

# Chapter 13 Solutions

**1.** There are four forces exerted on the beam.

- 1) The weight of the beam (392 N) directed downwards.
- 2) A tension force of the rope to the left ( $T_1$ ).
- 3) The tension of the rope that supports the 100 kg block. This tension is equal to the weight of the block, 980 N.
- 4) A tension force of the rope to the right ( $T_2$ ).

The sum of the y-components of the forces is

$$\begin{aligned}\sum F_y &= 0 \\ \rightarrow -392N + T_1 - 980N + T_2 &= 0\end{aligned}$$

The sum of torque is (with the axis on the left end of the beam with a positive direction in the clockwise direction)

$$\begin{aligned}\sum \tau &= 0 \\ \rightarrow 392N \cdot 2.5m + T_1 \cdot 0m + 980N \cdot 2m - T_2 \cdot 5m &= 0\end{aligned}$$

$T_2$  can now be found with this last equation.

$$\begin{aligned}392N \cdot 2.5m + 980N \cdot 2m - T_2 \cdot 5m &= 0 \\ T_2 &= 588N\end{aligned}$$

This value is then substituted in the sum of the y-components of the forces.

$$\begin{aligned}-392N + T_1 - 980N + T_2 &= 0 \\ -392N + T_1 - 980N + 588N &= 0 \\ T_1 &= 784N\end{aligned}$$

**2.** There are four forces exerted on the beam.

- 1) The weight of the beam (196 N).
- 2) The normal force exerted by the pivot (resolved in  $H$  and  $V$  components).
- 3) The tension force of the horizontal rope ( $T$ ).
- 4) The tension of the rope that supports the 30 kg street light. This tension is equal to the weight of the street light, 294 N.

The sums of the components of the forces on the beam are

$$\begin{aligned}\sum F_x &= 0 \\ &\rightarrow H - T = 0 \\ \sum F_y &= 0 \\ &\rightarrow -196N + V - 294N = 0\end{aligned}$$

The sum of torque is (with the axis at the pivot with a positive direction in the clockwise direction)

$$\begin{aligned}\sum \tau &= 0 \\ &\rightarrow +196N \cdot 3m \cdot \sin 53^\circ - T \cdot 3m + 294N \cdot 6m \cdot \sin 53^\circ = 0\end{aligned}$$

(In this last equation, the torque exerted by the tension was calculated with  $F_{r\perp}$ .)

$T$  is then found with the sum of torques.

$$\begin{aligned}196N \cdot 3m \cdot \sin 53^\circ - T \cdot 3m + 294N \cdot 6m \cdot \sin 53^\circ &= 0 \\ T &= 626.13N\end{aligned}$$

$H$  is then found with the sum of the  $x$ -components of the forces.

$$\begin{aligned}H - T &= 0 \\ H - 626.13N &= 0 \\ H &= 626.13N\end{aligned}$$

Finally,  $V$  is found with the sum of the  $y$ -components of the forces.

$$\begin{aligned}-196N + V - 294N &= 0 \\ V &= 490N\end{aligned}$$

The magnitude of the force exerted by the pivot is therefore

$$\begin{aligned}F &= \sqrt{H^2 + V^2} \\ &= \sqrt{(626.13N)^2 + (490N)^2} \\ &= 795.1N\end{aligned}$$

and its direction is

$$\begin{aligned}\theta &= \arctan \frac{V}{H} \\ &= \arctan \frac{490N}{626.13N} \\ &= 38.0^\circ\end{aligned}$$

**3.** There are four forces exerted on the beam.

- 1) The weight of the beam (196 N).
- 2) The normal force exerted by the pivot (resolved in  $H$  and  $V$  components).
- 3) The normal force exerted by the worker. This normal force is equal to the weight of the person (686 N).
- 4) The tension force of the rope that supports the beam ( $T$ ).

The sums of the components of the forces on the beam are

$$\begin{aligned}\sum F_x &= 0 \\ &\rightarrow H + T \cos(30^\circ) = 0 \\ \sum F_y &= 0 \\ &\rightarrow -196N + V - 686N + T \sin(30^\circ) = 0\end{aligned}$$

The sum of torque is (with the axis at the pivot with a positive direction in the clockwise direction)

$$\begin{aligned}\sum \tau &= 0 \\ &\rightarrow +196N \cdot 1.5m + 686N \cdot 1m - T \cdot 3m \cdot \sin 150^\circ = 0\end{aligned}$$

$T$  is then found with the sum of torques.

$$\begin{aligned}196N \cdot 1.5m + 686N \cdot 1m - T \cdot 3m \cdot \sin 150^\circ &= 0 \\ T &= 653.33N\end{aligned}$$

$H$  can be found with the sum of the  $x$ -components of the forces

$$\begin{aligned}H + T \cos(30^\circ) &= 0 \\ H + 653.33N \cdot \cos(30^\circ) &= 0 \\ H &= -565.8N\end{aligned}$$

and  $V$  can be found with the sum of the  $y$ -components of the forces.

