Chapter 12 Solutions

1. As the Earth makes one rotation in 24 hours, the angular velocity is

$$\begin{aligned}\omega &= \frac{\Delta\theta}{\Delta t}\\ &= \frac{2\pi}{24\cdot 60\cdot 60s}\\ &= 7,272\times 10^{-5}\,\tfrac{rad}{s}\end{aligned}$$

2. a) The angular velocity is

$$\begin{aligned}\omega &= \omega_0 + \alpha t\\ &= 0\,\tfrac{rad}{s} + 5\,\tfrac{rad}{s^2}\cdot 10s\\ &= 50\,\tfrac{rad}{s}\end{aligned}$$

b) The angular displacement is

$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2\\ &= 0rad + 0\,\tfrac{rad}{s}\cdot 10s + \frac{1}{2}\cdot 5\,\tfrac{rad}{s^2}\cdot(10s)^2\\ &= 250rad\end{aligned}$$

Therefore, the number of revolutions is

$$\frac{250rad}{2\pi\,\frac{rad}{revolution}} = 39.79\ revolutions$$

c) The time required to make 50 revolutions (100π rad) is

$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2\\ 100\pi rad &= 0rad + 0\,\tfrac{rad}{s}\cdot t + \frac{1}{2}\cdot 5\,\tfrac{rad}{s^2}\cdot t^2\\ 100\pi rad &= \frac{1}{2}\cdot 5\,\tfrac{rad}{s^2}\cdot t^2\\ t &= 11.21s\end{aligned}$$

3. First, let's find the angular velocity in rad/s.

$$\omega_0 = 120 \tfrac{\text{revolutions}}{\text{minute}} = 120 \tfrac{\text{revolutions}}{\text{minute}} \cdot \frac{2\pi rad}{1 \text{ revolution}} \cdot \frac{1 \text{ minute}}{60s} = 4\pi \tfrac{rad}{s}$$

$$\omega = 80 \tfrac{\text{revolutions}}{\text{minute}} = 80 \tfrac{\text{revolutions}}{\text{minute}} \cdot \frac{2\pi rad}{1 \text{ revolution}} \cdot \frac{1 \text{ minute}}{60s} = \tfrac{8\pi}{3} \tfrac{rad}{s}$$

Therefore

$$\begin{aligned}\theta &= \theta_0 + \frac{1}{2}(\omega + \omega_0)t \\ &= 0rad + \frac{1}{2} \cdot \left(4\pi \tfrac{rad}{s} + \tfrac{8\pi}{3} \tfrac{rad}{s}\right) \cdot 20s \\ &= \tfrac{200\pi}{3} rad\end{aligned}$$

Therefore, the number of revolutions is

$$\begin{aligned}\frac{\tfrac{200\pi}{3} rad}{2\pi \tfrac{rad}{revolution}} &= \tfrac{100}{3} revolutions \\ &= 33.33 revolutions\end{aligned}$$

4. a) The angular position at t = 5 s is

$$\begin{aligned}\theta &= 10 \tfrac{rad}{s} t - 0.5 \tfrac{rad}{s^2} t^2 \\ &= 10 \tfrac{rad}{s} \cdot 5s - 0.5 \tfrac{rad}{s^2} \cdot (5s)^2 \\ &= 37.5 rad\end{aligned}$$

Therefore, the number of revolutions is

$$\frac{37.5 rad}{2\pi \tfrac{rad}{revolution}} = 5.968 revolutions$$

b) The average angular velocity is

$$\begin{aligned}\bar{\omega} &= \frac{\Delta\theta}{\Delta t} \\ &= \frac{37.5rad}{5s} \\ &= 7.5\tfrac{rad}{s}\end{aligned}$$

c) First, the formula for the angular velocity is found by deriving the formula for the angular position

$$\begin{aligned}\omega &= \frac{d\theta}{dt} \\ &= \frac{d\left(10\tfrac{rad}{s}t - 0.5\tfrac{rad}{s^2}\cdot t^2\right)}{dt} \\ &= 10\tfrac{rad}{s} - 1\tfrac{rad}{s^2}\cdot t\end{aligned}$$

Therefore, the angular velocity at t = 4 s is

$$\begin{aligned}\omega &= 10\tfrac{rad}{s} - 1\tfrac{rad}{s^2}\cdot t \\ &= 10\tfrac{rad}{s} - 1\tfrac{rad}{s^2}\cdot 4s \\ &= 6\tfrac{rad}{s}\end{aligned}$$

d) First, the formula for the angular acceleration is found by deriving the formula for the angular velocity

$$\begin{aligned}\alpha &= \frac{d\omega}{dt} \\ &= \frac{d\left(10\tfrac{rad}{s} - 1\tfrac{rad}{s^2}\cdot t\right)}{dt} \\ &= -1\tfrac{rad}{s^2}\end{aligned}$$

The angular acceleration at t = 4 s is therefore - 1 rad/s².

e) We have

$$\begin{aligned}\omega &= 10\tfrac{rad}{s} - 1\tfrac{rad}{s^2}t \\ 0\tfrac{rad}{s} &= 10\tfrac{rad}{s} - 1\tfrac{rad}{s^2}\cdot t \\ t &= 10s\end{aligned}$$

5. a) The speed of the ends is

$$\begin{aligned} v &= \omega r \\ &= 2\pi \tfrac{rad}{s} \cdot 2m \\ &= 12.566 \tfrac{m}{s} \end{aligned}$$

b) The angular acceleration of the rod is found with

$$\begin{aligned} \omega &= \omega_0 + \alpha t \\ \tfrac{2\pi}{5} \tfrac{rad}{s} &= 2\pi \tfrac{rad}{s} + \alpha \cdot 0.2s \\ \alpha &= -25.13 \tfrac{rad}{s^2} \quad \left(8\pi \tfrac{rad}{s^2}\right) \end{aligned}$$

c) The centripetal acceleration of the ends of the rod is

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{\left(4\pi \tfrac{m}{s}\right)^2}{2m} = 78.96 \tfrac{m}{s^2} \end{aligned} \qquad or \qquad \begin{aligned} a_c &= \omega^2 r \\ &= \left(2\pi \tfrac{rad}{s}\right)^2 \cdot 2m \\ &= 78.96 \tfrac{m}{s^2} \end{aligned}$$

d) The tangential acceleration of the end of the rod is

$$\begin{aligned} a_t &= \alpha r \\ &= -25.13 \tfrac{m}{s^2} \cdot 2m \\ &= -50.27 \tfrac{m}{s^2} \end{aligned}$$

e) The acceleration of the end of the rod is

$$\begin{aligned} a &= \sqrt{a_c^2 + a_t^2} \\ &= \sqrt{\left(78.96 \tfrac{m}{s^2}\right)^2 + \left(50.27 \tfrac{m}{s^2}\right)^2} \\ &= 93.6 \tfrac{m}{s^2} \end{aligned}$$

6. As the rope tied to the 20 kg block is wound around the pulley at a distance of 25 cm from the axis, the displacement of the pulley at 25 cm from the axis of rotation must be the same as that of the block. This displacement is

$$\begin{aligned} \Delta s &= r\Delta\theta \\ &= 0.25m \cdot \left(200° \cdot \tfrac{\pi rad}{180°}\right) \\ &= 0.8727m \end{aligned}$$

According to the direction of rotation of the pulley, the block moves upwards.

As the rope tied to the 30 kg block is wound around the pulley at a distance of 50 cm from the axis, the displacement of the pulley at 50 cm from the axis of rotation must be the same as that of the block. This displacement is

$$\begin{aligned}\Delta s &= r\Delta\theta \\ &= 0.50m \cdot \left(200° \cdot \tfrac{\pi rad}{180°}\right) \\ &= 1.745m\end{aligned}$$

According to the direction of rotation of the pulley, the block moves downwards.

7. As the rope tied to the 20 kg block is wound around the pulley at a distance of 25 cm from the axis, the speed of the pulley at 25 cm from the axis of rotation must be the same as that of the block. This speed is

$$\begin{aligned}v &= \omega r \\ &= 8\tfrac{rad}{s} \cdot 0.25m \\ &= 2\tfrac{m}{s}\end{aligned}$$

According to the direction of rotation of the pulley, the block moves upwards.

As the rope tied to the 30 kg block is wound around the pulley at a distance of 50 cm from the axis, the speed of the pulley at 50 cm from the axis of rotation must be the same as that of the block. This speed is

$$\begin{aligned}v &= \omega r \\ &= 8\tfrac{rad}{s} \cdot 0.50m \\ &= 4\tfrac{m}{s}\end{aligned}$$

According to the direction of rotation of the pulley, the block moves downwards.

8. a)

The speed of the centre of mass of the wheels is obviously the same as that of the car. Therefore

$$\begin{aligned}\omega &= \frac{v_{cm}}{R} \\ &= \frac{40\frac{m}{s}}{0.40m} \\ &= 100\tfrac{rad}{s}\end{aligned}$$

b) The acceleration of the centre of mass of the wheels is obviously the same as that of the car. The car's acceleration is

$$\begin{aligned}2a\left(x - x_0\right) &= v^2 - v_0^2 \\ 2\cdot a\cdot\left(160m - 0m\right) &= 0 - \left(40\tfrac{m}{s}\right)^2 \\ a &= -5\tfrac{m}{s^2}\end{aligned}$$

Then

$$\begin{aligned}\alpha &= \frac{a_{cm}}{R} \\ &= \frac{-5\frac{m}{s^2}}{0.40m} \\ &= -12.5\tfrac{rad}{s^2}\end{aligned}$$

9. a) The angular acceleration of the pulley is

$$\begin{aligned}2\alpha\left(\theta - \theta_0\right) &= \omega^2 - \omega_0^2 \\ 2\cdot\alpha\cdot\left(120\pi rad - 0rad\right) &= \left(10\pi\tfrac{rad}{s}\right)^2 - \left(3\pi\tfrac{rad}{s}\right)^2 \\ \alpha &= 1.191\tfrac{rad}{s^2}\end{aligned}$$

b) The number of revolutions is found with

$$\begin{aligned}2\alpha\left(\theta - \theta_0\right) &= \omega^2 - \omega_0^2 \\ 2\cdot 1{,}191\tfrac{rad}{s^2}\cdot\left(\theta - 0rad\right) &= \left(3\pi\tfrac{rad}{s}\right)^2 - 0 \\ \theta = 37.28rad &= 5.934 revolutions\end{aligned}$$

10. a)

As the acceleration changes, the problem must be separated into parts where the acceleration is constant.

