations.

Both aspects of this duality can never be seen at the same time. Matter acts like particles **or** like waves, never with both aspects at the same time. That's what Bohr's complementarity principle is saying: particle and wave aspects complement each other. These two aspects are necessary to explain all the observations and they complement each other because they are never used at the same time to explain the same observation. Only the wave aspect is present during electron diffraction and only the corpuscular aspect is present in a particle collision. A situation cannot be analyzed using both wave and particle. It's one or the other.

A World of Probabilities

With the Copenhagen interpretation, physics is not deterministic anymore. In a deterministic physics, it is possible, in theory, to calculate exactly what will happen at a later time if the position and the velocity of every atom in the universe are known. Everything is predetermined.

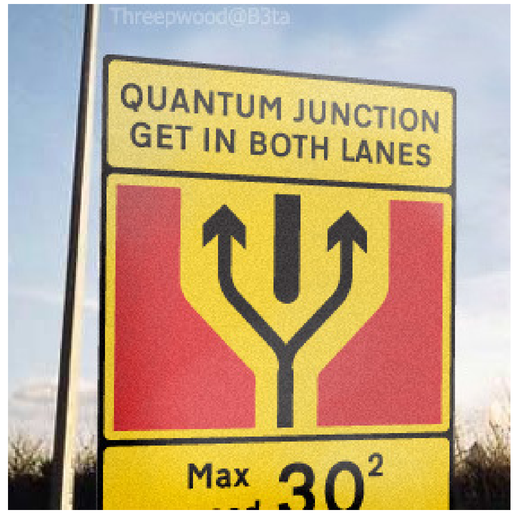
This quality is lost with the Copenhagen interpretation. If an electron is sent through two slits, the wave only gives the probability that the electron hits the screen at a specific place. It is impossible to know with any certainty where the electron will hit. Only the probability of hitting a specific location is known. It is, therefore, impossible to predict exactly what will happen. Only the probabilities can be known. Einstein was strongly opposed to this idea, and that's why he said that "*God does not play dice with the universe*".

The Copenhagen Interpretation Goes Farther

It was said earlier that the square of the amplitude of the wave function gives the probability of finding the particle at a specific location. In fact, the quantum theory can be used to calculate much more than just the position of a particle. For example, it can be used to predict the result of the measurement of the spin of a particle. However, in all cases, the interpretation remains the same: the theory only gives the probability of measuring a specific value. Here is an example to illustrate this concept. An electron is heading towards a detector measuring the spin of the particle. With an electron, there are only two possible outcomes: the spin can be either upwards (+) or downwards (–). With the quantum theory, the probability to measure + or – can be calculated depending on the situation. The result could be, for example, 70% chance of measuring + and 30% chance of measuring –.

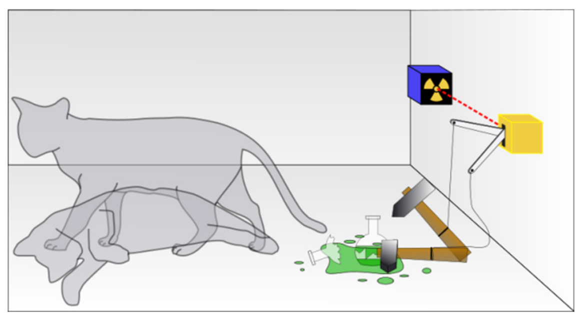
However, the Copenhagen interpretation goes way further than that. In this interpretation, the spin of the particle is not simply + or – before the measurement is made, it is in a state where the two possible outcomes exist at the same time. In our example, the spin of the particle is 70% + and 30% –. It is only when the measurement is made that the spin of the particle become either + or –. Before the measurement, the spin of the particle was both + and – at the same time, and, after the measurement, it is only + if + was measured or is only – if – was measured. This is called the *wave function collapse*.

Let's re-examine the two-slit experiment with this interpretation. We remember that in this experiment, we see electrons hitting a screen after going through 2 slits. Seeing the electron arriving at a specific location on the screen is a measure of its position. Before this measurement, the electron was everywhere at the same time (every place where ψ^2 is not zero). By striking the screen, the wave function collapsed, and it is only then that the position of the electron is randomly determined among all possibilities. As the electron was everywhere at the same time before the measurement, it is considered that the electron passed through both slits at the same time, as if, in a car, we were asked to go in both lanes at the same time.



www.ipod.org.uk/reality/reality_quantum_intro.asp

Many physicists, including Einstein and Schrödinger, disliked this interpretation. The latter invented in 1935 a tough experiment to illustrate the ridiculousness of this situation: Schrödinger's cat. A cat is locked up in a box with a randomly triggered device (poison) that can kill the cat. Suppose that after 2 hours, there is a 60% chance that the mechanism has been triggered and that the cat is dead. In this case, according to the Copenhagen interpretation, the cat is in a mixed state (60% dead, 40% living) and the fate of the poor feline will only be decided when the box is open to look at the cat. After the observation, the cat will be either only dead or only alive.

