

# Chapter 10 Solutions

1. a) The force of gravitation is

$$\begin{aligned}F_g &= mg \\ &= 120\text{kg} \cdot 9,8 \frac{\text{N}}{\text{kg}} \\ &= 1176\text{N}\end{aligned}$$

This force is directed downwards.

The components of the impulse made by gravity are

$$\begin{aligned}I_x &= F_x \Delta t & I_y &= F_y \Delta t \\ &= 0\text{N} \cdot 10\text{s} & &= -1176\text{N} \cdot 10\text{s} \\ &= 0 \frac{\text{kgm}}{\text{s}} & &= -11\,760 \frac{\text{kgm}}{\text{s}}\end{aligned}$$

b) Here, the normal force has the same magnitude as the force of gravity but directed upwards.

Thus, the components of the impulse made by the normal force are

$$\begin{aligned}I_x &= F_x \Delta t & I_y &= F_y \Delta t \\ &= 0\text{N} \cdot 10\text{s} & &= 1176\text{N} \cdot 10\text{s} \\ &= 0 \frac{\text{kgm}}{\text{s}} & &= 11\,760 \frac{\text{kgm}}{\text{s}}\end{aligned}$$

c) The force of friction is

$$\begin{aligned}F_f &= \mu_k F_N \\ &= 0,3 \cdot 1176\text{N} \\ &= 352,8\text{N}\end{aligned}$$

This force is directed towards the left (opposed to the motion).

The components of the impulse made by the friction force are

$$\begin{aligned}I_x &= F_x \Delta t & I_y &= F_y \Delta t \\ &= -352,8\text{N} \cdot 10\text{s} & &= 0\text{N} \cdot 10\text{s} \\ &= -3528 \frac{\text{kgm}}{\text{s}} & &= 0 \frac{\text{kgm}}{\text{s}}\end{aligned}$$

- d) The force exerted by Karim is an 80 N force directed towards the right. The components of the impulse made by Karim are

$$\begin{aligned} I_x &= F_x \Delta t & I_y &= F_y \Delta t \\ &= 800\text{N} \cdot 10\text{s} & &= 0\text{N} \cdot 10\text{s} \\ &= 8000 \frac{\text{kgm}}{\text{s}} & &= 0 \frac{\text{kgm}}{\text{s}} \end{aligned}$$

- 2.** To find the magnitude of the normal force and of the force exerted by Gilbert, the sum of the  $x$  and  $y$  components of the forces must be made.

There are 4 forces exerted on the crate.

- 1) The 294 N weight at  $-110^\circ$ .
- 2) A normal force towards the positive  $y$ -axis.
- 3) A 70 N friction force towards the negative  $x$ -axis.
- 4) The force exerted by Gilbert towards the positive  $x$ -axis.

Thus, the equations of force are

$$\begin{aligned} \sum F_x &= ma_x \\ &\rightarrow 294\text{N} \cdot \cos(-110^\circ) + F - 70\text{N} = 30\text{kg} \cdot 1 \frac{\text{m}}{\text{s}^2} \\ \sum F_y &= ma_y \\ &\rightarrow 294\text{N} \cdot \sin(-110^\circ) + F_N = 0 \end{aligned}$$

Using the force of the  $x$ -component of the forces, we find  $F = 200.55$  N.

Using the force of the  $y$ -component of the forces, we find  $F_N = 276.27$  N.

- a) The components of the impulse given by gravity are

$$\begin{aligned} I_x &= F_x \Delta t & I_y &= F_y \Delta t \\ &= 294\text{N} \cdot \cos(-110^\circ) \cdot 20\text{s} & &= 294\text{N} \cdot \sin(-110^\circ) \cdot 20\text{s} \\ &= -2011.1 \frac{\text{kgm}}{\text{s}} & &= -5525.4 \frac{\text{kgm}}{\text{s}} \end{aligned}$$

- b) The components of the impulse given by the friction force are

$$\begin{aligned} I_x &= F_x \Delta t & I_y &= F_y \Delta t \\ &= -70\text{N} \cdot 20\text{s} & &= 0\text{N} \cdot 20\text{s} \\ &= -1400 \frac{\text{kgm}}{\text{s}} & &= 0 \frac{\text{kgm}}{\text{s}} \end{aligned}$$

c) The components of the impulse given by Gilbert are

$$\begin{aligned} I_x &= F_x \Delta t & I_y &= F_y \Delta t \\ &= 200.55 N \cdot 20 s & &= 0 N \cdot 10 s \\ &= 4011.1 \frac{\text{kgm}}{s} & &= 0 \frac{\text{kgm}}{s} \end{aligned}$$

d) The components of the impulse given by the normal force are

$$\begin{aligned} I_x &= F_x \Delta t & I_y &= F_y \Delta t \\ &= 0 N \cdot 20 s & &= 276.27 N \cdot 20 s \\ &= 0 \frac{\text{kgm}}{s} & &= 5525.4 \frac{\text{kgm}}{s} \end{aligned}$$

e) The  $x$ -component of the net impulse is then

$$\begin{aligned} I_x &= -2011.1 \frac{\text{kgm}}{s} + -1400 \frac{\text{kgm}}{s} + 4011.1 \frac{\text{kgm}}{s} + 0 \frac{\text{kgm}}{s} \\ &= 600 \frac{\text{kgm}}{s} \end{aligned}$$

The  $y$ -component of the net impulse is then

$$\begin{aligned} I_y &= -5525.4 \frac{\text{kgm}}{s} + 0 \frac{\text{kgm}}{s} + 0 \frac{\text{kgm}}{s} + 5525.4 \frac{\text{kgm}}{s} \\ &= 0 \frac{\text{kgm}}{s} \end{aligned}$$

**3.** The components of the net impulse during the first 3 seconds are

$$\begin{aligned} I_x &= F_x \Delta t & I_y &= F_y \Delta t & I_z &= F_z \Delta t \\ &= 2 N \cdot 3 s & &= 1 N \cdot 3 s & &= -4 N \cdot 3 s \\ &= 6 \frac{\text{kgm}}{s} & &= 3 \frac{\text{kgm}}{s} & &= -12 \frac{\text{kgm}}{s} \end{aligned}$$

The components of the net impulse during the next 5 seconds are

$$\begin{aligned} I_x &= F_x \Delta t & I_y &= F_y \Delta t & I_z &= F_z \Delta t \\ &= -4 N \cdot 5 s & &= 5 N \cdot 5 s & &= 2 N \cdot 5 s \\ &= -20 \frac{\text{kgm}}{s} & &= 25 \frac{\text{kgm}}{s} & &= 10 \frac{\text{kgm}}{s} \end{aligned}$$

The components of the total net impulse are thus

$$\begin{aligned} I_{net\ x} &= 6 \frac{\text{kgm}}{s} - 20 \frac{\text{kgm}}{s} & I_{net\ y} &= 3 \frac{\text{kgm}}{s} + 25 \frac{\text{kgm}}{s} & I_{net\ z} &= -12 \frac{\text{kgm}}{s} + 10 \frac{\text{kgm}}{s} \\ &= -14 \frac{\text{kgm}}{s} & &= 28 \frac{\text{kgm}}{s} & &= -2 \frac{\text{kgm}}{s} \end{aligned}$$

- 4.** To find the impulse, the area under the curve from  $t = 0$  s and  $t = 8$  s must be found. First, there is a small triangle under the time axis. The area of this triangle is

$$\begin{aligned} Area_1 &= \frac{base \times height}{2} \\ &= \frac{2s \cdot 500N}{2} \\ &= 500 \frac{kgm}{s} \end{aligned}$$

As this triangle is under the time axis, the area corresponds to a negative impulse. Therefore  $I_{1x} = -500$  kgm/s.

Then, there is a large triangle above the time axis. The area of this triangle is

$$\begin{aligned} Area_2 &= \frac{base \times height}{2} \\ &= \frac{6s \cdot 2000N}{2} \\ &= 6000 \frac{kgm}{s} \end{aligned}$$

As this triangle is above the time axis, the area corresponds to a positive impulse. Therefore  $I_{2x} = 6000$  kgm/s.

The area on the small triangle to the left must not be counted as it is after  $t = 8$  s.

The total impulse is thus

$$I_{net\ x} = -500 \frac{kgm}{s} + 6000 \frac{kgm}{s} = 5500 \frac{kgm}{s}$$

- 5.** The impulse is

$$\begin{aligned}
 I_x &= \int_t^{t'} F_x dt \\
 &= \int_{0s}^{5s} 9 \frac{N}{s^2} \cdot t^2 dt \\
 &= \left[ 3 \frac{N}{s^2} \cdot t^3 \right]_{0s}^{5s} \\
 &= 3 \frac{N}{s^2} \cdot (5s)^3 - 3 \frac{N}{s^2} \cdot (0s)^3 \\
 &= 375 \frac{kgm}{s}
 \end{aligned}$$

**6.** The impulse given is

$$\begin{aligned}
 I_x &= F_x \Delta t \\
 &= 250N \cdot 20s \\
 &= 5000 \frac{kgm}{s}
 \end{aligned}$$

We thus have

$$\begin{aligned}
 I_{net\ x} &= \Delta p_x \\
 5000 \frac{kgm}{s} &= mv'_x - mv_x \\
 5000 \frac{kgm}{s} &= 2000kg \cdot v'_x - 2000kg \cdot \left(-5 \frac{m}{s}\right) \\
 v'_x &= -2.5 \frac{m}{s}
 \end{aligned}$$

**7.** Using an  $x$ -axis directed towards the motion of the arrow, the average force is

$$\begin{aligned}
 \bar{F}_x &= \frac{\Delta p_x}{\Delta t} \\
 &= \frac{mv'_x - mv_x}{\Delta t} \\
 &= \frac{0,1kg \cdot 150 \frac{m}{s} - 0,1kg \cdot 0 \frac{m}{s}}{0,05s} \\
 &= 300N
 \end{aligned}$$

**8.** Using an  $x$ -axis directed towards the motion of the bullet, the average force is

$$\begin{aligned}
 \bar{F}_x &= \frac{\Delta p_x}{\Delta t} \\
 &= \frac{mv'_x - mv_x}{\Delta t} \\
 &= \frac{0,02\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} - 0,02\text{kg} \cdot 900 \frac{\text{m}}{\text{s}}}{0,004\text{s}} \\
 &= -4500\text{N}
 \end{aligned}$$

The magnitude of the force is therefore 4500 N.

**9.** The average force on the car is

$$\begin{aligned}
 \bar{F}_x &= \frac{\Delta p_x}{\Delta t} \\
 &= \frac{mv'_x - mv_x}{\Delta t} \\
 &= \frac{1150\text{kg} \cdot 2.6 \frac{\text{m}}{\text{s}} - 1150\text{kg} \cdot (-15 \frac{\text{m}}{\text{s}})}{0.10\text{s}} \\
 &= 202\,400\text{N}
 \end{aligned}$$

As it is positive, the force exerted on the car by the wall is towards the right.

According to Newton's third law, the force exerted on the wall by the car is therefore 202 400 N towards the left.