First phase: α = 8 rad/s² (duration 4 s)

At the end of this phase, we have

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ &= 0\tfrac{rad}{s} + 8\tfrac{rad}{s^2}\cdot 4s \\ &= 32\tfrac{rad}{s}\end{aligned}$$

$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ &= 0rad + 0\tfrac{rad}{s}\cdot 4s + \tfrac{1}{2}8\tfrac{rad}{s^2}\cdot(4s)^2 \\ &= 64rad\end{aligned}$$

Second phase: α =0 rad/s² (duration 1 s)

The angular position at the end of this phase is

$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ &= 64rad + 32\tfrac{rad}{s}\cdot 1s + \frac{1}{2}\cdot 0\tfrac{rad}{s^2}\cdot(1s)^2 \\ &= 96rad\end{aligned}$$

Therefore, the number of revolutions is

$$\frac{96rad}{2\pi\frac{rad}{revolution}} = 15.28 revolutions$$

b) Since the rod turned only 64 rad = 10.2 revolutions during the first part, it is clear that the 100th revolution will be made during the second part. The exact moment is

$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \\ 200\pi rad &= 64rad + 32\tfrac{rad}{s}\cdot t + \frac{1}{2}\cdot 0\tfrac{rad}{s^2}\cdot t^2 \\ 564.32rad &= 32\tfrac{rad}{s}\cdot t \\ t &= 17.635s\end{aligned}$$

With the 4 seconds of the first phase, the total time is therefore 21.635 s.

11. a) We have

$$\omega_1 R_1 = \omega_2 R_2$$
$$40\pi \tfrac{rad}{s} \cdot 0.25m = \omega_2 \cdot 0.15m$$
$$\omega_2 = 209.44 \tfrac{rad}{s} = 2000RPM$$

b) The speed of the chain is the same as the speed of the edges of the pulley. You can take any of the two pulleys to make the calculate.

$$\begin{aligned} v &= \omega_1 R_1 \\ &= 125.66 \tfrac{rad}{s} \cdot 0.25m \\ &= 31.416 \tfrac{m}{s} \end{aligned} \qquad or \qquad \begin{aligned} v &= \omega_2 R_2 \\ &= 209.44 \tfrac{rad}{s} \cdot 0.15m \\ &= 31.416 \tfrac{m}{s} \end{aligned}$$

12. a) The moment of inertia is

$$\begin{aligned} I &= \sum mr^2 \\ &= 120kg \cdot (1.5m)^2 + 120kg \cdot (1.5m)^2 + 120kg \cdot (1.5m)^2 + 120kg \cdot (1.5m)^2 \\ &= 1080\, kgm^2 \end{aligned}$$

b) The kinetic energy is

$$\begin{aligned} E_k &= \frac{1}{2} I \omega^2 \\ &= \frac{1}{2} \cdot 1080\, kgm^2 \cdot \left(\tfrac{2}{5}\pi \tfrac{rad}{s}\right)^2 \\ &= 852.73J \end{aligned}$$

(We could also find the speed of the cars with $v = \omega r$ to get 1.885 m/s, and then the sum of energy $\frac{1}{2}mv^2$ of cars to obtain 852.73J.)

13. a) The moment of inertia is

$$\begin{aligned} I &= \sum mr^2 \\ &= 0.2kg \cdot (0m)^2 + 0.2kg \cdot (0m)^2 + 0.3kg \cdot (0.15m)^2 + 0.3kg \cdot (0.15m)^2 \\ &= 0.0135\, kgm^2 \end{aligned}$$

b) The kinetic energy is

$$\begin{aligned}E_k &= \frac{1}{2} I\omega^2 \\ &= \frac{1}{2}\cdot 0.0135kgm^2 \cdot \left(2\tfrac{rad}{s}\right)^2 \\ &= 0.027J\end{aligned}$$

14. a) The moment of inertia is

$$\begin{aligned}I &= \sum mr^2 \\ &= 1kg\cdot(0.2m)^2 + 2kg\cdot(0.2m)^2 + 3kg\cdot(0.2m)^2 + 4kg\cdot(0.2m)^2 \\ &= 0.4\ kgm^2\end{aligned}$$

b) The moment of inertia is

$$\begin{aligned}I &= \sum mr^2 \\ &= 1kg\cdot(0.2828m)^2 + 2kg\cdot(0m)^2 + 3kg\cdot(0.2828m)^2 + 4kg\cdot(0m)^2 \\ &= 0.32kgm^2\end{aligned}$$

15. First, the position of the centre of mass must be found. Let's take an axis that goes from one mass to another with $x = 0$ at the location of 100 g the mass. There we have

$$\begin{aligned}x_{cm} &= \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} \\ &= \frac{0m\cdot 0.1kg + 0.6m\cdot 0.2kg}{0.1kg + 0.2kg} \\ &= 0.4m\end{aligned}$$

The 100 g mass is therefore 40 cm from the axis of rotation, and the 200 g mass is 20 cm from the axis of rotation. The moment of inertia is then

$$\begin{aligned}I &= \sum mr^2 \\ &= 0.1kg\cdot(0.4m)^2 + 0.2kg\cdot(0.2m)^2 \\ &= 0.024kgm^2\end{aligned}$$

The total kinetic energy is then

$$\begin{aligned}E_k &= \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}\cdot 0.3kg \cdot \left(10\tfrac{m}{s}\right)^2 + \frac{1}{2}\cdot 0.024\,kgm^2 \cdot \left(20\tfrac{rad}{s}\right)^2 \\ &= 15J + 4.8J \\ &= 19.8J\end{aligned}$$

16. a) The rotational kinetic energy can be found with

$$E_{k\,rot} = \frac{1}{2}I\omega^2$$

First, the moment of inertia can be found. For a rod, it is

$$\begin{aligned}I &= \frac{1}{12}mL^2 \\ &= \frac{1}{12}\cdot 3kg \cdot (1m)^2 \\ &= 0.25kgm^2\end{aligned}$$

The angular speed must now be found. It is found with the speed of the two ends of the rod. This rod moves and turns at the same time. As the top of the rod goes faster, it turns in a clockwise direction. The speed due to translation is v_{cm}. The speed due to rotation is v_{rot}.

For the top of the rod, the speed due to translation is towards the right and the speed due the rotation is also towards the right. Thus, these two speeds add up.

$$3\tfrac{m}{s} = v_{cm} + v_{rot}$$

For the bottom of the rod, the speed due to translation is towards the right and the speed due the rotation is also towards the left. Thus, these two speeds are subtracted

$$1\tfrac{m}{s} = v_{cm} - v_{rot}$$

By subtracting these two equations, the speed due to rotation can be found.

$$\begin{aligned}3\tfrac{m}{s}-1\tfrac{m}{s}&=\left(v_{cm}+v_{rot}\right)-\left(v_{cm}-v_{rot}\right)\\2\tfrac{m}{s}&=2v_{rot}\\v_{rot}&=1\tfrac{m}{s}\end{aligned}$$

The angular velocity can now be found.

$$\begin{aligned}v_{rot}&=\omega r\\1\tfrac{m}{s}&=\omega\cdot 0.5m\\\omega&=2\tfrac{rad}{s}\end{aligned}$$

Therefore, the rotational kinetic energy is

$$\begin{aligned}E_{k\ rot}&=\frac{1}{2}I\omega^2\\&=\frac{1}{2}\cdot 0.25kgm^2\cdot\left(2\tfrac{rad}{s}\right)^2\\&=0.5J\end{aligned}$$

b) The kinetic energy due to the linear motion is

$$E_{k\ trans}=\frac{1}{2}mv_{cm}^2$$

To find it, the speed of the centre of mass is needed. It can be found with

$$\begin{aligned}3\tfrac{m}{s}&=v_{cm}+v_{rot}\\3\tfrac{m}{s}&=v_{cm}+1\tfrac{m}{s}\\v_{cm}&=2\tfrac{m}{s}\end{aligned}$$

Therefore, the kinetic energy due to the linear motion is

$$\begin{aligned}E_{k\ trans}&=\frac{1}{2}mv_{cm}^2\\&=\frac{1}{2}\cdot 3kg\cdot\left(2\tfrac{m}{s}\right)^2\\&=6J\end{aligned}$$

c) The total kinetic energy is

$$\begin{aligned} E_k &= E_{k\,rot} + E_{k\,trans} \\ &= 0.5J + 6J \\ &= 6.5J \end{aligned}$$

17. As the axis of rotation does not pass through the centre of mass of the sphere, the moment of inertia of this sphere is

$$\begin{aligned} I &= I_{cm\,sphere} + md^2 \\ &= \frac{2}{5}mR^2 + md^2 \\ &= \frac{2}{5}\cdot 2kg\cdot(0.06m)^2 + 2kg\cdot(0.03m)^2 \\ &= 0.00468kgm^2 \end{aligned}$$

18. This object consists of a rod and two spheres.

As the axis of rotation passes through the centre of mass of the rod, the moment of inertia of the rod is

$$\begin{aligned} I_1 &= I_{cm\,rod} \\ &= \frac{1}{12}mL^2 \\ &= \frac{1}{12}\cdot 15kg\cdot(0.8m)^2 \\ &= 0.8\ kgm^2 \end{aligned}$$

As the axis of rotation does not pass through the centre of mass of the sphere to the right, the moment of inertia of this sphere is

$$\begin{aligned} I_2 &= I_{cm\,sphere} + md^2 \\ &= \frac{2}{5}mR^2 + md^2 \\ &= \frac{2}{5}\cdot 10kg\cdot(0.12m)^2 + 10kg\cdot(0.52m)^2 \\ &= 2.7616\ kgm^2 \end{aligned}$$

As the axis of rotation does not pass through the centre of mass of the sphere to the left, the moment of inertia of this sphere is

$$\begin{aligned} I_3 &= I_{cm\ sphere} + md^2 \\ &= \frac{2}{5} mR^2 + md^2 \\ &= \frac{2}{5} \cdot 10kg \cdot (0.12m)^2 + 10kg \cdot (0.52m)^2 \\ &= 2.7616\ kgm^2 \end{aligned}$$

The total moment of inertia is therefore

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= 0.8kgm^2 + 2.7616\ kgm^2 + 2.7616\ kgm^2 \\ &= 6.3232\ kgm^2 \end{aligned}$$

19. This object consists of a rod and two disks.

As the axis of rotation passes through the centre of mass of the rod (which is considered to be a cylinder because the axis is in the direction of the rod), the moment of inertia of the rod is

$$\begin{aligned} I_1 &= I_{cm\ cylindre} \\ &= \frac{1}{2} mR^2 \end{aligned}$$

The mass of this piece must be found. The mass is

$$\begin{aligned} m &= \rho \cdot volume \\ &= \rho \cdot \pi r^2 l \\ &= 5000 \tfrac{kg}{m^3} \cdot \pi \cdot (0.05m)^2 \cdot 0.1m \\ &= 3.927kg \end{aligned}$$

The moment of inertia is therefore

$$\begin{aligned} I_1 &= \frac{1}{2} mR^2 \\ &= \frac{1}{2} \cdot 3.927kg \cdot (0.05m)^2 \\ &= 0.0049\ kgm^2 \end{aligned}$$