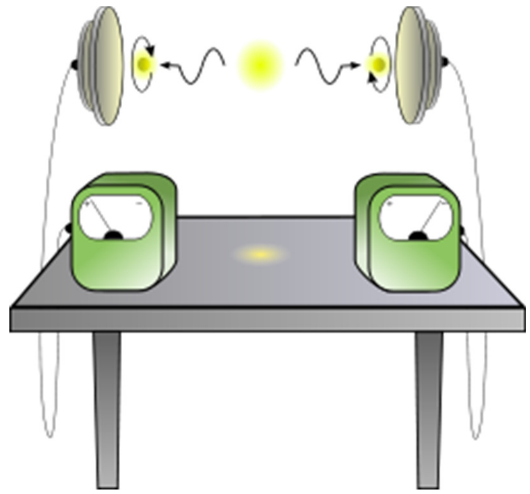


en.wikipedia.org/wiki/Schrödinger's_cat

For Einstein and Schrödinger, this superposition of states did not make any sense. For them, the cat cannot be dead and alive at the same time before the box is open. He is either dead or living at any moment in the box, even if nobody observed it. If only the probability that the cat is dead or alive after a while can be calculated, it is because not enough information is known. Therefore, this probability only represents the ignorance of the observers, and there is no real superposition of states. For Einstein and Schrödinger, when the spin of a particle is measured and + is obtained, it means that the spin was + before the measurement. If – is obtained, then the spin was – before the measurement. The fact that the quantum theory only enables us to calculate probabilities simply means that it is an incomplete theory. According to them, a more refined theory which ought to make more precise predictions must exist, a theory that would allow us to predict the results of the measurement, to know the spin of the particle before measuring it, rather than just giving probabilities.

The EPR Paradox

Another situation, imagined by Einstein, Podolsky, and Rosen (EPR) in 1935, was devised to show that the Copenhagen interpretation cannot be right. In this experiment, a particle with zero spin decays into two particles heading in opposite directions. If each of these particles have a spin, the spins of the two particles should have opposing signs according to the laws of conservation used in particle physics. This means that if the spin of a particle is measured and + is obtained, then the spin must be – for the other particle.



www.clker.com/clipart-15121.html

According to the Copenhagen interpretation, each particle is in a superimposed state (half +, half –) before the measurement of the spin. When one of the spins is measured, the wave function collapse, thus forcing the other particle to have a spin with the opposite sign. Suppose this is done with the setup shown in the above diagram and that – was measured on the sensor to the right.

Then, we know that the result of the measurement of the spin on the sensor to the left will give +. But then, how can the particle travelling towards the left knows that it must change form a mixed + and – state to the + only state? The answer seems to be simple at first glance: the measurement of the spin of the particle on the right sensor caused the collapse of the wave function, thereby changing the state of the particle to the left from a mixed state to the + only state. However, this collapse cannot be instantaneous since nothing can travel faster than light according to Einstein's relativity. At best, the collapse spreads at the speed of light.

This is where things get complicated. What will happen if the spin of the particle to the left was measured only a short time after the measurement of the spin of the particle to the right? In fact, the time between the measurements can be so small that a beam of light would not have enough time to move from one particle to the other during this time. Therefore, the collapse of the wave function does not have enough time to reach the particle to the left. Then, how can the second particle, which was in a mixed + and – state, know that the measurement must be + if this information doesn't have enough time to move from one particle to another? And yet, if this experiment is done, the result of the measurement will always be +!

Actually, the situation is even weirder if the point of view of another observer is taken according to relativity. Remember that the time at which events occur changes according to the observers. Here, there are even observers who will say that the spin of the particle on the left was measured before the spin of the particle to the right. For them, it is the measure on the particle to the left that caused the collapse of the wave function which then force the particle to the right to have a spin –, and this, even before the information had enough time to reach this particle!

For Einstein, the serious issues raised by this thought experiment showed that the Copenhagen interpretation could not be correct because it implied that the collapse of the wave function has to spread faster than the speed of light, which leads to all kinds of paradoxes. Einstein gave a different interpretation of this situation. As soon as the particles are emitted, they each have a specific value of spin that is measured a bit later. As they already have the spins that will be measured and there is no superposition of states at all, there is no need to have a collapse of the wave function. Spins are opposed because, right from the start, the particles had these opposite values of spin.