**10.** Using an  $x$ -axis towards the right and a  $y$ -axis upwards, the  $x$ -component of the average force is

$$\begin{aligned}
 \bar{F}_x &= \frac{\Delta p_x}{\Delta t} \\
 &= \frac{mv'_x - mv_x}{\Delta t} \\
 &= \frac{0.05\text{kg} \cdot 8 \frac{\text{m}}{\text{s}} \cdot \cos(25^\circ) - 0.05\text{kg} \cdot 12 \frac{\text{m}}{\text{s}} \cdot \cos(-45^\circ)}{0.06\text{s}} \\
 &= -1.029\text{N}
 \end{aligned}$$

The  $y$ -component of the average force is

$$\begin{aligned}
 \bar{F}_y &= \frac{\Delta p_y}{\Delta t} \\
 &= \frac{mv'_y - mv_y}{\Delta t} \\
 &= \frac{0.05\text{kg} \cdot 8 \frac{\text{m}}{\text{s}} \cdot \sin(25^\circ) - 0.05\text{kg} \cdot 12 \frac{\text{m}}{\text{s}} \cdot \sin(-45^\circ)}{0.06\text{s}} \\
 &= 9.889\text{N}
 \end{aligned}$$

The magnitude of force is therefore

$$\begin{aligned}
 \bar{F} &= \sqrt{\bar{F}_x^2 + \bar{F}_y^2} \\
 &= \sqrt{(-1.029\text{N})^2 + (9.889\text{N})^2} \\
 &= 9.942\text{N}
 \end{aligned}$$

and its direction is

$$\begin{aligned}
 \theta &= \arctan \frac{\bar{F}_y}{\bar{F}_x} \\
 \theta &= \arctan \frac{9.889\text{N}}{-1.029\text{N}} \\
 \theta &= 95.94^\circ
 \end{aligned}$$

**11.** The average force is

$$\begin{aligned}
 \bar{F}_x &= \frac{\Delta p_x}{\Delta t} \\
 &= \frac{I_x}{\Delta t}
 \end{aligned}$$

The impulse is found by calculating the area under the curve. As it is a triangle, the area is

$$\begin{aligned}
 \text{Area} &= \frac{\text{base} \times \text{height}}{2} \\
 &= \frac{30\text{s} \cdot 1000\text{N}}{2} \\
 &= 15\,000 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

The average force is therefore

$$\begin{aligned}\bar{F}_x &= \frac{I_x}{\Delta t} \\ &= \frac{15\,000 \frac{\text{kgm}}{\text{s}}}{30\text{s}} \\ &= 500\text{N}\end{aligned}$$

- 12.** An  $x$ -axis in the direction of the movement of the ball is used here. In the absence of external forces, the momentum is conserved.

Momentum at Instant 1 (Before the throwing of the ball by Edward)

$$\begin{aligned}p_{\text{tot } x} &= m_{\text{ball}}v_{\text{ball}} + m_{\text{Ed}}v_{\text{Ed}} \\ &= 0.8\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} + 65\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} \\ &= 0\end{aligned}$$

Momentum at Instant 2 (After the throwing of the ball by Edward)

$$\begin{aligned}p'_{\text{tot } x} &= m_{\text{ball}}v'_{\text{ball}} + m_{\text{Ed}}v'_{\text{Ed}} \\ &= 0.8\text{kg} \cdot 20 \frac{\text{m}}{\text{s}} + 65\text{kg} \cdot v'_{\text{Ed}} \\ &= 16 \frac{\text{kgm}}{\text{s}} + 65\text{kg} \cdot v'_{\text{Ed}}\end{aligned}$$

Momentum Conservation

$$\begin{aligned}p_{\text{tot } x} &= p'_{\text{tot } x} \\ 0 &= 16 \frac{\text{kgm}}{\text{s}} + 65\text{kg} \cdot v'_{\text{Ed}} \\ v'_{\text{Ed}} &= -0.246 \frac{\text{m}}{\text{s}}\end{aligned}$$

Edward thus goes at 0.246 m/s in the direction opposite to the movement of the ball.

- 13.** An  $x$ -axis in the direction of the movement of the ball is used here. In the absence of external forces, the momentum is conserved.

Momentum at Instant 1 (Before Marie-Sophie catches the ball)

$$\begin{aligned}p_{\text{tot } x} &= m_{\text{MS}}v_{\text{MS}} + m_{\text{ballon}}v_{\text{ballon}} \\ &= 60\text{kg} \cdot 0 \frac{\text{km}}{\text{h}} + 15\text{kg} \cdot 20 \frac{\text{km}}{\text{h}} \\ &= 15\text{kg} \cdot 20 \frac{\text{km}}{\text{h}}\end{aligned}$$

Momentum at Instant 2 (After Maris-Sophie has caught the ball)

$$\begin{aligned}
 p'_{tot\ x} &= m_{MS}v'_{MS} + m_{ballon}v'_{ballon} \\
 &= 60\text{kg} \cdot v' + 15\text{kg} \cdot v' \\
 &= 75\text{kg} \cdot v'
 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 P_{tot\ x} &= p'_{tot\ x} \\
 15\text{kg} \cdot 20 \frac{\text{km}}{\text{h}} &= 75\text{kg} \cdot v' \\
 v' &= 4 \frac{\text{km}}{\text{h}}
 \end{aligned}$$

Marie-Sophie (with the ball in her hands) thus goes at 4 km/h towards the right.

**14. First step: Yuri launches the ball.**

An  $x$ -axis in the direction of the movement of the ball is used here. In the absence of external forces, the momentum is conserved.

Momentum at Instant 1 (Before Youri throws the tank)

$$\begin{aligned}
 P_{tot\ x} &= m_{Youri}v_{Youri} + m_{tank}v_{tank} \\
 &= 80\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} + 20\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} \\
 &= 0
 \end{aligned}$$

Momentum at Instant 2 (After Youru has thrown the tank)

$$\begin{aligned}
 p'_{tot\ x} &= m_{Youri}v'_{Youri} + m_{tank}v'_{tank} \\
 &= 80\text{kg} \cdot v'_{Youri} + 20\text{kg} \cdot 5 \frac{\text{m}}{\text{s}} \\
 &= 80\text{kg} \cdot v'_{Youri} + 100 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 P_{tot\ x} &= p'_{tot\ x} \\
 0 &= 80\text{kg} \cdot v'_{Youri} + 100 \frac{\text{kgm}}{\text{s}} \\
 v'_{Youri} &= -1.25 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

Yuri thus goes at 1.25 m/s towards the left.

**Second step: Valentina catches the ball.**

In the absence of external force, the momentum is conserved

Momentum at Instant 1 (Before Valentina catches the tank)

$$\begin{aligned} p_{tot\ x} &= m_{Valentina} v_{Valentina} + m_{tank} v_{tank} \\ &= 70\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} + 20\text{kg} \cdot 5 \frac{\text{m}}{\text{s}} \\ &= 100 \frac{\text{kgm}}{\text{s}} \end{aligned}$$

Momentum at Instant 2 (After Valentina has caught the tank)

$$\begin{aligned} p'_{tot\ x} &= m_{Valentina} v'_{Valentina} + m_{tank} v_{tank} \\ &= 70\text{kg} \cdot v' + 20\text{kg} \cdot v' \\ &= 90\text{kg} \cdot v' \end{aligned}$$

Momentum Conservation

$$\begin{aligned} p_{tot\ x} &= p'_{tot\ x} \\ 100 \frac{\text{kgm}}{\text{s}} &= 90\text{kg} \cdot v' \\ v' &= 1.111 \frac{\text{m}}{\text{s}} \end{aligned}$$

Valentina (with the ball in the hands) thus goes at 1.111 m/s towards the right.

- 15.** An  $x$ -axis towards the right is used here. In the absence of external forces, the momentum is conserved.

Momentum at Instant 1 (Before Helmut pushes Brunnehilde)

$$\begin{aligned} p_{tot\ x} &= m_{Bru} v_{Bru} + m_{Hel} v_{Hel} \\ &= 25\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} + 70\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} \\ &= 0 \end{aligned}$$

Momentum at Instant 2 (After Helmut has pushed Brunnehilde)

$$\begin{aligned} p'_{tot\ x} &= m_{Bru} v'_{Bru} + m_{Hel} v'_{Hel} \\ &= 25\text{kg} \cdot 10 \frac{\text{m}}{\text{s}} + 70\text{kg} \cdot v'_{Hel} \\ &= 250 \frac{\text{kgm}}{\text{s}} + 70\text{kg} \cdot v'_{Hel} \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 0 &= 250 \frac{\text{kgm}}{\text{s}} + 70\text{kg} \cdot v'_{Hel} \\
 v'_{Hel} &= -3.571 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

Helmut thus travels at 3.571 m/s towards the left.

**16.** The time of arrival at the other end will be given by

$$t = \frac{L}{v_1 - v_2}$$

(time for Macpherson and the end of the buche to meet.) To find that time, we need the speed of the log. This speed is found with momentum conservation (which is constant here since there is no external force.)

An  $x$ -axis towards the right is used here.

Momentum at Instant 1 (MacPherson is not walking)

$$\begin{aligned}
 p_{tot\ x} &= m_{Mac} v_{Mac} + m_{log} v_{log} \\
 &= 94\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} + 345\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} \\
 &= 0
 \end{aligned}$$

Momentum at Instant 2 (MacPherson is now walking)

$$\begin{aligned}
 p'_{tot\ x} &= m_{Mac} v'_{Mac} + m_{log} v'_{log} \\
 &= 94\text{kg} \cdot 2.7 \frac{\text{m}}{\text{s}} + 345\text{kg} \cdot v'_{log} \\
 &= 253.8 \frac{\text{kgm}}{\text{s}} + 345\text{kg} \cdot v'_{log}
 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 0 &= 253.8 \frac{\text{kgm}}{\text{s}} + 345\text{kg} \cdot v'_{log} \\
 v'_{log} &= -0.736 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

The log therefore goes at 0.736 m/s towards the left.

The time required to reach the other end is therefore

$$\begin{aligned} t &= \frac{L}{v_1 - v_2} \\ &= \frac{5m}{2.7 \frac{m}{s} - -0.736 \frac{m}{s}} \\ &= 1.455s \end{aligned}$$

- 17.** An  $x$ -axis in the direction of the movement of the cannonballs is used here. In the absence of external forces, the momentum is conserved.

Momentum at Instant 1 (Before the firing)

$$\begin{aligned} p_{tot\ x} &= m_{ship}v_{ship} + m_{balls}v_{balls} \\ &= 1\,200\,000kg \cdot 0 \frac{m}{s} + 180kg \cdot 0 \frac{m}{s} \\ &= 0 \end{aligned}$$

Momentum at Instant 2 (After the firing)

$$\begin{aligned} p_{tot\ x} &= m_{ship}v'_{ship} + m_{balls}v'_{balls} \\ &= 1\,200\,000kg \cdot v'_{navire} + 180kg \cdot 425 \frac{m}{s} \\ &= 1\,200\,000kg \cdot v'_{navire} + 76\,500 \frac{kgm}{s} \end{aligned}$$

Momentum Conservation

$$\begin{aligned} p_{tot\ x} &= p'_{tot\ x} \\ 0 &= 1\,200\,000kg \cdot v'_{ship} + 76\,500 \frac{kgm}{s} \\ v'_{ship} &= -0.06375 \frac{m}{s} \end{aligned}$$

The ship will therefore move at 0.06375 m/s in the direction opposite to the movement of the cannonballs.

- 18.** An  $x$ -axis towards the right and a  $y$ -axis upwards is used here. In the absence of external forces, the momentum is conserved.