As the axis of rotation passes through the centre of mass of the disk to the right, the moment of inertia of the disk is

$$\begin{aligned} I_2 &= I_{cm\ cylindre} \\ &= \frac{1}{2} mR^2 \end{aligned}$$

The mass of this piece must be found. The mass is

$$\begin{aligned} m &= \rho \cdot volume \\ &= \rho \cdot \pi r^2 l \\ &= 5000 \tfrac{kg}{m^3} \cdot \pi \cdot (0.2m)^2 \cdot 0.04m \\ &= 25.13kg \end{aligned}$$

The moment of inertia is therefore

$$\begin{aligned} I_2 &= \frac{1}{2} mR^2 \\ &= \frac{1}{2} \cdot 25.13kg \cdot (0.2m)^2 \\ &= 0.5027\ kgm^2 \end{aligned}$$

The calculation is the same for the disk to the left.

$$I_3 = 0.5027\ kgm^2$$

The total moment of inertia is therefore

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= 0.0049kgm^2 + 0.5027kgm^2 + 0.5027kgm^2 \\ &= 1.0102\ kgm^2 \end{aligned}$$

20. This object consists of an empty cylinder (the side) and two disks (the top and the bottom).

As the axis of rotation passes through the centre of mass of the empty cylinder, the moment of inertia of the side of the tin can is

$$\begin{aligned} I_1 &= I_{cm\ \text{hollow cylindre}} \\ &= mR^2 \end{aligned}$$

The mass of this piece must be found. The mass is

$$\begin{aligned} m &= \sigma \cdot area \\ &= \sigma \cdot 2\pi rh \\ &= 10\tfrac{kg}{m^2} \cdot 2\pi \cdot 0.04m \cdot 0.09m \\ &= 0.2262kg \end{aligned}$$

The moment of inertia is therefore

$$\begin{aligned} I_1 &= mR^2 \\ &= 0.2262kg \cdot (0.04m)^2 \\ &= 3.619\times 10^{-4}\ kgm^2 \end{aligned}$$

As the axis of rotation passes through the centre of mass of the top, the moment of inertia of the disk is

$$\begin{aligned} I_2 &= I_{cm\ disk} \\ &= \frac{1}{2}mR^2 \end{aligned}$$

The mass of this piece must be found. The mass is

$$\begin{aligned} m &= \sigma \cdot area \\ &= \sigma \cdot \pi r^2 \\ &= 10\tfrac{kg}{m^2} \cdot \pi \cdot (0.04m)^2 \\ &= 0.05026kg \end{aligned}$$

The moment of inertia is therefore

$$\begin{aligned} I_2 &= \frac{1}{2}mR^2 \\ &= \frac{1}{2} \cdot 0.05026kg \cdot (0.04m)^2 \\ &= 4.021\times 10^{-5}\ kgm^2 \end{aligned}$$

The calculation is identical for the bottom disk.

$$I_3 = 4.021\times 10^{-5}\ kgm^2$$

The total moment of inertia is therefore

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= 3.619\times10^{-4}\ kgm^2 + 4.02\times10^{-5}\ kgm^2 + 4.02\times10^{-5}\ kgm^2 \\ &= 4.423\times10^{-4}\ kgm^2 \end{aligned}$$

21. As there is a single object that rotates around an axis without moving, the mechanical energy is

$$E_{mec} = \frac{1}{2} I\omega^2 + mgy$$

Instant 1: beam in the position shown in the figure.

Then, the rod does not rotate so that $\omega = 0$. It remains to find the height of the centre of mass, which is at the centre of the rod. This height is

$$y = 3m \cdot \sin 60° = 2.598m$$

The mechanical energy is therefore

$$\begin{aligned} E &= \frac{1}{2} I\omega^2 + mgy \\ &= 0 + 100kg \cdot 9.8\tfrac{N}{kg} \cdot 2.598m \\ &= 2546.1J \end{aligned}$$

Instant 2: just before the beam hits the ground.

Then, the height of the centre of mass is zero and the mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2} I\omega'^2 + mgy \\ &= \frac{1}{2} I\omega'^2 + 0 \end{aligned}$$

Mechanical Energy Conservation

According to the law of conservation, we have

$$E = E'$$

$$2546.1J = \frac{1}{2} I\omega'^2$$

To resolve this problem, the moment of inertia of the beam when the rotation axis is not at the centre of mass must be found. This moment of inertia is

$$\begin{aligned} I &= I_{cm} + md^2 \\ &= \frac{1}{12} mL^2 + md^2 \\ &= \frac{1}{12} \cdot 100kg \cdot (6m)^2 + 100kg \cdot (3m)^2 \\ &= 1200\ kgm^2 \end{aligned}$$

The mechanical energy conservation equation then becomes

$$2546.1J = \frac{1}{2} I \omega'^2$$

$$2546.1J = \frac{1}{2} \cdot 1200\ kgm^2 \cdot \omega'^2$$

$$\omega' = 2.06 \tfrac{rad}{s}$$

Therefore, the speed of the tip of the beam

$$\begin{aligned} v &= \omega r \\ &= 2.06 \tfrac{rad}{s} \cdot 6m \\ &= 12.36 \tfrac{m}{s} \end{aligned}$$

22. As there is a single object that revolves around its centre of mass while moving, the mechanical energy is

$$E_{mec} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 + mgy$$

As the ball is a sphere that rolls without slipping, the mechanical energy is

$$E_{mec} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 + mgy$$

$$E_{mec} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} \left(\frac{2}{5} m r^2 \right) \left(\frac{v_{cm}}{r} \right)^2 + mgy$$

$$E_{mec} = \frac{1}{2} m v_{cm}^2 + \frac{1}{5} m v_{cm}^2 + mgy$$

$$E_{mec} = \frac{7}{10} m v_{cm}^2 + mgy$$

Instant 1: ball at the position shown in the figure

Then, the ball does not move and so $v_{cm} = 0$. It remains to find the height of the centre of mass. The similarity with a pendulum motion allows us to find the height with the formula

$$y = R(1 - \cos\theta) = 1m(1 - \cos 45^\circ) = 0.2929m$$

(The origin $y = 0$ is thus at the height of the centre of the ball when it is at the bottom of the bowl.)

The mechanical energy is therefore

$$\begin{aligned} E &= \frac{7}{10} m v_{cm}^2 + mgy \\ &= 0 + 0.8kg \cdot 9.8 \tfrac{N}{kg} \cdot 0.2929m \\ &= 2.296J \end{aligned}$$

Instant 2: ball at the lowest point

Then, the height of the centre of mass is zero and the mechanical energy is

$$\begin{aligned} E' &= \frac{7}{10} m v_{cm}'^2 + mgy' \\ &= \frac{7}{10} \cdot 0.8kg \cdot v_{cm}'^2 + 0 \\ &= 0.56kg \cdot v_{cm}'^2 \end{aligned}$$

Mechanical Energy Conservation

According to the law of conservation, we have

$$E = E'$$
$$2.296J = 0.56kg \cdot v_{cm}'^2$$
$$v_{cm}' = 2.025 \tfrac{m}{s}$$

23. As there is a single object that revolves around its centre of mass while moving, the mechanical energy is

$$E_{mec} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 + mgy$$

As the log is a cylinder that rolls without slipping, we have

$$E_{mec} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 + mgy$$
$$E_{mec} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v_{cm}}{r}\right)^2 + mgy$$
$$E_{mec} = \frac{1}{2}mv_{cm}^2 + \frac{1}{4}mv_{cm}^2 + mgy$$
$$E_{mec} = \frac{3}{4}mv_{cm}^2 + mgy$$

Instant 1: log at the top of the slope

Then, the log moves at v_{cm} = 5 m/s.

The height of the centre of mass, 400 m uphill on a 40° slope, is

$$y = 400m \cdot \sin 40° = 257.1m$$

(The origin y = 0 is thus at the bottom of the slope.)

The mechanical energy is therefore

$$\begin{aligned} E &= \frac{3}{4} m v_{cm}^2 + mgy \\ &= \frac{3}{4} \cdot 4000kg \cdot \left(5 \tfrac{m}{s}\right)^2 + 4000kg \cdot 9.8 \tfrac{N}{kg} \cdot 257.1m \\ &= 75\,000J + 1.008 \times 10^7 J \\ &= 1.01539 \times 10^7 J \end{aligned}$$

Instant 2: log at the bottom of the slope

Then, the height of the centre of mass is zero and the energy is

$$\begin{aligned} E' &= \frac{3}{4} m v_{cm}'^2 + mgy' \\ &= \frac{3}{4} \cdot 4000kg \cdot v_{cm}'^2 + 0 \\ &= 3000kg \cdot v_{cm}'^2 \end{aligned}$$

Mechanical Energy Conservation

According to the law of conservation, we have

$$\begin{aligned} E &= E' \\ 1.01539 \times 10^7 J &= 3000kg \cdot v_{cm}'^2 \\ v_{cm}' &= 58.18 \tfrac{m}{s} \end{aligned}$$

24. There are two objects here. A mass which moves in a straight line and a pulley that revolves around its centre of mass without moving. The mechanical energy is therefore

$$E_{mec} = \frac{1}{2} m_1 v_{1cm}^2 + m_1 g y_1 + \frac{1}{2} I_{2cm} \omega_2^2 + m_2 g y_2$$

We now place the $y = 0$ for the pulley on the axis of rotation of the pulley, which eliminates the last term.

As the string must have the same speed as the 5 kg block, the edge of the pulley has the same speed as the 5 kg block. Therefore

$$\omega_2 = \frac{v_{1cm}}{r_2}$$

The mechanical energy is then

$$E_{mec} = \frac{1}{2}m_1 v_{1cm}^2 + m_1 g y_1 + \frac{1}{2}I_{2cm}\omega_2^2$$

$$E_{mec} = \frac{1}{2}m_1 v_{1cm}^2 + m_1 g y_1 + \frac{1}{2}\left(\frac{1}{2}m_2 r_2^2\right)\left(\frac{v_{1cm}}{r_2}\right)^2$$

$$E_{mec} = \frac{1}{2}m_1 v_{1cm}^2 + m_1 g y_1 + \frac{1}{4}m_2 v_{1cm}^2$$

<u>Instant 1: mass at rest</u>

At this instant, the mass does not move so that $v_{1cm} = 0$. We also choose the origin $y = 0$ for the mass at the initial position of the mass. The mechanical energy is therefore

$$\begin{aligned} E &= \frac{1}{2}m_1 v_{1cm}^2 + m_1 g y_1 + \frac{1}{4}m_2 v_{1cm}^2 \\ &= 0 + 0 + 0 \end{aligned}$$

<u>Instant 2: mass 8 m lower</u>

Then, the mass is 8 metres lower so that $y = -8$ m. The mechanical energy is therefore

$$\begin{aligned} E' &= \frac{1}{2}m_1 v_{1cm}'^2 + m_1 g y_1' + \frac{1}{4}m_2 v_{1cm}'^2 \\ &= \frac{1}{2}\cdot 5kg \cdot v_{1cm}'^2 + 5kg \cdot 9.8\tfrac{N}{kg}\cdot(-8m) + \frac{1}{4}\cdot 10kg \cdot v_{1cm}'^2 \\ &= 5kg \cdot v_{1cm}^2 - 392J \end{aligned}$$

<u>Mechanical Energy Conservation</u>

According to the law of conservation, we have

$$\begin{aligned} E &= E' \\ 0 &= 5kg \cdot v_{1cm}'^2 - 392J \\ v_{1cm} &= 8.854\tfrac{m}{s} \end{aligned}$$

25. As there is a single object that rotates around an axis without moving, the mechanical energy is

$$E_{mec} = \frac{1}{2} I \omega^2 + mgy$$

<u>Instant 1: object in the position shown in the figure.</u>

Then, the rod is not turning, and therefore $\omega = 0$. We now choose an origin $y = 0$ that runs along the rod in this position, so that the centre of mass is at $y = 0$ m initially. The mechanical energy is therefore

$$\begin{aligned} E &= \frac{1}{2} I \omega^2 + mgy \\ &= 0 + 0 \end{aligned}$$

<u>Instant 2: the rod is vertical</u>

Then, the mechanical energy is

$$E' = \frac{1}{2} I \omega'^2 + mgy'$$

There are two things that must be found: the moment of inertia of this object and the height of the centre of mass.

