Chapter 8 Solutions

1. a) The angle of the first minimum is

$$a\sin\theta = \lambda$$
$$0.01\times 10^{-3}m\sin\theta = 500\times 10^{-9}m$$
$$\theta = 2.866°$$

Therefore, the position on the screen is

$$\tan\theta = \frac{y}{L}$$
$$\tan 2.866° = \frac{y}{200cm}$$
$$y = 10.0cm$$

b) The angle of the second minimum is

$$a\sin\theta = 2\lambda$$
$$0.01\times 10^{-3}m\sin\theta = 2\cdot 500\times 10^{-9}m$$
$$\theta = 5.74°$$

Therefore, the position on the screen is

$$\tan\theta = \frac{y}{L}$$
$$\tan 5.74° = \frac{y}{200cm}$$
$$y = 20.1cm$$

2. If the central maximum is 4 cm wide, then the distance between the first minimum and the centre of the central maximum is 2 cm. So, we have

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{2cm}{500cm}$$
$$\theta = 0.2292°$$

Therefore,

$$a\sin\theta = \lambda$$
$$a\sin 0.2292° = 560\times10^{-9}m$$
$$a = 0.14mm$$

3. If the central maximum is 50 cm wide, then the distance between the first minimum and the centre of the central maximum is 25 cm. So, we have

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{25cm}{160cm}$$
$$\theta = 8.88°$$

Therefore,

$$a\sin\theta = 2\lambda$$
$$0.01m\sin 8.88° = \lambda$$
$$\lambda = 1.544mm$$

4. 0.5 cm from the centre of the central maximum, the angle is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{0.5cm}{200cm}$$
$$\theta = 0.1432°$$

Therefore, the value of α is

$$\begin{aligned}\alpha &= \frac{a\sin\theta}{\lambda}2\pi\\ &= \frac{0.1\times10^{-3}m\cdot\sin 0.1432°}{600\times10^{-9}m}2\pi\\ &= 2.618rad\end{aligned}$$

Thus, the intensity is

$$\begin{aligned} I &= I_0 \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)} \right)^2 \\ &= I_0 \left(\frac{\sin(1.309)}{(1.309)} \right)^2 \\ &= 0.5445 I_0 \end{aligned}$$

5. If the central maximum is 4 cm wide, then the distance between the first minimum and the centre of the central maximum is 2 cm. So, we have

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{2cm}{300cm}$$
$$\theta = 0.382°$$

Therefore,

$$a\sin\theta = \lambda$$
$$a\sin 0.382° = 450\times10^{-9} m$$
$$a = 6.75\times10^{-5} m$$

With the new wavelength, we have

$$a\sin\theta = \lambda$$
$$6.75\times10^{-5} m \sin\theta = 650\times10^{-9} m$$
$$\theta = 0.5517°$$

So, the position of the first minimum on the screen is

$$\tan\theta = \frac{y}{L}$$
$$\tan 0.5517° = \frac{y}{300cm}$$
$$y = 2.889cm$$

The width of the central maximum is twice this value, so it is 5.778 cm.

6. If the central maximum is 10 cm wide, then the distance between the first minimum and the centre of the central maximum is 5 cm. So, we have

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{5cm}{400cm}$$
$$\theta = 0.716°$$

Therefore,

$$\frac{\lambda}{a} = \sin\theta$$
$$= \sin 0.716°$$
$$= 0.0125$$

At the second maximum, we have

$$a\sin\theta = 2\lambda$$
$$\sin\theta = 2\frac{\lambda}{a}$$
$$\sin\theta = 2\cdot 0.0125$$
$$\sin\theta = 0.02499$$
$$\theta = 1.432°$$

So, the position of the second minimum on the screen is

$$\tan\theta = \frac{y}{L}$$
$$\tan 1.432° = \frac{y}{400cm}$$
$$y = 10.002cm$$

The distance between the second minimum and the first minimum is therefore

$$\Delta y = 10.002cm - 5cm - 5.002cm$$

7. The first minimum at 20° indicates that

$$a \sin 20° = \lambda$$
$$\sin 20° = \frac{\lambda}{a}$$

Therefore, the angle of the second minimum is

$$a \sin \theta = 2\lambda$$
$$\sin \theta = 2\frac{\lambda}{a}$$
$$\sin \theta = 2 \sin 20°$$
$$\sin \theta = 0,68404$$
$$\theta = 43,16°$$

For the 3rd minimum, we have

$$a \sin \theta = 3\lambda$$
$$\sin \theta = 3\frac{\lambda}{a}$$
$$\sin \theta = 3 \sin 20°$$
$$\sin \theta = 1,026$$

As there is no solution, there is no third minimum.