**x-component**

Momentum at Instant 1 (Before the explosion)

$$\begin{aligned}
 P_{tot\ x} &= m_{ball} v_{ball\ x} \\
 &= 10\text{kg} \cdot v_{ball\ x}
 \end{aligned}$$

Momentum at Instant 2 (After the explosion)

$$\begin{aligned}
 p'_{tot\ x} &= m_1 v'_{1x} + m_2 v'_{2x} + m_3 v'_{3x} \\
 &= 2\text{kg} \cdot 80 \frac{\text{m}}{\text{s}} \cdot \cos(80^\circ) + 5\text{kg} \cdot 50 \frac{\text{m}}{\text{s}} \cdot \cos(-60^\circ) + 3\text{kg} \cdot 60 \frac{\text{m}}{\text{s}} \cdot \cos(220^\circ) \\
 &= 14.896 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 P_{tot\ x} &= p'_{tot\ x} \\
 10\text{kg} \cdot v_{ball\ x} &= 14.896 \frac{\text{kgm}}{\text{s}} \\
 v_{ball\ x} &= 1.4896 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

**For the y-component**

Momentum at Instant 1 (Before the explosion)

$$\begin{aligned}
 P_{tot\ y} &= m_{ball} \cdot v_{ball\ y} \\
 &= 10\text{kg} \cdot v_{ball\ y}
 \end{aligned}$$

Momentum at Instant 2 (After the explosion)

$$\begin{aligned}
 p'_{tot\ y} &= m_1 v'_{1y} + m_2 v'_{2y} + m_3 v'_{3y} \\
 &= 2\text{kg} \cdot 80 \frac{\text{m}}{\text{s}} \cdot \sin(80^\circ) + 5\text{kg} \cdot 50 \frac{\text{m}}{\text{s}} \cdot \sin(-60^\circ) + 3\text{kg} \cdot 60 \frac{\text{m}}{\text{s}} \cdot \sin(220^\circ) \\
 &= -174.639 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 P_{tot\ y} &= p'_{tot\ y} \\
 10\text{kg} \cdot v_{ball\ y} &= -174.639 \frac{\text{kgm}}{\text{s}} \\
 v_{ball\ y} &= -17.4639 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

**Velocity of the cannonball**

The speed is

$$\begin{aligned}
 v &= \sqrt{v_x^2 + v_y^2} \\
 &= \sqrt{\left(1.4896 \frac{\text{m}}{\text{s}}\right)^2 + \left(-17.4639 \frac{\text{m}}{\text{s}}\right)^2} \\
 &= 17.53 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

and the direction of the velocity is

$$\begin{aligned}
 \theta &= \arctan \frac{v_y}{v_x} \\
 &= \arctan \frac{-17.4639 \frac{\text{m}}{\text{s}}}{1.4896 \frac{\text{m}}{\text{s}}} \\
 &= -85,1^\circ
 \end{aligned}$$

- 19.** An  $x$ -axis in the direction of the movement of the alpha particle is used here. In the absence of external forces, the momentum is conserved.

Momentum at Instant 1 (Before the decay)

$$\begin{aligned}
 P_{\text{tot } x} &= m_{\text{initial nucleus}} v_{\text{initial nucleus}} \\
 &= 0
 \end{aligned}$$

Momentum at Instant 2 (After the decay)

$$\begin{aligned}
 p'_{\text{tot } x} &= m_\alpha v'_\alpha + m_{\text{nucleus}} v'_{\text{nucleus}} \\
 &= 6.64 \times 10^{-27} \text{ kg} \cdot v'_\alpha + 388.6 \times 10^{-27} \text{ kg} \cdot v'_{\text{nucleus}}
 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 p_{\text{tot } x} &= p'_{\text{tot } x} \\
 0 &= 6.64 \times 10^{-27} \text{ kg} \cdot v'_\alpha + 388.6 \times 10^{-27} \text{ kg} \cdot v'_{\text{nucleus}} \\
 0 &= 6.64 \cdot v'_\alpha + 388.6 \cdot v'_{\text{nucleus}}
 \end{aligned}$$

To solve, you must use the fact that the sum of the kinetic energies is  $1.3 \times 10^{-13} \text{ J}$ . Therefore

$$\begin{aligned}
 E'_{k\alpha} + E'_{k\text{nucleus}} &= 1,3 \times 10^{-13} \text{ J} \\
 \frac{1}{2} \cdot 6.64 \times 10^{-27} \text{ kg} \cdot (v'_\alpha)^2 + \frac{1}{2} \cdot 388.6 \times 10^{-27} \text{ kg} \cdot (v'_{\text{nucleus}})^2 &= 1.3 \times 10^{-13} \text{ J} \\
 6.64 \cdot (v'_\alpha)^2 + 388.6 \cdot (v'_{\text{nucleus}})^2 &= 2.6 \times 10^{14} \frac{\text{m}^2}{\text{s}^2}
 \end{aligned}$$

We now have two equations and two unknowns.

We first solve for the speed of the alpha particle in the momentum equation

$$\begin{aligned}0 \frac{\text{kgm}}{\text{s}} &= 6.64 \cdot v'_{\alpha} + 388.6 \cdot v'_{\text{nucleus}} \\6.64 \cdot v'_{\alpha} &= -388.6 \cdot v'_{\text{nucleus}} \\v'_{\alpha} &= -58.52 \cdot v'_{\text{nucleus}}\end{aligned}$$

and substitute this value in the kinetic energy equation.

$$\begin{aligned}6.64 \cdot (v'_{\alpha})^2 + 388.6 \cdot (v'_{\text{nucleus}})^2 &= 2.6 \times 10^{14} \frac{\text{m}^2}{\text{s}^2} \\6.64 \cdot (-58.52 \cdot v'_{\text{nucleus}})^2 + 388.6 \cdot (v'_{\text{nucleus}})^2 &= 2.6 \times 10^{14} \frac{\text{m}^2}{\text{s}^2} \\22,742 \cdot (v'_{\text{nucleus}})^2 + 388.6 \cdot (v'_{\text{nucleus}})^2 &= 2.6 \times 10^{14} \frac{\text{m}^2}{\text{s}^2} \\23,131 \cdot (v'_{\text{nucleus}})^2 &= 2.6 \times 10^{14} \frac{\text{m}^2}{\text{s}^2} \\v'_{\text{nucleus}} &= \pm 1.06 \times 10^5 \frac{\text{m}}{\text{s}}\end{aligned}$$

According to the figure, it is quite clear that the negative answer is the right answer.

$$v'_{\text{nucleus}} = -1.06 \times 10^5 \frac{\text{m}}{\text{s}}$$

The other speed is therefore

$$\begin{aligned}v'_{\alpha} &= -58.52 \cdot v'_{\text{nucleus}} \\&= 6.20 \times 10^6 \frac{\text{m}}{\text{s}}\end{aligned}$$

**20.** An  $x$ -axis towards the right is used here. In the absence of external forces, the momentum is conserved.

Momentum at Instant 1 (Before Léon fires)

$$\begin{aligned}p_{\text{tot } x} &= m_{\text{carriage}} v_{\text{carriage } x} + m_{\text{ball}} v_{\text{ball } x} \\&= 150 \text{kg} \cdot 0 \frac{\text{m}}{\text{s}} + 0.03 \text{kg} \cdot 0 \frac{\text{m}}{\text{s}} \\&= 0\end{aligned}$$

Momentum at Instant 2 (After Léon has fired)

$$\begin{aligned}
 p'_{tot\ x} &= m_{\text{carriage}} v'_{\text{carriage}\ x} + m_{\text{ball}} v'_{\text{ball}\ x} \\
 &= 150\text{kg} \cdot v'_{\text{carriage}} + 0.03\text{kg} \cdot 900 \frac{\text{m}}{\text{s}} \cdot \cos 30^\circ \\
 &= 150\text{kg} \cdot v'_{\text{carriage}} + 23.383 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 0 &= 150\text{kg} \cdot v'_{\text{carriage}} + 23.383 \frac{\text{kgm}}{\text{s}} \\
 v'_{\text{carriage}} &= -0.1559 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

The trolley therefore moves at 0.1559 m/s towards the left.

- 21.** An  $x$ -axis towards the right is used here. In a collision, the momentum is conserved. Therefore

Momentum at Instant 1 (Before the collision)

$$\begin{aligned}
 p_{tot\ x} &= m_1 v_1 + m_2 v_2 \\
 &= 10\ 000\text{kg} \cdot 20 \frac{\text{m}}{\text{s}} + 20\ 000\text{kg} \cdot 2 \frac{\text{m}}{\text{s}} \\
 &= 240\ 000 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

Momentum at Instant 2 (After the collision)

$$\begin{aligned}
 p_{tot\ x} &= m_{tot} v'_1 \\
 &= 30\ 000\text{kg} \cdot v'
 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 240\ 000 \frac{\text{kgm}}{\text{s}} &= 30\ 000\text{kg} \cdot v' \\
 v' &= 8 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

- 22.** An  $x$ -axis towards the right is used here. In a collision, the momentum is conserved.

Momentum at Instant 1 (Before the collision)

$$\begin{aligned}
 p_{tot\ x} &= m_{Fabrice}v_{Fabrice} + m_{Raphael}v_{Raphael} \\
 &= 80\text{kg} \cdot 5\frac{\text{m}}{\text{s}} + 110\text{kg} \cdot \left(-4\frac{\text{m}}{\text{s}}\right) \\
 &= -40\frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

Momentum at Instant 2 (After the collision)

$$\begin{aligned}
 p'_{tot\ x} &= m_{Fabrice}v'_{Fabrice} + m_{Raphael}v'_{Raphael} \\
 &= 80\text{kg} \cdot v' + 110\text{kg} \cdot v' \\
 &= 190\text{kg} \cdot v'
 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 -40\frac{\text{kgm}}{\text{s}} &= 190\text{kg} \cdot v' \\
 v' &= -0.2105\frac{\text{m}}{\text{s}}
 \end{aligned}$$

**23.** An  $x$ -axis towards the right is used here. In a collision, the momentum is conserved.

Momentum at Instant 1 (Before the collision)

$$\begin{aligned}
 p_{tot\ x} &= m_1v_1 + m_2v_2 \\
 &= 5\text{kg} \cdot 10\frac{\text{m}}{\text{s}} + m_2 \cdot \left(-2\frac{\text{m}}{\text{s}}\right) \\
 &= 50\frac{\text{kgm}}{\text{s}} - m_2 \cdot 2\frac{\text{m}}{\text{s}}
 \end{aligned}$$

Momentum at Instant 2 (After the collision)

$$\begin{aligned}
 p'_{tot\ x} &= m_1v'_1 + m_2v'_2 \\
 &= 5\text{kg} \cdot \left(-1\frac{\text{m}}{\text{s}}\right) + m_2 \cdot \left(-1\frac{\text{m}}{\text{s}}\right) \\
 &= -5\frac{\text{kgm}}{\text{s}} - m_2 \cdot 1\frac{\text{m}}{\text{s}}
 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 50\frac{\text{kgm}}{\text{s}} - m_2 \cdot 2\frac{\text{m}}{\text{s}} &= -5\frac{\text{kgm}}{\text{s}} - m_2 \cdot 1\frac{\text{m}}{\text{s}} \\
 55\frac{\text{kgm}}{\text{s}} &= m_2 \cdot 1\frac{\text{m}}{\text{s}} \\
 m_2 &= 55\text{kg}
 \end{aligned}$$

## 24. First Collision

An  $x$ -axis towards the right is used here. In a collision, the momentum is conserved.

Momentum at Instant 1 (Before the first collision)

$$\begin{aligned} p_{tot\ x} &= m_1v_1 + m_2v_2 \\ &= 2\text{kg} \cdot 12\frac{\text{m}}{\text{s}} + 1\text{kg} \cdot 0\frac{\text{m}}{\text{s}} \\ &= 24\frac{\text{kgm}}{\text{s}} \end{aligned}$$

Momentum at Instant 2 (After the first collision)

$$\begin{aligned} p'_{tot\ x} &= m_1v'_1 + m_2v'_2 \\ &= 2\text{kg} \cdot v_2 + 1\text{kg} \cdot v_2 \\ &= 3\text{kg} \cdot v_2 \end{aligned}$$

Momentum Conservation

$$\begin{aligned} p_{tot\ x} &= p'_{tot\ x} \\ 24\frac{\text{kgm}}{\text{s}} &= 3\text{kg} \cdot v_2 \\ v_2 &= 8\frac{\text{m}}{\text{s}} \end{aligned}$$

## Second Collision

Momentum at Instant 1 (Before the second collision)

$$\begin{aligned} p_{tot\ x} &= m_1v_1 + m_2v_2 \\ &= 3\text{kg} \cdot 8\frac{\text{m}}{\text{s}} + 3\text{kg} \cdot 0\frac{\text{m}}{\text{s}} \\ &= 24\frac{\text{kgm}}{\text{s}} \end{aligned}$$

Momentum at Instant 2 (After the second collision)

$$\begin{aligned} p'_{tot\ x} &= m_1v'_1 + m_2v'_2 \\ &= 3\text{kg} \cdot v_3 + 3\text{kg} \cdot v_3 \\ &= 6\text{kg} \cdot v_3 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 24 \frac{\text{kgm}}{\text{s}} &= 6\text{kg} \cdot v_3 \\
 v_3 &= 4 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

## 25. Collision

An  $x$ -axis towards the right is used here. In a collision, the momentum is conserved. Therefore

Momentum at Instant 1 (Before the collision)

$$\begin{aligned}
 p_{tot\ x} &= m_{\text{bullet}} v_{\text{bullet}} + m_{\text{block}} v_{\text{block}} \\
 &= 0.02\text{kg} \cdot 800 \frac{\text{m}}{\text{s}} + 2\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} \\
 &= 16 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

Momentum at Instant 2 (Right after the collision)

$$\begin{aligned}
 p_{tot\ x} &= m_{tot} v'_{tot} \\
 &= 2.02\text{kg} \cdot v'
 \end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 16 \frac{\text{kgm}}{\text{s}} &= 2.02\text{kg} \cdot v' \\
 v' &= 7.921 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

### Rise of the Pendulum

We then have the rise of the pendulum. The maximum height reached by the block can be found with the conservation of mechanical energy. As the system is composed of a single object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

Mechanical Energy at Instant 2 (pendulum at its lowest point, immediately after the collision)

$$\begin{aligned}
 E_{mec} &= \frac{1}{2}mv^2 + mgy \\
 &= \frac{1}{2} \cdot 2.02\text{kg} \cdot \left(7.921 \frac{\text{m}}{\text{s}}\right)^2 + 0 \\
 &= 63.37\text{J}
 \end{aligned}$$

We have chosen to put the  $y = 0$  at the lowest point of the pendulum.

