Chapter 7 Solutions

1. The phase difference between the oscillation is

$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 \\ &= -1rad - 4rad \\ &= -5rad\end{aligned}$$

Thus, the amplitude is

$$\begin{aligned}A_{tot} &= \left|2A\cos\left(\frac{\Delta\phi}{2}\right)\right| \\ &= \left|2\cdot 0.2m\cos\left(\frac{-5rad}{2}\right)\right| \\ &= 0.3205m\end{aligned}$$

2. The phase difference between the oscillation is

$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 \\ &= -1.5rad - 2rad \\ &= -3.5rad\end{aligned}$$

Thus, the amplitude is

$$\begin{aligned}A_{tot} &= \sqrt{A_1^2 + 2A_1A_2\cos(\Delta\phi) + A_2^2} \\ &= \sqrt{(0.5m)^2 + 2\cdot 0.5m\cdot 0.4m\cos(-3.5) + (0.4m)^2} \\ &= 0.1882m\end{aligned}$$

3. a) To have constructive interference, we must have $\Delta\phi = 2m\pi$. Then

$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 \\ 2m\pi &= \phi_2 - 1 \\ \phi_2 &= 2m\pi + 1\end{aligned}$$

To obtain a value between 0 and 2π, m = 0 must be chosen. Then ϕ_2 = 1. The oscillation to add is thus

$$y_2 = 0.1m \sin\left(100\tfrac{rad}{s} t + 1\right)$$

b) To have destructive interference, we must have $\Delta\phi = (2m + 1)\pi$. Then

$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 \\ (2m+1)\pi &= \phi_2 - 1 \\ \phi_2 &= (2m+1)\pi + 1\end{aligned}$$

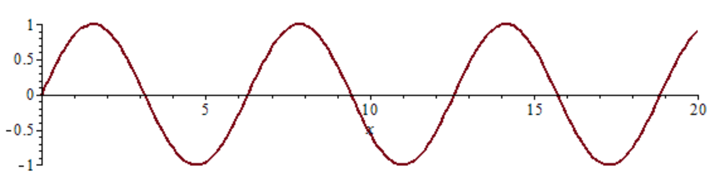
To obtain a value between 0 and 2π, m = 0 must be chosen. Then $\phi_2 = \pi + 1$. The oscillation to add is thus

$$y_2 = 0.1m \sin\left(100\tfrac{rad}{s} t + 4{,}142\right)$$

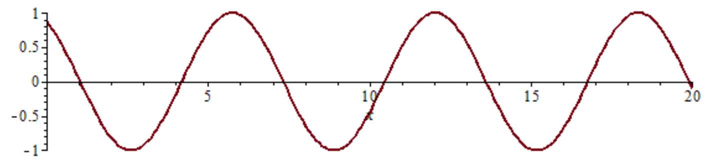
4. Here, the only phase difference is the one caused by the difference in distance $\Delta\phi_T$. Therefore, the phase shift is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{r_B - r_A}{\lambda}2\pi \\ &= -\frac{3.6m - 5.2m}{0.5m}2\pi \\ &= \tfrac{32}{5}\pi\end{aligned}$$

5. Here, we have only the phase difference due to the sources $\Delta\phi_S$. Let's assume that source A has a vanishing constant phase



If source B is ahead by a third of a cycle on source A, then its graph must be



In this case, the phase constant is

$$\phi_{\text{source 2}} = \frac{2\pi}{3}$$

Thus, $\Delta\phi_S$ is

$$\begin{aligned}\Delta\phi_S &= \phi_{source\ 2} - \phi_{source\ 1} \\ &= \frac{2\pi}{3} - 0 \\ &= \frac{2\pi}{3}\end{aligned}$$