This object consists of a rod and a disk. In both cases, the axis of rotation is not at the centre of mass. Therefore

$$\begin{aligned} I_{rod} &= I_{cm\,rod} + md^2 \\ &= \frac{1}{12} mL^2 + md^2 \\ &= \frac{1}{12} \cdot 0.25kg \cdot (0.8m)^2 + 0.25kg \cdot (0.4m)^2 \\ &= 0.05333 kgm^2 \end{aligned}$$

$$\begin{aligned} I_{disk} &= I_{cm\,disk} + md^2 \\ &= \frac{1}{2}mr^2 + md^2 \\ &= \frac{1}{2}\cdot 0.6kg\cdot(0.2m)^2 + 0.6kg\cdot(1m)^2 \\ &= 0.612kgm^2 \end{aligned}$$

Therefore, the total moment of inertia is

$$\begin{aligned} I &= I_{rod} + I_{disk} \\ &= 0.05333kgm^2 + 0.612kgm^2 \\ &= 0.66533kgm^2 \end{aligned}$$

For the height of the centre of mass when the rod is vertical, the rod is replaced by a point mass at the centre of mass of the rod and the disk is replaced by a point mass at the centre of the disk. With the origin $y = 0$ at the axis of rotation, the position of the centre of mass is

$$\begin{aligned} y_{cm} &= \frac{y_1m_1 + y_2m_2}{m_1+m_2} \\ &= \frac{(-0.4m)\cdot 0.25kg + (-1m)\cdot 0.6kg}{0.25kg + 0.6kg} \\ &= -0.8235m \end{aligned}$$

The mechanical energy is therefore

$$\begin{aligned} E' &= \frac{1}{2}\cdot 0.66533kgm^2\cdot\omega^2 + 0.85kg\cdot 9.8\tfrac{N}{kg}\cdot(-0.8235m) \\ &= 0.33267kgm^2\cdot\omega^2 - 6.86J \end{aligned}$$

Mechanical Energy Conservation

According to the law of conservation, we have

$$\begin{gathered} E = E' \\ 0 = 0.33267kgm^2\cdot\omega'^2 - 6.86J \\ \omega' = 4.541\tfrac{rad}{s} \end{gathered}$$

26. There are four objects here. Two masses that move in a straight line, a pulley that revolves around its centre of mass without moving and a spring. The mechanical energy is therefore

$$E_{mec} = \frac{1}{2}m_1 v_{1cm}^2 + m_1 g y_1 + \frac{1}{2}m_2 v_{2cm}^2 + m_2 g y_2 + \frac{1}{2}I_{3cm}\omega_3^2 + m_3 g y_3 + \frac{1}{2}kx^2$$

We now place the $y = 0$ for the pulley on the axis of rotation of the pulley, which eliminates the sixth term.

As the two blocks are connected by a rope, they must have the same speed. Let's call this velocity v. This means that

$$v_{1cm} = v_{2cm} = v$$

The mechanical energy is now

$$E_{mec} = \frac{1}{2}m_1 v^2 + m_1 g y_1 + \frac{1}{2}m_2 v^2 + m_2 g y_2 + \frac{1}{2}I_{3cm}\omega_3^2 + \frac{1}{2}kx^2$$

As the rope must have the same speed as the blocks, the edge of the pulley has the same speed as the blocks. Therefore

$$\omega_3 = \frac{v}{r_3}$$

The mechanical energy then becomes

$$E_{mec} = \frac{1}{2}m_1 v^2 + m_1 g y_1 + \frac{1}{2}m_2 v^2 + m_2 g y_2 + \frac{1}{2}\left(\frac{1}{2}m_3 r_3^2\right)\left(\frac{v}{r_3}\right)^2 + \frac{1}{2}kx^2$$

$$E_{mec} = \frac{1}{2}m_1 v^2 + m_1 g y_1 + \frac{1}{2}m_2 v^2 + m_2 g y_2 + \frac{1}{4}m_3 v^2 + \frac{1}{2}kx^2$$

Instant 1: 36 kg block at its highest point

Then, the blocks do not move and therefore $v = 0$. We then choose an origin $y = 0$ for each block located at the initial position of each block. Finally, spring is not stretched, which means that $x = 0$. The mechanical energy is therefore

$$\begin{aligned} E &= \frac{1}{2}m_1 v^2 + m_1 g y_1 + \frac{1}{2}m_2 v^2 + m_2 g y_2 + \frac{1}{4}m_3 v^2 + \frac{1}{2}kx^2 \\ &= 0 \end{aligned}$$

Instant 2: 36 kg block 1 m lower

Then, block 2 is 1 m lower, whereas block 1 remained at its $y = 0$ m. The spring is now stretched 1 m. The mechanical energy is thus

$$\begin{aligned}
E' &= \frac{1}{2}m_1 v_{1cm}'^2 + m_1 g y_1' + \frac{1}{2}m_2 v_{1cm}'^2 + m_2 g y_2' + \frac{1}{4}m_3 v_{1cm}'^2 + \frac{1}{2}kx'^2 \\
&= \frac{1}{2}\cdot 12kg \cdot v'^2 + 0 + \frac{1}{2}\cdot 36kg \cdot v'^2 + 36kg \cdot 9.8\tfrac{N}{kg}\cdot(-1m) + \frac{1}{4}\cdot 20kg \cdot v'^2 + \frac{1}{2}\cdot 200\tfrac{N}{m}\cdot(1m)^2 \\
&= 6kg \cdot v'^2 + 18kg \cdot v'^2 - 352.8J + 5kg \cdot v'^2 + 100J \\
&= 29kg \cdot v'^2 - 252.8J
\end{aligned}$$

Mechanical Energy Conservation

According to the law of conservation, we have

$$\begin{aligned}
E &= E' \\
0J &= 29kg \cdot v'^2 - 252.8J \\
v' &= 2.952\tfrac{m}{s}
\end{aligned}$$

27. Taking the clockwise direction as the positive direction, the net torque is

$$\begin{aligned}
\tau_{net} &= \tau_1 + \tau_2 + \tau_3 \\
&= -30N \cdot 2m \cdot \sin 135° + 25N \cdot 0m + 10N \cdot 2m \cdot \sin 160° \\
&= -35.586Nm
\end{aligned}$$

28. Taking the clockwise direction as the positive direction, the net torque is

$$\begin{aligned}
\tau_{net} &= \tau_1 + \tau_2 + \tau_3 \\
&= -12N \cdot 0.15m \cdot \sin 90° + 10N \cdot 0.35m \cdot \sin 90° + 9N \cdot 0.35m \cdot \sin 90° \\
&= 4.85Nm
\end{aligned}$$

29. Taking the clockwise direction as the positive direction, the torque exerted by the 160 N force is

$$\begin{aligned}\tau_{net} &= Fr_{\perp} \\ &= -160N \cdot 0.165m \\ &= -26.4Nm\end{aligned}$$

30. The angular acceleration is found with

$$\tau_{net} = I\alpha$$

First, the moment of inertia of the rod must be found. The rod rotates around an axis that is not at the centre of mass. The moment of inertia is therefore

$$\begin{aligned}I &= I_{cm} + md^2 \\ &= \frac{1}{12} mL^2 + md^2 \\ &= \frac{1}{12} \cdot 100kg \cdot (6m)^2 + 100kg \cdot (3m)^2 \\ &= 1200kgm^2\end{aligned}$$

Once the rope broke, gravitation is the only force that exerts a non-vanishing torque. With a positive direction in the clockwise direction, the torque is

$$\begin{aligned}\tau &= Fr\sin\theta \\ &= 980N \cdot 3m \cdot \sin 30° \\ &= 1470Nm\end{aligned}$$

Thus, the angular acceleration is

$$\begin{aligned}\tau_{net} &= I\alpha \\ 1470Nm &= 1200kgm^2 \cdot \alpha \\ \alpha &= 1.225 \tfrac{rad}{s^2}\end{aligned}$$

31. a) The angular acceleration is found with

$$\tau_{net} = I\alpha$$

With a positive direction in the counterclockwise direction, the torque is

$$\begin{aligned}\tau &= \tau_1 + \tau_2 \\ &= -100N \cdot 0.3m \cdot \sin 90° + 120N \cdot 0.3m \cdot \sin 90° \\ &= 6Nm\end{aligned}$$

Thus, the acceleration is

$$\begin{aligned}\tau_{net} &= I\alpha \\ 6Nm &= 0.5kgm^2 \cdot \alpha \\ \alpha &= 12 \tfrac{rad}{s^2}\end{aligned}$$

b) The angular speed of the pulley is

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ &= \tfrac{50}{3}\pi \tfrac{rad}{s} + 12 \tfrac{rad}{s^2} \cdot 10s \\ &= 172,36 \tfrac{rad}{s} \\ &= 1646RPM\end{aligned}$$

32. At equilibrium, there is no angular acceleration and we have

$$\begin{aligned}\tau_{net} &= I\alpha \\ \tau_{net} &= 0\end{aligned}$$

With a positive direction in the clockwise direction, the net torque is

$$\begin{aligned}\tau &= \tau_1 + \tau_2 \\ &= \left(120kg \cdot 9.8 \tfrac{N}{kg}\right) \cdot 1m \cdot \sin 90° - F_R \cdot 0.5m \cdot \sin 90° \\ &= 1176Nm - F_R \cdot 0.5m\end{aligned}$$

Since the net torque must be zero, we have

$$\begin{aligned}0 &= 1176Nm - F_R \cdot 0.5m \\ F_R &= 2352N\end{aligned}$$

Therefore, stretching the spring is

$$\begin{aligned}F_R &= kx \\ 2352N &= 1000 \tfrac{N}{m} \cdot x \\ x &= 2.352m\end{aligned}$$

33. The angular acceleration is found with

$$\tau_{net} = I\alpha$$

The only forces that acts on the wheel somewhere else than at the centre of the wheel are the normal force and friction force (both made by the axe), which are exerted on the circumference of the wheel. The normal force has a magnitude of 160 N and the friction force has a magnitude of 96 N (0.6 · 160 N).

