Chapter 2 Solutions

1. a) The x-component of the average velocity is

$$\begin{aligned}\overline{v}_x &= \frac{\Delta x}{\Delta t} \\ &= \frac{8m - 0m}{2s} \\ &= 4\tfrac{m}{s}\end{aligned}$$

The y-component of the average velocity is

$$\begin{aligned}\overline{v}_y &= \frac{\Delta y}{\Delta t} \\ &= \frac{6m - 0m}{2s} \\ &= 3\tfrac{m}{s}\end{aligned}$$

The average velocity is therefore

$$\vec{\overline{v}} = \left(4\vec{i} + 3\vec{j}\right)\tfrac{m}{s}$$

b) To find the average acceleration, the velocity must be resolved in components.

At $t = 0$ s, the components of the velocity are

$$\begin{aligned}v_x &= v\cos\theta = 15\tfrac{m}{s}\cdot\cos 30° = 12.99\tfrac{m}{s} \\ v_y &= v\sin\theta = 15\tfrac{m}{s}\cdot\sin 30° = 7.5\tfrac{m}{s}\end{aligned}$$

At $t = 2$ s, the components of the velocity are

$$\begin{aligned}v_x &= v\cos\theta = 20\tfrac{m}{s}\cdot\cos 120° = -10\tfrac{m}{s} \\ v_y &= v\sin\theta = 20\tfrac{m}{s}\cdot\sin 120° = 17.32\tfrac{m}{s}\end{aligned}$$

The x-component of the average acceleration is

$$\begin{aligned}\overline{a}_x &= \frac{\Delta v_x}{\Delta t} \\ &= \frac{-10\frac{m}{s} - 12.99\frac{m}{s}}{2s} \\ &= -11.495\frac{m}{s^2}\end{aligned}$$

The y-component of the average acceleration is

$$\begin{aligned}\overline{a}_y &= \frac{\Delta v_y}{\Delta t} \\ &= \frac{17.32\frac{m}{s} - 7.5\frac{m}{s}}{2s} \\ &= 4.91\frac{m}{s^2}\end{aligned}$$

The average acceleration is therefore

$$\vec{\overline{a}} = \left(-11.495\vec{i} + 4.91\vec{j}\right)\frac{m}{s^2}$$

2. a) The coordinates at points 1 and 2 are (If the Sun is at (0,0))

Position 1	$x = 1.5 \times 10^{11}$ m	$y = 0$ m
Position 2	$x = 0$ m	$y = 1.5 \times 10^{11}$ m

The components of the displacement are

$$\begin{aligned}\Delta x &= x_2 - x_1 = 0m - 1.5\times10^{11}m = -1.5\times10^{11}m \\ \Delta y &= y_2 - y_1 = 1.5\times10^{11}m - 0m = 1.5\times10^{11}m\end{aligned}$$

The displacement is then

$$\overrightarrow{\Delta s} = \left(-1.5\times10^{11}\vec{i} + 1.5\times10^{11}\vec{j}\right)m$$

b) The distance travelled is the length of the arc of circle between points 1 and 2, which is a quarter of the circumference

$$\text{distance} = \frac{1}{4}2\pi r = \frac{\pi r}{2} = \frac{\pi \cdot 1.5\times10^{11}m}{2} = 2.3562\times10^{11}m$$

c) The components of the average velocity are

$$\overline{v}_x = \frac{\Delta x}{\Delta t} = \frac{0m - 1.5\times10^{11}m}{7.8894\times10^6 s} = -19,013\tfrac{m}{s}$$
$$\overline{v}_y = \frac{\Delta y}{\Delta t} = \frac{1.5\times10^{11}m - 0m}{7.8894\times10^6 s} = 19,013\tfrac{m}{s}$$

The average velocity is then

$$\vec{\overline{v}} = \left(-19,013\vec{i} + 19,013\vec{j}\right)\tfrac{m}{s}$$

d) The components of the average acceleration are

$$\overline{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{-30,000\tfrac{m}{s} - 0\tfrac{m}{s}}{7.8894\times10^6 s} = -0.0038\tfrac{m}{s^2}$$
$$\overline{a}_y = \frac{\Delta v_y}{\Delta t} = \frac{0\tfrac{m}{s} - 30,000\tfrac{m}{s}}{7.8894\times10^6 s} = -0.0038\tfrac{m}{s^2}$$

The average acceleration is then

$$\vec{\overline{a}} = \left(-0.0038\vec{i} - 0.0038\vec{j}\right)\tfrac{m}{s^2}$$

3. a)

The components of the velocity are found by deriving the components of the position.

$$v_x = \frac{dx}{dt} = \frac{d\left(-3\tfrac{m}{s^2}\cdot t^2 + 2\tfrac{m}{s}\cdot t - 4m\right)}{dt} = -6\tfrac{m}{s^2}\cdot t + 2\tfrac{m}{s}$$
$$v_y = \frac{dy}{dt} = \frac{d\left(-2\tfrac{m}{s^3}\cdot t^3 + 6\tfrac{m}{s^2}\cdot t^2 + 1m\right)}{dt} = -6\tfrac{m}{s^3}\cdot t^2 + 12\tfrac{m}{s^2}\cdot t$$

At t = 1 s, these components are

$$v_x = -6\tfrac{m}{s^2}\cdot 1s + 2\tfrac{m}{s} = -4\tfrac{m}{s}$$
$$v_y = -6\tfrac{m}{s^3}\cdot (1s)^2 + 12\tfrac{m}{s^2}\cdot 1s = 6\tfrac{m}{s}$$

The speed is then

$$v = \sqrt{v_x^2 + v_y^2} = 7.21\tfrac{m}{s}$$

and the direction of the velocity is

$$\theta = \arctan\frac{v_y}{v_x} = \arctan\frac{6}{-4} = 123.7°$$

b) The components of the acceleration are found by deriving the components of the velocity.

$$a_x = \frac{dv_x}{dt} = \frac{d\left(-6\tfrac{m}{s^2}\cdot t + 2\tfrac{m}{s}\right)}{dt} = -6\tfrac{m}{s^2}$$
$$a_y = \frac{dv_y}{dt} = \frac{d\left(-6\tfrac{m}{s^3}\cdot t^2 + 12\tfrac{m}{s^2}\cdot t\right)}{dt} = -12\tfrac{m}{s^3}\cdot t + 12\tfrac{m}{s^2}$$

At t = 1 s, these components are

$$a_x = -6\tfrac{m}{s^2}$$
$$a_y = -12\tfrac{m}{s^3}\cdot 1s + 12\tfrac{m}{s^2} = 0\tfrac{m}{s^2}$$

The magnitude of the acceleration is therefore

$$a = \sqrt{a_x^2 + a_y^2} = 6\tfrac{m}{s^2}$$

and its direction is

$$\theta = \arctan\frac{a_y}{a_x} = \arctan\frac{0}{-6} = 180°$$

4. The position is found with

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 2m + 2\tfrac{m}{s}\cdot 5s + \frac{1}{2}\cdot 1\tfrac{m}{s^2}\cdot(5s)^2 = 24.5m$$
$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = -2m + 4\tfrac{m}{s}\cdot 5s + \frac{1}{2}\cdot\left(-2\tfrac{m}{s^2}\right)\cdot(5s)^2 = -7m$$
$$z = z_0 + v_{0z}t + \frac{1}{2}a_z t^2 = 2m + \left(-1\tfrac{m}{s}\right)\cdot 5s + \frac{1}{2}\cdot 0\tfrac{m}{s^2}\cdot(5s)^2 = -3m$$

The object is therefore at the position (24.5 m, -7 m, -3 m) (we could also write $\vec{r} = \left(24.5\vec{i} - 7\vec{j} - 3\vec{k}\right)m$).