At this point, you surely agree with Einstein…

Bell's Theorem

For a few years, there were several confrontations between Bohr and Einstein in which the latter presented different scenarios to show that the Copenhagen interpretation had to be false. Each time, Bohr was able to find a way to explain the situation using the Copenhagen interpretation. For example, Bohr claimed that the collapse of the wave function actually travels faster than light for the EPR paradox, and that this is possible since no information can be transmitted by this process. A (friendly) war between the two clans continued until exhaustion, and none had provided the decisive argument in favour of its interpretation.

The situation changed in 1964 when John Stewart Bell discovered a way to determine experimentally which interpretation was right (unfortunately, Bohr and Einstein were both dead by then). It can be done with an experiment very similar to the one described for the EPR paradox, except that the two spins are measured along different axes. Bell determined that the results of this experiment should respect certain inequalities (Bell's inequalities) if Einstein's interpretation is correct while those inequalities would not be true with the Copenhagen interpretation. The details are a little complicated but what really matters here is that they finally had a way to experimentally determine who was right. The experiment was conducted for the first time in the early 1970s and again several times later, getting more accurate every time. These experiments proved without a shadow of a doubt that Einstein's interpretation <u>cannot</u> be correct. The Copenhagen interpretation is correct. An object is really in a mixed state before measurements are made and there is a collapse of the wave function when a measurement is made!

Going back to what has been said about the EPR paradox, then this means that the collapse of the wave function actually travels faster than light.

What Causes the Wave Function to Collapse?

The Copenhagen interpretation brings some conceptual difficulties. For example, what happens if there is a fly with Schrödinger's cat in the box? Will the observation of the cat by the fly causes the collapse of the wave function and prevents the cat from being in a superposition of states? Some would say no by asserting that only conscious human beings can cause the collapse of the wave function! Apart from the problem of deciding which living beings are conscious, or even deciding what consciousness is, this conception brings us to ask ourselves what would happen if all the conscious beings of the universe were to disappear. Would the universe then be sentenced to stay forever in a superposition of many states?

Actually, no conscious beings are required. In fact, as soon as there is interaction with the environment, the wave function collapse. If the interaction is stronger, then collapse is faster. The collapse of the wave function, therefore, does not only occur during a measurement but can also happen spontaneously if the system interacts with the environment. One could, therefore, conclude that a simple photon locked up with the cat in the box causes the collapse of the wave function. And even if there is no photon, the interactions of the atoms of the cat among themselves will cause the collapse of the wave function. The very large number of atoms in a cat implies that it cannot be in a superposition of dead and alive states for more than 10^{-23} seconds. There is, therefore, no chance of seeing a real cat in a superposition of dead and alive states. To keep the cat in a superimposed state longer, it would be necessary to put the cat in a vacuum (to minimize interactions with the atoms surrounding the cat) and to cool the cat so that its temperature is close to absolute zero (to minimize interactions between the atoms of the cat). Obviously, the cat would then be dead, and the poison would have nothing to do with it... However, simpler systems can remain in a superposition of state for a much longer time, thereby allowing physicists to observe systems in superimposed states experimentally. This was done by Serge Haroche team in 1996 (Nobel Prize 2012), among others.

The theory is still unable to determine what will happen to the cat. The cat can’t be observed in a superposition of dead and alive states, but the ultimate fate of the cat (dead or alive) cannot be known. The theory is unable to predict the outcome of this experiment; it can only give the probability of each outcome.

Everett’s Many Worlds Interpretation

In 1957, a surprising interpretation was given by Hugh Everett, then a student at Princeton University. He was trying to give an explanation to the fact that, among all possible outcomes of a measurement, only one is observed. What happened to all the other possible states?

Everett’s answer is surprising: each time an observation of a phenomenon is made, the universe splits into several universes in which each possible outcome is observed.