With a positive direction in the direction of the rotation of the wheel, the net torque is

$$\begin{aligned}\tau &= \tau_1 + \tau_2 \\ &= 160N \cdot 0.35m \cdot \sin 0° - 96N \cdot 0.35m \cdot \sin 90° \\ &= -33.6Nm\end{aligned}$$

Thus, the angular acceleration is

$$\begin{aligned}\tau_{net} &= I\alpha \\ \tau_{net} &= \frac{1}{2}mr^2\alpha \\ -33.6Nm &= \frac{1}{2} \cdot 50kg \cdot (0.35m)^2 \cdot \alpha \\ \alpha &= -10.97 \tfrac{rad}{s^2}\end{aligned}$$

The angular speed of the pulley will be 10 rad/s after

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ 10 \tfrac{rad}{s} &= 30 \tfrac{rad}{s} + \left(-10.97 \tfrac{rad}{s^2}\right) \cdot t \\ t &= 1.823s\end{aligned}$$

34. While the speed of the wheel increases, there is the applied force (which is a torque that we will call τ_{ext}) and friction force that acts on the wheel. Therefore

$$\begin{aligned}\tau_{net} &= I\alpha \\ \tau_{ext} + \tau_f &= I\alpha\end{aligned}$$

For the moment, we cannot do much with this equation because there are two unknowns (the two torques).

Then the deceleration phase of the wheel follows. During, this phase, only the friction force acts on the wheel.

$$\tau_{net} = I\alpha$$
$$\tau_f = I\alpha$$

With this last equation, we can know the torque exerted by the friction force. With a positive direction in the direction of rotation of the wheel, the acceleration during the slowing-down phase is found with

$$\omega = \omega_0 + \alpha t$$
$$0 \tfrac{rad}{s} = 30 \tfrac{rad}{s} + \alpha \cdot 120s$$
$$\alpha = -0.25 \tfrac{rad}{s}$$

The torque exerted by the friction force is therefore

$$\begin{aligned}\tau_f &= I\alpha \\ &= 0.5kgm^2 \cdot \left(-0.25 \tfrac{rad}{s^2}\right) \\ &= -0.125Nm\end{aligned}$$

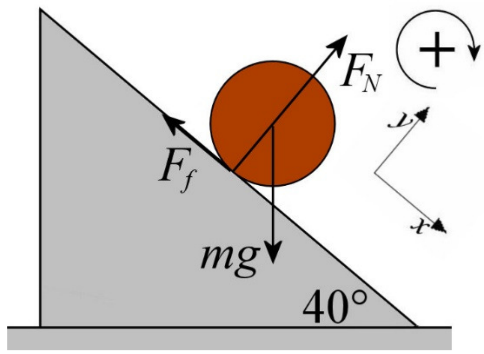
We can then return to the first phase of the motion. During this phase, the acceleration is

$$\omega = \omega_0 + \alpha t$$
$$30 \tfrac{rad}{s} = 0 \tfrac{rad}{s} + \alpha \cdot 10s$$
$$\alpha = 3 \tfrac{rad}{s}$$

Therefore

$$\tau_{ext} + \tau_f = I\alpha$$
$$\tau_{ext} + -0.125Nm = 0.5kgm^2 \cdot 3 \tfrac{rad}{s^2}$$
$$\tau_{ext} = 1.625Nm$$

35. a) The forces on the log are: the weight, the normal force and the friction force.



As the log make a translation motion and a rotating motion, the sum of the forces and the sum of the torques must be made. Here, the centre of mass of the object must be taken as the axis of rotation since the axis is not fixed. The equations are therefore

$$\sum F_x = mg\cos(-50°) - F_f = ma$$
$$\sum F_y = mg\sin(-50°) + F_N = 0$$
$$\sum \tau = \cancel{mg\times 0} + \cancel{F_N R\sin 0°} + F_f R\sin 90° = I\alpha$$

In this last equation, the distance is zero for the torque exerted by gravity, because force is applied at the centre of mass and the axis of rotation is also at the centre of mass. The angle is zero for the torque exerted by the normal force, because the force is directed exactly towards the centre of mass. (You can also see that in this case, the lever arm is zero when the force is directed towards the axis of rotation.) The only remaining torque is due to the force of friction. The friction force is the only force which makes the log spins. If the friction is removed, the log will simply slide without revolving.

Using the rolling without slipping condition $a_{cm} = \alpha R$ and the moment of inertia of a cylinder, the sum of torques becomes

$$F_f R\sin 90° = I\alpha$$
$$F_f R = \left(\frac{1}{2}mR^2\right)\left(\frac{a}{R}\right)$$
$$F_f = \frac{1}{2}ma$$

It is the force of friction required so that the log rolls without slipping.

The acceleration can then be found using the sum of the *x*-components of the forces.

$$mg\cos(-50°) - F_f = ma$$
$$\cancel{m}g\cos(-50°) - \frac{1}{2}\cancel{m}a = \cancel{m}a$$
$$g\cos(-50°) = a + \frac{1}{2}a$$
$$g\cos(-50°) = \frac{3}{2}a$$
$$a = \frac{2}{3}g\cos(-50°)$$

Since cos (-50°) = cos (50°), the acceleration is

$$\begin{aligned} a &= \frac{2}{3}g\cos(50°) \\ &= \frac{2}{3}\cdot 9.8\tfrac{m}{s^2}\cdot\cos(50°) \\ &= 4.2\tfrac{m}{s^2} \end{aligned}$$

b) The force of friction on the log is

$$F_f = \frac{1}{2}ma$$
$$F_f = \frac{1}{2}m\frac{2}{3}g\cos(50°)$$
$$F_f = \frac{1}{3}mg\cos(50°)$$

As this is static friction, this force must be less than the static friction maximum. This means that

$$\frac{1}{3}mg\cos(50°) \le \mu_s F_N$$

The normal force can be found with the sum of the *y*-components of the forces.

$$mg\sin(-50°) + F_N = 0$$
$$F_N = -mg\sin(-50°)$$
$$F_N = mg\sin(50°)$$

Therefore

$$\frac{1}{3}\cancel{mg}\cos(50°) \le \mu_s \cancel{mg}\sin(50°)$$
$$\mu_s \ge \frac{1}{3}\frac{\cos(50°)}{\sin(50°)}$$
$$\mu_s \ge 0.2797$$

Therefore, the minimum friction coefficient is

$$\mu_{s\,\min} = 0.2797$$

36. As the block moves in a straight line and does not rotate, only the sum of the forces on the object must be done. The forces on the object are the weight, the normal force, and the tension of the rope. With an x-axis directed downhill, the equations are there

$$\sum F_x = m_1 a_x$$
$$\to 49N \cdot \cos(-60°) - T = m_1 a_x$$
$$\sum F_y = m_1 a_y$$
$$\to 49N \cdot \sin(-60°) + F_N = 0$$

(Actually, the sum of the y-components of the forces there will be useless here, it only allows to find the normal force and there is no need to know this force here.)

The cylinder rotates and does not move is a straight line. In this case, only the sum of the torques must be done. The forces on the cylinder are the weight, the normal force made by the axle and the tension of the rope. As the weight and the normal force are exerted on the axis of rotation, these two forces do not exert any torque. Only the tension force exerts a torque. With a positive direction in the anti-clockwise direction, the sum of torques is

$$\tau_{net} = I\alpha$$
$$Tr\sin 90° = I\alpha$$

The edge of the cylinder must have the same acceleration as the rope, which must have the same acceleration as the block. Therefore

$$\alpha = \frac{a}{r}$$

Using the moment of inertia of the cylinder, the sum of torque on the cylinder becomes

$$Tr = I\alpha$$
$$Tr = \frac{1}{2}m_2 r^2 \frac{a}{r}$$
$$T = \frac{1}{2}m_2 a$$

Our two equations are thus

$$49N \cdot \cos(-60°) - T = m_1 a \quad \text{and} \quad T = \frac{1}{2}m_2 a$$

Substituting the second equation for T in the first, we obtain

$$49N \cdot \cos(-60°) - \frac{1}{2}m_2 a = m_1 a$$
$$m_1 a + \frac{1}{2}m_2 a = 49N \cdot \cos(-60°)$$
$$\left(m_1 + \frac{1}{2}m_2\right) \cdot a = 49N \cdot \cos(-60°)$$
$$\left(5kg + \frac{1}{2} \cdot 10kg\right) \cdot a = 49N \cdot \cos(-60°)$$
$$a = 2.45 \tfrac{m}{s^2}$$

37. As the block moves and does not rotate, only the sum of the forces on the object must be done. The forces on the object are the weight and the tension of the rope. With a y-axis directed upwards, the sum is

$$\sum F_y = ma_y$$
$$\rightarrow -49N + T = 5kg \cdot a$$

The pulley rotates and does not move in a straight line. In this case, only the sum of the torques must be done. The forces on the cylinder are the weight, the normal force exerted by the axle and the tension of the strings. As the weight and the normal force acts on the axis of rotation, these two forces do not exert any torque. Only the torques exerted by the tensions of the strings remains. With a positive direction in the anti-clockwise direction, the sum of torque is

$$\tau_{net} = I\alpha$$
$$-T \cdot 0.5m \cdot \sin 90° + 200N \cdot 0.25m \cdot \sin 90° = 8kgm^2 \cdot \alpha$$
$$-T \cdot 0.5m + 200N \cdot 0.25m = 8kgm^2 \cdot \alpha$$

As the rope connected to the block is wrapped 50 cm from the axis of the pulley, the tangential acceleration of the pulley at 50 cm from the axis must be the same as the acceleration of the block. Therefore

$$\alpha = \frac{a}{0.5m}$$

The sum of torques on the cylinder then becomes

$$\begin{aligned} -T \cdot 0.5m + 200N \cdot 0.25m &= 8kgm^2 \cdot \alpha \\ -T \cdot 0.5m + 200N \cdot 0.25m &= 8kgm^2 \cdot \frac{a}{0.5m} \\ -T \cdot 0.5m + 50Nm &= 16kgm \cdot a \end{aligned}$$

Our two equations are thus

$$-49N + T = 5kg \cdot a \qquad \text{and} \qquad -T \cdot 0.5m + 50Nm = 16kgm \cdot a$$

If we solve for *T* in the first equation and substitute in the second equation, we obtain

$$\begin{aligned} -\left(5kg \cdot a + 49N\right) \cdot 0.5m + 50Nm &= 16kgm \cdot a \\ -2.5kgm \cdot a - 24.5Nm + 50Nm &= 16kgm \cdot a \\ 25.5Nm &= 18.5kgm \cdot a \\ a &= 1.378 \tfrac{m}{s^2} \end{aligned}$$