5. The x-component of the acceleration is found with

$$2a_x\left(x-x_0\right)=v_x^2-v_{x0}^2$$
$$2\cdot a_x\cdot\left(2m-1m\right)=\left(5\tfrac{m}{s}\right)^2-\left(-2\tfrac{m}{s}\right)^2$$
$$a_x=10.5\tfrac{m}{s^2}$$

The y-component of the acceleration is found with

$$2a_y\left(y-y_0\right)=v_y^2-v_{y0}^2$$
$$2\cdot a_y\cdot\left(1m--1m\right)=\left(6\tfrac{m}{s}\right)^2-\left(0\tfrac{m}{s}\right)^2$$
$$a_y=9\tfrac{m}{s^2}$$

The z-component of the acceleration is found with

$$2a_z\left(z-z_0\right)=v_x^2-v_{x0}^2$$
$$2\cdot a_z\cdot\left(-4m--2m\right)=\left(-7\tfrac{m}{s}\right)^2-\left(1\tfrac{m}{s}\right)^2$$
$$a_z=-12\tfrac{m}{s^2}$$

The acceleration is therefore

$$\vec{a}=\left(10,5\vec{i}+9\vec{j}-12\vec{k}\right)\tfrac{m}{s^2}$$

6. a) The components of the initial velocity are

$$v_{0x}=50\tfrac{m}{s}\cdot\cos 32°=42.4\tfrac{m}{s}$$
$$v_{0y}=50\tfrac{m}{s}\cdot\sin 32°=26.5\tfrac{m}{s}$$

The flight time is found with

$$y=y_0+v_{0y}t-\tfrac{1}{2}gt^2$$
$$0m=0m+26.5\tfrac{m}{s}\cdot t-\tfrac{1}{2}\cdot 9.8\tfrac{m}{s^2}\cdot t^2$$
$$26.5\tfrac{m}{s}\cdot t=4.9\tfrac{m}{s^2}\cdot t^2$$
$$26.5\tfrac{m}{s}=4.9\tfrac{m}{s^2}\cdot t$$
$$t=5.407s$$

b) The maximum height is found using the fact that the y-component of the velocity is zero at the highest point.

$$-2g\left(y-y_0\right)=v_y^2-v_{0y}^2$$
$$-2\cdot 9.8\tfrac{m}{s^2}\cdot\left(y-0m\right)=\left(0\tfrac{m}{s}\right)^2-\left(26.5\tfrac{m}{s}\right)^2$$
$$y=35.82m$$

c) The range is

$$\begin{aligned}x&=x_0+vt\\&=0m+42.4\tfrac{m}{s}\cdot 5.407s\\&=229.3m\end{aligned}$$

7. a) The components of the initial velocity are

$$v_{0x}=45\tfrac{m}{s}\cdot\cos 60°=22.5\tfrac{m}{s}$$
$$v_{0y}=45\tfrac{m}{s}\cdot\sin 60°=39.0\tfrac{m}{s}$$

The maximum height is found using the fact that the y-component of the velocity is zero at the highest point.

$$-2g\left(y-y_0\right)=v_y^2-v_{0y}^2$$
$$-2\cdot 9.8\tfrac{m}{s^2}\cdot\left(y-0m\right)=\left(0\tfrac{m}{s}\right)^2-\left(39.0\tfrac{m}{s}\right)^2$$
$$y=77.49m$$

b) The flight time is found with

$$y=y_0+v_{0y}t-\tfrac{1}{2}gt^2$$
$$-265m=0m+39.0\tfrac{m}{s}\cdot t-\tfrac{1}{2}\cdot 9.8\tfrac{m}{s^2}\cdot t^2$$
$$265m+39\tfrac{m}{s}\cdot t-4.9\tfrac{m}{s^2}\cdot t^2=0$$
$$t=12.34s\ \text{ and }\ t=-4.38s$$

Only the positive response is good.

c) The range is

$$\begin{aligned} x &= x_0 + vt \\ &= 0m + 22.5\tfrac{m}{s} \cdot 12.34s \\ &= 277.6m \end{aligned}$$

d) The x-component of the velocity always remains the same.

$$v_x = v_{0x} = 22.5\tfrac{m}{s}$$

At the ground, the y-component of the velocity is

$$\begin{aligned} v_y &= v_{0y} - gt \\ &= 39.0\tfrac{m}{s} - 9.8\tfrac{m}{s^2} \cdot 12.34s \\ &= -81.9\tfrac{m}{s} \end{aligned}$$

The speed is then

$$v = \sqrt{v_x^2 + v_y^2} = 85.0\tfrac{m}{s}$$

and the direction of the velocity is

$$\theta = \arctan\frac{v_y}{v_x} = \arctan\frac{-81,9}{22,5} = -74.6°$$

8. The speed is found with

$$y = (\tan\theta)x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$

$$1.05m = (\tan 40°)\cdot 10m - \left(\frac{9.8\frac{m}{s^2}}{2\cdot v_0^2\cdot\cos^2 40°}\right)\cdot(10m)^2$$

$$v_0 = 10.67\tfrac{m}{s}$$

9. With the maximum height, the y-component of the initial velocity can be found.

$$-2g(y - y_0) = v_y^2 - v_{0y}^2$$

$$-19.6\tfrac{m}{s^2}(0.42m - 0m) = \left(0\tfrac{m}{s}\right)^2 - v_{0y}^2$$

$$v_{0y} = 2.87\tfrac{m}{s}$$

With the range, the x-component of the initial velocity can be found.

$$x = x_0 + v_{0x}t$$
$$1.06m = 0m + v_{0x} \cdot 0.5s$$
$$v_{0x} = 2.12\tfrac{m}{s}$$

The initial speed is therefore

$$v = \sqrt{v_x^2 + v_y^2} = 3.57\tfrac{m}{s}$$

and the direction of the initial velocity is

$$\theta = \arctan\frac{v_y}{v_x} = \arctan\frac{2.87}{2.12} = 53.5°$$

10. As Ruprecht falls back to the ground at the same height, the angle can be found with the range formula.

$$R = \frac{v_0^2 \sin 2\theta}{g}$$
$$70m = \frac{\left(35\tfrac{m}{s}\right)^2 \cdot \sin 2\theta}{9.8\tfrac{m}{s^2}}$$
$$\theta = 17.03° \quad \text{or} \quad \theta = 72.97°$$

The picture and common sense suggest that 17.03° is the correct answer.

