Chapter 9 Solutions

1. The intensity after the first polarizer is

$$\begin{aligned} I &= \frac{1}{2} I_0 \\ &= \frac{1}{2} 50 \tfrac{W}{m^2} \\ &= 25 \tfrac{W}{m^2} \end{aligned}$$

The intensity after the second polarizer is

$$\begin{aligned} I &= I_0 \cos^2 \theta \\ &= 25 \tfrac{W}{m^2} \cos^2 25° \\ &= 20.53 \tfrac{W}{m^2} \end{aligned}$$

2. The intensity after the first polarizer is

$$\begin{aligned} I &= \frac{1}{2} I_0 \\ &= \frac{1}{2} 40 \tfrac{W}{m^2} \\ &= 20 \tfrac{W}{m^2} \end{aligned}$$

The intensity after the second polarizer is

$$\begin{aligned} I &= I_0 \cos^2 \theta \\ &= 20 \tfrac{W}{m^2} \cos^2 75° \\ &= 1.34 \tfrac{W}{m^2} \end{aligned}$$

The intensity after the third polarizer is

$$\begin{aligned} I &= I_0 \cos^2 \theta \\ &= 1.34 \tfrac{W}{m^2} \cos^2 15° \\ &= 1.25 \tfrac{W}{m^2} \end{aligned}$$

3. The intensity after the first polarizer is

$$
\begin{aligned}
I &= \frac{1}{2} I_0 \\
&= \frac{1}{2} 20 \tfrac{W}{m^2} \\
&= 10 \tfrac{W}{m^2}
\end{aligned}
$$

The angle is found with the formula of the intensity after the second polarizer.

$$
\begin{aligned}
I &= I_0 \cos^2 \theta \\
4 \tfrac{W}{m^2} &= 10 \tfrac{W}{m^2} \cos^2 \theta \\
0.4 &= \cos^2 \theta
\end{aligned}
$$

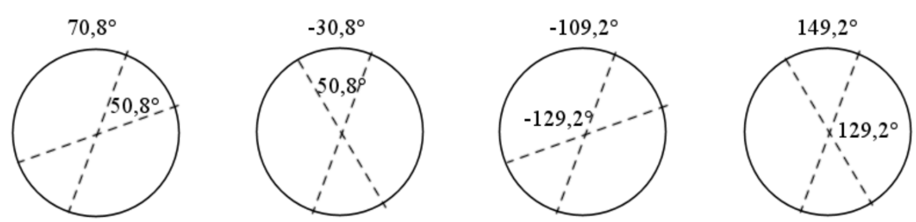
There are 2 solutions to the square root:

$$\sqrt{0.4} = \cos\theta \qquad \text{and} \qquad -\sqrt{0.4} = \cos\theta$$

As there are two solutions to inverse cosine function, the answers are

$$
\begin{array}{ccc}
\sqrt{0.4} = \cos\theta & \text{and} & -\sqrt{0.4} = \cos\theta \\
\theta = \pm 50.8° & \text{and} & \theta = \pm 129.2°
\end{array}
$$

These angles are the angles between the current polarizer and the previous polarizer. As the angle of the previous polarizer is 20°, these solutions are



The first solution is identical to the third, and the second solution is identical to the fourth. The two solutions are therefore 70.8° and 149.2°.

4. After the passage through the polarizer at an angle θ, we have

$$5\tfrac{W}{m^2} = I_0 \cos^2 \theta$$

Since the intensity decreases to 3 W/m² if the polarizer is turned by 20°, we have

$$3\tfrac{W}{m^2} = I_0 \cos^2 \left(\theta + 20°\right)$$

So we have 2 equations and 2 unknowns. If the second equation is divided by the first, we have

$$\frac{3}{5} = \frac{I_0 \cos^2 \left(\theta + 20°\right)}{I_0 \cos^2 \theta}$$

$$\sqrt{\frac{3}{5}} = \frac{\cos\left(\theta + 20°\right)}{\cos\theta}$$

Since $\cos(a+b) = \cos a \cos b - \sin a \sin b$, this equation is

$$\sqrt{\frac{3}{5}} = \frac{\cos\theta \cos 20° - \sin\theta \sin 20°}{\cos\theta}$$

$$\sqrt{\frac{3}{5}} = \cos 20° - \tan\theta \sin 20°$$

$$\theta = 25,767°$$

Thus, the initial intensity is

$$5\tfrac{W}{m^2} = I_0 \cos^2 25,767°$$

$$I_0 = 6,165\tfrac{W}{m^2}$$

5. The angle of polarization is

$$\tan\theta_p = \frac{n_2}{n_1}$$

$$\tan\theta = \frac{1.55}{1}$$

$$\theta_p = 57.2°$$

6. The angle of polarization is

$$\tan\theta_p = \frac{n_2}{n_1}$$
$$\tan\theta = \frac{1.2}{1.6}$$
$$\theta_p = 36.9°$$