Mechanical Energy at Instant 3 (pendulum at its highest point)

$$\begin{aligned}
 E_{mec} &= \frac{1}{2}mv^2 + mgy \\
 &= 0\text{J} + 2.02\text{kg} \cdot 9.8 \frac{\text{N}}{\text{kg}} \cdot y \\
 &= 19.796\text{N} \cdot y
 \end{aligned}$$

Conservation of mechanical energy

$$\begin{aligned}
 E &= E' \\
 63.37\text{J} &= 19.796\text{N} \cdot y \\
 y &= 3.201\text{m}
 \end{aligned}$$

The angle is thus

$$\begin{aligned}
 \cos \theta &= \frac{L - y}{L} \\
 \cos \theta &= \frac{8\text{m} - 3.201\text{m}}{8\text{m}} \\
 \theta &= 53.1^\circ
 \end{aligned}$$

**26.** An  $x$ -axis towards the right is used here. In a collision, the momentum is conserved.

Momentum at Instant 1 (Before the collision)

$$\begin{aligned}
 p_{tot\ x} &= m_{ball}v_{ball} + m_{carriage}v_{carriage} \\
 &= 10\text{kg} \cdot 8 \frac{\text{m}}{\text{s}} \cdot \cos(-70^\circ) + 200\text{kg} \cdot 4 \frac{\text{m}}{\text{s}} \\
 &= 827.36 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

Momentum at Instant 2 (Right after the collision)

$$\begin{aligned} p'_{tot\ x} &= m_{tot} v'_{tot} \\ &= 210\text{kg} \cdot v' \end{aligned}$$

Momentum Conservation

$$\begin{aligned} p_{tot\ x} &= p'_{tot\ x} \\ 827.36 \frac{\text{kgm}}{\text{s}} &= 210\text{kg} \cdot v' \\ v' &= 3.94 \frac{\text{m}}{\text{s}} \end{aligned}$$

The carriage therefore moves at 3.94 m/s towards the right.

- 27.** An  $x$ -axis towards the right is used here. In a collision, the momentum is conserved. Therefore

$$\begin{aligned} p_{tot\ x} &= p'_{tot\ x} \\ m_1 v_1 + m_2 v_2 &= m_1 v'_1 + m_2 v'_2 \\ 0.2\text{kg} \cdot 10 \frac{\text{m}}{\text{s}} + 0.55\text{kg} \cdot \left(-4 \frac{\text{m}}{\text{s}}\right) &= 0.2\text{kg} \cdot v'_1 + 0.55\text{kg} \cdot v'_2 \\ -0.2 \frac{\text{kgm}}{\text{s}} &= 0.2\text{kg} \cdot v'_1 + 0.55\text{kg} \cdot v'_2 \\ -1 \frac{\text{m}}{\text{s}} &= v'_1 + 2.75 \cdot v'_2 \end{aligned}$$

In an elastic collision, kinetic energy is also conserved.

$$\begin{aligned} E_{k\ tot} &= E'_{k\ tot} \\ \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 &= \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2 \\ \frac{1}{2} \cdot 0.2\text{kg} \cdot \left(10 \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2} \cdot 0.55\text{kg} \cdot \left(-4 \frac{\text{m}}{\text{s}}\right)^2 &= \frac{1}{2} \cdot 0.2\text{kg} \cdot v'^2_1 + \frac{1}{2} \cdot 0.55\text{kg} \cdot v'^2_2 \\ 14.4\text{J} &= \frac{1}{2} 0.2\text{kg} \cdot v'^2_1 + \frac{1}{2} 0.55\text{kg} \cdot v'^2_2 \\ 144 \frac{\text{m}^2}{\text{s}^2} &= v'^2_1 + 2.75 \cdot v'^2_2 \end{aligned}$$

We can solve for  $v_1$  in the first equation

$$v'_1 = -1 \frac{\text{m}}{\text{s}} - 2.75 \cdot v'_2$$

And substitute in the second equation to get

$$\begin{aligned}
 144 \frac{m^2}{s^2} &= \left(-1 \frac{m}{s} - 2.75 \cdot v_2'\right)^2 + 2.75 \cdot v_2'^2 \\
 144 \frac{m^2}{s^2} &= 1 \frac{m^2}{s^2} + 5.5 \frac{m}{s} \cdot v_2' + 7.5625 \cdot v_2'^2 + 2.75 \cdot v_2'^2 \\
 0 &= -143 \frac{m^2}{s^2} + 5.5 \frac{m}{s} \cdot v_2' + 10.3125 \cdot v_2'^2 \\
 v_2' &= -4 \frac{m}{s} \quad \text{and} \quad v_2' = 3.467 \frac{m}{s}
 \end{aligned}$$

The first solution is obviously the speed before the collision. The other solution is the speed after the collision. Thus

$$v_2' = 3.467 \frac{m}{s}$$

The speed of the other ball is

$$\begin{aligned}
 v_1' &= -1 \frac{m}{s} - 2.75 \cdot v_2' \\
 &= -10.53 \frac{m}{s}
 \end{aligned}$$

## 28. a)

An  $x$ -axis towards the right is used here. In a collision, the momentum is conserved. Therefore

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 m_A v_A + m_B v_B &= m_A v_A' + m_B v_B' \\
 500\text{kg} \cdot 20 \frac{m}{s} + 250\text{kg} \cdot 10 \frac{m}{s} &= 500\text{kg} \cdot v_1' + 250\text{kg} \cdot 12 \frac{m}{s} \\
 v_1' &= 19 \frac{m}{s}
 \end{aligned}$$

b) The kinetic energy before the collision is

$$\begin{aligned}
 E_k &= \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \\
 &= \frac{1}{2} \cdot 500\text{kg} \cdot \left(20 \frac{m}{s}\right)^2 + \frac{1}{2} \cdot 250\text{kg} \cdot \left(10 \frac{m}{s}\right)^2 \\
 &= 112\ 500\text{J}
 \end{aligned}$$

The kinetic energy after the collision is

$$\begin{aligned}
 E_k' &= \frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_B v_B'^2 \\
 &= \frac{1}{2} \cdot 500\text{kg} \cdot \left(19 \frac{m}{s}\right)^2 + \frac{1}{2} \cdot 250\text{kg} \cdot \left(12 \frac{m}{s}\right)^2 \\
 &= 108\ 250\text{J}
 \end{aligned}$$

The change of kinetic energy is thus

$$\begin{aligned}\Delta E_k &= 108\,250J - 112\,500J \\ &= -4250J\end{aligned}$$

Therefore, the fraction lost is

$$\frac{-4250J}{112\,500J} = -0.0378$$

Therefore, 3.78% of the kinetic energy is lost.

c) The change of momentum is

$$\begin{aligned}\Delta p_x &= p'_x - p_x \\ &= 500kg \cdot 19 \frac{m}{s} - 500kg \cdot 20 \frac{m}{s} \\ &= -500 \frac{kgm}{s}\end{aligned}$$

d) As momentum of the system is conserved and the 500 kg asteroid lost 500 kgm/s, then the momentum of the 250 kg asteroid must increase by 500 kgm/s. You can easily check this with

$$\begin{aligned}\Delta p_x &= p'_x - p_x \\ &= 250kg \cdot 12 \frac{m}{s} - 250kg \cdot 10 \frac{m}{s} \\ &= 500 \frac{kgm}{s}\end{aligned}$$

**29.** An  $x$ -axis towards the right is used here. In a collision, the momentum is conserved.

Momentum at Instant 1 (Before the collision)

$$\begin{aligned}p_{tot\ x} &= m_{bullet}v_{bullet} + m_{block}v_{block} \\ &= 0.004kg \cdot 750 \frac{m}{s} + 1.15kg \cdot 0 \frac{m}{s} \\ &= 3 \frac{kgm}{s}\end{aligned}$$

Momentum at Instant 2 (Right after the collision)

$$\begin{aligned}p'_{tot\ x} &= m_{bullet}v'_{bullet} + m_{block}v'_{block\ x} \\ &= 0.004kg \cdot 320 \frac{m}{s} + 1.15kg \cdot v'_{block} \\ &= 1.28 \frac{kgm}{s} + 1.15kg \cdot v'_{block}\end{aligned}$$

Momentum Conservation

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 3 \frac{\text{kgm}}{\text{s}} &= 1.28 \frac{\text{kgm}}{\text{s}} + 1.15\text{kg} \cdot v'_{block} \\
 v'_{block} &= 1.496 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

**30. First Part: The Descent of the Block**

The speed of the block at the end of its descent will be found with the conservation of mechanical energy. As the system is composed of a single object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

Energy at Instant 1 (configuration shown in the figure)

The mechanical energy is

$$\begin{aligned}
 E &= \frac{1}{2}mv^2 + mgy \\
 &= 0\text{J} + 2\text{kg} \cdot 9.8 \frac{\text{N}}{\text{kg}} \cdot 4\text{m} \\
 &= 78.4\text{J}
 \end{aligned}$$

We have chosen to set the origin  $y = 0$  at the level of the horizontal surface.

Energy at Instant 2 (just before the collision with the 3 kg block)

The mechanical energy is

$$\begin{aligned}
 E' &= \frac{1}{2}mv'^2 + mgy' \\
 &= \frac{1}{2} \cdot 2\text{kg} \cdot v'^2 + 0 \\
 &= 1\text{kg} \cdot v'^2
 \end{aligned}$$

Conservation of mechanical energy

$$E = E'$$

$$78.4J = 1kg \cdot v'^2$$

$$v' = 8.854 \frac{m}{s}$$

### Second Part: The Collision

In a collision, momentum is conserved.

#### Momentum at Instant 1 (Before the collision)

$$p_{tot\ x} = m_{block\ 1}v_{block\ 1} + m_{block\ 2}v_{block\ 2}$$

$$= 2kg \cdot 8.854 \frac{m}{s} + 3kg \cdot 0 \frac{m}{s}$$

$$= 17.709 \frac{kgm}{s}$$

#### Momentum at Instant 2 (After the collision)

$$p'_{tot\ x} = m_{block\ 1}v'_{block\ 1} + m_{block\ 2}v'_{block\ 2}$$

$$= 2kg \cdot v' + 3kg \cdot v'$$

$$= 5kg \cdot v'$$

#### Momentum Conservation

$$p_{tot\ x} = p'_{tot\ x}$$

$$17.709 \frac{m}{s} = 5kg \cdot v'$$

$$v' = 3.542 \frac{m}{s}$$

This is the speed of the blocks after the collision.