The flight time is then

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$
$$0m = 0m + \left(35\tfrac{m}{s} \cdot \sin 17.03°\right) \cdot t - \tfrac{1}{2} \cdot 9.8\tfrac{m}{s^2} \cdot t^2$$
$$10.25\tfrac{m}{s} \cdot t = 4.9\tfrac{m}{s^2} \cdot t^2$$
$$10.25\tfrac{m}{s} = 4.9\tfrac{m}{s^2} \cdot t$$
$$t = 2.09s$$

11. a) The speed is found with

$$y = (\tan\theta)\,x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$
$$-20m = (\tan 35°)\cdot 350m - \left(\frac{9.8\,\frac{m}{s^2}}{2\cdot v_0^2\cdot\cos^2 35°}\right)\cdot(350m)^2$$
$$v_0 = 58.09\,\tfrac{m}{s}$$

b) The flight time is

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$
$$-20m = 0m + \left(58.09\,\tfrac{m}{s}\cdot\sin 35°\right)\cdot t - \tfrac{1}{2}\cdot 9.8\,\tfrac{m}{s^2}\cdot t^2$$
$$20m + 33.32\,\tfrac{m}{s}\cdot t - 4.9\,\tfrac{m}{s^2}\cdot t^2 = 0$$
$$t = 7.355s \quad \text{and} \quad t = -0.555s$$

Only the positive answer is good.

c) The maximal height is

$$-2g\,(y - y_0) = v_y^2 - v_{0y}^2$$
$$-2\cdot 9.8\,\tfrac{m}{s^2}\cdot(y - 0m) = \left(0\,\tfrac{m}{s}\right)^2 - \left(33.32\,\tfrac{m}{s}\right)^2$$
$$y = 56.65m$$

d) The x-component of the velocity always remains the same.

$$v_x = v_{0x} = 58.09\,\tfrac{m}{s}\cdot\cos 35° = 47.59\,\tfrac{m}{s}$$

At the arrival at the ground, the y-component of the velocity is

$$\begin{aligned} v_y &= v_{0y} - gt \\ &= 33,32\,\tfrac{m}{s} - 9,8\,\tfrac{m}{s^2}\cdot 7,355s \\ &= -38,76\,\tfrac{m}{s} \end{aligned}$$

The speed is

$$v = \sqrt{v_x^2 + v_y^2} = 61,37\,\tfrac{m}{s}$$

and the direction of the velocity is

$$\theta = \arctan\frac{v_y}{v_x} = \arctan\frac{-38.76}{47.59} = -39.16°$$

12. If the velocity is horizontal at $t = 3$ s, then the object has reached its maximum height at this instant. Since the object falls back at the same height it was launched, the total flight time is twice as large as the time required to reach the maximum height. The flight time is therefore 6 s.

As for the x-component of the velocity, it is always the same. Since it is 20 m/s at the highest point, it is also 20 m/s at the start.

Then, the range is

$$\begin{aligned} x &= x_0 + v_{0x}t \\ &= 0m + 20\tfrac{m}{s}\cdot 6s \\ &= 120m \end{aligned}$$

b) Since the y-component of the velocity is zero at the maximum height, the y-component of the initial velocity is

$$\begin{aligned} v_y &= v_{0y} - gt \\ 0\tfrac{m}{s} &= v_{0y} - 9.8\tfrac{m}{s^2}\cdot 3s \\ v_{oy} &= 29.4\tfrac{m}{s} \end{aligned}$$

The x-component of the velocity remains the same

$$v_{0x} = v_x = 20\tfrac{m}{s}$$

The initial speed is then

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = 35.56\tfrac{m}{s}$$

c) The launch angle is

$$\theta = \arctan\frac{v_{0y}}{v_{0x}} = \arctan\frac{29.4}{20} = 55.8°$$

13. a)

The time is found with the change in the *y*-component of the velocity. The components of the initial velocity are

$$v_{0x} = v_0 \cos\theta = 30\tfrac{m}{s} \cdot \cos 60° = 15\tfrac{m}{s}$$
$$v_{0y} = v_0 \sin\theta = 30\tfrac{m}{s} \cdot \sin 60° = 25{,}98\tfrac{m}{s}$$

The angle at position 2 can be found with

$$v_{0x} = v_0 \cos\theta$$
$$15\tfrac{m}{s} = 20\tfrac{m}{s} \cdot \cos\theta$$
$$\theta = 41.41°$$

The *y*-component of the velocity at position 2 is

$$v_y = v \sin\theta = 20\tfrac{m}{s} \cdot \sin 41.41° = 13.23\tfrac{m}{s}$$

The time is then

$$v_y = v_{0y} - gt$$
$$13.23\tfrac{m}{s} = 25.98\tfrac{m}{s} - 9.8\tfrac{m}{s^2} \cdot t$$
$$t = 1.301s$$

b) During this time of 1.301 s, the *x*-component of the displacement is

$$x = x_0 + v_{0x}t$$
$$x = 0m + 15\tfrac{m}{s} \cdot 1.301s$$
$$x = 19.52m$$

and the *y*-component of the displacement is

$$\begin{aligned} y &= y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \\ &= 0m + 25.98\tfrac{m}{s} \cdot 1.301s - \tfrac{1}{2} \cdot 9.8\tfrac{m}{s^2} \cdot (1.301s)^2 \\ &= 25.51m \end{aligned}$$

The distance is therefore

$$d = \sqrt{x^2 + y^2} = 32.1m$$

14. The range is found with a bit of trigonometry.

$$\tan 30° = \frac{y}{x}$$
$$\tan 30° = \frac{40m}{x}$$
$$x = 69.28m$$

Knowing the range, the initial speed can be found, provided the flight time is known. This time can be found with the *y*-component of the motion.

