# Chapter 3 Solutions

**1.** Using an axis pointing towards the right, the acceleration is

$$\sum F_x = ma_x$$
$$120N = 80kg \cdot a_x$$
$$a_x = 1.5\tfrac{m}{s^2}$$

**2.** The force will be found with

$$\sum F_x = ma_x$$

To calculate the force, the acceleration must be known. Using an axis in the direction of the velocity, the acceleration of the car is

$$2a\left(x - x_0\right) = v^2 - v_0^2$$
$$2 \cdot a \cdot \left(80m - 0m\right) = \left(0\tfrac{m}{s}\right)^2 - \left(27.78\tfrac{m}{s}\right)^2$$
$$a = -4.823\tfrac{m}{s^2}$$

The force is therefore

$$\sum F_x = ma_x$$
$$F = 1200kg \cdot \left(-4.823\tfrac{m}{s^2}\right)$$
$$F = -5787N$$

The answer is negative because it is in the opposite direction to the velocity, which we had set as positive. The magnitude of the force is therefore 5787 N.

**3.** The time will be found with the acceleration. To find this acceleration, we need to know how strongly the truck can pull. This information can be found with what is known about the truck when there is no trailer.

Without the trailer, the acceleration is (using an axis in the direction of the velocity)

$$v = v_0 + at$$
$$8\tfrac{m}{s} = 0\tfrac{m}{s} + a \cdot 1s$$
$$a = 8\tfrac{m}{s^2}$$

Therefore, the force that accelerates the truck is

$$\sum F_x = ma_x$$
$$F = 1800kg \cdot 8\tfrac{m}{s^2}$$
$$F = 14\,400N$$

With the trailer, the total mass is now 134 800 kg. Therefore, the acceleration is

$$\sum F_x = ma$$
$$14\,400N = 134\,800kg \cdot a$$
$$a = 0.1068\tfrac{m}{s^2}$$

The time required to reach 10 km/h is therefore

$$v = v_0 + at$$
$$2.778\tfrac{m}{s} = 0\tfrac{m}{s} + 0.1068\tfrac{m}{s^2} \cdot t$$
$$t = 26s$$

**4.** The acceleration of the plane is

$$\sum F_x = ma_x$$
$$48\,900N \cdot 2 = 23\,500kg \cdot a$$
$$a = 4.16\tfrac{m}{s^2}$$

The length of the runway is therefore

$$2a_x\left(x - x_0\right) = v_x^2 - v_{x0}^2$$
$$2 \cdot 4.16\tfrac{m}{s^2} \cdot \left(x - 0m\right) = \left(80\tfrac{m}{s}\right)^2 - \left(0\tfrac{m}{s}\right)^2$$
$$x = 768.9m$$

**5.** a) We're going to find the velocity from the acceleration. The acceleration is found with Newton's 2nd law.

*Forces Acting on the Object*

The only force in the direction of the *x*-axis is $F_x = 120\frac{N}{s} \cdot t + 500N$.

*Sum of the Forces*

The *x*-component of the net force is

$$\sum F_x = 120\tfrac{N}{s} \cdot t + 500N$$

*Newton's Second Law*

Newton's second law gives

$$\sum F_x = ma_x$$
$$120\tfrac{N}{s} \cdot t + 500N = 1000kg \cdot a_x$$

*Solving the Equation*

The formula for acceleration is

$$120\tfrac{N}{s} \cdot t + 500N = 1000kg \cdot a_x$$
$$a_x = 0.12\tfrac{m}{s^3} \cdot t + 0.5\tfrac{m}{s^2}$$

As $a_x = dv_x/dt$, we have

$$\frac{dv_x}{dt} = 0.12\tfrac{m}{s^3} \cdot t + 0.5\tfrac{m}{s^2}$$

With an integral, the formula for the velocity is obtained.

$$v_x = \int \left(0.12\tfrac{m}{s^3} \cdot t + 0.5\tfrac{m}{s^2}\right) dt$$
$$= 0.06\tfrac{m}{s^3} \cdot t^2 + 0.5\tfrac{m}{s^2} \cdot t + constant$$

The constant can be found because we know that the velocity at $t$ = 0 s is 5 m/s.

$$5\tfrac{m}{s} = 0.06\tfrac{m}{s^3} \cdot (0s)^2 + 0.5\tfrac{m}{s^2} \cdot 0s + constant$$
$$5\tfrac{m}{s} = constant$$