38. As the 20 kg block moves and does not rotate, only the sum of the forces on the object must be done. The forces on the object are the weight and the tension of the rope. With a *y*-axis directed upwards, the sum of forces is

$$\begin{aligned} \sum F_y &= ma_y \\ &\to -196N + T_1 = 20kg \cdot a_1 \end{aligned}$$

As the 30 kg block moves and does not rotate, only the sum of the forces on the object must be done. The forces on the object are the weight and the tension of the rope. With a *y*-axis directed downwards, the sum of forces is

$$\begin{aligned} \sum F_y &= ma_y \\ &\to 294N - T_2 = 30kg \cdot a_2 \end{aligned}$$

The pulley rotates and does not move in a straight line. In this case, only the sum of the torques must be done. The forces on the cylinder are the weight, the normal force exerted by the axle and the tensions of the strings. As the weight and the normal force are exerted on the axis of rotation, these two forces do not exert any torque. The tensions of the strings are thus the only forces exerting a torque. With a positive direction in the clockwise direction, the sum of torque is

$$\tau_{net} = I\alpha$$
$$-T_1 \cdot 0.25m \cdot \sin 90° + T_2 \cdot 0.5m \cdot \sin 90° = 8kgm^2 \cdot \alpha$$
$$-T_1 \cdot 0.25m + T_2 \cdot 0.5m = 8kgm^2 \cdot \alpha$$

As the rope connected to the 20 kg block is wrapped 25 cm from the axis of the pulley, the tangential acceleration of the pulley at 25 cm from the axis must be the same as the acceleration of the 20 kg block. This means that

$$\alpha = \frac{a_1}{0.25m}$$

The equation of the forces on the 20 kg block then becomes

$$-196N + T_1 = 20kg \cdot a_1$$
$$-196N + T_1 = 20kg \cdot \alpha \cdot 0.25m$$
$$-196N + T_1 = 5kgm \cdot \alpha$$

As the rope connected to the 30 kg block is wrapped 50 cm from the axis of the pulley, the tangential acceleration of the pulley at 50 cm from the axis must be the same as the acceleration of the 30 kg block. This means that

$$\alpha = \frac{a_2}{0.5m}$$

The equation of the forces on the 30 kg block then becomes

$$294N - T_2 = 30kg \cdot a_2$$
$$294N - T_2 = 30kg \cdot \alpha \cdot 0.5m$$
$$294N - T_2 = 15kgm \cdot \alpha$$

We thus have the 3 following equations.

$$-196N + T_1 = 5kgm \cdot \alpha$$
$$294N - T_2 = 15kgm \cdot \alpha$$
$$-T_1 \cdot 0.25m + T_2 \cdot 0.5m = 8kgm^2 \cdot \alpha$$

We can then solve for both tensions in the first and second equations and substitute the results in the third equation. We then obtain

$$
\begin{gathered}
-T_1 \cdot 0.25m + T_2 \cdot 0.5m = 8kgm^2 \cdot \alpha \\
-\left(5kgm \cdot \alpha + 196N\right) \cdot 0.25m + \left(294N - 15kgm \cdot \alpha\right) \cdot 0.5m = 8kgm^2 \cdot \alpha \\
-1.25kgm^2 \cdot \alpha - 49Nm + 147Nm - 7.5kgm^2 \cdot \alpha = 8kgm^2 \cdot \alpha \\
98Nm = 16.75kgm^2 \cdot \alpha \\
\alpha = 5.851 \tfrac{rad}{s^2}
\end{gathered}
$$

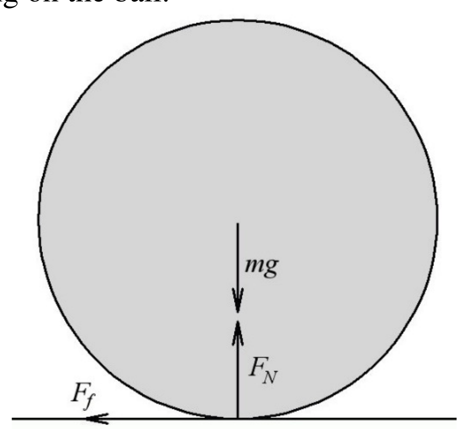
Thus, the accelerations are

$$
\begin{gathered}
\alpha = \frac{a_1}{0.25m} \\
5.851 \tfrac{rad}{s^2} = \frac{a_1}{0.25m} \\
a_1 = 1.463 \tfrac{m}{s^2}
\end{gathered}
$$

and

$$
\begin{gathered}
\alpha = \frac{a_2}{0.5m} \\
5.851 \tfrac{rad}{s^2} = \frac{a_2}{0.5m} \\
a_2 = 2.925 \tfrac{m}{s^2}
\end{gathered}
$$

39. Here are the forces acting on the ball.



Only the friction force exerts a torque on the ball. Therefore,

$$\tau_{net} = I\alpha$$
$$F_f R = \left(\frac{2}{5}mR^2\right)\alpha$$
$$\mu_k F_N R = \left(\frac{2}{5}mR^2\right)\alpha$$
$$\mu_k mgR = \left(\frac{2}{5}mR^2\right)\alpha$$
$$\alpha = \frac{5}{2}\frac{\mu_k g}{R}$$

The friction also slows down the ball. Therefore,

$$-F_f = ma$$
$$-\mu_k F_N = ma$$
$$-\mu_l mg = ma$$
$$a = -\mu_k g$$

Thus, the angular speed as a function of time is

$$\omega = \omega_0 + \alpha t$$
$$\omega = 0 + \frac{5\mu_k g}{2R}t$$
$$\omega = \frac{5\mu_k g}{2R}t$$

and the speed of the ball as a function of time is

$$v = v_0 + at$$
$$v = v_0 - \mu_k gt$$

When the ball rolls without slipping, the following condition applies.

$$v = \omega R$$

Let’s find when this condition will be met.

$$v = \omega R$$
$$v_0 - \mu_k gt = \frac{5\mu_k g}{2R} tR$$
$$v_0 - \mu_k gt = \frac{5\mu_k g}{2} t$$
$$v_0 = \frac{5\mu_k g}{2} t + \mu_k gt$$
$$v_0 = \left(\frac{5\mu_k g}{2} + \mu_k g \right) t$$
$$v_0 = \frac{7\mu_k g}{2} t$$
$$t = \frac{2}{7\mu_k g} v_0$$

At that time, the speed of the ball will be

$$v = v_0 - \mu_k gt$$
$$v = v_0 - \mu_c g \frac{2}{7\mu_k g} v_0$$
$$= v_0 - \frac{2}{7} v_0$$
$$= \left(1 - \frac{2}{7} \right) v_0$$
$$= \frac{5}{7} v_0$$
$$= \frac{5}{7} \cdot 3.5 \tfrac{m}{s}$$
$$= 2.5 \tfrac{m}{s}$$

40. a) First, the angular acceleration of the sphere is found.

$$\tau_{net} = I\alpha$$
$$0.45Nm = \left(\frac{2}{5} mr^2 \right) \alpha$$
$$0.45Nm = \left(\frac{2}{5} \cdot 10kg \cdot (0.20m)^2 \right) \cdot \alpha$$
$$\alpha = 2.8125 \tfrac{rad}{s^2}$$

The angular displacement is then

$$\begin{aligned}\theta &= \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \\ &= 0 + 0 + \frac{1}{2}\cdot 2.8125\tfrac{rad}{s^2}\cdot(10s)^2 \\ &= 140.625rad\end{aligned}$$

b) The work done by the torque is

$$\begin{aligned}W &= \tau\Delta\theta \\ &= 0.45Nm\cdot 140.625rad \\ &= 63.28J\end{aligned}$$

c) The angular velocity can then be found with the work-energy theorem.

$$\begin{aligned}W &= \Delta E_k \\ &= \frac{1}{2}I\omega'^2 - \frac{1}{2}I\omega^2\end{aligned}$$

With a vanishing initial angular speed, we have

$$\begin{aligned}W &= \frac{1}{2}I\omega'^2 \\ W &= \frac{1}{2}\cdot\frac{2}{5}\cdot mr^2\omega'^2 \\ W &= \frac{1}{5}mr^2\omega'^2 \\ 63.28J &= \frac{1}{5}\cdot 10kg\cdot(0.2m)^2\cdot\omega'^2 \\ 63.28J &= 0.08kgm^2\cdot\omega'^2 \\ \omega' &= 28.125\tfrac{rad}{s}\end{aligned}$$

Note that we can also use the angular acceleration to get

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ &= 0\tfrac{rad}{s} + 2.8125\tfrac{rad}{s^2}\cdot 10s \\ &= 28.125\tfrac{rad}{s}\end{aligned}$$

41. The work is found with the work-energy theorem.

$$\begin{aligned} W &= \Delta E_k \\ &= \frac{1}{2} I \omega'^2 - \frac{1}{2} I \omega^2 \end{aligned}$$

With a vanishing initial angular speed, we have

$$\begin{aligned} W &= \frac{1}{2} I \omega'^2 \\ &= \frac{1}{2} \cdot \frac{2}{5} \cdot m r^2 \omega'^2 \\ &= \frac{1}{5} m r^2 \omega'^2 \\ &= \frac{1}{5} \cdot 20kg \cdot (0.1m)^2 \cdot \left(80\pi \tfrac{rad}{s}\right)^2 \\ &= 2526.6J \end{aligned}$$

The average power is therefore

$$\begin{aligned} \overline{P} &= \frac{W}{\Delta t} \\ &= \frac{2526.6J}{30s} \\ &= 84.22W \end{aligned}$$

42. We have

$$\begin{aligned} P &= \tau \omega \\ 274 \cdot 746W &= \tau \cdot \left(200\pi \tfrac{rad}{s}\right) \\ \tau &= 325.32Nm \end{aligned}$$

43. a) The work done by the motor corresponds to the energy change of the system

$$W_{ext} = \Delta E_{mec}$$

This system is composed of a mass that moves in a straight line and a rotating pulley. The mechanical energy of this system is thus

$$E_{mec} = \frac{1}{2}m_1v^2 + m_1gy + \frac{1}{2}I\omega^2$$

(The first two terms are for the block and the last one is for the pulley.)