### Third Part: The Sliding

According to the sum of the  $x$ -components of the force, the acceleration is

$$\sum F_x = ma_x$$

$$-F_f = ma_x$$

$$-\mu_k F_N = ma_x$$

$$-\mu_k mg = ma_x$$

$$a_x = -\mu_k g$$

$$a_x = -2.45 \frac{m}{s^2}$$

With an initial velocity of 3.542 m/s, the stopping distance is

$$2a_x(x - x_0) = v_x^2 - v_{0x}^2$$

$$2 \cdot \left(-2.45 \frac{m}{s^2}\right) \cdot (x - 0m) = \left(0 \frac{m}{s}\right)^2 - \left(3.542 \frac{m}{s}\right)^2$$

$$x = 2.56m$$

**31.** An  $x$ -axis towards the right is used here.

Momentum conservation gives

$$p_{tot\ x} = p'_{tot\ x}$$

$$m_1v_1 + m_2v_2 = m_1v'_1 + m_2v'_2$$

$$0.2kg \cdot 10 \frac{m}{s} + 0.55kg \cdot \left(-4 \frac{m}{s}\right) = 0.2kg \cdot v'_1 + 0.55kg \cdot v'_2$$

$$-0.2 \frac{kgm}{s} = 0.2kg \cdot v'_1 + 0.55kg \cdot v'_2$$

$$-1 \frac{m}{s} = v'_1 + 2.75 \cdot v'_2$$

The initial kinetic energy is

$$E_k = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2} \cdot 0.2kg \cdot \left(10 \frac{m}{s}\right)^2 + \frac{1}{2} \cdot 0.55kg \cdot \left(4 \frac{m}{s}\right)^2$$

$$= 14.4J$$

Since 40% of the kinetic energy is lost in the collision, 60% of energy remains after the collision. The kinetic energy after the collision is therefore of 8.64 J. Thus,

$$E'_k = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

$$8.64J = \frac{1}{2} \cdot 0.2kg \cdot v_1'^2 + \frac{1}{2} \cdot 0.55kg \cdot v_2'^2$$

$$8.64J = 0.1kg \cdot v_1'^2 + 0.275kg \cdot v_2'^2$$

$$86.4 \frac{m^2}{s^2} = v_1'^2 + 2.75v_2'^2$$

We can solve for  $v_1$  in the first equation (momentum conservation)

$$v'_1 = -1 \frac{m}{s} - 2.75 \cdot v'_2$$

and substitute in the second equation (tootal energy after the collision) to get

$$\begin{aligned}
 86.4 \frac{m^2}{s^2} &= \left(-1 \frac{m}{s} - 2.75 \cdot v_2'\right)^2 + 2.75 \cdot v_2'^2 \\
 86.4 \frac{m^2}{s^2} &= 1 \frac{m^2}{s^2} + 5.5 \frac{m}{s} \cdot v_2' + 7,5625 \cdot v_2'^2 + 2.5 \cdot v_2'^2 \\
 0 &= -85.4 \frac{m^2}{s^2} + 5.5 \frac{m}{s} \cdot v_2' + 10.3125 \cdot v_2'^2
 \end{aligned}$$

This quadratic equation can be solved to obtain

$$v_2' = 2.623 \frac{m}{s} \quad \text{and} \quad v_2' = -3.157 \frac{m}{s}$$

The velocity of the other ball is then calculated with  $-1 \frac{m}{s} = v_1' + 2.75v_2'$ . The results are

$$\begin{aligned}
 v_1' &= -8.214 \frac{m}{s} & \text{and} & & v_1' &= 7.681 \frac{m}{s} \\
 v_2' &= 2.623 \frac{m}{s} & & & v_2' &= -3.157 \frac{m}{s}
 \end{aligned}$$

In these two solutions, there is one (the second) where the signs of the velocities are identical to the signs before the collision. As it is impossible to have velocities after the collision that are both in the same direction as the initial velocities, this solution must be rejected.

Thus the answer is

$$\begin{aligned}
 v_1' &= -8.214 \frac{m}{s} \\
 v_2' &= 2.623 \frac{m}{s}
 \end{aligned}$$

**32.** In a collision, momentum is conserved. Therefore,

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

Since ball 2 is at rest, the equation becomes

$$m_1 v_1 = m_1 v_1' + m_2 v_2'$$

In an elastic collision, kinetic energy is conserved. Therefore,

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

Since ball 2 is at rest, the equation becomes

$$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

Solving for  $v_1'$  in the first equation

$$v_1' = v_1 - \frac{m_2}{m_1}v_2'$$

and using this results in the second equation yields

$$\begin{aligned} \frac{1}{2}m_1v_1^2 &= \frac{1}{2}m_1\left(v_1 - \frac{m_2}{m_1}v_2'\right)^2 + \frac{1}{2}m_2v_2'^2 \\ m_1v_1^2 &= m_1\left(v_1 - \frac{m_2}{m_1}v_2'\right)^2 + m_2v_2'^2 \\ m_1v_1^2 &= m_1v_1^2 - 2m_2v_1v_2' + \frac{m_2^2}{m_1}v_2'^2 + m_2v_2'^2 \\ 0 &= -2m_2v_1v_2' + \frac{m_2^2}{m_1}v_2'^2 + m_2v_2'^2 \\ 2m_2v_1v_2' &= \frac{m_2^2}{m_1}v_2'^2 + m_2v_2'^2 \\ 2m_2v_1 &= \frac{m_2^2}{m_1}v_2' + m_2v_2' \\ 2m_2m_1v_1 &= m_2^2v_2' + m_1m_2v_2' \\ 2m_1v_1 &= m_2v_2' + m_1v_2' \\ 2m_1v_1 &= (m_2 + m_1)v_2' \\ v_2' &= \frac{2m_1v_1}{m_1 + m_2} \end{aligned}$$

The speed of the other ball is thus

$$\begin{aligned} v_1' &= v_1 - \frac{m_2}{m_1}v_2' \\ &= v_1 - \frac{m_2}{m_1} \frac{2m_1v_1}{m_1 + m_2} \\ &= v_1 - \frac{2m_2v_1}{m_1 + m_2} \\ &= \frac{m_1 + m_2}{m_1 + m_2}v_1 - \frac{2m_2v_1}{m_1 + m_2} \\ &= \frac{m_1 + m_2 - 2m_2}{m_1 + m_2}v_1 \\ &= \frac{m_1 - m_2}{m_1 + m_2}v_1 \end{aligned}$$

**33.** In a collision, momentum is conserved. Therefore,

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

In an elastic collision, kinetic energy is conserved. Therefore,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

This last equation gives

$$\begin{aligned} m_1v_1^2 + m_2v_2^2 &= m_1v_1'^2 + m_2v_2'^2 \\ m_1v_1^2 - m_1v_1'^2 &= m_2v_2'^2 - m_2v_2^2 \\ m_1(v_1^2 - v_1'^2) &= m_2(v_2'^2 - v_2^2) \\ m_1(v_1 - v_1')(v_1 + v_1') &= m_2(v_2' - v_2)(v_2' + v_2) \end{aligned}$$

But the first equation gives

$$\begin{aligned} m_1v_1 + m_2v_2 &= m_1v_1' + m_2v_2' \\ m_1v_1 - m_1v_1' &= m_2v_2' - m_2v_2 \\ m_1(v_1 - v_1') &= m_2(v_2' - v_2) \end{aligned}$$

Thus  $m_1(v_1 - v_1')$  can be replaced in our energy equation to obtain

$$\begin{aligned} m_1(v_1 - v_1')(v_1 + v_1') &= m_2(v_2' - v_2)(v_2' + v_2) \\ m_2(v_2' - v_2)(v_1 + v_1') &= m_2(v_2' - v_2)(v_2' + v_2) \\ v_1 + v_1' &= v_2' + v_2 \end{aligned}$$

We now have these 2 equations.

$$\begin{aligned} m_1v_1 + m_2v_2 &= m_1v_1' + m_2v_2' \\ v_1 + v_1' &= v_2' + v_2 \end{aligned}$$

Solving for  $v_2'$  in the 2<sup>nd</sup> equation

$$v_2' = v_1 + v_1' - v_2$$

and substituting in the 1<sup>st</sup> equation, gives

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 (v_1 + v'_1 - v_2)$$

It only remains to solve for  $v'_1$ .

$$\begin{aligned} m_1 v_1 + m_2 v_2 &= m_1 v'_1 + m_2 v_1 + m_2 v'_1 - m_2 v_2 \\ m_1 v_1 + m_2 v_2 - m_2 v_1 + m_2 v_2 &= m_1 v'_1 + m_2 v'_1 \\ (m_1 - m_2) v_1 + 2m_2 v_2 &= (m_1 + m_2) v'_1 \\ v'_1 &= \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 \end{aligned}$$

The speed of the other ball is thus

$$\begin{aligned} v'_2 &= v_1 + v'_1 - v_2 \\ &= v_1 + \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 - v_2 \\ &= \frac{m_1 + m_2}{m_1 + m_2} v_1 + \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 - \frac{m_1 + m_2}{m_1 + m_2} v_2 \\ &= \frac{m_1 + m_2 + m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2 - m_1 - m_2}{m_1 + m_2} v_2 \\ &= \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2 \end{aligned}$$

**34.** The variation of kinetic energy is

$$\begin{aligned} \Delta E_k &= E'_k - E_k \\ &= \frac{1}{2} m_{tot} v'^2 - \left( \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right) \end{aligned}$$

The second ball has no speed before the collision. Thus,

$$\Delta E_k = \frac{1}{2} m_{tot} v'^2 - \frac{1}{2} m_1 v_1^2$$

The speed of the object after the collision is found with momentum conservation.

$$m_1 v_1 = (m_1 + m_2) v'$$

$$v' = \frac{m_1 v_1}{m_1 + m_2}$$

Therefore, the kinetic energy variation is

$$\begin{aligned} \Delta E_k &= \frac{1}{2} m_{tot} v'^2 - \frac{1}{2} m_1 v_1^2 \\ &= \frac{1}{2} (m_1 + m_2) \left( \frac{m_1 v_1}{m_1 + m_2} \right)^2 - \frac{1}{2} m_1 v_1^2 \\ &= \frac{1}{2} \frac{m_1^2 v_1^2}{m_1 + m_2} - \frac{1}{2} m_1 v_1^2 \\ &= \frac{1}{2} \left( \frac{m_1^2}{m_1 + m_2} - m_1 \right) v_1^2 \\ &= \frac{1}{2} \left( \frac{m_1^2}{m_1 + m_2} - \frac{m_1 (m_1 + m_2)}{m_1 + m_2} \right) v_1^2 \\ &= \frac{1}{2} \left( \frac{m_1^2}{m_1 + m_2} - \frac{m_1^2 + m_1 m_2}{m_1 + m_2} \right) v_1^2 \\ &= \frac{1}{2} \left( \frac{m_1^2 - m_1^2 - m_1 m_2}{m_1 + m_2} \right) v_1^2 \\ &= -\frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} v_1^2 \end{aligned}$$

**35.** An  $x$ -axis towards the right and a  $y$ -axis upwards is used here. In the absence of external forces, the momentum is conserved. We thus have, for the  $x$ -component

$$p_{tot\ x} = p'_{tot\ x}$$

$$m_1 v_{1x} + m_2 v_{2x} = m_{tot} v'_x$$

$$3\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} + 2\text{kg} \cdot -3 \frac{\text{m}}{\text{s}} = 5\text{kg} \cdot v'_x$$

$$v'_x = -1.2 \frac{\text{m}}{\text{s}}$$

For the  $y$ -component, we have

$$\begin{aligned}
 p_{tot\ y} &= p'_{tot\ y} \\
 m_1 v_{1y} + m_2 v_{2y} &= m_{tot} v'_y \\
 3\text{kg} \cdot 2 \frac{\text{m}}{\text{s}} + 2\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} &= 5\text{kg} \cdot v'_y \\
 v'_y &= 1.2 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