$$\begin{aligned}
 -196N + V - 686N + T \sin(30^\circ) &= 0 \\
 -196N + V - 686N + 653.33N \cdot \sin(30^\circ) &= 0 \\
 V &= 553.33N
 \end{aligned}$$

The magnitude of the force exerted by the pivot is therefore

$$\begin{aligned}
 F &= \sqrt{H^2 + V^2} \\
 &= \sqrt{(-565.8N)^2 + (553.83N)^2} \\
 &= 792.8N
 \end{aligned}$$

and its direction is

$$\begin{aligned}
 \theta &= \arctan \frac{V}{H} \\
 &= \arctan \frac{553,83N}{-565,8N} \\
 &= 135,5^\circ
 \end{aligned}$$

**4.** There are three forces exerted on the beam.

- 1) The weight (980 N).
- 2) The force exerted by Sam ( $F_1$ ).
- 3) The force exerted by Joe ( $F_2$ ).

The sum of the y-components of the forces is

$$\begin{aligned}
 \sum F_y &= 0 \\
 \rightarrow -980N + F_1 + F_2 &= 0
 \end{aligned}$$

The sum of torque is (with the axis where Sam exerts a force and with a positive direction in the clockwise direction)

$$\begin{aligned}
 \sum \tau &= 0 \\
 \rightarrow +980N \cdot 2.8m - F_2 \cdot 4.6m &= 0
 \end{aligned}$$

$F_2$  can be found with the sum of torques.

$$+980N \cdot 2.8m - F_2 \cdot 4.6m = 0$$

$$F_2 = 596.5N$$

$F_1$  can then be found with the sum of the  $y$ -components of the forces.

$$-980N + F_1 + F_2 = 0$$

$$-980N + F_1 + 596.5N = 0$$

$$F_1 = 383.5N$$

**5.** There are three forces exerted on the beam.

- 1) The weight (98 N).
- 2) The force exerted by the right hand ( $F_1$ ).
- 3) The force exerted by the left hand ( $F_2$ ).

The sum of the  $y$ -components of the forces is

$$\sum F_y = 0$$

$$\rightarrow -98N + F_1 + F_2 = 0$$

The sum of torque is (with the axis where the right hand exerts a force and with a positive direction in the clockwise direction)

$$\sum \tau = 0$$

$$\rightarrow -F_2 \cdot 0.7m + 98N \cdot 1.7m = 0$$

$F_2$  can be found with the sum of torques.

$$-F_2 \cdot 0.7m + 98N \cdot 1.7m = 0$$

$$F_2 = 238N$$

$F_1$  can then be found with the sum of the  $y$ -components of the forces is

$$-98N + F_1 + F_2 = 0$$

$$-98N + F_1 + 238N = 0$$

$$F_1 = -140N$$

As it is negative, this force is directed downwards.

**6.** There are four forces exerted on the board.

- 1) The weight of the board ( $w$ ).
- 2) The normal force exerted by the support on the left ( $N_1$ ).
- 3) The normal force exerted by the support on the right ( $N_2$ ).
- 4) The normal force exerted by the cat, which is equal to the weight of the cat (49 N).

As the board is about to tip, the normal exerted by the support on the left ( $N_1$ ) is zero.

The sum of the  $y$ -components of the forces is

$$\begin{aligned}\sum F_y &= 0 \\ \rightarrow -w + N_2 - 49N &= 0\end{aligned}$$

The sum of torque is (with the axis where the force  $N_2$  is exerted with a positive direction in the clockwise direction)

$$\begin{aligned}\sum \tau &= 0 \\ \rightarrow -w \cdot 1m + 49N \cdot 1.5m &= 0\end{aligned}$$

$w$  can be found with the sum of torques.

$$\begin{aligned}-w \cdot 1m + 49N \cdot 1.5m &= 0 \\ w &= 73.5N\end{aligned}$$

The mass is therefore

$$\begin{aligned}w &= mg \\ 73.5N &= m \cdot 9.8 \frac{N}{kg} \\ m &= 7.5kg\end{aligned}$$

**7.** There are three forces exerted on the bridge.

- 1) The weight of the bridge (1960 N).
- 2) The force exerted by the pivot (components  $H$  and  $V$ ).
- 3) The tension force made by the rope.

The sums of the components of the forces on the bridge are

$$\begin{aligned}\sum F_x &= 0 \\ &\rightarrow H + T \cos(115.8^\circ) = 0 \\ \sum F_y &= 0 \\ &\rightarrow -1960N + V + T \sin(115.8^\circ) = 0\end{aligned}$$

The sum of torque is (with the axis at the pivot with a positive direction in the clockwise direction)

$$\begin{aligned}\sum \tau &= 0 \\ &\rightarrow 1960N \cdot 4.5m \cdot \sin(110^\circ) - T \cdot 5m \cdot \sin(44.2^\circ) = 0\end{aligned}$$

$T$  can be found with the sum of torques.

$$\begin{aligned}1960N \cdot 4.5m \cdot \sin(110^\circ) - T \cdot 5m \cdot \sin(44.2^\circ) &= 0 \\ T &= 2377.7N\end{aligned}$$