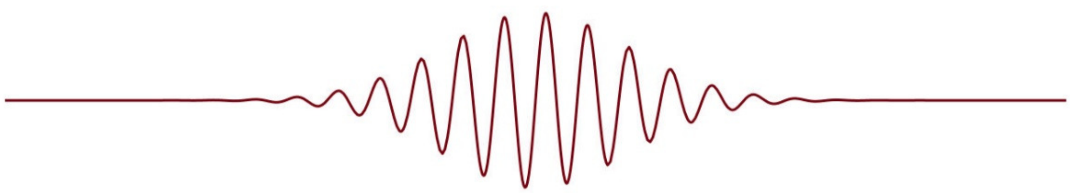
Let’s take an example to clarify this situation. Suppose the spin of a particle is measured and the quantum theory tells us that there is a 50% probability of measuring + and a 50% probability of measuring –. When the measurement is made, the universe splits in two universes: a universe in which the measured value is + and another universe in which measured value is –. In each of these worlds, the observer wonders what happened to the other possibility that disappeared during the measurement. In this interpretation, the two possibilities did not disappear. They both occurred but in different universes!

The cat experiment can then be reinterpreted. If the cat in a superposition of dead and alive states is observed, then, according to Everett, the universe splits into two universes. In one universe, the cat is dead, and in the other universe, the cat is alive.

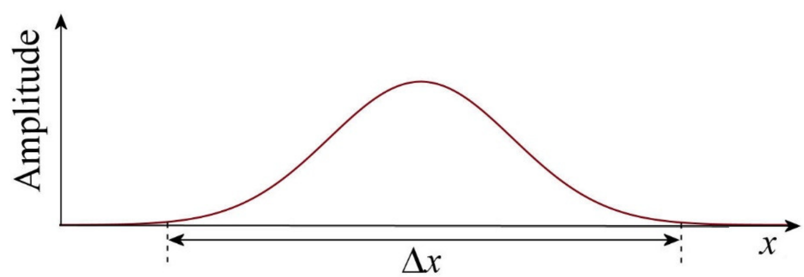
10.7 HEISENBERG INDETERMINACY PRINCIPLE

An Indeterminate Position

This is how the wave of a particle moving at a certain speed might look like.

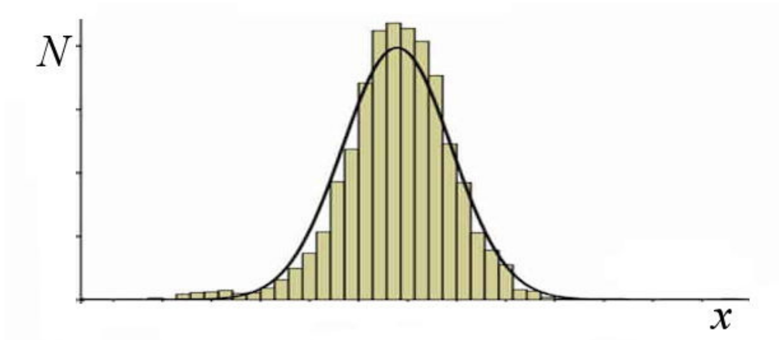


It is clear that the amplitude of this wave changes according to the position.



Since the probability of finding the particle at a certain location depends on the amplitude, the particle can only be found in the region of length Δx. If the position is measured, a value of x within this range will be obtained. It is impossible to determine in advance what value will be measured. One of the possible values of x in the interval Δx will be obtained, but it is absolutely impossible to predict which one. It is only possible to know that there is a greater probability to measure a position near the center of the interval since the amplitude is greater there.

If the same experiment were to be repeated several times and the position of the particle measured with exactly the same conditions and the same very precise devices, a different result would be obtained each time. If a graph that shows the number of times a position located on a small interval was obtained, a graph that looks like this would probably be obtained.



www.ztable.net/normal-distribution/

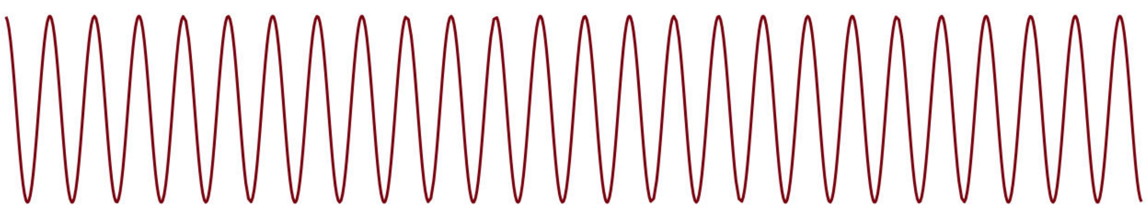
The more measurement is made, the closer the graph obtained would be to the black curve that gives the theoretical probability.

Since it is impossible to know exactly where the particle will be found for a specific measurement, we speak of an *indeterminate position*. Note that once the position is measured, the position of the particle is exactly known, and its position is no longer indeterminate. The indeterminacy refers only to an indeterminacy before measurement.