The speed is therefore

$$\begin{aligned}
 v &= \sqrt{v_x^2 + v_y^2} \\
 &= \sqrt{\left(-1.2 \frac{\text{m}}{\text{s}}\right)^2 + \left(1.2 \frac{\text{m}}{\text{s}}\right)^2} \\
 &= 1.697 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

and the direction of the velocity is

$$\begin{aligned}
 \theta &= \arctan \frac{v_y}{v_x} \\
 &= \arctan \frac{1.2 \frac{\text{m}}{\text{s}}}{-1.2 \frac{\text{m}}{\text{s}}} \\
 &= 135^\circ
 \end{aligned}$$

**36.** An  $x$ -axis towards the right and a  $y$ -axis upwards is used here. In the absence of external forces, the momentum is conserved. We thus have, for the  $x$ -component

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 m_1 v_{Ax} + m_2 v_{Bx} &= m_{tot} v'_x \\
 1500\text{kg} \cdot v_A \cdot \cos(45^\circ) + 2000\text{kg} \cdot (-v_B) &= 3500\text{kg} \cdot 12 \frac{\text{m}}{\text{s}} \cdot \cos(60^\circ) \\
 1060.7\text{kg} \cdot v_A - 2000\text{kg} \cdot v_B &= 21\,000 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

For the  $y$ -component, we have

$$\begin{aligned}
 p_{tot\ y} &= p'_{tot\ y} \\
 m_1 v_{1y} + m_2 v_{2y} &= m_{tot} v'_y \\
 1500\text{kg} \cdot v_A \cdot \sin(45^\circ) + 2000\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} &= 3500\text{kg} \cdot 12 \frac{\text{m}}{\text{s}} \cdot \sin(60^\circ) \\
 1060.7\text{kg} \cdot v_A &= 36\,373 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

This equation allows us to find the speed of car A.

$$1060.7 \text{ kg} \cdot v_A = 36,373 \frac{\text{kgm}}{\text{s}}$$

$$v_A = 34.29 \frac{\text{m}}{\text{s}}$$

With this information, the speed of car B can be found with the equation for the  $x$ -component.

$$1060.7 \text{ kg} \cdot v_A + 2000 \text{ kg} \cdot -v_B = 21,000 \frac{\text{kgm}}{\text{s}}$$

$$1060.7 \text{ kg} \cdot 34.29 \frac{\text{m}}{\text{s}} + 2000 \text{ kg} \cdot -v_B = 21,000 \frac{\text{kgm}}{\text{s}}$$

$$36,373 \frac{\text{kgm}}{\text{s}} + 2000 \text{ kg} \cdot -v_B = 21,000 \frac{\text{kgm}}{\text{s}}$$

$$2000 \text{ kg} \cdot -v_B = -15,373 \frac{\text{kgm}}{\text{s}}$$

$$v_B = 7.69 \frac{\text{m}}{\text{s}}$$

**37.** An  $x$ -axis towards the right and a  $y$ -axis upwards is used here. In the absence of external forces, the momentum is conserved. We thus have, for the  $x$ -component

$$p_{\text{tot } x} = p'_{\text{tot } x}$$

$$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}$$

$$1 \text{ kg} \cdot 5 \frac{\text{m}}{\text{s}} + 3 \text{ kg} \cdot 0 \frac{\text{m}}{\text{s}} = 1 \text{ kg} \cdot v'_{1x} + 3 \text{ kg} \cdot v'_2 \cdot \cos(-50^\circ)$$

$$5 \frac{\text{kgm}}{\text{s}} = 1 \text{ kg} \cdot v'_{1x} + 3 \text{ kg} \cdot v'_2 \cdot \cos(-50^\circ)$$

$$5 \frac{\text{m}}{\text{s}} = v'_{1x} + 3 \cdot v'_2 \cdot \cos(-50^\circ)$$

For the  $y$ -component, we have

$$p_{\text{tot } y} = p'_{\text{tot } y}$$

$$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}$$

$$1 \text{ kg} \cdot 0 \frac{\text{m}}{\text{s}} + 3 \text{ kg} \cdot 0 \frac{\text{m}}{\text{s}} = 1 \text{ kg} \cdot v'_{1y} + 3 \text{ kg} \cdot v'_2 \cdot \sin(-50^\circ)$$

$$0 \frac{\text{kgm}}{\text{s}} = 1 \text{ kg} \cdot v'_{1y} + 3 \text{ kg} \cdot v'_2 \cdot \sin(-50^\circ)$$

$$0 \frac{\text{m}}{\text{s}} = v'_{1y} + 3 \cdot v'_2 \cdot \sin(-50^\circ)$$

As this is an elastic collision, the kinetic energy is also conserved. Therefore

$$\begin{aligned}
 E_{k\ tot} &= E'_{k\ tot} \\
 \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 &= \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \\
 m_1v_1^2 + m_2v_2^2 &= m_1v_1'^2 + m_2v_2'^2 \\
 1kg \cdot \left(5\frac{m}{s}\right)^2 + 3kg \cdot \left(0\frac{m}{s}\right)^2 &= 1kg \cdot v_1'^2 + 3kg \cdot v_2'^2 \\
 25J &= 1kg \cdot v_1'^2 + 3kg \cdot v_2'^2 \\
 25\frac{m^2}{s^2} &= v_1'^2 + 3v_2'^2
 \end{aligned}$$

Since  $v_1'^2 = v_{1x}'^2 + v_{1y}'^2$ , this equation becomes

$$\begin{aligned}
 25\frac{m^2}{s^2} &= v_1'^2 + 3v_2'^2 \\
 25\frac{m^2}{s^2} &= v_{1x}'^2 + v_{1y}'^2 + 3v_2'^2
 \end{aligned}$$

If we solve for  $v_{1x}'$  in the  $x$ -component of the momentum equation, we have

$$\begin{aligned}
 5\frac{m}{s} &= v_{1x}' + 3v_2' \cos(-50^\circ) \\
 v_{1x}' &= 5\frac{m}{s} - 3v_2' \cos(-50^\circ)
 \end{aligned}$$

If we solve for  $v_{1y}'$  in the  $y$ -component of the momentum equation, we have

$$\begin{aligned}
 0\frac{m}{s} &= v_{1y}' + 3v_2' \sin(-50^\circ) \\
 v_{1y}' &= -3v_2' \sin(-50^\circ)
 \end{aligned}$$

We then substitute these values in the conservation of kinetic energy equation.

$$\begin{aligned}
 25\frac{m^2}{s^2} &= v_{1x}'^2 + v_{1y}'^2 + 3v_2'^2 \\
 25\frac{m^2}{s^2} &= \left(5\frac{m}{s} - 3v_2' \cos(-50^\circ)\right)^2 + \left(-3v_2' \sin(-50^\circ)\right)^2 + 3v_2'^2
 \end{aligned}$$

It remains to solve for  $v_2'$ .

$$\begin{aligned}
 25\frac{m^2}{s^2} &= \left(5\frac{m}{s} - 3v_2' \cos(-50^\circ)\right)^2 + \left(-3v_2' \sin(-50^\circ)\right)^2 + 3v_2'^2 \\
 25\frac{m^2}{s^2} &= 25\frac{m^2}{s^2} - 30\frac{m}{s} \cdot v_2' \cos(-50^\circ) + 9v_2'^2 \cos^2(-50^\circ) + 9v_2'^2 \sin^2(-50^\circ) + 3v_2'^2 \\
 0 &= -30\frac{m}{s} \cdot v_2' \cos(-50^\circ) + 9v_2'^2 + 3v_2'^2 \\
 0 &= -30\frac{m}{s} \cdot v_2' \cos(-50^\circ) + 12v_2'^2 \\
 0 &= -30\frac{m}{s} \cdot \cos(-50^\circ) + 12v_2' \\
 v_2' &= 1.607\frac{m}{s}
 \end{aligned}$$

With this answer  $v'_{1x}$  can be found

$$\begin{aligned}v'_{1x} &= 5 \frac{m}{s} - 3v'_2 \cos(-50^\circ) \\ &= 1.901 \frac{m}{s}\end{aligned}$$

and  $v'_{1y}$  can be found

$$\begin{aligned}v'_{1y} &= -3v'_2 \sin(-50^\circ) \\ &= 3.693 \frac{m}{s}\end{aligned}$$

The speed is therefore

$$\begin{aligned}v'_1 &= \sqrt{v'^2_{1x} + v'^2_{1y}} \\ &= \sqrt{\left(1.901 \frac{m}{s}\right)^2 + \left(3.693 \frac{m}{s}\right)^2} \\ &= 4.154 \frac{m}{s}\end{aligned}$$

and the direction of the velocity is

$$\begin{aligned}\theta &= \arctan \frac{v_y}{v_x} \\ &= \arctan \frac{3.693 \frac{m}{s}}{1.901 \frac{m}{s}} \\ &= 62.8^\circ\end{aligned}$$

The two answers are therefore

$$\begin{aligned}v'_1 &= 4.154 \frac{m}{s} \text{ at } 62.8^\circ \\ v'_2 &= 1.607 \frac{m}{s}\end{aligned}$$

### 38. a)

An  $x$ -axis towards the right and a  $y$ -axis upwards is used here. In the absence of external forces, the momentum is conserved. We thus have, for the  $x$ -component

$$\begin{aligned}
 p_{tot\ x} &= p'_{tot\ x} \\
 m_1 v_{1x} + m_2 v_{2x} &= m_1 v'_{1x} + m_2 v'_{2x} \\
 4kg \cdot 8 \frac{m}{s} + 6kg \cdot 0 \frac{m}{s} &= 4kg \cdot 6 \frac{m}{s} \cdot \cos(30^\circ) + 6kg \cdot v'_{2x} \\
 32 \frac{kgm}{s} &= 20.785 \frac{kgm}{s} + 6kg \cdot v'_{2x} \\
 v'_{2x} &= 1.869 \frac{m}{s}
 \end{aligned}$$

For the y-component, we have

$$\begin{aligned}
 p_{tot\ y} &= p'_{tot\ y} \\
 m_1 v_{1y} + m_2 v_{2y} &= m_1 v'_{1y} + m_2 v'_{2y} \\
 4kg \cdot 0 \frac{m}{s} + 6kg \cdot 0 \frac{m}{s} &= 4kg \cdot 6 \frac{m}{s} \cdot \sin(30^\circ) + 6kg \cdot v'_{2y} \\
 0 \frac{kgm}{s} &= 12 \frac{kgm}{s} + 6kg \cdot v'_{2y} \\
 v'_{2y} &= -2 \frac{m}{s}
 \end{aligned}$$

The speed is therefore

$$\begin{aligned}
 v'_2 &= \sqrt{v'^2_{2x} + v'^2_{2y}} \\
 &= \sqrt{\left(1.869 \frac{m}{s}\right)^2 + \left(2 \frac{m}{s}\right)^2} \\
 &= 2.738 \frac{m}{s}
 \end{aligned}$$

And the direction of the velocity is

$$\begin{aligned}
 \theta &= \arctan \frac{v_y}{v_x} \\
 &= \arctan \frac{-2 \frac{m}{s}}{1.869 \frac{m}{s}} \\
 &= -46.9^\circ
 \end{aligned}$$

b) The kinetic energy before the collision is

$$\begin{aligned}
 E_{k\ tot} &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \\
 &= \frac{1}{2} \cdot 4kg \cdot \left(8 \frac{m}{s}\right)^2 + \frac{1}{2} \cdot 6kg \cdot \left(0 \frac{m}{s}\right)^2 \\
 &= 128J
 \end{aligned}$$

The kinetic energy after the collision is

$$\begin{aligned}
 E'_{k\text{ tot}} &= \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \\
 &= \frac{1}{2} \cdot 4\text{kg} \cdot \left(6 \frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2} \cdot 6\text{kg} \cdot \left(2.738 \frac{\text{m}}{\text{s}}\right)^2 \\
 &= 94.48\text{J}
 \end{aligned}$$

The energy loss is therefore

$$\begin{aligned}
 \Delta E_k &= 94.48\text{J} - 128\text{J} \\
 &= -33.52\text{J}
 \end{aligned}$$

c) No, because there is a loss of kinetic energy in the collision.