$H$  can then be found with the sum of the  $x$ -components of the forces.

$$\begin{aligned}H + T \cos(115.8^\circ) &= 0 \\ H + 2377.7 \cdot \cos(115.8^\circ) &= 0 \\ H &= 1034.8N\end{aligned}$$

$V$  can be found with the sum of the  $y$ -components of the forces.

$$\begin{aligned}-1960N + V + T \sin(115.8^\circ) &= 0 \\ -1960N + V + 2377.7N \cdot \sin(115.8^\circ) &= 0 \\ V &= -180.6N\end{aligned}$$

The magnitude of the force exerted by the pivot is therefore

$$\begin{aligned}F &= \sqrt{H^2 + V^2} \\ &= \sqrt{(1034.8N)^2 + (-180.6N)^2} \\ &= 1050.5N\end{aligned}$$

and its direction is

$$\begin{aligned}\theta &= \arctan \frac{V}{H} \\ &= \arctan \frac{-180.6\text{N}}{1034.8\text{N}} \\ &= -9.9^\circ\end{aligned}$$

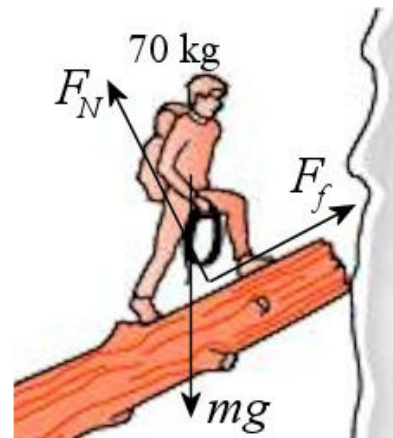
- 8.** Let's assume that Gaëlle is at the end of the trunk. The frictional force required for the trunk to not slip will then be found. This value will then be compared to the maximum value of static friction force to see the required friction force can be reached.

There are five forces exerted on the trunk.

- 1) The weight of the trunk (2940 N), downwards.
- 2) The normal force exerted by the ground ( $N_1$ ), upwards.
- 3) The friction force exerted by the ground ( $F_f$ ), towards the right.
- 4) The normal force exerted by the cliff ( $N_2$ ), towards the left.
- 5) The total force exerted by Gaëlle directed downwards, which is equal to her weight (686 N).

This 5<sup>th</sup> force is actually the sum of the normal force and the frictional force. If we look at the forces acting on Gaëlle, we have the forces shown on the right.

Since Gaëlle does not accelerate, the sum of these 3 forces must be zero. This means that the sum of the normal and the friction forces must cancel out the weight. The sum of the normal and the friction forces on Gaëlle is therefore a force directed upwards, and whose magnitude is equal to her weight (686 N). This is the total force made by the trunk on Gaëlle.



If the trunk exerts a total force of 686 N upwards on Gaëlle, then Gaëlle exerts a total force of 686 N downwards on the trunk according to Newton's 3<sup>rd</sup> law.

The sums of the components of the forces on the trunk are

$$\begin{aligned}\sum F_x &= 0 \\ &\rightarrow F_f - N_2 = 0 \\ \sum F_y &= 0 \\ &\rightarrow N_1 - 2940\text{N} - 686\text{N} = 0\end{aligned}$$

The sum of torque is (with the axis at the left end of the trunk with a positive direction in the clockwise direction)

$$\begin{aligned}\sum \tau &= 0 \\ \rightarrow 2940N \cdot 2m \cdot \sin(63^\circ) + 686N \cdot 5m \cdot \sin(63^\circ) - N_2 \cdot 5m \cdot \sin(27^\circ) &= 0\end{aligned}$$

$N_2$  can be found with the sum of torques.

$$\begin{aligned}2940N \cdot 2m \cdot \sin(63^\circ) + 686N \cdot 5m \cdot \sin(63^\circ) - N_2 \cdot 5m \cdot \sin(27^\circ) &= 0 \\ N_2 &= 3654.4N\end{aligned}$$

$F_f$  can then be found with the sum of the  $x$ -components of the forces.

$$\begin{aligned}F_f - N_2 &= 0 \\ F_f - 3654.4N &= 0 \\ F_f &= 3654.4N\end{aligned}$$

Let see now if the frictional force can be as large as that. This maximum friction force is

$$F_{f \max} = \mu_s N_1$$

The normal force can be found with the sum of the  $y$ -components of the forces.

$$\begin{aligned}N_1 - 2940N - 686N &= 0 \\ N_1 &= 3626N\end{aligned}$$

The maximum friction force is therefore

$$\begin{aligned}F_{f \max} &= \mu_s N_1 \\ &= 0.8 \cdot 3626N \\ &= 2900.8N\end{aligned}$$

As a frictional force of 3654.4 N is required and as the maximum static friction force is only 2900.8 N, the trunk will slip.

**9.** a) There are four forces exerted on the board

1) The weight of the board (49 N).

- 2) The normal force exerted by the scale on the left (380 N).
- 3) The normal force exerted by the scale on the right (280 N).
- 4) The normal force exerted by Annabelle, which is equal to its weight (called  $w$ ). This force is exerted all over Annabelle's body, but the effect is the same if we consider that it is exerted right below Annabelle's centre of mass.