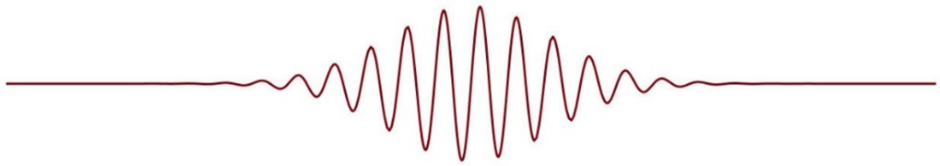
Some will talk about an uncertainty on the position rather than an indeterminacy. It is better to use the term indeterminacy to avoid confusion with the uncertainty on the position that would come from a measuring device. The indeterminacy does not come from a measuring device. Even with the best measuring device in the world is used to measure the position, the measured position would not always be the same if the same exact experiment were to be repeated under the same conditions. These variations would not be caused by the inaccuracy of the measuring apparatus, but by the indeterminacy of the position which is a consequence of the wave nature of matter.

An Indeterminate Momentum

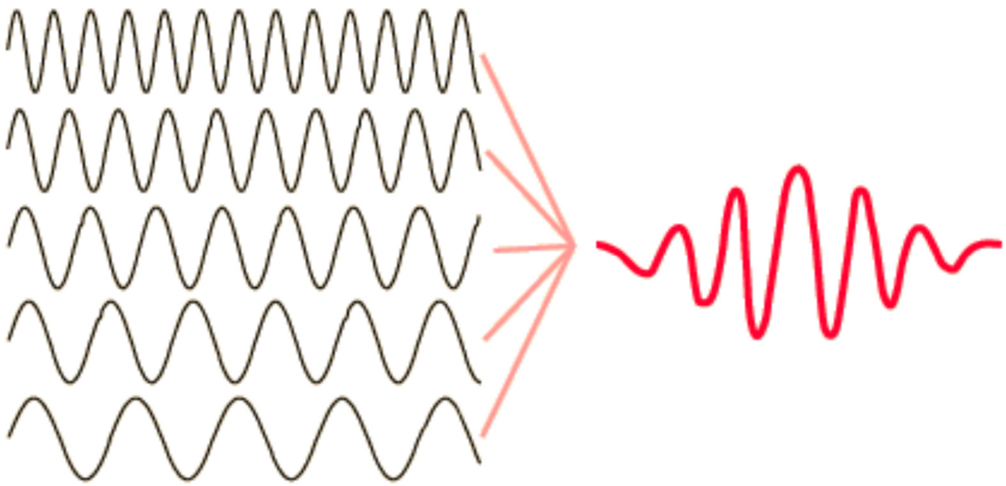
If a wave has a very precise wavelength, then the result is a sine wave with a constant amplitude.



However, this wave extends to infinity towards the left and the right with a constant amplitude. This means that it is possible to find the particle anywhere in the universe. To obtain a particle that is localized in a certain place, the wave must look like this.

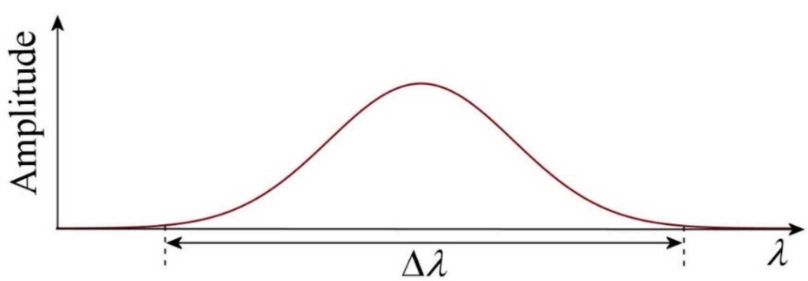


However, to get a wave with this shape, there is only one way: several sine functions must be added together.



hyperphysics.phy-astr.gsu.edu/hbase/waves/wpack.html

Actually, an infinite number of sine function must be added to form such a wave packet. All the waves of wavelengths lying in a $\Delta\lambda$ wavelength interval must be added together, and the amplitude of these waves must vary with the wavelength according to the following graph.



The wave does not have only one wavelength. The wave contains several wavelengths simultaneously. This is not very surprising. A beam of light of a certain color, say red at 650 nm, is actually a superposition of several waves having wavelengths close to 650 nm.