So, the velocity formula is

$$v_x = 0.06\tfrac{m}{s^3} \cdot t^2 + 0.5\tfrac{m}{s^2} \cdot t + 5\tfrac{m}{s}$$

At $t$ = 30 s, the velocity is

$$\begin{aligned} v_x &= 0.06\tfrac{m}{s^3}\cdot(30s)^2 + 0.5\tfrac{m}{s^2}\cdot 30 + 5\tfrac{m}{s} \\ &= 74\tfrac{m}{s} \end{aligned}$$

b) The velocity formula is

$$v_x = 0.06\tfrac{m}{s^3}\cdot t^2 + 0.5\tfrac{m}{s^2}\cdot t + 5\tfrac{m}{s}$$

Since $v_x = dx/dt$, we have

$$\frac{dx}{dt} = 0.06\tfrac{m}{s^3}\cdot t^2 + 0.5\tfrac{m}{s^2}\cdot t + 5\tfrac{m}{s}$$

With an integral, the formula for the position is obtained.

$$\begin{aligned} x &= \int\left(0.06\tfrac{m}{s^3}\cdot t^2 + 0.5\tfrac{m}{s^2}\cdot t + 5\tfrac{m}{s}\right)dt \\ &= 0.02\tfrac{m}{s^3}\cdot t^3 + 0.25\tfrac{m}{s^2}\cdot t^2 + 5\tfrac{m}{s}\cdot t + constant \end{aligned}$$

The constant can be found because we know that the position at $t$ = 0 s is 0 m.

$$0m = 0.02\tfrac{m}{s^3}\cdot(0s)^3 + 0.25\tfrac{m}{s^2}\cdot(0s)^2 + 5\tfrac{m}{s}\cdot 0s + constant$$
$$0m = constant$$

Therefore, the formula for the position is

$$x = 0.02\tfrac{m}{s^3}\cdot t^3 + 0.25\tfrac{m}{s^2}\cdot t^2 + 5\tfrac{m}{s}\cdot t$$

At $t$ = 30 s, the position is

$$\begin{aligned} x &= 0.02\tfrac{m}{s^3}\cdot(30s)^3 + 0.25\tfrac{m}{s^2}\cdot(30s)^2 + 5\tfrac{m}{s}\cdot 30s \\ &= 915m \end{aligned}$$

c) Since we have the formula for the velocity as a function of time, we need to know the value of $t$ when the car is at $x$ = 400 m. To find this time, we need to solve the equation

$$400m = 0.02\tfrac{m}{s^3}\cdot t^3 + 0.25\tfrac{m}{s^2}\cdot t^2 + 5\tfrac{m}{s}\cdot t$$

In Wolfram, just write

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solve 400=0.02*t^3+0.25*t^2+5*t
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and the only real solution is $t$ = 20.989 s.

The velocity at that time is

$$\begin{aligned} v_x &= 0.06\tfrac{m}{s^3}\cdot(20.989s)^2 + 0,5\tfrac{m}{s^2}\cdot 20.989s + 5\tfrac{m}{s} \\ &= 41.93\tfrac{m}{s} \end{aligned}$$

**6.** We will resolve into *x* and *y* components, using an *x*-axis to the right and a *y*-axis upwards.

*Sum of the Forces*

The components of the 25 N force at 0° are

$$\begin{aligned} F_{1x} &= 25N\cdot\cos(0°) = 25N \\ F_{1y} &= 25N\cdot\sin(0°) = 0N \end{aligned}$$

The components of the 30 N force at 45° are

$$\begin{aligned} F_{2x} &= 30N\cdot\cos(45°) = 15\sqrt{2}N \\ F_{2y} &= 30N\cdot\sin(45°) = 15\sqrt{2}N \end{aligned}$$

The components of the 20 N force at 90° are

$$\begin{aligned} F_{3x} &= 20N\cdot\cos(90°) = 0N \\ F_{3y} &= 20N\cdot\sin(90°) = 20N \end{aligned}$$

The components of the 25 N force at 225° are

$$\begin{aligned} F_{4x} &= 20N\cdot\cos(225°) = -10\sqrt{2}N \\ F_{4y} &= 20N\cdot\sin(225°) = -10\sqrt{2}N \end{aligned}$$