The sum of the y-components of the forces is

$$\begin{aligned}\sum F_y &= 0 \\ \rightarrow -49N + 380N + 280N - w &= 0\end{aligned}$$

Therefore, the weight of the person is

$$\begin{aligned}-49N + 380N + 280N - w &= 0 \\ w &= 611N\end{aligned}$$

The mass of the person is, therefore,  $611 \text{ N} / 9.8 \text{ N/kg} = 62.3 \text{ kg}$ .

- b) The sum of torque is (with the axis where the force exerted by the scale on the left is and with a positive direction in the clockwise direction)

$$\begin{aligned}\sum \tau &= 0 \\ \rightarrow +49N \cdot 1.25m + 611N \cdot d - 280N \cdot 2.5m &= 0\end{aligned}$$

The distance between the scale on the left and centre of mass of the person is found by solving for  $d$ .

$$\begin{aligned}49N \cdot 1.25m + 611N \cdot d - 280N \cdot 2.5m &= 0 \\ d &= 1.045m\end{aligned}$$

Annabelle's centre of mass is therefore 1.045 m from her feet.

## 10. There are 3 forces exerted on the lever.

- 1) The tension force exerted by the rope that supports the 125 kg mass. This tension force is equal to the weight of the mass, 1225 N.
- 2) The tension force exerted by the rope that supports the 275 kg mass. This tension force is equal to the weight of the mass, 2695 N.
- 3) The normal force exerted by the fulcrum ( $F_N$ ).

The sum of torque is (with the axis at the fulcrum with a positive direction in the clockwise direction)

$$\begin{aligned}\sum \tau &= 0 \\ \rightarrow -1225N \cdot x + 2695N \cdot (4m - x) &= 0\end{aligned}$$

The solution of this equation is

$$\begin{aligned}-1225N \cdot x + 2695N \cdot (4m - x) &= 0 \\ -1225N \cdot x + 10\,780Nm - 2695N \cdot x &= 0 \\ -3920N \cdot x + 10\,780Nm &= 0 \\ x &= 2.75m\end{aligned}$$

**11.** There are 3 forces exerted on the beam.

- 1) The weight of the beam (490 N).
- 2) The tension force exerted by the rope that supports the 200 kg mass. This tension force is equal to the weight of the mass, 1960 N.
- 3) The force and the torque exerted by the screws.

Using an axis system located at the point where the beam is fixed to the wall and using a positive direction in the anti-clockwise direction, the table of force is

Forces	$x$	$y$	$\tau$
Weight of the beam	0	-490 N	$-490\text{ N} \cdot 1\text{ m} \cdot \sin 90^\circ$ $= -490\text{ Nm}$
Tension of the rope	0	1960 N	$1960\text{ N} \cdot 2\text{ m} \cdot \sin 90^\circ$ $= 3920\text{ Nm}$
Screws	$H$	$V$	$\tau$

The equations are therefore

$$\begin{aligned}\sum F_x &= 0 \\ \rightarrow H &= 0 \\ \sum F_y &= 0 \\ \rightarrow -490N + 1960N + V &= 0 \\ \sum \tau &= 0 \\ \rightarrow -490Nm + 3920Nm + \tau &= 0\end{aligned}$$

$H$  is found with the sum of the  $x$ -components of the forces.

$$H = 0$$

$V$  is found with the sum of the  $y$ -components of the forces.

$$\begin{aligned} -490N + 1960N + V &= 0 \\ V &= -1470N \end{aligned}$$

And the torque can be found with the sum of torques.

$$\begin{aligned} -490Nm + 3920Nm + \tau &= 0 \\ \tau &= -3430Nm \end{aligned}$$

**12.** There are 3 forces on the beam.

- 1) The weight of the beam (392 N).
- 2) The normal force exerted by the 30 kg block. This normal force is equal to the weight of the block, 294 N.
- 3) The force and the torque exerted by the screws.

Using an axis system located at the point where the beam is fixed to the wall and using a positive direction in the anti-clockwise direction, the table of force is

Forces	$x$	$y$	$\tau$
Weight of the beam	0	-392 N	$-392 \text{ N} \cdot 1 \text{ m} \cdot \sin 90^\circ$ $= -392 \text{ Nm}$
Block	0	-294 N	$-294 \text{ N} \cdot x \sin 90^\circ$ $= -294 \text{ N} \cdot x$
Screws	$H$	$V$	$\tau$

The equations are therefore

$$\begin{aligned} \sum F_x &= 0 \\ &\rightarrow H = 0 \\ \sum F_y &= 0 \\ &\rightarrow -490N - 392N + V = 0 \\ \sum \tau &= 0 \\ &\rightarrow -392Nm - 294N \cdot x + \tau = 0 \end{aligned}$$

a) Since the torque exerted by the screws must be less than 800 Nm, the third equation tells us that

$$\begin{aligned}
 -392Nm - 294N \cdot x + \tau &= 0 \\
 392Nm + 294N \cdot x &= \tau \\
 392Nm + 294N \cdot x = \tau &\leq 800Nm \\
 392Nm + 294N \cdot x &\leq 800Nm \\
 294N \cdot x &\leq 408Nm \\
 x &\leq 1.388m
 \end{aligned}$$