The rope passing over the edge of the pulley, the edge of the pulley must have the same speed as the block. This means that

$$v = \omega r$$

Using the formula for the moment of inertia of a disk, the mechanical energy becomes

$$E_{mec} = \frac{1}{2}m_1v^2 + mgy + \frac{1}{2}I\omega^2$$

$$E_{mec} = \frac{1}{2}m_1v^2 + m_1gy + \frac{1}{2}\left(\frac{1}{2}m_2r^2\right)\left(\frac{v}{r}\right)^2$$

$$E_{mec} = \frac{1}{2}m_1v^2 + m_1gy + \frac{1}{4}m_2v^2$$

Putting the origin $y = 0$ at the initial position of the block, the initial mechanical energy is zero since the speeds are also zero at the beginning.

$$E = 0$$

Once the block has risen 4.5 m, the mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2}m_1v^2 + m_1gy + \frac{1}{4}m_2v^2 \\ &= \frac{1}{2}\cdot 5kg\cdot\left(3\tfrac{m}{s}\right)^2 + 5kg\cdot 9.8\tfrac{N}{kg}\cdot 4.5m + \frac{1}{4}\cdot 10kg\cdot\left(3\tfrac{m}{s}\right)^2 \\ &= 265.5J \end{aligned}$$

The work is therefore

$$\begin{aligned} W_{ext} &= \Delta E_{mec} \\ &= E' - E \\ &= 265.5J - 0J \\ &= 265.5J \end{aligned}$$

b) The average power is

$$\begin{aligned}\overline{P} &= \frac{W}{\Delta t} \\ &= \frac{265.5J}{3s} \\ &= 88.5W\end{aligned}$$

c) The instantaneous power is

$$P_{mot} = \tau_{mot}\omega$$

The angular speed is found with

$$\begin{aligned}v &= \omega r \\ 3\tfrac{m}{s} &= \omega \cdot 0.4m \\ \omega &= 7.5\tfrac{rad}{s}\end{aligned}$$

To find the torque, we will use

$$\tau_{net} = I\alpha$$

The angular acceleration is

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ 7.5\tfrac{rad}{s} &= 0\tfrac{rad}{s} + \alpha \cdot 3s \\ \alpha &= 2.5\tfrac{rad}{s^2}\end{aligned}$$

This means that the net torque is

$$\begin{aligned}\tau_{net} &= I\alpha \\ &= \frac{1}{2}mr^2\alpha \\ &= \frac{1}{2}\cdot 10kg \cdot (0.4m)^2 \cdot 2.5\tfrac{rad}{s^2} \\ &= 2Nm\end{aligned}$$

This net torque comes from two torques: the torque exerted by the motor and the torque exerted by the tension of the rope.

$$\begin{aligned}\tau_{net} &= \tau_{mot} + \tau_T \\ &= \tau_{mot} - T \cdot 0.4m \cdot \sin 90° \\ &= \tau_{mot} - T \cdot 0.4m\end{aligned}$$

(A positive direction in the direction of rotation of the pulley is used.)

The tension can be found with the equation of the forces on the block.

$$T - 49N = 5kg \cdot a$$

As this block has reached a speed of 3 m/s in 3 seconds, its acceleration is 1 m/s² and the tension is

$$\begin{aligned} T - 49N &= 5kg \cdot 1\tfrac{m}{s^2} \\ T &= 54N \end{aligned}$$

Therefore

$$\begin{aligned} \tau_{net} &= \tau_{mot} - T \cdot 0.4m \\ 2Nm &= \tau_{mot} - 54N \cdot 0.4m \\ \tau_{mot} &= 23.6Nm \end{aligned}$$

The power is therefore

$$\begin{aligned} P_{mot} &= \tau_{mot}\omega \\ &= 23.6Nm \cdot 7.5\tfrac{rad}{s} \\ &= 177W \end{aligned}$$

44. The angular momentum is

$$\begin{aligned} L &= I\omega \\ &= \frac{1}{12}mL^2\omega \\ &= \frac{1}{12} \cdot 0.3kg \cdot (2m)^2 \cdot \left(4\pi \tfrac{rad}{s}\right) \\ &= 1.2566\tfrac{kgm^2}{s} \end{aligned}$$

45. *<u>Angular Momentum at Instant 1</u>*

$$\begin{aligned} L &= L_{disk\,1} + L_{disk\,2} \\ &= I_1\omega_1 + I_2\omega_2 \\ &= \frac{1}{2}m_1r_1^2\omega_1 + \frac{1}{2}m_1r_1^2\omega_2 \\ &= 0 + \frac{1}{2}\cdot 0.5kg\cdot(0.15m)^2\cdot 3.5\tfrac{rad}{s} \\ &= 0.01969\tfrac{kgm^2}{s} \end{aligned}$$

(The positive direction is in the direction of rotation of the bottom disk.)

Angular Momentum at Instant 2

$$\begin{aligned} L' &= L'_{disk\,1} + L'_{disk\,2} \\ &= I_1\omega' + I_2\omega' \\ &= \frac{1}{2}m_1r_1^2\omega' + \frac{1}{2}m_1r_1^2\omega' \\ &= \frac{1}{2}\cdot 0.1kg\cdot(0.14m)^2\,\omega' + \frac{1}{2}\cdot 0.5kg\cdot(0.15m)^2\cdot\omega' \\ &= 0.00098kgm^2\cdot\omega' + 0.005625kgm^2\cdot\omega' \\ &= 0.006605kgm^2\cdot\omega' \end{aligned}$$

Angular Momentum Conservation

$$\begin{aligned} L &= L' \\ 0.01969\tfrac{kgm^2}{s} &= 0.006605kgm^2\cdot\omega' \\ \omega' &= 2.98\tfrac{rad}{s} \end{aligned}$$

46. *Angular Momentum at Instant 1*

$$\begin{aligned} L &= L_{rod} + L_{bullet} \\ &= I\omega + mvr \\ &= 0 + 0.01kg\cdot 400\tfrac{m}{s}\cdot 1m \\ &= 4\tfrac{kgm^2}{s} \end{aligned}$$

(Our positive direction is in the clockwise direction.)

Angular Momentum at Instant 2

$$\begin{aligned}
L' &= L'_{rod} \\
&= I'\omega' \\
&= \left(\frac{1}{12}m_{rod}L^2 + m_{bullet}(1m)^2\right)\omega' \\
&= \left(\frac{1}{12}\cdot 0.6kg\cdot(4m)^2 + 0.01kg\cdot(1m)^2\right)\cdot\omega' \\
&= 0.81kgm^2\cdot\omega'
\end{aligned}$$

Angular Momentum Conservation

$$\begin{aligned}
L &= L' \\
4\tfrac{kgm^2}{s} &= 0.81kgm^2\cdot\omega' \\
\omega' &= 4.938\tfrac{rad}{s}
\end{aligned}$$

47. *Angular Momentum at Instant 1*

$$\begin{aligned}
L &= I\omega \\
&= 35kgm^2\cdot 4\pi\tfrac{rad}{s} \\
&= 140\pi\tfrac{kgm^2}{s}
\end{aligned}$$

(Our positive direction is in the direction of the rotation of the figure skater.)

Angular Momentum at Instant 2

$$\begin{aligned}
L' &= I'\omega' \\
&= 8kgm^2\cdot\omega'
\end{aligned}$$

Angular Momentum Conservation

$$\begin{aligned}
L &= L' \\
140\pi\tfrac{kgm^2}{s} &= 8kgm^2\cdot\omega' \\
\omega' &= 54.98\tfrac{rad}{s} \\
\omega' &= 8.75\tfrac{revolutions}{s}
\end{aligned}$$

48. a)

To find the moment of inertia, the position of the centre of mass of the two astronauts must be found. Let's take an axis going from Buzz to Allan with the origin $x = 0$ where Buzz is. The position of the centre of mass is therefore

$$\begin{aligned} x_{cm} &= \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} \\ &= \frac{0 + 10m \cdot 80kg}{120kg + 80Kg} \\ &= 4m \end{aligned}$$

This means that Buzz is 4 m from the axis of rotation and that Alan is 6 m from the axis of rotation. The moment of inertia is therefore

$$\begin{aligned} I &= m_1 r_1^2 + m_2 r_2^2 \\ &= 120kg \cdot (4m)^2 + 80kg \cdot (6m)^2 \\ &= 4800kgm^2 \end{aligned}$$

b) When the rope is pulled, the centre of mass remains at the same position. As the distance between the two astronauts is 4 m, we can say that Buzz is at position x and that Allan is at position x + 4 m. Therefore

$$\begin{aligned} x_{cm} &= \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} \\ 4m &= \frac{x \cdot 120kg + (x + 4m) \cdot 80kg}{120kg + 80Kg} \\ 800kgm &= x \cdot 120kg + (x + 4m) \cdot 80kg \\ 800kgm &= x \cdot 120kg + x \cdot 80kg + \cdot 320kgm^2 \\ 480kgm^2 &= x \cdot 200kg \\ x &= 2.4m \end{aligned}$$

Buzz is therefore at x = 2.4 m, which means that he is 1.6 m from the axis of rotation (which is at the centre of mass). Allan is at x = 6.4 m, 2.4 m from the axis of rotation. The new moment of inertia is therefore

$$\begin{aligned} I &= m_1 r_1^2 + m_2 r_2^2 \\ &= 120kg \cdot (1.6m)^2 + 80kg \cdot (2.4m)^2 \\ &= 768kgm^2 \end{aligned}$$

c) The angular momentum when the astronauts are 10 m from each other is

$$\begin{aligned} L &= I\omega \\ &= 4800kgm^2 \cdot 0.8\tfrac{rad}{s} \\ &= 3840\tfrac{kgm^2}{s} \end{aligned}$$

When astronauts are 4 m from each other, the angular momentum is

$$\begin{aligned} L' &= I'\omega' \\ &= 768kgm^2 \cdot \omega' \end{aligned}$$

From the conservation of angular momentum, we have

$$\begin{aligned} L &= L' \\ 3840\tfrac{kgm^2}{s} &= 768kgm^2 \cdot \omega' \\ \omega' &= 5\tfrac{rad}{s} \end{aligned}$$

d) Buzz's speed is

$$\begin{aligned} v &= \omega r \\ &= 5\tfrac{rad}{s} \cdot 1.6m \\ &= 8\tfrac{m}{s} \end{aligned}$$

Allan's speed is

$$\begin{aligned} v &= \omega r \\ &= 5\tfrac{rad}{s} \cdot 2.4m \\ &= 12\tfrac{m}{s} \end{aligned}$$

e) Buzz makes a circular motion, and the tension of the rope is the centripetal force. Therefore

$$\begin{aligned} T &= \frac{mv^2}{r} \\ &= \frac{120kg \cdot \left(8\tfrac{m}{s}\right)^2}{1.6m} \\ &= 4800N \end{aligned}$$

Note that it can also be done with Allan

$$\begin{aligned} T &= \frac{mv^2}{r} \\ &= \frac{80kg \cdot \left(12\tfrac{m}{s}\right)^2}{2.4m} \\ &= 4800N \end{aligned}$$

f) The kinetic energy when the astronauts are 10 m from each other is

$$\begin{aligned} E_k &= \frac{1}{2} I\omega^2 \\ &= \frac{1}{2} \cdot 4800kgm^2 \cdot \left(0.8\tfrac{rad}{s}\right)^2 \\ &= 1536J \end{aligned}$$

When the astronauts are 4 m from each other, the kinetic energy is

$$\begin{aligned} E_k &= \frac{1}{2} I\omega^2 \\ &= \frac{1}{2} \cdot 768kgm^2 \cdot \left(5\tfrac{rad}{s}\right)^2 \\ &= 9600J \end{aligned}$$

The energy therefore increases by 8064 J.

g) Buzz does the work by pulling on the rope. Since

$$W = \Delta E_k$$

the work done by Buzz is 8064 J.