7. If the reflected ray is polarized, then the angle of incidence is equal to the angle of polarization. This angle is

$$\tan\theta_p = \frac{n_2}{n_1}$$
$$\tan\theta = \frac{1.7}{1}$$
$$\theta_p = 59.53°$$

The angle of refraction is thus

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1\sin 59.53° = 1.7\sin\theta_2$$
$$\theta_2 = 30.46°$$

8. a) With a 48° critical angle, we have

$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\sin 48° = \frac{n_2}{n_1}$$

The individual values of the indices of refraction cannot be found, but the value of n_2/n_1 can be found. Its value is

$$\sin 48° = \frac{n_2}{n_1}$$

$$\frac{n_2}{n_1} = 0.743$$

The angle of polarization is therefore

$$\tan\theta_p = \frac{n_2}{n_1}$$

$$\tan\theta = 0.743$$

$$\theta_p = 36.6°$$

b) No, since the angle of polarization is 36.6° and the total reflection begins at 48°.

9. Let's separate this beam into two parts. The first part is the beam with intensity I_{max} after its passage through the polarizer. To have this intensity, the axis of the polarizer must be aligned with the direction of the most intense polarization. If the polarizer is turned by an angle θ, then the intensity that passes for this most intense wave is

$$I_{max} = I_{max}\cos^2\theta$$

The second is the beam with intensity I_{min} after its passage through the polarizer. This component is perpendicular to the most intense component. Thus, the angle between this component and the polarization axis is 90° – θ. The intensity of the light of this component is, therefore,

$$I_{min} = I_{min}\cos^2(90-\theta)$$

The total intensity is the sum of these two intensities.

$$I = I_{max}\cos^2\theta + I_{min}\cos^2(90-\theta)$$

But since

$$p = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

The intensity I_{min} is

$$\left(I_{max}+I_{min}\right)p=I_{max}-I_{min}$$
$$I_{max}p+I_{min}p=I_{max}-I_{min}$$
$$I_{min}p+I_{min}=I_{max}-I_{max}p$$
$$I_{min}\left(p+1\right)=\left(1-p\right)I_{max}$$
$$I_{min}=\frac{1-p}{1+p}I_{max}$$

Thus, the total intensity becomes

$$\begin{aligned}I&=I_{max}\cos^2\theta+I_{min}\cos^2\left(90-\theta\right)\\&=I_{max}\left(\cos^2\theta+\frac{1-p}{1+p}\cos^2\left(90-\theta\right)\right)\\&=\frac{I_{max}}{1+p}\left(\left(1+p\right)\cos^2\theta+\left(1-p\right)\cos^2\left(90-\theta\right)\right)\end{aligned}$$

But since $\cos\left(90-\theta\right)=\sin\theta$, the intensity becomes

$$\begin{aligned}I&=\frac{I_{max}}{1+p}\left(\left(1+p\right)\cos^2\theta+\left(1-p\right)\sin^2\theta\right)\\&=\frac{I_{max}}{1+p}\left(\cos^2\theta+p\cos^2\theta+\sin^2\theta-p\sin^2\theta\right)\\&=\frac{I_{max}}{1+p}\left(\cos^2\theta+\sin^2\theta+p\left(\cos^2\theta-\sin^2\theta\right)\right)\\&=\frac{I_{max}}{1+p}\left(1+p\left(\cos^2\theta-\sin^2\theta\right)\right)\end{aligned}$$

Finally, since $\cos^2\theta-\sin^2\theta=\cos2\theta$, we obtain

$$\begin{aligned}I&=\frac{I_{max}}{1+p}\left(1+p\cos2\theta\right)\\&=\frac{1+p\cos2\theta}{1+p}I_{max}\end{aligned}$$

This is what we were supposed to demonstrate.