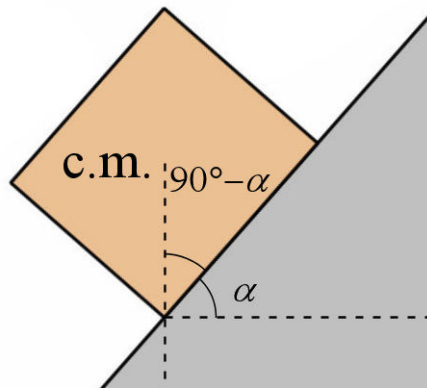
b) With the sum of the  $x$ -components of the forces, we can find that

$$H = 0$$

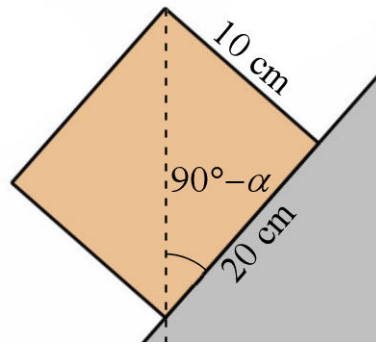
With the sum of the  $y$ -components of the forces, we can find that

$$\begin{aligned}
 -490N + -392N + V &= 0 \\
 V &= 686N
 \end{aligned}$$

- 13.** For the block to fall, the centre of mass of the block, which is at the centre of the block, must not be above the area of contact between the block and the slope. The limit, in the following situation, happens when the centre of mass is on the edge of the contact area.



We see the angle between the diagonal of the box and the bottom of the box is  $90^\circ - \alpha$ . We thus have the following situation.



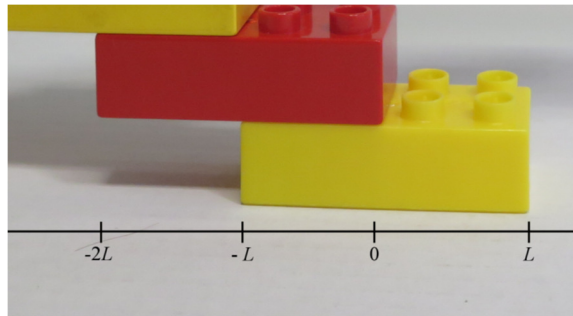
The angle is therefore

$$\tan(90^\circ - \alpha) = \frac{10\text{cm}}{20\text{cm}}$$

$$\alpha = 63.4^\circ$$

The box will tip when an angle of  $63.4^\circ$  is reached.

- 14.** The tower does not fall over if the centre of mass is above the surface of the first block in contact with the table. According to the ruler shown in the figure, the centre of mass must be between  $-L$  and  $L$ .



Let's find the position of the centre of mass of this tower. First the  $x$ -coordinate of the centre of mass of each block must be found.

1 <sup>st</sup> block	$x_{cm} = 0$ (This is the block in contact with the table)
2 <sup>nd</sup> block	$x_{cm} = -L$
3 <sup>rd</sup> block	$x_{cm} = -2L$
4 <sup>th</sup> block	$x_{cm} = -3L$
5 <sup>th</sup> block	$x_{cm} = -2L$
6 <sup>th</sup> block	$x_{cm} = -L$
7 <sup>th</sup> block	$x_{cm} = 0$
8 <sup>th</sup> block	$x_{cm} = L$

$$\begin{array}{ll} 9^{\text{th}} \text{ block} & x_{cm} = 2L \\ 10^{\text{th}} \text{ block} & x_{cm} = 3L \end{array}$$

Let's find the position of the centre of mass after the first 7 blocks were added. (The centre of mass surely does not exceed  $x = L$  at this point because the position of the centre of mass of all these blocks is zero or negative.)

The position is

$$\begin{aligned} x_{cm7} &= \frac{m \cdot (0 - L - 2L - 3L - 2L - L - 0)}{7m} \\ &= \frac{-9L}{7} \end{aligned}$$

(By the way, this shows that the tower is not in equilibrium with only 7 blocks, because the centre of mass is not between  $-L$  and  $L$ . It would fall over to the left.)