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$
$$-40m = 0m + 0\tfrac{m}{s}\cdot t - \tfrac{1}{2}\cdot 9.8\tfrac{m}{s^2}\cdot t^2$$
$$-40m = -4.9\tfrac{m}{s^2}\cdot t^2$$
$$t = 2.86s$$

The initial speed is therefore found with

$$x = x_0 + v_{0x}t$$
$$69.28m = 0m + v_{0x}\cdot 2.857s$$
$$v_{0x} = 24.25\tfrac{m}{s}$$

15. If the projectile is 2 m below its initial height, then the flight time is

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$
$$-2m = 0m + 0\tfrac{m}{s}\cdot t - \tfrac{1}{2}\cdot 9.8\tfrac{m}{s^2}\cdot t^2$$
$$-2m = -4.9\tfrac{m}{s^2}\cdot t^2$$
$$t = 0.6389s$$

This means that the *x*-component of the displacement is

$$\begin{aligned} x &= x_0 + v_{0x}t \\ &= 0m + 30\tfrac{m}{s}\cdot 0.6389s \\ &= 19.17m \end{aligned}$$

Now, we want to hit this target at 19.17 m, at the same height as the cannon. Since the height is the same, the formula of the range can be used to find the angle.

$$R = \frac{v_0^2 \sin 2\theta}{g}$$
$$19.17m = \frac{\left(30\tfrac{m}{s}\right)^2 \cdot \sin 2\theta}{9.8\tfrac{m}{s^2}}$$
$$\theta = 6.02° \quad \text{or} \quad \theta = 83.98°$$

Both answers are good.

16. a) The height can be found with

$$\begin{aligned} y &= (\tan\theta)\,x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2 \\ &= (\tan 30°)\cdot 10m - \left(\frac{9.8\tfrac{m}{s^2}}{2\cdot\left(30\tfrac{m}{s}\right)^2\cdot\cos^2 30°}\right)\cdot(10m)^2 \\ &= 5.048m \end{aligned}$$

This is the height from the starting point. As this point is at 1 m above ground, the height above ground is 6.048 m.

b) The *x*-component of the velocity is equal to the *x*-component of the initial velocity.

$$v_x = v_{0x} = 30\tfrac{m}{s}\cdot\cos 30° = 25.98\tfrac{m}{s}$$

The *y*-component of the velocity is

$$-2g\,(y - y_0) = v_y^2 - v_{0y}^2$$
$$-2\cdot 9.8\tfrac{m}{s^2}\cdot(5.048m - 0m) = v^2 - \left(30\tfrac{m}{s}\cdot\sin 30°\right)^2$$
$$v_y = 11.23\tfrac{m}{s}$$

The speed is

$$v = \sqrt{v_x^2 + v_y^2} = 28.30\tfrac{m}{s}$$

and the direction of the velocity is

$$\theta = \arctan\frac{v_y}{v_x} = \arctan\frac{11.23}{25.98} = 23.4°$$

17. Let's find the height of the ball when it arrives at the wall. We start by finding the time to get to the wall with

$$\begin{aligned} x &= x_0 + v_{0x}t \\ 67.5m &= 0m + \left(20\tfrac{m}{s}\cdot\cos 35°\right)\cdot t \\ t &= 4.12s \end{aligned}$$

At this time, the height of the ball is

$$\begin{aligned} y &= y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \\ &= 0m + \left(20\tfrac{m}{s}\cdot\sin 35°\right)\cdot 4.12s - \tfrac{1}{2}\cdot 9.8\tfrac{m}{s^2}\cdot\left(4.12s\right)^2 \\ &= -35.92m \end{aligned}$$

Since the height is negative, the ball would pass under the wall if there were no ground. Obviously, the ball fell to the ground before reaching towards the wall.

18. The distance is found with

$$\begin{aligned} y &= \left(\tan\theta\right)x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2 \\ 8m &= \left(\tan 40°\right)\cdot x - \left(\frac{9.8\frac{m}{s^2}}{2\cdot\left(24\frac{m}{s}\right)^2\cdot\cos^2 40°}\right)\cdot x^2 \\ 8m - 0.839\cdot x + 0.0145m^{-1}\cdot x^2 &= 0 \\ x = 12.04m \quad &\text{and} \quad x = 45.85m \end{aligned}$$

19. The components of the initial velocity are

$$\begin{aligned} v_{0x} &= 10\tfrac{m}{s}\cdot\cos\left(-30°\right) = 8.66\tfrac{m}{s} \\ v_{0y} &= 10\tfrac{m}{s}\cdot\sin\left(-30°\right) = -5\tfrac{m}{s} \end{aligned}$$

At the lowest point, the y-component of the velocity is found with

$$v = \sqrt{v_x^2 + v_y^2}$$
$$30\tfrac{m}{s} = \sqrt{\left(8.66\tfrac{m}{s}\right)^2 + v_y^2}$$
$$v_y = -28.72\tfrac{m}{s}$$

(With the square root, the answer can be positive or negative. The situation here forces us to keep the negative response.)

Then, the height is found

$$-2g\left(y - y_0\right) = v_y^2 - v_{0y}^2$$
$$-2 \cdot 9.8\tfrac{m}{s^2} \cdot \left(y - 0m\right) = \left(-28.72\tfrac{m}{s}\right)^2 - \left(-5\tfrac{m}{s}\right)^2$$
$$y = -40.82m$$

The height of the cliff is therefore 40.82 m.

20. The equation of the trajectory of the projectile launched at 45° is

$$y = \left(\tan 45°\right)x - \left(\frac{g}{2v_0^2 \cos^2 45°}\right)x^2$$
$$y = x - \left(\frac{g}{v_0^2}\right)x^2$$

The equation of the trajectory of the projectile launched at 60° is

$$y = \left(\tan 60°\right)x - \left(\frac{g}{2v_0^2 \cos^2 60°}\right)x^2$$
$$y = \sqrt{3}x - \left(\frac{2g}{v_0^2}\right)x^2$$

Since the two projectiles hit the ground at the same height y, the following equation must be true

$$x - \left(\frac{g}{v_0^2}\right)x^2 = \sqrt{3}x - \left(\frac{2g}{v_0^2}\right)x^2$$

The solution to this equation is

$$x-\left(\frac{g}{v_0^2}\right)x^2=\sqrt{3}x-\left(\frac{2g}{v_0^2}\right)x^2$$
$$\left(\frac{2g}{v_0^2}\right)x^2-\left(\frac{g}{v_0^2}\right)x^2=\sqrt{3}x-x$$
$$\left(\frac{2g}{v_0^2}-\frac{g}{v_0^2}\right)x^2=\left(\sqrt{3}-1\right)x$$
$$\left(\frac{g}{v_0^2}\right)x^2=\left(\sqrt{3}-1\right)x$$
$$\left(\frac{g}{v_0^2}\right)x=\left(\sqrt{3}-1\right)$$
$$x=\frac{\left(\sqrt{3}-1\right)v_0^2}{g}$$

Since the speed is 50 m/s, x is

$$\begin{aligned}x&=\frac{\left(\sqrt{3}-1\right)\cdot\left(50\,\tfrac{m}{s}\right)^2}{9.8\,\tfrac{m}{s^2}}\\&=186.74m\end{aligned}$$

Then, the height can be found with one of the two equations of the trajectory.

$$\begin{aligned}y&=x-\left(\frac{g}{v_0^2}\right)x^2\\&=186.747m-\frac{9.8\,\tfrac{m}{s^2}}{\left(50\,\tfrac{m}{s}\right)^2}\cdot\left(186.747m\right)^2\\&=50.04m\end{aligned}$$