### **39.** Momentum at Instant 1 ( $t = 0$ on the diagram)

$$\begin{aligned}
 p_x &= m_{\text{block 1}} v_{\text{block 1}} + m_{\text{block 2}} v_{\text{block 2}} \\
 &= 10\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} + 20\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} \\
 &= 0
 \end{aligned}$$

### Momentum at Instant 2 ( $t = 3$ s on the diagram)

$$\begin{aligned}
 p'_x &= m_{\text{block 1}} v'_{\text{block 1}} + m_{\text{block 2}} v'_{\text{block 2}} \\
 &= 10\text{kg} \cdot v' + 20\text{kg} \cdot v' \\
 &= 30\text{kg} \cdot v'
 \end{aligned}$$

### Impulse given by the 50 N force

$$\begin{aligned}
 I_x &= F_x \Delta t \\
 &= 50\text{N} \cdot 3\text{s} \\
 &= 150 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

### Impulse-Momentum Theorem

$$\begin{aligned}
 p_x + I_x &= p'_x \\
 0 + 150 \frac{\text{kgm}}{\text{s}} &= 30\text{kg} \cdot v' \\
 v' &= 5 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

### **40.** Momentum at Instant 1 (Configuration shown on the diagram)

$$\begin{aligned}
 p_y &= m_{\text{block}} v_{\text{block}} + m_{\text{bullet}} v_{\text{bullet}} \\
 &= 1\text{kg} \cdot 0 \frac{\text{m}}{\text{s}} + 0.020\text{kg} \cdot 500 \frac{\text{m}}{\text{s}} \\
 &= 10 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

Momentum at Instant 2 ( $t = 0.5$  s after the collision)

$$\begin{aligned}
 p'_y &= m_{\text{tot}} v' \\
 &= 1.02\text{kg} \cdot v'
 \end{aligned}$$

Impulse given by gravity during 0.5 s

$$\begin{aligned}
 I_y &= F_y \Delta t \\
 &= -mg \Delta t \\
 &= -1.02\text{kg} \cdot 9.8 \frac{\text{N}}{\text{kg}} \cdot 0.5\text{s} \\
 &= -4.998 \frac{\text{kgm}}{\text{s}}
 \end{aligned}$$

Impulse-Momentum Theorem

$$\begin{aligned}
 p_y + I_y &= p'_y \\
 10 \frac{\text{kgm}}{\text{s}} - 4.998 \frac{\text{kgm}}{\text{s}} &= 1.02\text{kg} \cdot v' \\
 v' &= 4.904 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

**41.** a) The thrust force is

$$\begin{aligned}
 F_{\text{thrust}} &= v_{\text{exp}} R \\
 &= 3200 \frac{\text{m}}{\text{s}} \cdot 3400 \frac{\text{kg}}{\text{s}} \\
 &= 1.088 \times 10^7 \text{ N}
 \end{aligned}$$

b) The speed will be

$$\begin{aligned}
 v' &= v + v_{\text{exp}} \ln \frac{m}{m - RT} \\
 &= 0 \frac{\text{m}}{\text{s}} + 3200 \frac{\text{m}}{\text{s}} \cdot \ln \frac{710\,000\text{kg}}{710\,000\text{kg} - 3400 \frac{\text{kg}}{\text{s}} \cdot 60\text{s}} \\
 &= 0 \frac{\text{m}}{\text{s}} + 3200 \frac{\text{m}}{\text{s}} \cdot \ln \frac{710\,000\text{kg}}{506\,000\text{kg}} \\
 &= 1084 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

c) The speed will be

$$\begin{aligned}
 v' &= v + v_{\text{exp}} \ln \frac{m}{m - RT} - gt \\
 &= 0 \frac{\text{m}}{\text{s}} + 3200 \frac{\text{m}}{\text{s}} \cdot \ln \frac{710\,000\text{kg}}{710\,000\text{kg} - 3400 \frac{\text{kg}}{\text{s}} \cdot 30\text{s}} - 9.8 \frac{\text{m}}{\text{s}^2} \cdot 30\text{s} \\
 &= 0 \frac{\text{m}}{\text{s}} + 3200 \frac{\text{m}}{\text{s}} \cdot \ln \frac{710\,000\text{kg}}{608\,000\text{kg}} - 294 \frac{\text{m}}{\text{s}} \\
 &= 202.3 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

**42.** We have

$$\begin{aligned}
 v' &= v + v_{\text{exp}} \ln \frac{m}{m - RT} \\
 18\,000 \frac{\text{m}}{\text{s}} &= 15\,000 \frac{\text{m}}{\text{s}} + v_{\text{exp}} \ln \frac{100\,000\text{kg}}{100\,000\text{kg} - 750 \frac{\text{kg}}{\text{s}} \cdot 60\text{s}} \\
 3000 \frac{\text{m}}{\text{s}} &= v_{\text{exp}} \ln \frac{100\,000\text{kg}}{55\,000\text{kg}} \\
 v_{\text{exp}} &= 5018 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

**43.** The thrust force is

$$F_t = (v_{\text{exp}} - v) R$$

Using the data given, we have

$$431\,000\text{N} = (166.7 \frac{\text{m}}{\text{s}} - 0) \cdot R$$

$$431\,000\text{N} = 166.7 \frac{\text{m}}{\text{s}} \cdot R$$

$$\frac{431\,000\text{N}}{166.7 \frac{\text{m}}{\text{s}}} = R$$

$$2586 \frac{\text{kg}}{\text{s}} = R$$

**44.** The thrust force is

$$F_t = (v_{\text{exp}} - v)R$$

Using the data given, we have

$$200\,000\text{N} = \left(260 \frac{\text{m}}{\text{s}} - 230 \frac{\text{m}}{\text{s}}\right) \cdot R$$

$$200\,000\text{N} = 30 \frac{\text{m}}{\text{s}} \cdot R$$

$$6667 \frac{\text{kg}}{\text{s}} = R$$

**45.** The thrust force is

$$F_t = \frac{1}{2}(v_{\text{exp}}^2 - v^2)\rho A$$

The area is

$$\begin{aligned} A &= \pi R^2 \\ &= \pi \cdot (1\text{m})^2 \\ &= 3.14\text{m}^2 \end{aligned}$$

Therefore, the thrust force is

$$\begin{aligned} F_t &= \frac{1}{2}(v_{\text{exp}}^2 - v^2)\rho A \\ &= \frac{1}{2}\left(\left(50 \frac{\text{m}}{\text{s}}\right)^2 - 0\right) \cdot 1.2 \frac{\text{kg}}{\text{m}^3} \cdot 3.14\text{m}^2 \\ &= 4712\text{N} \end{aligned}$$

**46.** The thrust force is

$$F_t = \frac{1}{2}(v_{\text{exp}}^2 - v^2)\rho A$$

The area is

$$\begin{aligned} A &= \pi R^2 \\ &= \pi \cdot (1.5\text{m})^2 \\ &= 7.07\text{m}^2 \end{aligned}$$

Therefore,

$$\begin{aligned}
 F_t &= \frac{1}{2}(v_{\text{exp}}^2 - v^2)\rho A \\
 &= \frac{1}{2}\left((80\frac{\text{m}}{\text{s}})^2 - (75\frac{\text{m}}{\text{s}})^2\right) \cdot 0.9\frac{\text{kg}}{\text{m}^3} \cdot 7.07\text{m}^2 \\
 &= 2465\text{N}
 \end{aligned}$$

**47.** The thrust force is

$$F_t = \frac{1}{2}(v_{\text{exp}}^2 - v^2)\rho A$$

The area is

$$\begin{aligned}
 A &= \pi R^2 \\
 &= \pi \cdot (1\text{m})^2 \\
 &= 3.14\text{m}^2
 \end{aligned}$$

Therefore

$$\begin{aligned}
 F_t &= \frac{1}{2}(v_{\text{exp}}^2 - v^2)\rho A \\
 500\text{N} &= \frac{1}{2}\left(v_{\text{exp}}^2 - (75\frac{\text{m}}{\text{s}})^2\right) \cdot 0.9\frac{\text{kg}}{\text{m}^3} \cdot 3.14\text{m}^2 \\
 77.3\frac{\text{m}}{\text{s}} &= v_{\text{exp}}
 \end{aligned}$$

**48.** When the spring is compressed at the maximum, the two masses have the same speed. Thus, according to the law of conservation of momentum, we have

$$\begin{aligned}
 p_{\text{tot } x} &= p'_{\text{tot } x} \\
 m_1v_1 + m_2v_2 &= m_1v'_1 + m_2v'_2 \\
 1\text{kg} \cdot 30\frac{\text{m}}{\text{s}} &= 1\text{kg} \cdot v' + 2\text{kg} \cdot v' \\
 v' &= 10\frac{\text{m}}{\text{s}}
 \end{aligned}$$

In addition, the mechanical energy must be conserved. With two masses and a spring, the mechanical energy is

$$E_{mec} = \frac{1}{2} m_1 v_1^2 + m_1 g y_1 + \frac{1}{2} m_2 v_2^2 + m_2 g y_2 + \frac{1}{2} k x^2$$

Before the collision, the mechanical energy is (using a  $y = 0$  on the ground)

$$\begin{aligned} E_{mec} &= \frac{1}{2} m_1 v_1^2 + m_1 g y_1 + \frac{1}{2} m_2 v_2^2 + m_2 g y_2 + \frac{1}{2} k x^2 \\ &= \frac{1}{2} \cdot 1 \text{kg} \cdot (30 \frac{\text{m}}{\text{s}})^2 + 0 + 0 + 0 + 0 \\ &= 450 \text{J} \end{aligned}$$

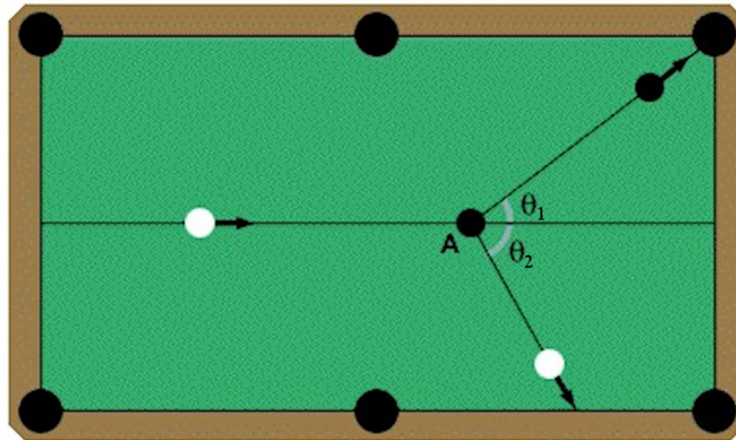
When the spring is compressed at its maximum, the mechanical energy is

$$\begin{aligned} E'_{mec} &= \frac{1}{2} m_1 v_1'^2 + m_1 g y_1' + \frac{1}{2} m_2 v_2'^2 + m_2 g y_2' + \frac{1}{2} k x'^2 \\ &= \frac{1}{2} 1 \text{kg} \cdot (10 \frac{\text{m}}{\text{s}})^2 + 0 + \frac{1}{2} 2 \text{kg} \cdot (10 \frac{\text{m}}{\text{s}})^2 + 0 + \frac{1}{2} k x'^2 \\ &= 50 \text{J} + 100 \text{J} + \frac{1}{2} k x'^2 \\ &= 150 \text{J} + \frac{1}{2} k x'^2 \end{aligned}$$

Then, the conservation of mechanical energy gives

$$\begin{aligned} E_{mec} &= E'_{mec} \\ 450 \text{J} &= 150 \text{J} + \frac{1}{2} k x'^2 \\ 300 \text{J} &= \frac{1}{2} \cdot 9600 \frac{\text{N}}{\text{m}} x'^2 \\ x' &= 0.25 \text{m} \end{aligned}$$