8. a) We have

$$\frac{d}{a} = \frac{0.2mm}{0.04mm} = 5$$

This means that $m_d = 4$. The number of maximum is therefore 2 x 4 + 1 = 9.

b) 3 cm from the centre of the central maximum, the angle is

$$\tan \theta = \frac{y}{L}$$
$$\tan \theta = \frac{3cm}{240cm}$$
$$\theta = 0.7162°$$

The value of $\Delta\phi$ is thus

$$\begin{aligned}\Delta\phi &= \frac{d\sin\theta}{\lambda}2\pi \\ &= \frac{0.2\times10^{-3}m\cdot\sin 0.7162°}{600\times10^{-9}m}2\pi \\ &= 26.178rad\end{aligned}$$

The value of α is

$$\begin{aligned}\alpha &= \frac{a\sin\theta}{\lambda}2\pi \\ &= \frac{0.04\times10^{-3}m\cdot\sin 0.7162°}{600\times10^{-9}m}2\pi \\ &= 5.236rad\end{aligned}$$

Therefore, the intensity is

$$\begin{aligned}I_{tot} &= 4I_0\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2\cos^2\frac{\Delta\phi}{2} \\ &= 4I_0\left(\frac{\sin\left(\frac{5.236}{2}\right)}{\left(\frac{5.236}{2}\right)}\right)^2\cos^2\frac{26.178}{2} \\ &= 0.1097I_0\end{aligned}$$

c) The angle of the first interference maximum is

$$\begin{aligned}d\sin\theta &= \lambda \\ \sin\theta &= \frac{\lambda}{d}\end{aligned}$$

The value of $\Delta\phi$ is therefore

$$\begin{aligned}\Delta\phi &= \frac{d\sin\theta}{\lambda}2\pi \\ &= \frac{d\,\frac{\lambda}{d}}{\lambda}2\pi \\ &= 2\pi\end{aligned}$$

The value of α is

$$\begin{aligned}\alpha &= \frac{a\sin\theta}{\lambda}2\pi \\ &= \frac{a\frac{\lambda}{d}}{\lambda}2\pi \\ &= \frac{a}{d}2\pi \\ &= \frac{0.04mm}{0.2mm}2\pi \\ &= \frac{2\pi}{5}\end{aligned}$$

Thus, the intensity is

$$\begin{aligned}I_{tot} &= 4I_0\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)}\right)^2\cos^2\frac{\Delta\phi}{2} \\ &= 4I_0\left(\frac{\sin\left(\frac{\pi}{5}\right)}{\left(\frac{\pi}{5}\right)}\right)^2\cos^2\frac{2\pi}{2} \\ &= 3.5I_0\end{aligned}$$

As the intensity of the central interference maximum is 4 I_0, the ratio of intensity is

$$ratio = \frac{3.5I_0}{4I_0} = 0.875$$

The intensity is thus 87.5% of the intensity of the central interference maximum.

9. a) We notice that the 8th-order interference maximum is close to y = 5 cm. (Any maximum or minimum can be used). At this position, the angle is

$$\begin{aligned}\tan\theta &= \frac{y}{L} \\ \tan\theta &= \frac{5cm}{200cm} \\ \theta &= 1.432°\end{aligned}$$

For the 8th-order maximum, we have

$$d\sin\theta = 8\lambda$$
$$d\sin 1.432° = 8\cdot 650\times 10^{-9}m$$
$$d = 2.0806\times 10^{-4}m = 0.20806mm$$

As this is a little approximate, let's say 0.2 mm.

b) We notice that the 1st-order diffraction minimum is close to y = 3.2 cm. (Any minimum can be used). At this position, the angle is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{3.2cm}{200cm}$$
$$\theta = 0.9167°$$

For the first-order minimum, we have

$$a\sin\theta = \lambda$$
$$a\sin 0.9167° = 650\times 10^{-9}m$$
$$a = 4.063\times 10^{-5}m = 0.04063mm$$

As this is a little approximate, let's say 0.04 mm.

10. The angle of the first-order minimum is

$$\sin\theta = 1,22\frac{\lambda}{a}$$
$$\sin\theta = 1.22\frac{560\times 10^{-9}m}{0,1\times 10^{-3}m}$$
$$\theta = 0.39145°$$

Therefore, the distance between the centre of the diffraction pattern and the first minimum on the screen is

$$\tan\theta = \frac{y}{L}$$
$$\tan 0.3914° = \frac{y}{200cm}$$
$$y = 1.366cm$$

11. If the central maximum has a 6 mm diameter, then the distance between the first minimum and the centre of the central maximum is 3 mm. So, we have

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{0.3cm}{180cm}$$
$$\theta = 0.0955°$$

Therefore,

$$\sin\theta = 1.22\frac{\lambda}{a}$$
$$\sin 0.0955° = 1.22\frac{620\times10^{-9}m}{a}$$
$$a = 4.538\times10^{-4}m = 0.4538mm$$

12. According to the Babinet's principle, the diffraction pattern obtained with a hair is identical to the pattern obtained with a slit. Thus, the width of the hair is the same as the width of the slit that corresponds to the same diffraction pattern. So, this problem will be treated as a slit problem.