21. The speed is found with

$$y=\left(\tan\theta\right)x-\left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$
$$155m=\left(\tan 60°\right)\cdot\left(195m\right)-\left(\frac{9.8\,\tfrac{m}{s^2}}{2\cdot v_0^2\cdot\cos^2 60°}\right)\cdot\left(195m\right)^2$$
$$v_0=63.86\,\tfrac{m}{s}$$

22. The angle is found with

$$y = (\tan\theta)\,x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$

$$3000m = (\tan\theta)\cdot 5000m - \left(\frac{9.8\,\frac{m}{s^2}}{2\cdot\left(760\,\frac{m}{s}\right)^2\cdot\cos^2\theta}\right)\cdot(5000m)^2$$

$$3000m = (\tan\theta)\cdot 5000m - 212.08m\cdot\frac{1}{\cos^2\theta}$$

$$3000 = (\tan\theta)\cdot 5000 - 212.08\cdot\sec^2\theta$$

It remains to solve this equation for θ. To do so, it must be remembered that $\sec^2\theta = 1+\tan^2\theta$. Therefore

$$3000 = 5000\cdot\tan\theta - 212.084\cdot\left(1+\tan^2\theta\right)$$

$$3000 = 5000\cdot\tan\theta - 212.084 - 212.084\cdot\tan^2\theta$$

$$3212.084 - 5000\cdot\tan\theta + 212.084\cdot\tan^2\theta = 0$$

This is a quadratic equation of $\tan^2\theta$. The solutions to this equation are

$$\tan\theta = 0.6609 \quad \text{and} \quad 22.91$$

The two solutions are therefore

$$\theta = 33.46° \quad \text{and} \quad \theta = 87.50°$$

23. We have those 2 formulas when the projectile is thrown at 28° and 32°.

$$40m = \frac{v_0^2\sin(2\cdot 28°)}{g} \qquad\qquad R = \frac{v_0^2\sin(2\cdot 32°)}{g}$$

Dividing the 2nd equation by the 1st, the result is

$$\frac{R}{40m} = \frac{\frac{v_0^2\sin(64°)}{g}}{\frac{v_0^2\sin(56°)}{g}}$$

$$\frac{R}{40m} = \frac{\sin 64°}{\sin 56°}$$

$$R = 43.37m$$

24. The maximum height is given by

$$h_{\max} = \frac{v_0^2 \sin^2\theta}{2g}$$

And the range is given by

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

Dividing the maximum height by the range, the result is

$$\begin{aligned}\frac{h_{\max}}{R} &= \frac{\dfrac{v_0^2 \sin^2\theta}{2g}}{\dfrac{v_0^2 \sin 2\theta}{g}} \\ &= \frac{\sin^2\theta}{2\sin 2\theta}\end{aligned}$$

Since $\sin 2\theta = 2\sin\theta\cos\theta$, the equation becomes

$$\begin{aligned}\frac{h_{\max}}{R} &= \frac{\sin^2\theta}{4\sin\theta\cos\theta} \\ &= \frac{\sin\theta}{4\cos\theta} \\ &= \frac{\tan\theta}{4}\end{aligned}$$

Then, the answer can be found

$$\begin{aligned}\frac{15m}{40m} &= \frac{\tan\theta}{4} \\ \theta &= 56.3°\end{aligned}$$

25. In x, the grenade travels 20 m in 4 seconds. Therefore, its horizontal velocity is

$$v_x = \frac{20m}{2s} = 10\,\tfrac{m}{s}$$

In y, the grenade must be $y = 4$ m at $t = 4$ s. Therefore,

$$\begin{aligned} y &= y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \\ 4m &= 0m + v_{0y} \cdot 2s - \tfrac{1}{2} \cdot 9.8\tfrac{m}{s^2} \cdot (2s)^2 \\ v_{0y} &= 11.8\tfrac{m}{s} \end{aligned}$$

Thus, the speed is

$$v_0 = \sqrt{\left(10\tfrac{m}{s}\right)^2 + \left(11.8\tfrac{m}{s}\right)^2} = 15.47\tfrac{m}{s}$$

And the angle is

$$\begin{aligned} \theta &= \arctan\frac{v_y}{v_x} \\ &= \arctan\frac{11.8}{10} \\ &= 49.72° \end{aligned}$$

26. Using a $y = 0$ at the ground, the height as a function of time is given by

$$y = v_{0y}t - \tfrac{1}{2}gt^2$$

At $t = 1$, we have

$$h = v_{0y} \cdot 1s - \tfrac{1}{2}g(1s)^2$$

and at $t = 5$ s, we have

$$h = v_{0y} \cdot 5s - \tfrac{1}{2}g(5s)^2$$

Then we have two equations and two unknowns. By multiplying the first equation by 5, our two equations are

$$\begin{aligned} 5h &= v_{0y} \cdot 5s - \tfrac{1}{2}g \cdot 5s^2 \\ h &= v_{0y} \cdot 5s - \tfrac{1}{2}g \cdot 25s^2 \end{aligned}$$

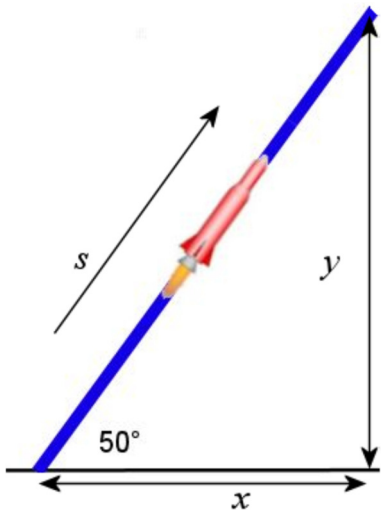
By subtracting these two equations, the result is

$$4h = g \cdot 10s^2$$

Thus, the height is 24.5 m.

27. This movement must be split into two parts: a motion in a straight line for 20 seconds and a free-fall motion.

A s-axis pointing in the direction of the motion will be used for the first part.