Using the same formula, the position of the centre of mass with more blocks is found.

$$x_{cm8} = \frac{m \cdot (0 - L - 2L - 3L - 2L - L - 0 + L)}{8m} = \frac{-8L}{8}$$

$$x_{cm9} = \frac{m \cdot (0 - L - 2L - 3L - 2L - L - 0 + L + 2L)}{9m} = \frac{-6L}{9}$$

$$x_{cm10} = \frac{m \cdot (0 - L - 2L - 3L - 2L - L - 0 + L + 2L + 3L)}{10m} = \frac{-3L}{10}$$

$$x_{cm11} = \frac{m \cdot (0 - L - 2L - 3L - 2L - L - 0 + L + 2L + 3L + 4L)}{11m} = \frac{L}{11}$$

$$x_{cm12} = \frac{m \cdot (0 - L - 2L - 3L - 2L - L - 0 + L + 2L + 3L + 4L + 5L)}{12m} = \frac{6L}{12}$$

$$x_{cm13} = \frac{m \cdot (0 - L - 2L - 3L - 2L - L - 0 + L + 2L + 3L + 4L + 5L + 6L)}{13m} = \frac{12L}{13}$$

$$x_{cm14} = \frac{m \cdot (0 - L - 2L - 3L - 2L - L - 0 + L + 2L + 3L + 4L + 5L + 6L + 7L)}{14m} = \frac{19L}{14}$$

We have just passed  $L$ , which means that the tower falls with 14 blocks. The maximum number of block is thus 13.

**15. a)**

The height of the centre of mass is 1 m, and the width of the box is 0.5 m. Therefore, the maximum acceleration is

$$\begin{aligned} a_{\max} &= \frac{gD}{2H} \\ &= \frac{9.8 \frac{N}{kg} \cdot 0.5m}{2 \cdot 1m} \\ &= 2.45 \frac{m}{s^2} \end{aligned}$$

b) The force of static friction is the force that gives this acceleration to the box. Thus,

$$F_f = ma_{\max}$$

As the maximum value of the force of friction is  $\mu_s F_N$ , we must have

$$\begin{aligned} \mu_s F_N &> F_f = ma_{\max} \\ \mu_s F_N &> ma_{\max} \end{aligned}$$

Since the normal force is equal to the weight in this situation, the equation becomes

$$\begin{aligned} \mu_s mg &> ma_{\max} \\ \mu_s g &> a_{\max} \\ \mu_s &> \frac{a_{\max}}{g} \\ \mu_s &> \frac{2.45 \frac{m}{s^2}}{9.8 \frac{N}{kg}} \\ \mu_s &> 0.25 \end{aligned}$$

Therefore, the coefficient must be greater than 0.25 so that the box tips over before sliding. So, the minimum coefficient is 0.25.

**16.** Here, we don't have a formula for an object accelerating on a tilted surface, but we know that the apparent weight must point towards the area bounded by the points of

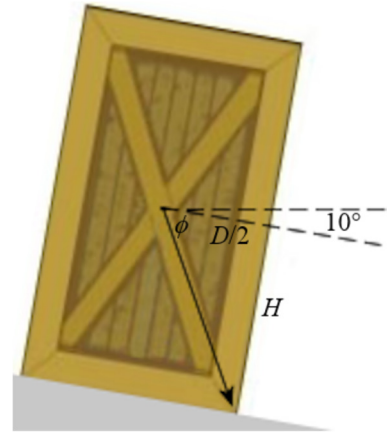
support, which corresponds here to the base of the box. At the largest acceleration, the apparent weight will point towards the edge of the base of the box.

In this case, the direction of the apparent weight is

$$\theta = \phi + 10^\circ$$

With the dimensions of the box, the angle  $\phi$  can be found so that

$$\begin{aligned}\theta &= \arctan \frac{H}{D/2} + 10^\circ \\ &= \arctan \frac{1m}{0.5m} + 10^\circ \\ &= 73.435^\circ\end{aligned}$$



This means that at the maximum acceleration, the direction of the apparent weight vector is  $-73.435^\circ$ .

The direction of the apparent weight is found with its components. Since the acceleration is in the direction  $\theta = 170^\circ$ , these components are

$$\begin{aligned}w_{app\ x} &= -ma_x = -ma \cos 170^\circ \\ w_{app\ y} &= -mg - ma_y = -mg - ma \sin 170^\circ\end{aligned}$$

Therefore,

$$\begin{aligned}\tan \theta &= \frac{w_{app\ y}}{w_{app\ x}} \\ \tan(-73.435^\circ) &= \frac{-mg - ma \sin 170^\circ}{ma \cos 170^\circ} \\ \tan(-73.435^\circ) &= \frac{g + a \sin 170^\circ}{a \cos 170^\circ}\end{aligned}$$

It only remains to solve for  $a$ .

$$a \cdot \cos 170^\circ \cdot \tan(-73.435^\circ) = g + a \cdot \sin 170^\circ$$

$$a \cdot \cos 170^\circ \cdot \tan(-73.435^\circ) - a \cdot \sin 170^\circ = g$$

$$a \cdot (\cos 170^\circ \cdot \tan(-73.435^\circ) - \sin 170^\circ) = g$$

$$a = \frac{g}{\cos 170^\circ \cdot \tan(-73.435^\circ) - \sin 170^\circ}$$

$$a = 3.124 \frac{m}{s^2}$$

**17.** a) As demonstrated in the notes, the tilt angle of the bike is given by

$$\tan \theta = \frac{v^2}{rg}$$

Therefore,

$$\tan 40^\circ = \frac{v^2}{100m \cdot 9.8 \frac{N}{kg}}$$

$$v = 28.68 \frac{m}{s} = 103.2 \frac{km}{h}$$

b) The force of static friction is the force that gives this acceleration to the bike. Thus,