The components of the 50 N force at 270° are

$$\begin{aligned} F_{5x} &= 50N\cdot\cos(270°) = 0N \\ F_{5y} &= 50N\cdot\sin(270°) = -50N \end{aligned}$$

The *x*-component of the total force is then

$$\begin{aligned}\sum F_x &= F_{1x} + F_{2x} + F_{3x} + F_{4x} + F_{5x} \\ &= 25N + 15\sqrt{2}N + 0N + -10\sqrt{2}N + 0N \\ &= 32.07N\end{aligned}$$

The $y$-component of the total force is then

$$\begin{aligned}\sum F_y &= F_{1y} + F_{2y} + F_{3y} + F_{4y} + F_{5y} \\ &= 0N + 15\sqrt{2}N + 20N + -10\sqrt{2}N + -50N \\ &= -22.93N\end{aligned}$$

*Newtons' Second Law*

The $x$-component of the acceleration is then

$$\begin{aligned}\sum F_x &= ma_x \\ 32.07N &= 5kg \cdot a_x \\ a_x &= 6.414\tfrac{m}{s^2}\end{aligned}$$

The $y$-component of the acceleration is then

$$\begin{aligned}\sum F_y &= ma_y \\ -22.93N &= 5kg \cdot a_y \\ a_y &= -4.586\tfrac{m}{s^2}\end{aligned}$$

The magnitude of the acceleration is

$$\begin{aligned}a &= \sqrt{a_x^2 + a_y^2} \\ &= \sqrt{\left(6.414\tfrac{m}{s^2}\right)^2 + \left(-4.586\tfrac{m}{s^2}\right)^2} \\ &= 7.885\tfrac{m}{s^2}\end{aligned}$$

and its direction is

$$\begin{aligned}\theta &= \arctan\frac{a_y}{a_x} \\ &= \arctan\frac{-4.586\frac{m}{s^2}}{6.414\frac{m}{s^2}} \\ &= -35.56°\end{aligned}$$

**7.** Let's find first the sum of the forces on the sled and Aaron. We will resolve into *x* and *y* components using an *x*-axis to the right and a *y*-axis upwards.

*Sum of the Forces*

The components of the 57 N force to the right are

$$F_{1x} = 57N \cdot \cos(0°) = 57N$$
$$F_{1y} = 57N \cdot \sin(0°) = 0N$$

The components of the force exerted by the mom are

$$F_{2x} = 55N \cdot \cos(145°) = -45.05N$$
$$F_{2y} = 55N \cdot \sin(145°) = 31.55N$$

The components of the force exerted by the dad are

$$F_{3x} = 55N \cdot \cos(215°) = -45.05N$$
$$F_{3y} = 55N \cdot \sin(215°) = -31.55N$$

The *x*-component of the total force is

$$\begin{aligned} \sum F_x &= F_{1x} + F_{2x} + F_{3x} \\ &= 57N + -45.05N + -45.05N \\ &= -33.11N \end{aligned}$$

The *y*-component of the total force is

$$\begin{aligned} \sum F_y &= F_{1y} + F_{2y} + F_{3y} \\ &= 0N + 31.55N + -31.55N \\ &= 0N \end{aligned}$$

*Newton's Second Law*

To find the mass with Newton's second law, the acceleration must be found. If the sled moves 6 m in 2 seconds, its acceleration is

$$x = x_0 + v_0 t + \tfrac{1}{2} a_x t^2$$
$$-6m = 0m + 0\tfrac{m}{s} \cdot 2s + \tfrac{1}{2} \cdot a_x \cdot (2s)^2$$
$$a_x = -3\tfrac{m}{s^2}$$

The mass is therefore

$$\sum F_x = ma_x$$
$$-33.11N = m \cdot \left(-3\tfrac{m}{s^2}\right)$$
$$m = 11.04kg$$

Aaron's mass is therefore

$$m = m_{tot} - m_{sled}$$
$$= 11.04kg - 2kg$$
$$= 9.04kg$$

**8.** Let's find first the sum of the forces on the box. We will resolve into *x* and *y* components using an *x*-axis to the right and a *y*-axis upwards.