49. *Angular Momentum at Instant 1*

$$\begin{aligned} L &= I\omega \\ &= \left(I_{disque} + I_{marj}\right)\omega \\ &= \left(\frac{1}{2}m_{disque}r_{disque}^2 + m_{marj}r_{marj}^2\right)\omega \\ &= \left(\frac{1}{2}\cdot 200kg\cdot(1.5m)^2 + 60kg\cdot(1.5m)^2\right)\cdot 1.3\tfrac{rad}{s} \\ &= 468\tfrac{kgm^2}{s} \end{aligned}$$

(Our positive direction is in the direction of rotation of the circular plate.)

Angular Momentum at Instant 2

$$\begin{aligned} L' &= I'\omega' \\ &= \left(I_{disque} + I'_{marj}\right)\omega' \\ &= \left(\frac{1}{2}m_{disque}r_{disque}^2 + m_{marj}r_{marj}'^2\right)\omega' \\ &= \left(\frac{1}{2}\cdot 200kg\cdot(1.5m)^2 + 60kg\cdot(0m)^2\right)\cdot\omega' \\ &= 225kgm^2\cdot\omega' \end{aligned}$$

Angular Momentum Conservation

$$\begin{aligned} L &= L' \\ 468\tfrac{kgm^2}{s} &= 225kgm^2\cdot\omega' \\ \omega' &= 2.08\tfrac{rad}{s} \end{aligned}$$

50. a)

Two equations are used to solve this problem. The first is the equation for the mechanical energy. The mechanical energy is the sum of the kinetic energy of rotation and the energy of the spring.

$$126J = \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

The second equation is the equation of the sum of the forces acting on the masses. As the spring makes the centripetal force, the force equation is

$$kx = m\frac{v^2}{r}$$
$$kx = m\omega^2 r$$

First, we solve for the angular speed in the energy formula

$$126J = \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$
$$\omega^2 = \frac{252J - kx^2}{I}$$

Since the moment of inertia is

$$I = \sum mr^2 = 2mr^2$$

we have

$$\omega^2 = \frac{252J - kx^2}{2mr^2}$$

By substituting this value in the centripetal force equation, we obtain

$$kx = m\omega^2 r$$
$$kx = m\frac{252J - kx^2}{2mr^2}r$$
$$kx = \frac{252J - kx^2}{2r}$$
$$2kxr = 252J - kx^2$$

The radius is equal to half of the length of the spring. Since the length of the spring is equal to its original length plus its stretching, we have

$$\begin{aligned} r &= \frac{1}{2}(L_0 + x) \\ &= \frac{1}{2}\cdot(0.1m + x) \\ &= 0.05m + \frac{x}{2} \end{aligned}$$

Our equation then becomes

$$2kxr = 252J - kx^2$$
$$2kx \cdot \left(0.05m + \frac{x}{2}\right) = 252J - kx^2$$

It only remains to solve this equation for *x*.

$$kx \cdot 0.1m + kx^2 = 252J - kx^2$$
$$2kx^2 + kx \cdot 0.1m - 252J = 0$$
$$2 \cdot 1200\tfrac{N}{m} \cdot x^2 + 1200\tfrac{N}{m} \cdot x \cdot 0.1m - 252J = 0$$
$$2400\tfrac{N}{m} \cdot x^2 + 120N \cdot x - 252J = 0$$

The positive solution of the quadratic equation is

$$x = 0.3m$$

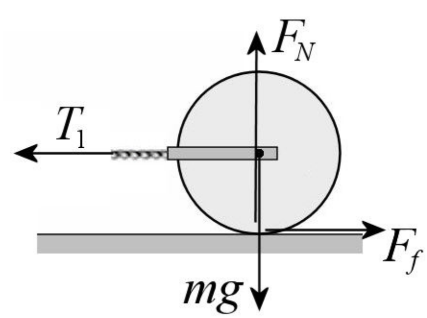
Therefore, the spring is stretched 30 cm.

b) If the stretching is 30 cm, then the total length of the spring is 40 cm, and the radius of the path of each mass is 20 cm.

The angular speed is then easily found with

$$kx = m\omega^2 r$$
$$1200\tfrac{N}{m} \cdot 0,3m = 0.5kg \cdot \omega^2 \cdot 0.2m$$
$$\omega^2 = 3600\tfrac{rad^2}{s^2}$$
$$\omega = 60\tfrac{rad}{s}$$

51. There are 4 forces acting on cylinder 1.



1) The weight directed downwards
2) A normal force directed upwards
3) The tension of the rope towards the left
4) The force of friction towards the right

With an *x*-axis towards the left, the sum of the *x*-component of the forces is

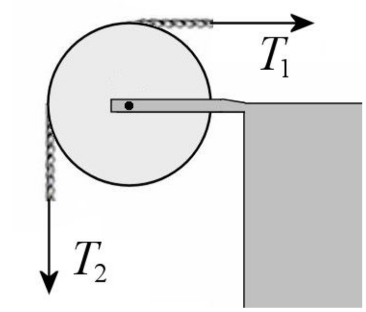
$$T_1 - F_f = ma$$

As this cylinder must also roll, the torque acting on the cylinder must also be considered. As the friction force is the only force making a torque, the sum of the torques is

$$F_f R = I\alpha$$

The moment of inertia is ½mR^2 and the angular acceleration must be a/R (rolling condition). Thus, the equation becomes

$$F_f R = I\alpha$$
$$F_f R = \frac{1}{2} mR^2 \frac{a}{R}$$
$$F_f = \frac{1}{2} ma$$

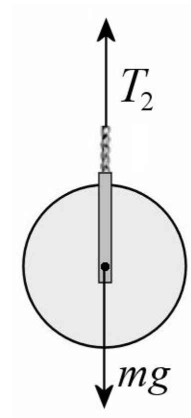


Cylinder 2 will only rotate. The tension forces of the ropes will make this cylinder spin. The sum of the torque (using a positive direction in the counterclockwise direction) is

$$-T_1 R + T_2 R = I\alpha$$

The moment of inertia is ½mR^2 and the angular acceleration must be a/R (rope over a pulley condition). Thus, the equation becomes

$$-T_1 R + T_2 R = I\alpha$$
$$-T_1 R + T_2 R = \frac{1}{2} mR^2 \frac{a}{R}$$
$$-T_1 + T_2 = \frac{1}{2} ma$$



There are 2 forces acting on cylinder 3.

1) The weight directed downwards
2) A tension force directed upwards

With an x-axis directed downwards, the sum of the x-component of the forces is

$$mg - T_2 = ma$$

As this cylinder will not spin, there is need to make the sum of the torques acting on this cylinder.

Our equations are

$$T_1 - F_f = ma$$
$$F_f = \frac{1}{2}ma$$
$$-T_1 + T_2 = \frac{1}{2}ma$$
$$mg - T_2 = ma$$

Finding the value of the force of friction from the 2nd equation to substituting it in the 1st equation, we obtain

$$T_1 - F_f = ma$$
$$T_1 - \frac{1}{2}ma = ma$$
$$T_1 = \frac{3}{2}ma$$

We now have these 3 equations.

$$T_1 = \frac{3}{2}ma$$
$$-T_1 + T_2 = \frac{1}{2}ma$$
$$mg - T_2 = ma$$

If these 3 equations are added together, we get

$$T_1 + \left(-T_1 + T_2\right) + \left(mg - T_2\right) = \frac{3}{2}ma + \frac{1}{2}ma + ma$$
$$mg = 3ma$$

Therefore, the acceleration is

$$a = \frac{g}{3}$$

As 1.8 m has to be travelled with this acceleration from a zero initial speed, the final speed is

$$2a\left(x-x_0\right)=v^2-v_0^2$$
$$2\cdot\frac{9.8\frac{m}{s^2}}{3}\cdot\left(1.8m-0m\right)=v^2-0$$
$$v^2=11.76\tfrac{m^2}{s^2}$$
$$v=3.429\tfrac{m}{s}$$

<u>*Alternate Solution*</u>

The mechanical energy of this system is

$$E_{mec}=\frac{1}{2}mv_1^2+\frac{1}{2}I\omega_1^2+mgy_1+\frac{1}{2}mv_2^2+\frac{1}{2}I\omega_2^2+mgy_2+\frac{1}{2}mv_3^2+\frac{1}{2}I\omega_3^2+mgy_3$$

<u>*Energy at Instant 1 (at rest)*</u>

In the initial configuration, the speed and the angular speeds are zero. Taking $y = 0$ at the original height of each cylinder, the mechanical energy is zero.

$$E_{mec}=0$$

<u>*Energy at Instant 2 (just before cylinder 3 hits the ground)*</u>

At this moment, cylinder 2 does not move and cylinder 3 does not rotate. Cylinders 1 and 2 remained at their $y = 0$, but cylinder 3 is now 1.8 m under its $y = 0$. Thus, the mechanical energy is

$$E'_{mec}=\frac{1}{2}mv_1'^2+\frac{1}{2}I\omega_1'^2+\frac{1}{2}I\omega_2'^2+\frac{1}{2}mv_3'^2+mgy_3'$$

For cylinder 1, the moment of inertia is $\frac{1}{2}mR^2$ and the angular speed must be v/R (rolling condition).

For cylinder 2, the moment of inertia is $\frac{1}{2}mR^2$ and the angular speed must be v/R (rope over a pulley condition).

The energy then becomes

$$\begin{aligned} E'_{mec} &= \frac{1}{2}mv'^2 + \frac{1}{2}\cdot\frac{1}{2}\cdot m\left(\frac{v}{r}\right)^2 + \frac{1}{2}\cdot\frac{1}{2}\cdot mr^2\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv'^2 + mgy'_3 \\ &= \frac{1}{2}mv'^2 + \frac{1}{4}mv'^2 + \frac{1}{4}mv'^2 + \frac{1}{2}mv'^2 + mgy'_3 \\ &= \frac{3}{2}mv'^2 + mgy'_3 \end{aligned}$$

Mechanical Energy Conservation

The speed is finally obtained with the conservation of energy.

$$E_{mec} = E'_{mec}$$

$$0J = \frac{3}{2}mv'^2 + mgy'_3$$

$$\frac{3}{2}mv'^2 = -mgy'_3$$

$$v'^2 = -\frac{2}{3}gy'_3$$

$$v'^2 = -\frac{2}{3}\cdot 9.8\tfrac{N}{kg}\cdot(-1.8m)$$

$$v'^2 = 11.76\tfrac{m^2}{s^2}$$

$$v' = 3.429\tfrac{m}{s}$$