The angle of the first minimum is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{0,065m}{9,67m}$$
$$\theta = 0,3851°$$

The width of the hair is then found with

$$a\sin\theta = \lambda$$
$$a\sin 0,3851° = 523\times10^{-9}m$$
$$a = 7,78\times10^{-5}m$$
$$a = 77,8\mu m$$

13. The critical angle is

$$\sin\theta_c = 1.22\frac{\lambda}{a}$$
$$\sin\theta_c = 1.22\frac{550\times10^{-9}m/1.33}{3\times10^{-3}m}$$
$$\theta_c = 0.009635°$$

Therefore, the distance is

$$\theta_c(rad) = \frac{d}{L}$$
$$1.6817\times10^{-4}rad = \frac{0.02m}{L}$$
$$L = 118.9m$$

14. The critical angle is

$$\sin\theta_c = 1.22\frac{\lambda}{a}$$
$$\sin\theta_c = 1.22\frac{550\times10^{-9}m}{0.25m}$$
$$\theta_c = 1.538\times10^{-4}°$$

Therefore,

$$\theta_c(rad) = \frac{d}{L}$$
$$2.684\times10^{-6}rad = \frac{d}{200{,}000m}$$
$$d = 0.5368m$$

15. The angle is

$$\begin{aligned}\theta_c(rad) &= \frac{d}{L}\\ &= \frac{8\times10^{7}km}{4.73\times10^{13}km}\\ &= 1.691\times10^{-6}rad\end{aligned}$$

Therefore,

$$\sin\theta_c = 1.22\frac{\lambda}{a}$$
$$\sin 1.691\times 10^{-4}rad = 1.22\frac{550\times 10^{-9}m}{a}$$
$$a = 0.3967m$$

16. The intensity is

$$I = I_0\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}\right)^2$$

At a maximum (or a minimum), we must have $dI/d\alpha = 0$. Thus, we must have

$$\frac{dI}{d\alpha} = 0$$
$$\frac{d}{d\alpha}\left(I_0\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}\right)^2\right) = 0$$
$$I_0 2\left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}\right)\left(\frac{\cos\left(\frac{\alpha}{2}\right)\frac{1}{2}}{\frac{\alpha}{2}} - \frac{\sin\left(\frac{\alpha}{2}\right)\cdot\frac{1}{2}}{\left(\frac{\alpha}{2}\right)^2}\right) = 0$$

There are two possibilities for this derivative to vanish. First possibility: the first term in parentheses vanishes.

$$\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} = 0$$

The solution of this equation is

$$\frac{\alpha}{2} = M\pi$$

where M = 1, 2, 3,,,. We recognize this solution: those are the minimum of intensity.

Second possibility: the second term in parentheses vanishes.

$$\frac{\cos\left(\frac{\alpha}{2}\right)\frac{1}{2}}{\frac{\alpha}{2}} - \frac{\sin\left(\frac{\alpha}{2}\right)\cdot\frac{1}{2}}{\left(\frac{\alpha}{2}\right)^2} = 0$$

The solution leads to

$$\frac{\cos\left(\frac{\alpha}{2}\right)\frac{1}{2}}{\frac{\alpha}{2}} = \frac{\sin\left(\frac{\alpha}{2}\right)\cdot\frac{1}{2}}{\left(\frac{\alpha}{2}\right)^2}$$

$$\cos\left(\frac{\alpha}{2}\right) = \frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}$$

$$\frac{\alpha}{2} = \tan\left(\frac{\alpha}{2}\right)$$

This equation is not easy to solve. Among other things, it can be solved with a software like Maple or with the given internet site.

Here is the solution according to the internet site.

Input:

$x = \tan(x)$

Open code

Alternate forms:

$x = \frac{\sin(x)}{\cos(x)}$

$x = \frac{i\left(e^{-ix} - e^{ix}\right)}{e^{-ix} + e^{ix}}$

Alternate form assuming x is real:

$x = \frac{\sin(2x)}{\cos(2x) + 1}$

Numerical solutions: More digits

$x \approx \pm 10.9041216594289\ldots$

$x \approx \pm 7.72525183693771\ldots$

$x \approx \pm 4.49340945790906\ldots$

$x = 0$

$x \approx 14.0661939128315\ldots$

The first maximum is thus at $x = 4.49341$. (The approximation in which the maxima were assumed to be exactly between the minima gives 4.7124) Thus,

$$\frac{\alpha}{2} = 4.49341$$
$$\alpha = 8.98682$$

Since

$$\alpha = \frac{a \sin\theta}{\lambda} 2\pi$$

the angle is

$$\frac{a \sin\theta}{\lambda} 2\pi = 8.98682$$
$$\frac{0.1 \times 10^{-3} m \sin\theta}{600 \times 10^{-9} m} 2\pi = 8.98682$$
$$\sin\theta = 0.00858178$$
$$\theta = 0.4917°$$

Therefore, the distance is

$$\tan\theta = \frac{y}{L}$$
$$\tan 0.4917° = \frac{y}{2m}$$
$$y = 0.01716m$$
$$y = 1.716cm$$