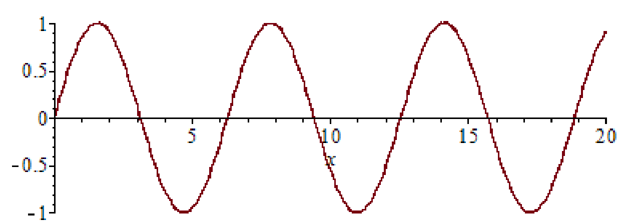
6. The total phase difference is

$$\Delta\phi = \Delta\phi_T + \Delta\phi_S + \Delta\phi_R$$

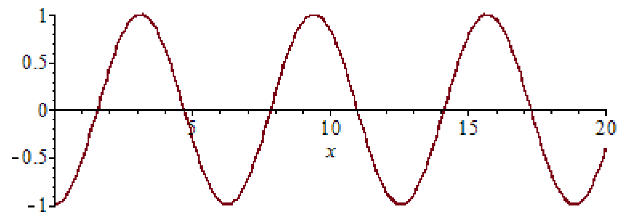
$\Delta\phi_T$ is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda} 2\pi \\ &= -\frac{r_B - r_A}{\lambda} 2\pi \\ &= -\frac{3m - 5m}{0.2m} 2\pi \\ &= 20\pi\end{aligned}$$

Let's assume that source A has a zero phase constant.



If source B lags by a quarter cycle on source A, then the graph must be



In this case, the phase constant is

$$\phi_{\text{source 2}} = -\frac{\pi}{2}$$

Thus, $\Delta\phi_S$ is

$$\begin{aligned}\Delta\phi_S &= \phi_{source\ 2} - \phi_{source\ 1}\\ &= -\frac{\pi}{2} - 0\\ &= -\frac{\pi}{2}\end{aligned}$$

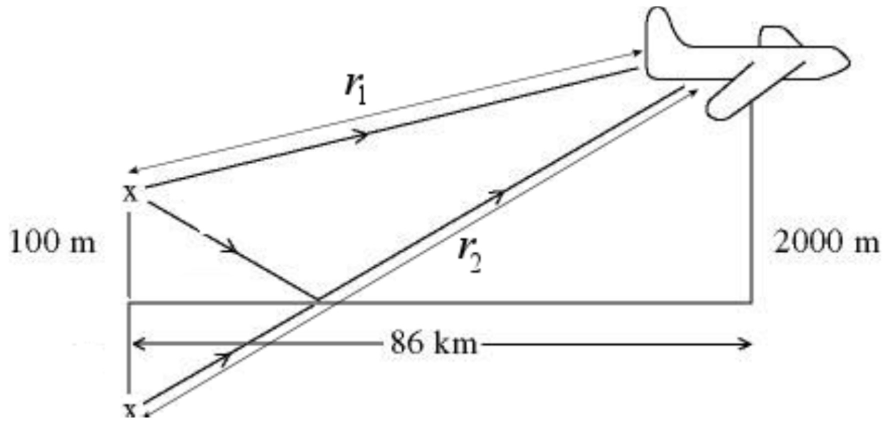
The total phase difference is then

$$\begin{aligned}\Delta\phi &= \Delta\phi_T + \Delta\phi_S + \Delta\phi_R\\ &= 20\pi + -\frac{\pi}{2} + 0\\ &= \frac{39\pi}{2}\\ &= 61.26 rad\end{aligned}$$

7. First, let's find the wavelength of the signal. This wavelength is

$$\lambda = \frac{c}{f} = \frac{3\times10^8 \frac{m}{s}}{120\times10^6 Hz} = 2.5m$$

The distances are (assuming that wave 2 is the reflected wave)



$$r_1 = \sqrt{(86000m)^2 + (1900m)^2}$$
$$r_2 = \sqrt{(86000m)^2 + (2100m)^2}$$

The path length difference is thus

$$\begin{aligned}\Delta r &= \sqrt{(86000m)^2 + (2100m)^2} - \sqrt{(86000m)^2 + (1900m)^2} \\ &= 4.6499m\end{aligned}$$

Then, $\Delta\phi_T$ is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{-4.6499m}{2,5m} \\ &= -11.686rad\end{aligned}$$

Since the reflected wave is inverted, $\Delta\phi_R$ is

$$\Delta\phi_R = \pi$$

The total phase difference is thus

$$\begin{aligned}\Delta\phi &= -11.686rad + \pi \\ &= -8.545rad\end{aligned}$$

8. Since the path length difference is d, $\Delta\phi_T$ is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{d}{0.25m}2\pi\end{aligned}$$

(We'll assume that source 2 is the speaker farthest from the observer and that source 1 is the speaker closer to the observer. The d must be positive since $d = r_2 - r_1$.)