$$F_f = ma$$

As the maximum value of the force of friction is  $\mu_s F_N$ , we must have

$$\mu_s F_N > F_f$$

$$\mu_s F_N > ma$$

Since the normal force is equal to the weight in this situation, the equation becomes

$$\mu_s mg > ma$$

$$\mu_s g > a$$

As the acceleration is  $v^2/r$ , the result is

$$\mu_s > \frac{v^2}{rg}$$

But since

$$\tan \theta = \frac{v^2}{rg}$$

The equation becomes

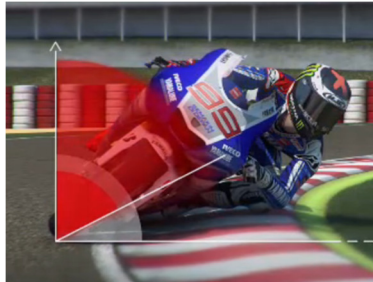
$$\mu_s > \tan \theta$$

Here, the result is

$$\begin{aligned}\mu_s &> \tan 40^\circ \\ \mu_s &> 0.839\end{aligned}$$

Therefore, the minimum coefficient is 0.839.

(This equation is always true for a motorcycle on a horizontal surface, the coefficient of friction must always be greater than the tangent of the tilt angle of the bike. So it can be quickly estimated that in the following situation

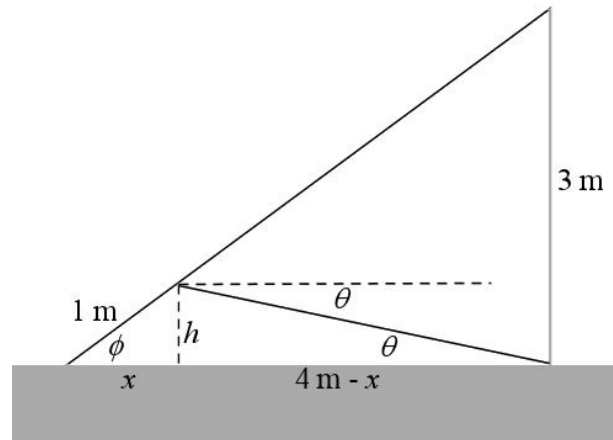


where the angle is  $64^\circ$ , that the coefficient of friction must be at least 2.05 to prevent any slippage.)

**18.** There are four forces exerted on the beam.

- 1) The weight of the beam (490 N), directed downwards.
- 2) The tension of the string towards the left ( $T$ ).
- 3) The normal force exerted by the ground, directed upwards ( $N_1$ ).
- 4) The normal force exerted by the wall, towards the left ( $N_2$ ).

Before performing the sum of the forces, we need to find the direction of the tension force.



The angle  $\phi$  is

$$\tan \phi = \frac{3m}{4m}$$

$$\phi = 36.87^\circ$$

Thus, the values of  $h$  and  $x$  are

$$x = 1m \cdot \cos(36.87^\circ) \qquad h = 1m \cdot \sin(36.87^\circ)$$

$$= 0.8m \qquad \qquad \qquad = 0.6m$$

Therefore, the value of  $\theta$  is

$$\tan \theta = \frac{0.6m}{3.2m}$$

$$\theta = 10.62^\circ$$

Thus, the sum of the forces exerted on the beam is

$$\sum F_y = 0$$

$$\rightarrow T \cos(-10.62^\circ) - N_2 = 0$$

$$\sum F_x = 0$$

$$\rightarrow -490N + T \sin(-10.62^\circ) + N_1 = 0$$

and the sum of the torques (with the axis at the end of the beam in contact with the ground with a clockwise positive direction) is

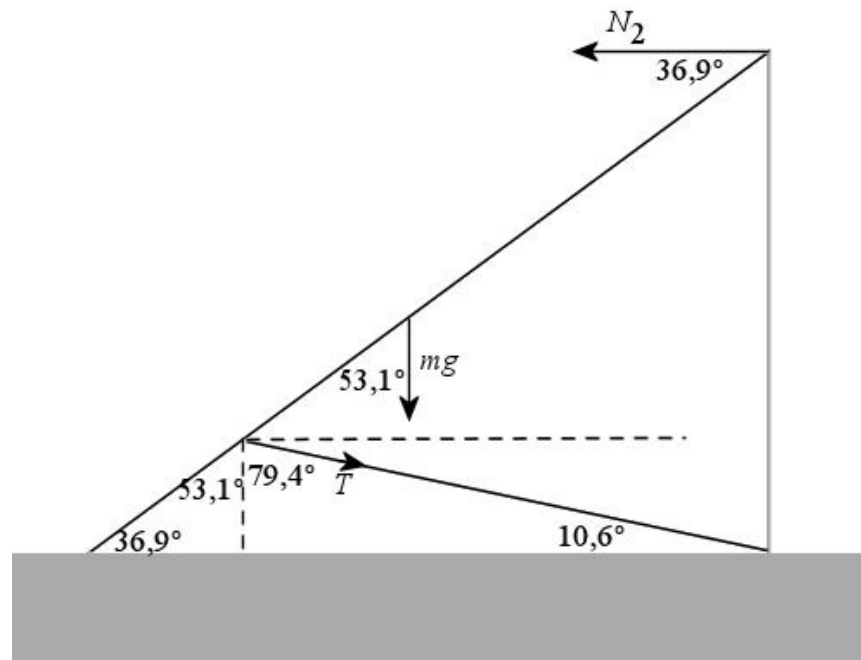
$$\sum \tau = 0$$

$$\rightarrow 490N \cdot 2.5m \cdot \sin(53.13^\circ) + T \cdot 1m \cdot \sin(132.51^\circ) - N_2 \cdot 5m \sin(36.87^\circ) = 0$$