The components of the 400 N force are

$$F_{1x} = 400N \cdot \cos(30°) = 346.4N$$
$$F_{1y} = 400N \cdot \sin(30°) = 200N$$

The components of the 600 N force are

$$F_{2x} = 600N \cdot \cos(140°) = -459.6N$$
$$F_{2y} = 600N \cdot \sin(140°) = 385.7N$$

The components of the unknown force *F* are

$$F_{3x} \text{ and } F_{3y}$$

The sum of the *x*-components is 0 N. Therefore, we have

$$\sum F_x = F_{1x} + F_{2x} + F_{3x}$$
$$0N = 346.4N + -459.6N + F_{3x}$$
$$F_{3x} = 113.2N$$

The sum of the $y$-components is 850 N. Therefore, we have

$$\sum F_y = F_{1y} + F_{2y} + F_{3y}$$
$$850N = 200N + 385.7N + F_{3y}$$
$$F_{3y} = 264.3N$$

The magnitude of the force is thus

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = 287.5N$$

and its direction is

$$\theta = \arctan\frac{F_{3y}}{F_{3x}} = 66.8°$$

**9.** a) The acceleration of the spaceship is

$$a = \frac{F}{m} = \frac{-200N}{2500kg} = -0.08\tfrac{m}{s^2}$$

The velocity after 0.5 s is therefore

$$v = at = -0.08\tfrac{m}{s^2} \cdot 0.5s = -0.04\tfrac{m}{s}$$

b) The acceleration of the astronaut is

$$a = \frac{F}{m} = \frac{200N}{100kg} = 2\tfrac{m}{s^2}$$

The velocity after 0.5 s is therefore

$$v = at = 2\tfrac{m}{s^2} \cdot 0.5s = 1\tfrac{m}{s}$$

**10.** The acceleration of the object is

$$a = \frac{F}{m} = \frac{-8\frac{N}{m} \cdot x}{2kg} = -4\tfrac{1}{s^2} \cdot x$$

Since $a = dv/dt$, this equation becomes

$$\frac{dv}{dt} = -4\tfrac{1}{s^2} \cdot x$$

And now, the magic trick to solve.

$$\frac{dv}{dx}\frac{dx}{dt} = -4\tfrac{1}{s^2} \cdot x$$

$$\frac{dv}{dx}v = -4\tfrac{1}{s^2} \cdot x$$

$$vdv = -4\tfrac{1}{s^2} \cdot xdx$$

Integrating each side, the equation becomes

$$\int vdv = \int -4\tfrac{1}{s^2} \cdot xdx$$

$$\frac{v^2}{2} = -\frac{4\tfrac{1}{s^2}}{2} \cdot x^2 + Cst$$

Knowing that the speed is 10 m/s at $x = 0$ the constant can be found.

$$\frac{\left(10\tfrac{m}{s}\right)^2}{2} = 0 + Cst$$

$$Cst = 50\tfrac{m^2}{s^2}$$

Therefore, the speed is given by

$$\frac{v^2}{2} = -\frac{4\tfrac{1}{s^2}}{2} \cdot x^2 + 50\tfrac{m^2}{s^2}$$

$$v^2 = -4\tfrac{1}{s^2} \cdot x^2 + 100\tfrac{m^2}{s^2}$$

The position when the speed is zero can now be found.

$$0 = -4\tfrac{1}{s^2} \cdot x^2 + 100\tfrac{m^2}{s^2}$$

$$4\tfrac{1}{s^2} \cdot x^2 = 100\tfrac{m^2}{s^2}$$

$$x = 5m$$

**11.** The acceleration of the object is

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \frac{d(2\frac{1}{sm}\cdot x^2)}{dt} \\ &= \frac{d(2\frac{1}{sm}\cdot x^2)}{dx}\frac{dx}{dt} \\ &= 4\tfrac{1}{sm}\cdot x\frac{dx}{dt} \end{aligned}$$

Since *dx/dt* = *v*, the equation becomes

$$a = 4\tfrac{1}{sm}\cdot x\cdot v$$

Since

$$v = 2\tfrac{1}{sm}\cdot x^2$$

The acceleration becomes

$$\begin{aligned} a &= 4\tfrac{1}{sm}\cdot x\cdot 2\tfrac{1}{sm}\cdot x^2 \\ &= 8\tfrac{1}{s^2m^2}\cdot x^3 \end{aligned}$$

Therefore, the force is

$$\begin{aligned} F &= ma \\ &= 2kg\cdot 8\tfrac{1}{s^2m^2}\cdot x^3 \\ &= 16\tfrac{kg}{s^2m^2}\cdot x^3 \end{aligned}$$