There is no other phase difference since there is no reflection, and the sources are in phase.

The total phase difference is thus

$$\Delta\phi = -\frac{d}{0.25m}2\pi$$

To obtain destructive interference, we must have

$$\Delta\phi = (2m+1)\pi$$

Then

$$-\frac{d}{0.25m}2\pi = (2m+1)\pi$$

If this equation is solved for d, we obtain

$$\begin{aligned}-\frac{d}{0.25m}2\pi &= (2m+1)\pi \\ d &= -\frac{(2m+1)}{2}\cdot 0.25m\end{aligned}$$

Positive or vanishing values of m give negative values of d, which are unacceptable. The smallest value of d is found with m = -1. This value is

$$\begin{aligned}d &= -\frac{(2(-1)+1)}{2}\cdot 0.25m \\ &= 12.5cm\end{aligned}$$

9. The distances are (assuming that wave 2 is the one coming from speaker B)

$$r_1 = \sqrt{(1m)^2 + (2.4m)^2}$$
$$r_2 = 2.4m$$

The path length difference is

$$\begin{aligned}\Delta r &= 2.4m - \sqrt{(1m)^2 + (2.4m)^2} \\ &= -0.2m\end{aligned}$$

Thus, $\Delta\phi_T$ is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{-0.2m}{\lambda}2\pi \\ &= \frac{0.2m}{\lambda}2\pi\end{aligned}$$

As there is no phase difference between the sources and no reflection, this phase difference is the total phase difference.

$$\Delta\phi = \frac{0.2m}{\lambda}2\pi$$

To obtain destructive interference, the phase difference must be

$$\Delta\phi = (2m+1)\pi$$

Then

$$\frac{0.2m}{\lambda}2\pi = (2m+1)\pi$$

If this equation is solved for λ, we arrive at

$$\begin{aligned}\frac{0.2m}{\lambda}2 &= (2m+1) \\ \lambda &= \frac{0.4m}{2m+1}\end{aligned}$$

To get the minimum frequency, the longest wavelength is needed. Thus, we need the smallest numerator, which we obtain with m = 0. Then, the maximum wavelength is

$$\lambda_{\max} = \frac{0.4m}{2\cdot 0+1} = 0.4m$$

The minimum frequency is thus

$$f_{\min} = \frac{v}{\lambda_{\max}} = \frac{340\frac{m}{s}}{0.4m} = 850Hz$$

10. First, let's find the wavelength of the signal. This wavelength is

$$\lambda = \frac{v}{f} = \frac{343\frac{m}{s}}{490Hz} = 0.7m$$

The distances are (assuming that wave 1 is the wave reflected by the wall)

$$r_1 = 2\sqrt{(3m)^2 + d^2}$$
$$r_2 = 6m$$

The path length difference is thus

$$\Delta r = 6m - 2\sqrt{(3m)^2 + d^2}$$

Then, $\Delta\phi_T$ is

$$\begin{aligned}\Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{6m - 2\sqrt{(3m)^2 + d^2}}{0.7m}2\pi \\ &= \frac{2\sqrt{(3m)^2 + d^2} - 6m}{0.7m}2\pi\end{aligned}$$

As the sound reflected on a wall is inverted, $\Delta\phi_R$ is

$$\Delta\phi_R = \pi$$

As $\Delta\phi_S = 0$, the total phase difference is

$$\Delta\phi = \frac{2\sqrt{(3m)^2 + d^2} - 6m}{0.7m} 2\pi + \pi$$

To have constructive interference, the phase difference must be

$$\Delta\phi = 2m\pi$$

Then

$$\frac{2\sqrt{(3\text{m})^2 + d^2} - 6\text{m}}{0.7\text{m}} 2\pi + \pi = 2m\pi$$

Note here that since the two terms on the left are positive, the term on the right must also be positive, thereby eliminating all the negative values of *m*.