This last equation gives

$$980Nm + T \cdot 0.73716m - N_2 \cdot 3m = 0$$

The following figure explains the value of these angles.



From the sum of the  $x$ -component of the forces, the normal force  $N_2$  is obtained

$$N_2 = T \cos(-10.62^\circ)$$

If this value is substituted in the sum of torques, we arrive at

$$980Nm + T \cdot 0.73716m - T \cos(-10.62^\circ) \cdot 3m = 0$$

It only remains to solve for  $T$

$$\begin{aligned}
 980Nm + T \cdot 0.73716m - T \cos(-10.62^\circ) \cdot 3m &= 0 \\
 980Nm &= -T \cdot 0.73716m + T \cos(-10.62^\circ) \cdot 3m \\
 980Nm &= T \cdot (-0.73716m + \cos(-10.62^\circ) \cdot 3m) \\
 980Nm &= T \cdot 2.211m \\
 T &= 443.1N
 \end{aligned}$$

**19.** There are three forces exerted on the beam.

- 1) The weight (980 N).
- 2) The tension of the rope ( $T$ ).
- 3) The force made by the load ( $F$ ).
- 4) The force made by the pivot ( $H$  and  $V$ ).

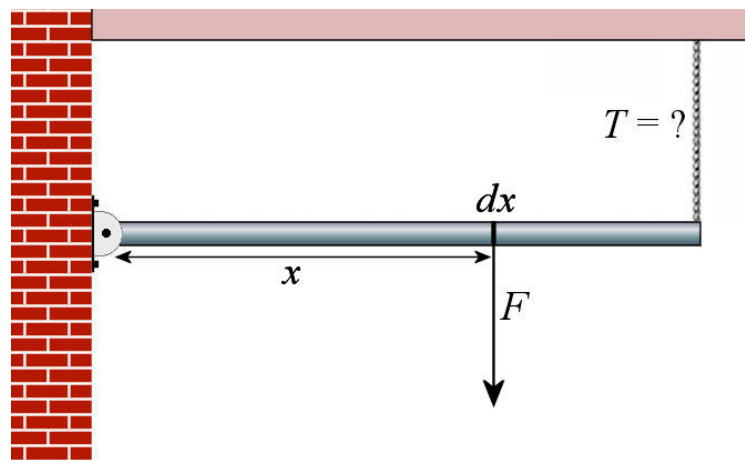
Therefore, the sum of the forces exerted on the beam

$$\begin{aligned}
 \sum F_y &= 0 \\
 \rightarrow -980N - F + F_T + V &= 0
 \end{aligned}$$

and the sum of the torques (with the axis at the pivot with a clockwise positive direction) is

$$\begin{aligned}
 \sum \tau &= 0 \\
 \rightarrow \tau_{load} + 980N \cdot 2.5m - T \cdot 5m &= 0
 \end{aligned}$$

The tension of the rope can be found with the torque equation, but the force exerted by the load must be found first. To obtain it, let's look at a small piece of the beam of length  $dx$ .



The force on the piece is

$$dF = f dx$$

( $f$  is the force per unit length) and the torque made by the force exerted on the piece is

$$d\tau = x dF$$

$$d\tau = f x dx$$

It is known that  $f$  changes from 0 N/m at  $x = 0$  m to 900 N/m at  $x = 5$  m. The formula for  $f$  is thus

$$f = 180 \frac{N}{m^2} \cdot x$$

Therefore, the torque made by the force acting on the small piece of length  $dx$  is

$$d\tau = \left(180 \frac{N}{m^2} \cdot x\right) x dx$$

$$d\tau = 180 \frac{N}{m^2} \cdot x^2 dx$$

All these torques are then summed with an integral.

$$\begin{aligned} \tau_{charge} &= \int_{0m}^{5m} 180 \frac{N}{m^2} \cdot x^2 dx \\ &= \left[ 60 \frac{N}{m^2} \cdot x^3 \right]_{0m}^{5m} \\ &= 60 \frac{N}{m^2} \cdot (5m)^3 \\ &= 7500 Nm \end{aligned}$$

The sum of torques equation then becomes

$$\begin{aligned} \sum \tau &= 0 \\ &\rightarrow 7500 Nm + 980 N \cdot 2.5m - T \cdot 5m = 0 \end{aligned}$$

Solving for the tension, the result is  $T = 1990$  N.