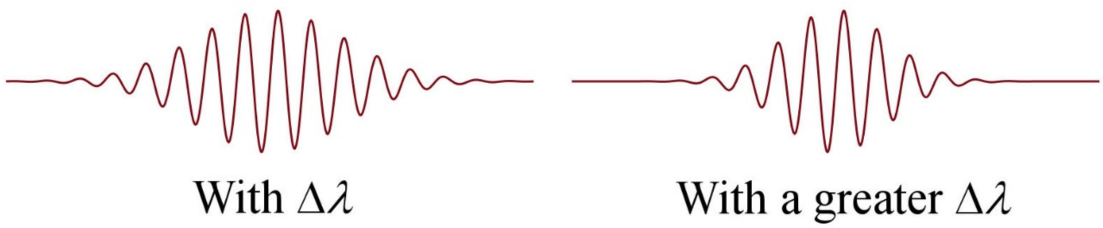
If there are many wavelengths simultaneously in the wave, then the particle has many momenta simultaneously since the wavelength and momentum are related by de Broglie formula.

$$\lambda = \frac{h}{p}$$

This means that if the momentum is measured with a very precise device, the result will be one of the possible values of the momentum. It is impossible to determine in advance what value will be obtained. One of the possible values on the interval Δp will be obtained, but it is absolutely impossible to predict which one. It is only possible to know that there is a greater probability to measure a momentum near the center of the interval since the amplitude is greater there. Again, there is some indeterminacy on the value of p that will be measured, and the indeterminacy comes from the wave nature of matter.

The Indeterminacy Principle

There is a link between Δx and Δp. If more wavelengths are added (large $\Delta\lambda$), the wave is smaller (small Δx).



Advanced calculations (using Fourier integrals…) show that the following relation exists.

$$\Delta x \Delta p \approx h$$

(This is approximately equal because there are different ways to define where exactly the intervals Δx and Δp begin and end.) If Δp is large, Δx is small. This actually agrees with what was said earlier: if Δp is large, then several momenta are present at the same time. This means that many wavelengths are present at the same time, and the wave packet is smaller.

The product $\Delta x \Delta p$ is approximately equal to h in the best of cases. If another way to vary the amplitude with x and λ is used, then the value of $\Delta x \Delta p$ is greater than h. So, we have the following relation, which is called Heisenberg's indeterminacy (or uncertainty) principle.

Heisenberg Indeterminacy Principle (With *p* and *x*)

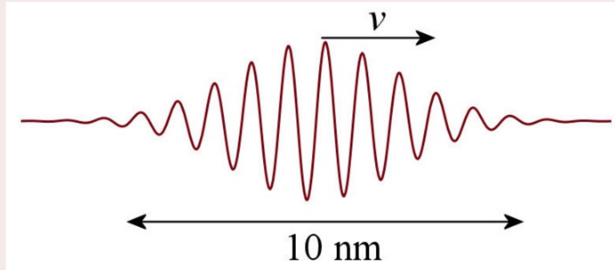
$$\Delta x \Delta p \geq h$$

Here, we will often use $\Delta x \Delta p = h$.

The principle says that if the position can be predicted with great precision (small Δx), then the momentum can no longer be predicted with much precision (large Δp), and that if the momentum can be predicted with great precision (small Δp), then the position can no longer be predicted with much precision (large Δx). It is impossible to have a system in which the value of the position and the momentum that will be measured can be predicted at the same time with great precision. If one value can be predicted with a lot of precision, then the other value cannot be predicted accurately.

Example 10.7.1

An electron is confined in a region 10 nm wide. What is the value of Δp?



We have

$$\Delta p \Delta x = h$$

$$\Delta p = \frac{h}{\Delta x}$$

$$\Delta p = \frac{6.626\times 10^{-34} Js}{10\times 10^{-9} m}$$

$$\Delta p = 6.626\times 10^{-26} \tfrac{kgm}{s}$$

This could mean, for example, that the momentum of the particle can range from 10^{-24} kgm/s to 1.06626 x 10^{-24} kgm/s (the gap between these two is Δp). If the momentum of such a particle is measured, any value of p between these two extreme values can be obtained. If the momentum of several of these particles is measured, many different results between these two values would be obtained, even if the conditions are exactly the same.

Heisenberg's indeterminacy principle has no real effect on macroscopic objects. The value of h is so small that the Δx and Δp would be really small in this case. In theory, slightly different values of the position would be obtained if the position of a baseball were to be measured several times in identical experiments, but these variations would undoubtedly be billions of times smaller than the uncertainty of the device measuring the position. The variations due to the wave nature of the matter would be completely hidden by the variations caused by the uncertainty of the apparatus.