Solving for *d,* we arrive at

$$\begin{aligned}
\frac{2\sqrt{(3\text{m})^2 + d^2} - 6\text{m}}{0.7\text{m}} 2 + 1 &= 2m \\
\frac{2\sqrt{(3\text{m})^2 + d^2} - 6\text{m}}{0.7\text{m}} 2 &= 2m - 1 \\
2\sqrt{(3\text{m})^2 + d^2} - 6\text{m} &= \frac{2m-1}{2} 0.7\text{m} \\
2\sqrt{(3\text{m})^2 + d^2} &= \frac{2m-1}{2} 0.7\text{m} + 6\text{m} \\
\sqrt{(3\text{m})^2 + d^2} &= \frac{2m-1}{4} 0.7\text{m} + 3\text{m} \\
(3\text{m})^2 + d^2 &= \left(\frac{2m-1}{4} 0.7\text{m} + 3\text{m}\right)^2 \\
d^2 &= \left(\frac{2m-1}{4} 0.7\text{m} + 3\text{m}\right)^2 - (3\text{m})^2
\end{aligned}$$

Here's what you get for different values of *m*.

$m = 0$	d^2 is negative, which is unacceptable
$m = 1$	$d^2 = 1.080625 \text{ m}^2$
$m = 2$ and more	d^2 is greater than 1.080625 m^2

The minimum value is, therefore, found with m = 1. The distance is then

$$\begin{aligned} d &= \sqrt{1.080625m^2} \\ &= 1.0395m \end{aligned}$$

11. Assuming that the speaker B is source 2, $\Delta\phi_T$ is

$$\begin{aligned} \Delta\phi_T &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{\Delta r}{0.32m}2\pi \end{aligned}$$

$\Delta\phi_S$ is

$$\Delta\phi_S = \frac{\pi}{4}$$

The value is positive since source 2 is ahead of source 1.

As there is no reflection, the total phase difference is

$$\Delta\phi = -\frac{\Delta r}{\lambda}2\pi + \frac{\pi}{4}$$

With destructive interference, we must have

$$\Delta\phi = (2m+1)\pi$$

Thus

$$-\frac{\Delta r}{\lambda}2\pi + \frac{\pi}{4} = (2m+1)\pi$$

Solving for the path length difference, we have

$$-\frac{\Delta r}{\lambda}2+\frac{1}{4}=2m+1$$
$$-\frac{\Delta r}{\lambda}2=2m+\frac{3}{4}$$
$$-\frac{\Delta r}{\lambda}=m+\frac{3}{8}$$
$$\Delta r=-\left(m+\frac{3}{8}\right)\lambda$$

If $m = 0$, then $\Delta r = -3\lambda/ 8$.
If $m = 1$, then $\Delta r = -11\lambda/ 8$.
As m increases from $m = 1$, the path length difference increases (in absolute value).
If $m = -1$, then $\Delta r = 5\lambda/ 8$.
As m decreases from $m = -1$, the path length difference increases.

The smallest path length difference (in absolute value) is thus

$$\Delta r=-\frac{3}{8}\lambda=-\frac{3}{8}32cm=-12cm$$

Therefore

$$r_2-r_1=-12cm$$

Source 2 must be 12 cm closer than source 1.