During the motion in a straight line, the displacement is

$$\begin{aligned} s &= s_0 + v_0 t + \tfrac{1}{2} a t^2 \\ &= 0m + 0\tfrac{m}{s} \cdot 20s + \tfrac{1}{2} \cdot 20\tfrac{m}{s^2} \cdot (20s)^2 \\ &= 4000m \end{aligned}$$

The x and y positions are then

$$\begin{aligned} x &= s\cos 50° = 4000m \cdot \cos 50° = 2571.2m \\ y &= s\sin 50° = 4000m \cdot \sin 50° = 3064.2m \end{aligned}$$

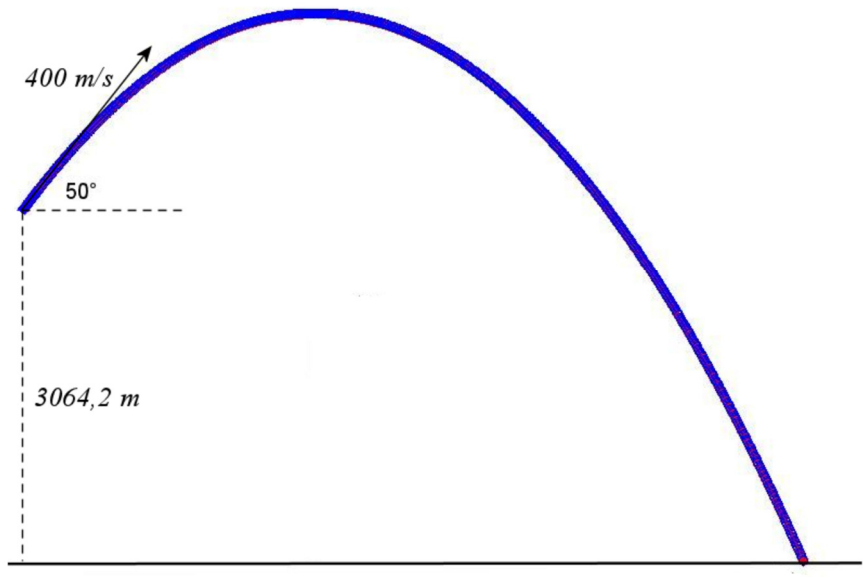
The speed at the end of this part is

$$\begin{aligned} v &= v_0 + at \\ &= 0\tfrac{m}{s} + 20\tfrac{m}{s^2} \cdot 20s \\ &= 400\tfrac{m}{s} \end{aligned}$$

This velocity is in the direction of the trajectory. If it is resolved into x and y components, we obtain

$$\begin{aligned} v_x &= v\cos 50° = 400\tfrac{m}{s} \cdot \cos 50° = 257.12\tfrac{m}{s} \\ v_y &= v\sin 50° = 400\tfrac{m}{s} \cdot \sin 50° = 306.42\tfrac{m}{s} \end{aligned}$$

For the free-fall part, the initial positions and velocities of this part are the value at the end of the first part. We then have the following situation.



a) The maximal height is found with

$$-2g\left(y-y_0\right)=v_y^2-v_{0y}^2$$
$$-2\cdot 9.8\tfrac{m}{s^2}\cdot\left(y-3064.2m\right)=\left(0\tfrac{m}{s}\right)^2-\left(306.42\tfrac{m}{s}\right)^2$$
$$y=7855m$$

b) The flight time for this part is found with

$$y=y_0+v_{0y}t-\tfrac{1}{2}gt^2$$
$$0m=3064.2m+306.42\tfrac{m}{s}\cdot t-\tfrac{1}{2}\cdot 9.8\tfrac{m}{s^2}\cdot t^2$$
$$t=71.30s \quad \text{and} \quad t=-8.77s$$

If the 20 seconds of the first part is added, the total flight time is 91.30 s.

c) The x position at the end of the second phase is

$$\begin{aligned} x &= x_0+v_{0x}t \\ &= 2571.2m+257.12\tfrac{m}{s}\cdot 71.30s \\ &= 20{,}905m \end{aligned}$$

28. The acceleration is

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{\left(1024\,\tfrac{m}{s}\right)^2}{3.844\times10^8\,m} \\ &= 0.002728\,\tfrac{m}{s^2} \end{aligned}$$

29. The radius of the path is

$$\begin{aligned} r &= \frac{circumference}{2\pi} \\ &= 7.958m \end{aligned}$$

The acceleration is

$$\begin{aligned} a_c &= \frac{4\pi^2 r}{T^2} \\ &= \frac{4\pi^2 \cdot 7.958m}{(5s)^2} \\ &= 12.57\,\tfrac{m}{s^2} \end{aligned}$$

30. The period of the train is

$$\begin{aligned} a_c &= \frac{4\pi^2 r}{T^2} \\ 20\,\tfrac{m}{s^2} &= \frac{4\pi^2 \cdot 0.5m}{T^2} \\ T &= 0.9935s \end{aligned}$$

The time to make 50 revolutions is therefore

$$\begin{aligned} t_{tot} &= 50 \cdot 0.9935s \\ &= 49.67s \end{aligned}$$

31. The radius of the path is found with

$$v = \frac{2\pi r}{T}$$
$$36\tfrac{m}{s} = \frac{2\pi r}{24s}$$
$$r = 137.5m$$

The acceleration is therefore

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{\left(36\tfrac{m}{s}\right)^2}{137.5m} \\ &= 9.425\tfrac{m}{s^2} \end{aligned}$$

32. At point A, the acceleration is

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{\left(20\tfrac{m}{s}\right)^2}{10m} \\ &= 40\tfrac{m}{s^2} \end{aligned}$$

This acceleration is directed upwards.

At point B, the acceleration is

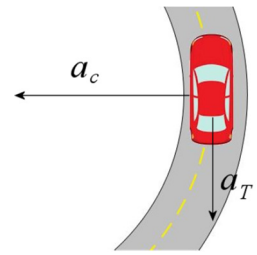
$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{\left(12\tfrac{m}{s}\right)^2}{15m} \\ &= 9.6\tfrac{m}{s^2} \end{aligned}$$

This acceleration is directed downwards.

33. a) The tangential acceleration is

$$2a_T\left(s-s_0\right)=v^2-v_0^2$$
$$2\cdot a_T\cdot\left(4\pi r-0m\right)=\left(0\tfrac{m}{s}\right)^2-\left(25\tfrac{m}{s}\right)^2$$
$$2\cdot a_T\cdot\left(4\pi\cdot 50m-0m\right)=\left(0\tfrac{m}{s}\right)^2-\left(25\tfrac{m}{s}\right)^2$$
$$a_T=-0.4974\tfrac{m}{s^2}$$

As the positive direction has been put in the direction of the car's movement, a negative value means that acceleration is in the opposite direction to the velocity of the car.