**49.** The collision is as follows.



The equation of conservation of the  $x$ -component of the momentum is

$$\begin{aligned}
 p_x &= p'_x \\
 mv_{1x} + \cancel{mv_{2x}} &= mv'_{1x} + mv'_{2x} \\
 v_{1x} &= v'_{1x} + v'_{2x} \\
 v_1 &= v'_1 \cos \theta_1 + v'_2 \cos \theta_2
 \end{aligned}$$

The equation of conservation of the  $y$ -component of the momentum is

$$\begin{aligned}
 p_y &= p'_y \\
 \cancel{mv_{1y}} + \cancel{mv_{2y}} &= mv'_{1y} + mv'_{2y} \\
 0 &= v'_{1y} + v'_{2y} \\
 0 &= v'_1 \sin \theta_1 - v'_2 \sin \theta_2
 \end{aligned}$$

The equation for the conservation of kinetic energy is

$$\begin{aligned}
 E_k &= E'_k \\
 \frac{1}{2}mv_1^2 + \cancel{\frac{1}{2}mv_2^2} &= \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 \\
 v_1^2 &= v_1'^2 + v_2'^2
 \end{aligned}$$

First,  $\theta_2$  is eliminated with  $\cos^2 \theta_2 + \sin^2 \theta_2 = 1$ . The cosine comes from the conservation of the  $x$ -component of the momentum

$$v'_2 \cos \theta_2 = v_1 - v'_1 \cos \theta_1$$

The sine comes from the conservation of the  $y$ -component of the momentum

$$v'_2 \sin \theta_2 = v'_1 \sin \theta_1$$

Then

$$\begin{aligned} (v'_2 \cos \theta_2)^2 + (v'_2 \sin \theta_2)^2 &= (v_1 - v'_1 \cos \theta_1)^2 + (v'_1 \sin \theta_1)^2 \\ v_2'^2 \cos^2 \theta_2 + v_2'^2 \sin^2 \theta_2 &= v_1^2 - 2v_1 v'_1 \cos \theta_1 + v_1'^2 \cos^2 \theta_1 + v_1'^2 \sin^2 \theta_1 \\ v_2'^2 (\cos^2 \theta_2 + \sin^2 \theta_2) &= v_1^2 - 2v_1 v'_1 \cos \theta_1 + v_1'^2 (\cos^2 \theta_1 + \sin^2 \theta_1) \\ v_2'^2 &= v_1^2 - 2v_1 v'_1 \cos \theta_1 + v_1'^2 \end{aligned}$$

Using the conservation of energy formula  $v_1^2 = v_1'^2 + v_2'^2$ , the equation becomes

$$\begin{aligned} v_2'^2 &= v_1^2 - 2v_1 v'_1 \cos \theta_1 + v_1'^2 \\ v_1^2 - v_1'^2 &= v_1^2 - 2v_1 v'_1 \cos \theta_1 + v_1'^2 \\ -v_1'^2 &= -2v_1 v'_1 \cos \theta_1 + v_1'^2 \\ -2v_1'^2 &= -2v_1 v'_1 \cos \theta_1 \\ v_1' &= v_1 \cos \theta_1 \end{aligned}$$

Then  $v'_2$  is easily found with  $v_1^2 = v_1'^2 + v_2'^2$ .

$$\begin{aligned} v_2'^2 &= v_1^2 - v_1'^2 \\ v_2'^2 &= v_1^2 - v_1^2 \cos^2 \theta_1 \\ v_2'^2 &= v_1^2 (1 - \cos^2 \theta_1) \\ v_2'^2 &= v_1^2 \sin^2 \theta_1 \\ v_2' &= v_1 \sin \theta_1 \end{aligned}$$

Finally,  $\theta_2$  is found with  $0 = v'_1 \sin \theta_1 - v'_2 \sin \theta_2$ .

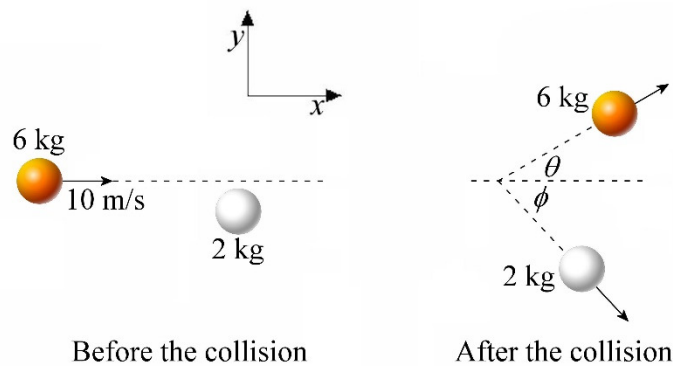
$$\begin{aligned} \sin \theta_2 &= \frac{v'_1 \sin \theta_1}{v'_2} \\ \sin \theta_2 &= \frac{(v_1 \cos \theta_1) \sin \theta_1}{v_1 \sin \theta_1} \\ \sin \theta_2 &= \cos \theta_1 \end{aligned}$$

Since  $\cos \theta = \sin (90^\circ - \theta)$ , this equation becomes

$$\begin{aligned}\sin \theta_2 &= \cos \theta_1 \\ \sin \theta_2 &= \sin (90^\circ - \theta_1) \\ \theta_2 &= 90^\circ - \theta_1 \\ \theta_1 + \theta_2 &= 90^\circ\end{aligned}$$

Since the sum of the angles is  $90^\circ$ , the trajectories are perpendicular.

**50.** The collision is as follow.



The equation of conservation of the  $x$ -component of the momentum is

$$\begin{aligned}p_x &= p'_x \\ m_1 v_{1x} + \cancel{m_2 v_{2x}} &= m_1 v'_{1x} + m_2 v'_{2x} \\ m_1 v_{1x} &= m_1 v'_{1x} + m_2 v'_{2x} \\ m_1 v_{1x} &= m_1 v'_1 \cos \theta_1 + m_2 v'_2 \cos \theta_2 \\ 60 \frac{\text{kgm}}{\text{s}} &= 6\text{kg} \cdot v'_1 \cos \theta_1 + 2\text{kg} \cdot v'_2 \cos \theta_2 \\ 30 \frac{\text{m}}{\text{s}} &= 3 \cdot v'_1 \cos \theta_1 + v'_2 \cos \theta_2\end{aligned}$$

The equation of conservation of the  $y$ -component of the momentum is

$$\begin{aligned}p_y &= p'_y \\ \cancel{m_1 v_{1y}} + \cancel{m_2 v_{2y}} &= m_1 v'_{1y} + m_2 v'_{2y} \\ 0 &= m_1 v'_{1y} + m_2 v'_{2y} \\ 0 &= m_1 v'_1 \sin \theta_1 - m_2 v'_2 \sin \theta_2 \\ 0 &= 6\text{kg} \cdot v'_1 \sin \theta_1 - 2\text{kg} \cdot v'_2 \sin \theta_2 \\ 0 &= 3v'_1 \sin \theta_1 - v'_2 \sin \theta_2\end{aligned}$$

The equation for the conservation of kinetic energy is

$$\begin{aligned}
 E_k &= E'_k \\
 \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 &= \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \\
 m_1v_1^2 &= m_1v_1'^2 + m_2v_2'^2 \\
 600 \frac{\text{kgm}^2}{\text{s}^2} &= 6\text{kg} \cdot v_1'^2 + 2\text{kg} \cdot v_2'^2 \\
 300 \frac{\text{m}^2}{\text{s}^2} &= 3v_1'^2 + v_2'^2
 \end{aligned}$$

First,  $\theta_2$  is eliminated with  $\cos^2 \theta_2 + \sin^2 \theta_2 = 1$ . The cosine comes from the conservation of the  $x$ -component of the momentum

$$v_2' \cos \theta_2 = 30 \frac{\text{m}}{\text{s}} - 3v_1' \cos \theta_1$$

The sine comes from the conservation of the  $y$ -component of the momentum

$$v_2' \sin \theta_2 = 3v_1' \sin \theta_1$$

Thus

$$\begin{aligned}
 (v_2' \cos \theta_2)^2 + (v_2' \sin \theta_2)^2 &= (30 \frac{\text{m}}{\text{s}} - 3v_1' \cos \theta_1)^2 + (3v_1' \sin \theta_1)^2 \\
 v_2'^2 \cos^2 \theta_2 + v_2'^2 \sin^2 \theta_2 &= 900 \frac{\text{m}^2}{\text{s}^2} - 180 \frac{\text{m}}{\text{s}} v_1' \cos \theta_1 + 9v_1'^2 \cos^2 \theta_1 + 9v_1'^2 \sin^2 \theta_1 \\
 v_2'^2 (\cos^2 \theta_2 + \sin^2 \theta_2) &= 900 \frac{\text{m}^2}{\text{s}^2} - 180 \frac{\text{m}}{\text{s}} v_1' \cos \theta_1 + 9v_1'^2 (\cos^2 \theta_1 + \sin^2 \theta_1) \\
 v_2'^2 &= 900 \frac{\text{m}^2}{\text{s}^2} - 180 \frac{\text{m}}{\text{s}} v_1' \cos \theta_1 + 9v_1'^2
 \end{aligned}$$

Using the conservation of energy formula  $300 \frac{\text{m}^2}{\text{s}^2} = 3 \cdot v_1'^2 + v_2'^2$ , the equation becomes

$$\begin{aligned}
 v_2'^2 &= 900 \frac{\text{m}^2}{\text{s}^2} - 180 \frac{\text{m}}{\text{s}} v_1' \cos \theta_1 + 9v_1'^2 \\
 300 \frac{\text{m}^2}{\text{s}^2} - 3 \cdot v_1'^2 &= 900 \frac{\text{m}^2}{\text{s}^2} - 180 \frac{\text{m}}{\text{s}} v_1' \cos \theta_1 + 9v_1'^2 \\
 -3 \cdot v_1'^2 &= 600 \frac{\text{m}^2}{\text{s}^2} - 180 \frac{\text{m}}{\text{s}} v_1' \cos \theta_1 + 9v_1'^2 \\
 -v_1'^2 &= 200 \frac{\text{m}^2}{\text{s}^2} - 60 \frac{\text{m}}{\text{s}} v_1' \cos \theta_1 + 3v_1'^2 \\
 0 &= 200 \frac{\text{m}^2}{\text{s}^2} - 60 \frac{\text{m}}{\text{s}} v_1' \cos \theta_1 + 4v_1'^2
 \end{aligned}$$

Solving for the cosine, the result is

$$0 = 200 \frac{m^2}{s^2} - 60 \frac{m}{s} v_1' \cos \theta_1 + 4v_1'^2$$

$$60 \frac{m}{s} v_1' \cos \theta_1 = 200 \frac{m^2}{s^2} + 4v_1'^2$$

$$\cos \theta_1 = \frac{10 \frac{m}{s}}{3} \frac{1}{v_1'} + \frac{1}{15 \frac{m}{s}} v_1'$$

The value of  $v_1'$  can vary depending on how the collision occurs. For a specific value of  $v_1'$  there's a maximum angle. Since the derivative of a function is zero at an extremum, the maximum value of the angle is found with

$$\frac{d(\cos \theta_1)}{dv_1'} = 0$$

Thus, at the maximum angle

$$\frac{d(\cos \theta_1)}{dv_1'} = 0$$

$$\frac{-10 \frac{m}{s}}{3} \frac{1}{v_1'^2} + \frac{1}{15 \frac{m}{s}} = 0$$

$$\frac{1}{15 \frac{m}{s}} = \frac{10 \frac{m}{s}}{3} \frac{1}{v_1'^2}$$

$$v_1'^2 = 50 \frac{m^2}{s^2}$$

Therefore, the angle is

$$\cos \theta_1 = \frac{10 \frac{m}{s}}{3} \frac{1}{v_1'} + \frac{1}{15 \frac{m}{s}} v_1'$$

$$\cos \theta_1 = \frac{10 \frac{m}{s}}{3} \sqrt{\frac{1}{50 \frac{m^2}{s^2}}} + \frac{1}{15 \frac{m}{s}} \sqrt{50 \frac{m^2}{s^2}}$$

$$\cos \theta_1 = 0,94281$$

$$\theta_1 = 19,47^\circ$$