12. The angle is

$$\tan\theta=\frac{y}{L}$$
$$\tan\theta=\frac{1cm}{200cm}$$
$$\theta=0.2865°$$

The distance is then found with

$$d\sin\theta=m\lambda$$
$$d\sin\left(0.2865°\right)=4\cdot 600nm$$
$$d=4.8\times 10^{-4}m=0.48mm$$

13. The angle is given by

$$d\sin\theta = m\lambda$$
$$0.1\times10^{-3}m\sin\theta = 5\cdot 500nm$$
$$\theta = 1.4325°$$

The position of the maximum is thus

$$\tan\theta = \frac{y}{L}$$
$$\tan 1.4325° = \frac{y}{160cm}$$
$$y = 4.001cm$$

14. With a 550 nm wavelength, we have

$$d\sin\theta = m\lambda$$
$$d\sin\theta = 5\cdot 550nm$$
$$d\sin\theta = 2750nm$$

With the other wavelength, we have

$$d\sin\theta = m\lambda$$
$$d\sin\theta = 4\lambda$$

As these maxima are at the same position, so at the same angle, the two $d\sin\theta$ of these two equations are equal. Therefore

$$2750nm = 4\lambda$$
$$\lambda = 687.5nm$$

15. The figure shows that the third minimum (m = 2) is 11.5 mm from the centre of the central maximum. The angle is

$$d\sin\theta = \left(m+\tfrac{1}{2}\right)\lambda$$
$$0.2\times10^{-3}m\sin\theta = \left(2+\tfrac{1}{2}\right)632\times10^{-9}m$$
$$\theta = 0.4526°$$

Therefore

$$\tan\theta = \frac{y}{L}$$
$$\tan 0.4526° = \frac{1.15cm}{L}$$
$$L = 145.6cm$$

16. In Young's experiment, the path length difference can be found from the phase difference with

$$\Delta\phi = -\frac{\Delta r}{\lambda}2\pi$$
$$2 = \left| -\frac{\Delta r}{450\times 10^{-9}m}2\pi \right|$$
$$\Delta r = 143.24nm$$

As the path length difference is

$$\Delta r = d\sin\theta$$

We have

$$143.24\times 10^{-9}m = 0.2\times 10^{-3}m\sin\theta$$
$$\theta = 0.041°$$

The position is then found with

$$\tan\theta = \frac{y}{L}$$
$$\tan 0.041° = \frac{y}{240cm}$$
$$y = 0.1719cm$$

17. The angle 2 cm away from the central maximum is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{2cm}{200cm}$$
$$\theta = 0.5729°$$

The path length difference is then

$$\begin{aligned}\Delta r &= d\sin\theta \\ &= 0.2\times10^{-3}m\sin 0.5729° \\ &= 1.9999\times10^{-6}m\end{aligned}$$

The phase difference is thus

$$\begin{aligned}\Delta\phi &= -\frac{\Delta r}{\lambda}2\pi \\ &= -\frac{1.9999\times10^{-6}m}{600\times10^{-9}m}2\pi \\ &= -20.9429rad\end{aligned}$$

Therefore, the intensity is

$$\begin{aligned}I_{tot} &= 4I\cos^2\left(\frac{\Delta\phi}{2}\right) \\ &= 1.0018I\end{aligned}$$

18. If the intensity is 50% of the maximum intensity, then the intensity is

$$I_{tot} = 0.5I_{\max} = 0.5\times4I = 2I$$

The phase difference is thus

$$\begin{aligned}I_{tot} &= 4I\cos^2\left(\frac{\Delta\phi}{2}\right) \\ 2I &= 4I\cos^2\left(\frac{\Delta\phi}{2}\right) \\ \frac{1}{2} &= \cos^2\left(\frac{\Delta\phi}{2}\right) \\ \pm\frac{1}{\sqrt{2}} &= \cos\left(\frac{\Delta\phi}{2}\right) \\ \pm\frac{\pi}{4} &= \frac{\Delta\phi}{2}\end{aligned}$$

(Actually, there are many other solutions to an inverse cosine function, but they are all larger (in absolute value)). As we want the smallest distance from the central maximum, the smallest value of the phase difference is used. The phase difference is thus

$$\Delta\phi = \pm\frac{\pi}{2}$$

Therefore

$$\begin{aligned}\Delta\phi &= -\frac{\Delta r}{\lambda}2\pi \\ \pm\frac{\pi}{2} &= -\frac{\Delta r}{500\times 10^{-9}m}2\pi \\ \Delta r &= \pm 125nm\end{aligned}$$