Quite a similar reasoning can be done with the energy of a wave and the duration of the wave. The result is, at best,

$$\Delta E \Delta t \approx h$$

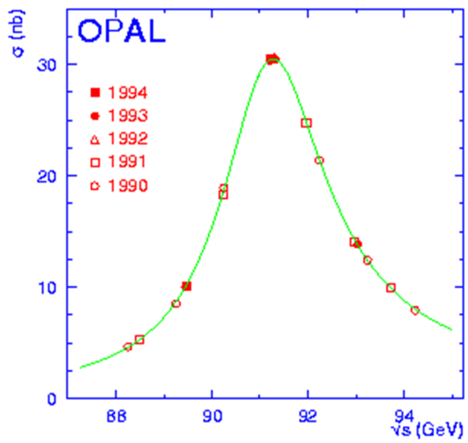
Therefore, we have

Heisenberg Indeterminacy Principle (With *E* and *t*)

$$\Delta E \Delta t \geq h$$

This kind of uncertainty can be seen in the measurement of the mass of very short-lived particles. Here is the graph obtained when the mass of several Z bosons, which live for just 3 x 10^{-25} sec (which gives an uncertainty of 13 GeV for the energy), is measured.

The spread of the measured value can clearly be seen.



www.etp.physik.uni-muenchen.de/opal/opal_en.html

There are some subtleties with Heisenberg's uncertainty principle that you can discover in this document.
https://physique.merici.ca/waves/heisenberg-eng.pdf

SUMMARY OF EQUATIONS

Planck Constant

$$h = 6.626 \times 10^{-34} J \cdot s$$

Energy Emitted by an Atom in Oscillation

$$E = nhf$$

Einstein's Photon Hypothesis

Light is composed of particles (photons) whose energy is

$$E_\gamma = hf = \frac{1240 eVnm}{\lambda}$$

Photoelectric Effect

$$E_{k\,\max} = hf - \phi$$

Threshold Frequency for the Photoelectric Effect

$$f_0 = \frac{\phi}{h}$$

Compton Effect

$$\Delta\lambda = \frac{h}{mc}\left(1-\cos\theta\right)$$

$$\Delta\lambda = 0.0024263nm\cdot\left(1-\cos\theta\right)$$

Equations of Conservation for the Compton Effect

Energy conservation	$E_\gamma = E'_\gamma + E_{ke}$
x-component of momentum conservation	$p_\gamma = p'_\gamma\cos\theta + p'_e\cos\phi$
y-component of momentum conservation	$0 = p'_\gamma\sin\theta - p'_e\sin\phi$

De Broglie Wavelength

$$\lambda = \frac{h}{p}$$

Copenhagen Interpretation

The square of the amplitude (ψ^2) at a location is proportional to the probability of finding the particle at this location.

Heisenberg Indeterminacy Principle (With *p* and *x*)

$$\Delta x\Delta p \geq h$$

Heisenberg Indeterminacy Principle (With *E* and *t*)

$$\Delta E\Delta t \geq h$$

EXERCISES

Use the following values for the exercises.

Electron	$m_e = 9.1094 \times 10^{-31}$ kg
Proton	$m_p = 1.6726 \times 10^{-27}$ kg
Neutron	$m_n = 1.6749 \times 10^{-27}$ kg
	$1\ eV = 1.602 \times 10^{-19}$ J

10.1 Photons

1. What is the energy of a photon of green light having a 550 nm wavelength?

2. A laser emits red light having a 632 nm wavelength and a power of 1 mW. How many photons per second are emitted by the laser?

3. Yellow light (λ = 585 nm) with an intensity of 50 W/m² arrives on a wall with a surface area of 3 m². How many photons hit the wall in 20 seconds?

4. Blue light (λ = 470 nm) with an intensity of 200 W/m² gets into an eye. How many photons enter the eye each second if the pupil has a diameter of 5 mm?

10.2 Photoelectric Effect

5. Ultraviolet light with a 150 nm wavelength is incident on a drop of mercury. What is the maximum energy of the ejected electrons (in eV) if the work function of mercury is 4.5 eV?

6. The threshold wavelength of cesium is 686 nm. What is the maximum energy of the ejected electrons if the incident light on a piece of cesium has a wavelength of…

   a) 690 nm
   b) 450 nm

7. The work function of a metal is 3.2 eV.

   a) What is the threshold wavelength of this metal?
   b) What is the maximum speed of the ejected electrons if ultraviolet light with a 250 nm wavelength is incident on the metal?