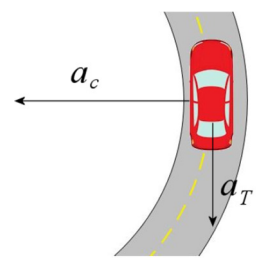
b) After one lap, the speed of the car is

$$2a_T\left(s-s_0\right)=v^2-v_0^2$$
$$2\cdot\left(-0.4974\tfrac{m}{s^2}\right)\cdot\left(2\pi r-0m\right)=v^2-\left(25\tfrac{m}{s}\right)^2$$
$$2\cdot\left(-0.4974\tfrac{m}{s^2}\right)\cdot\left(2\pi\cdot 50m-0m\right)=v^2-\left(25\tfrac{m}{s}\right)^2$$
$$v=17.68\tfrac{m}{s}$$

The centripetal acceleration is

$$\begin{aligned}a_c&=\frac{v^2}{r}\\&=\frac{\left(17.68\tfrac{m}{s}\right)^2}{50m}\\&=6.25\tfrac{m}{s^2}\end{aligned}$$

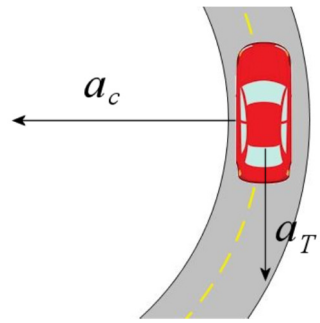
As always, the centripetal acceleration is directed towards the center of the circle.



c) The magnitude of the total acceleration is

$$a=\sqrt{a_c^2+a_T^2}$$
$$a=\sqrt{\left(6.25\tfrac{m}{s^2}\right)^2+\left(0.4974\tfrac{m}{s^2}\right)^2}=6.27\tfrac{m}{s^2}$$

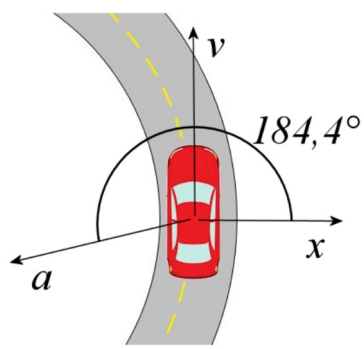
The directions of the accelerations are



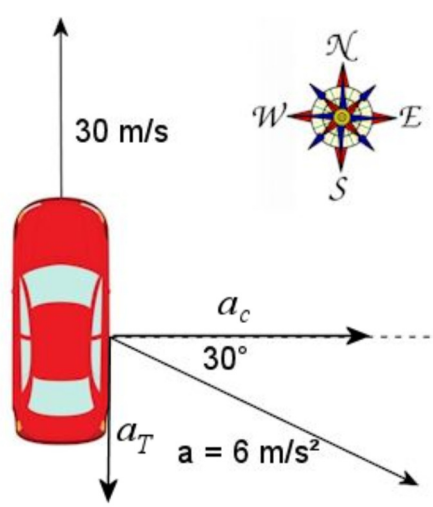
With conventional, x and y axes, the direction is

$$\theta = \arctan\frac{v_y}{v_x} = \arctan\frac{-0.4974}{-6.25} = 184.4°$$

That means that there is an angle of 94,4° with the speed.



34. The components of the acceleration are



a) As the tangential acceleration is opposed to the speed, this car slows down.

b) In this exercise, the tangential acceleration is the y-component of the acceleration. This component is

$$\begin{aligned} a_T &= a\sin(-30°) \\ &= 6\tfrac{m}{s^2}\cdot\sin(-30°) \\ &= -3\tfrac{m}{s^2} \end{aligned}$$

c) As the centripetal acceleration is towards the East, this car turns East.

d) The radius of curvature is found with the centripetal acceleration. In this exercise, the centripetal acceleration is the x-component of the acceleration. This component is

$$\begin{aligned} a_c &= a\cos(-30°) \\ &= 6\tfrac{m}{s^2}\cdot\cos(-30°) \\ &= 5.20\tfrac{m}{s^2} \end{aligned}$$

The radius is therefore

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ 5.20\tfrac{m}{s^2} &= \frac{\left(30\tfrac{m}{s}\right)^2}{r} \\ r &= 173.2m \end{aligned}$$

35. To find d, the point of intersection of the parabola and the slope must be found. The equation of the parabola is

$$y = (\tan\theta)\,x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$

$$y = (\tan 60°)\cdot x - \left(\frac{9.8\tfrac{m}{s^2}}{2\cdot 2500\tfrac{m^2}{s^2}\cdot\cos^2 60°}\right)\cdot x^2$$

The equation for the slope is just the equation of a line which passes through the origin.

$$y = mx$$

As the slope is equal to the tangent of the angle of inclination, the equation is

$$y = (\tan 20°) \cdot x$$

At the point of intersection, the values of y are identical. This means that

$$y_{parabole} = y_{droite}$$

$$(\tan 20°) \cdot x = (\tan 60°) \cdot x - \left(\frac{9.8 \frac{m}{s^2}}{2 \cdot 2500 \frac{m^2}{s^2} \cdot \cos^2 60°} \right) \cdot x^2$$

The value of x is, therefore,

$$(\tan 20°) \cdot x = (\tan 60°) \cdot x - \left(\frac{9.8 \frac{m}{s^2}}{2 \cdot 2500 \frac{m^2}{s^2} \cdot \cos^2 60°} \right) \cdot x^2$$

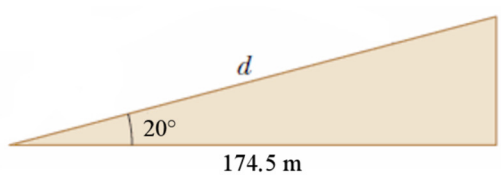
$$(\tan 20°) = (\tan 60°) - \left(\frac{9.8 \frac{m}{s^2}}{2 \cdot 2500 \frac{m^2}{s^2} \cdot \cos^2 60°} \right) \cdot x$$

$$\left(\frac{9.8 \frac{m}{s^2}}{2 \cdot 2500 \frac{m^2}{s^2} \cdot \cos^2 60°} \right) \cdot x = (\tan 60°) - (\tan 20°)$$

$$x = \frac{((\tan 60°) - (\tan 20°)) \cdot 2 \cdot 2500 \frac{m^2}{s^2} \cdot \cos^2 60°}{9.8 \frac{m}{s^2}}$$

$$x = 174.5m$$

Then, d is



$$\cos 20° = \frac{174.5m}{d}$$

$$d = 185.7m$$

36. To find d, the point of intersection of the parabola and the slope must be found. The equation of the parabola is

$$y = (\tan \theta) x - \left(\frac{g}{2v_0^2 \cos^2 \theta} \right) x^2$$

The equation for the slope is just the equation of a line which passes through the origin.

$$y = mx$$

As the slope is equal to the tangent of the angle of inclination, the equation is

$$y = (\tan\alpha)\,x$$

At the point of intersection, the values of y are identical. This means that

$$y_{parabole} = y_{droite}$$

$$(\tan\alpha)\,x = (\tan\theta)\,x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$

Therefore, the value of x is

$$(\tan\alpha) = (\tan\theta) - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x$$

$$\left(\frac{g}{2v_0^2\cos^2\theta}\right)x = (\tan\theta - \tan\alpha)$$

$$x = (\tan\theta - \tan\alpha)\frac{2v_0^2\cos^2\theta}{g}$$

$$x = \frac{v_0^2}{g}\left(2\sin\theta\cos\theta - 2\cos^2\theta\tan\alpha\right)$$

Since $2\sin\theta\cos\theta = \sin 2\theta$, x is

$$x = \frac{v_0^2}{g}\left(\sin 2\theta - 2\cos^2\theta\tan\alpha\right)$$

When d is at its maximum value, x is also at its maximum value. So we're looking for the maximum value of x. At the maximum value, we have

$$\frac{dx}{d\theta} = 0$$

Therefore,

$$0 = \frac{d}{d\theta}\left(\frac{v_0^2}{g}\left(\sin 2\theta - 2\cos^2\theta\tan\alpha\right)\right)$$

$$0 = \frac{v_0^2}{g}\left(2\cos 2\theta - -4\sin\theta\cos\theta\tan\alpha\right)$$

$$0 = 2\cos 2\theta + 4\sin\theta\cos\theta\tan\alpha$$

It only remains to solve for θ.

$$\begin{aligned}
0 &= 2\cos 2\theta + 4\sin\theta\cos\theta\tan\alpha \\
0 &= 2\cos 2\theta + 2\sin 2\theta\tan\alpha \\
2\cos 2\theta &= -2\sin 2\theta\tan\alpha \\
1 &= -\tan 2\theta\tan\alpha \\
-\cot\alpha &= \tan 2\theta
\end{aligned}$$

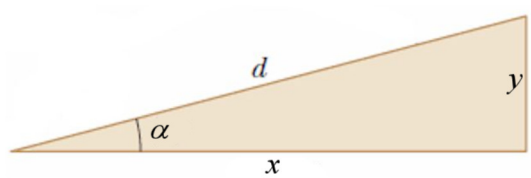
But

$$\begin{aligned}
-\cot\alpha &= -\frac{\cos\alpha}{\sin\alpha} \\
&= \frac{\cos\alpha}{-\sin\alpha} \\
&= \frac{\cos(-\alpha)}{\sin(-\alpha)} \\
&= \frac{\sin(90° - (-\alpha))}{\cos(90° - (-\alpha))} \\
&= \tan(90° + \alpha)
\end{aligned}$$

Therefore

$$\begin{aligned}
\cot\alpha &= \tan 2\theta \\
\tan(90° + \alpha) &= \tan 2\theta \\
90° + \alpha &= 2\theta \\
\theta &= \frac{1}{2}(90° + \alpha)
\end{aligned}$$

The distance is thus



$$d = \frac{x}{\cos\alpha}$$

With the value of x, d is

$$d = \frac{v_0^2}{g\cos\alpha}\left(\sin 2\theta - 2\cos^2\theta\tan\alpha\right)$$

The maximum value of d is reach when θ is the maximum angle. Thus

$$\begin{aligned} d_{\max} &= \frac{v_0^2}{g\cos\alpha}\left(\sin 2\tfrac{1}{2}(90°+\alpha) - 2\cos^2\tfrac{1}{2}(90°+\alpha)\tan\alpha\right) \\ &= \frac{v_0^2}{g\cos\alpha}\left(\sin(90°+\alpha) - 2\cos^2\tfrac{1}{2}(90°+\alpha)\tan\alpha\right) \end{aligned}$$

As

$$\sin(90°+\alpha) = \cos\alpha$$

$d_{\max}$ is

$$d_{\max} = \frac{v_0^2}{g\cos\alpha}\left(\cos\alpha - 2\cos^2\tfrac{1}{2}(90°+\alpha)\tan\alpha\right)$$

As

$$\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2}$$

$d_{\max}$ becomes

$$\begin{aligned} d_{\max} &= \frac{v_0^2}{g\cos\alpha}\left(\cos\alpha - 2\frac{1+\cos(90°+\alpha)}{2}\tan\alpha\right) \\ &= \frac{v_0^2}{g\cos\alpha}\left(\cos\alpha - \left(1+\cos(90°+\alpha)\right)\tan\alpha\right) \end{aligned}$$

As

$$\cos(90°+\alpha) = -\sin\alpha$$

$d_{\max}$ is

$$d_{\max} = \frac{v_0^2}{g\cos\alpha}\left(\cos\alpha - (1-\sin\alpha)\tan\alpha\right)$$

Therefore,

$$\begin{aligned}
d_{\max} &= \frac{v_0^2}{g\cos\alpha}\left(\cos\alpha - (1-\sin\alpha)\tan\alpha\right) \\
&= \frac{v_0^2}{g\cos^2\alpha}\left(\cos^2\alpha - (1-\sin\alpha)\sin\alpha\right) \\
&= \frac{v_0^2}{g\cos^2\alpha}\left(\cos^2\alpha - \sin\alpha + \sin^2\alpha\right) \\
&= \frac{v_0^2}{g\cos^2\alpha}(1-\sin\alpha)
\end{aligned}$$

37. Let's find the speed by solving

$$y = (\tan\theta)x - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$

for v_0. We then obtain

$$\begin{aligned}
(\tan\theta)x - y &= \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2 \\
v_0^2 &= \frac{gx^2}{2\cos^2\theta\left((\tan\theta)x - y\right)} \\
v_0^2 &= \frac{gx^2}{2x\cos^2\theta\tan\theta - 2y\cos^2\theta} \\
v_0^2 &= \frac{gx^2}{2x\cos\theta\sin\theta - 2y\cos^2\theta} \\
v_0^2 &= \frac{gx^2}{x\sin 2\theta - 2y\cos^2\theta}
\end{aligned}$$

If the speed is minimal, then the divisor is maximum. Therefore, we need to find the maximum value of

$$x\sin 2\theta - 2y\cos^2\theta$$

Of course, the value is maximum when the derivative is zero. This means that

$$\frac{d\left(x\sin 2\theta - 2y\cos^2\theta\right)}{d\theta} = 0$$
$$2x\cos 2\theta - -4y\cos\theta\sin\theta = 0$$
$$2x\cos 2\theta + 2y\sin 2\theta - = 0$$
$$x\cos 2\theta + y\sin 2\theta = 0$$
$$y\sin 2\theta = -x\cos 2\theta$$
$$y\tan 2\theta = -x$$
$$\tan 2\theta = \frac{-x}{y}$$

With x = 195 m and y = 155 m, the solutions are

$$\tan 2\theta = \frac{-195m}{155m}$$
$$2\theta = -51.52° \quad \text{and} \quad 2\theta = 128.48°$$

(The second solution is found by adding 180 ° to the first.) Obviously, the first solution must be rejected. The remaining solution is θ = 64.24 °.

Thus, the minimal speed is

$$\begin{aligned} v_0 &= \sqrt{\frac{gx^2}{x\sin 2\theta - 2y\cos^2\theta}} \\ &= \sqrt{\frac{9.8\frac{m}{s^2}\cdot(195m)^2}{(195m)\cdot\sin(128.48°) - 2\cdot(155m)\cdot\cos^2(64.24°)}} \\ &= \sqrt{3960.2\frac{m^2}{s^2}} \\ &= 62.93\frac{m}{s} \end{aligned}$$