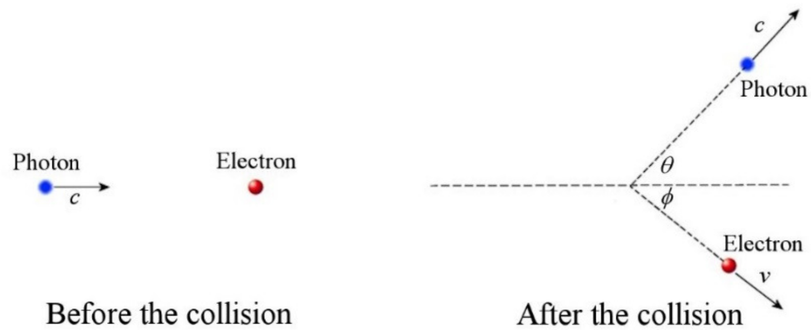
8. Electrons having a maximum speed of 500 000 m/s are ejected from a metal when light having a 400 nm wavelength is incident on the metal. What is the threshold wavelength of this metal?

9. Light having a wavelength of 450 nm and an intensity of 40 W/m² is incident on a metal. How many electrons are ejected per second and per square centimetre of the surface if only 3% of the photons arriving on the metal eject an electron?

10.3 Compton Effect

10. Photons with a 62 keV energy are scattered by electrons.

a) What is the wavelength shift of the photons scattered at 45°?
b) What is the wavelength of the photons scattered at 45°?
c) What is the energy (in keV) of the photons scattered at 45°?
d) What is the kinetic energy of the electrons after they have caused the scattering of the photons at 45°?
e) At what angle are projected the electrons after they have caused the scattering of the photons at 45°? (Angle ϕ in the following diagram.)



11. A photon having an initial energy of 50 keV is scattered by an electron. After the scattering, the energy of the photon is 49.5 keV. At what angle was the photon scattered?

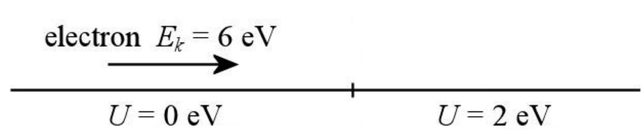
10.5 De Broglie Waves

12. What is the wavelength of a proton travelling at 10^4 m/s?

13. What is the wavelength of a proton travelling at 2×10^8 m/s?

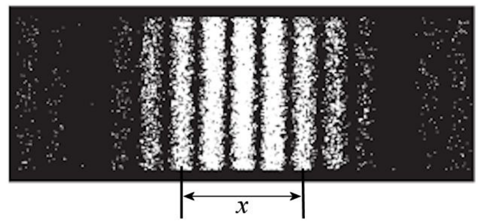
14. What is the wavelength of an electron if its kinetic energy is 10 eV?

15. An electron has a kinetic energy of 6 eV when it is in a region where $U = 0$ eV. The electron is travelling towards a region where $U = 2$ eV. By how much will the wavelength of the electron change when it enters the region where $U = 2$ eV?



10.6 Interpretation of the Wave

16. Young's experiment is done with electrons whose kinetic energy is 2 eV. The electrons pass through two slits 0.1 µm apart and the arrival of the electrons is observed on a screen located 3 m from the slits. The diagram shows what is then observed. What is the distance *x* in the diagram?



web.utk.edu/~cnattras/Phys250Fall2012/modules/module%202/matter_waves.htm

10.7 Heisenberg Indeterminacy Principle

17. The momentum of an electron ranges from 2 x 10^{-23} kgm/s to 2.05 x 10^{-23} kgm/s. What is the of Δx for this electron?

18. The lifetime of an excited state in an atom is 10^{-8} s. What is the indeterminacy of the photon energy (ΔE, in eV) emitted during a transition between this excited level and the fundamental level?

ANSWERS

10.1 Photons

1. 2.25 eV
2. 3.182 x 10^{15}
3. 8.835 x 10^{21}
4. 9.291 x 10^{15}

10.2 Photoelectric Effect

5. 3.767 eV
6. a) No ejected electrons b) 0.948 eV
7. a) 387.5 nm b) 7.868 x 10^{5} m/s
8. 519 nm
9. 2.718 x 10^{14}

10.3 Compton Effect

10. a) 0.0007106 nm b) 0.0207106 nm c) 59.873 keV d) 2127 eV

e) 65.1°

11. 26.3°

10.5 de Broglie Waves

12. 0.0396nm
13. 1.476 x 10^{-15} m
14. 0.3879 nm
15. 0.1125 nm

10.6 Interpretation of the Wave

16. 10.41 cm

10.7 Heisenberg Indeterminacy Principle

17. 1.325 nm
18. 4.136 x 10^{-7} eV