The angle is thus

$$\begin{aligned}\Delta r &= d\sin\theta \\ 125\times 10^{-9}m &= 0{,}15\times 10^{-3}m\sin\theta \\ \theta &= 0.04775°\end{aligned}$$

Therefore, the position on the screen is

$$\begin{aligned}\tan\theta &= \frac{y}{L} \\ \tan 0.04775° &= \frac{y}{300cm} \\ y &= 0.25cm\end{aligned}$$

19. First, let's find the wavelength of the signal. This wavelength is

$$\lambda = \frac{v}{f} = \frac{343\frac{m}{s}}{245Hz} = 1.4m$$

The distances are (assuming that wave 2 is the wave reflected by the wall)

$$\begin{aligned}r_1 &= 6m \\ r_2 &= 2\sqrt{(3m)^2 + (2m)^2}\end{aligned}$$

The path length difference is thus

$$\Delta r = 2\sqrt{(3m)^2 + (2m)^2} - 6m$$
$$= 1.211m$$

Therefore, $\Delta\phi_T$ is

$$\Delta\phi_T = -\frac{\Delta r}{\lambda} 2\pi$$
$$= -\frac{1.211m}{1.4m} 2\pi$$
$$= -5.435 rad$$

As the sound reflected on a wall is inverted, $\Delta\phi_R$ is

$$\Delta\phi_R = \pi$$

As $\Delta\phi_S = 0$, the total phase difference is

$$\Delta\phi = -5.435 + \pi = -2.294 rad$$

Therefore, the intensity is

$$I_{tot} = I_1 \left(1 + 2\tfrac{A_2}{A_1}\cos(\Delta\phi) + \tfrac{A_2^2}{A_1^2}\right)$$
$$= I_1 \left(1 + 2\tfrac{0,7A_1}{A_1}\cos(-2,294 rad) + \tfrac{(0,7A_1)^2}{A_1^2}\right)$$
$$= I_1 \left(1 + 1,4\cos(-2,294 rad) + 0,49\right)$$
$$= 0,5637 I_1$$

Therefore, the intensity is 56.37% of the intensity we would have if there was only the wave arriving directly from the source.

20. We know that the intensity is given by

$$I_{tot} = I_A \left(1 + 2\tfrac{A_B}{A_A}\cos(\Delta\phi) + \tfrac{A_B^2}{A_A^2}\right)$$

To know the total intensity, the ratio of the amplitudes and phase difference must be known.

As the intensity is proportional to the square of the amplitude, we have

$$\frac{I_B}{I_A} = \left(\frac{A_B}{A_A}\right)^2$$

$$\frac{\frac{P}{4\pi r_B^2}}{\frac{P}{4\pi r_A^2}} = \left(\frac{A_B}{A_A}\right)^2$$

$$\frac{r_A^2}{r_B^2} = \left(\frac{A_B}{A_A}\right)^2$$

$$\frac{r_A}{r_B} = \frac{A_B}{A_A}$$

The ratio of the amplitudes is therefore equal to the inverse of the ratio of the distances. This ration is

$$\begin{aligned}\frac{A_B}{A_A} &= \frac{r_A}{r_B} \\ &= \frac{300m}{\sqrt{(300m)^2 + (200m)^2}} \\ &= \frac{3}{\sqrt{13}}\end{aligned}$$

It remains to find the phase difference. As we have only the phase difference caused by the difference in distance, the phase shift is

$$\begin{aligned}\Delta\phi &= -\frac{r_B - r_A}{\lambda} 2\pi \\ &= \frac{\sqrt{(300m)^2 + (200m)^2} - 300m}{3\times 10^8 \frac{m}{s} / 100\,\text{MHz}} 2\pi \\ &= \frac{60.555m}{3m} 2\pi \\ &= 126.826 rad\end{aligned}$$

Therefore, the intensity is

$$\begin{aligned}I_{tot} &= I_A \left(1 + 2\frac{A_B}{A_A}\cos(\Delta\phi) + \frac{A_B^2}{A_A^2}\right) \\ &= 0.001\tfrac{W}{m^2}\left(1 + 2\cdot\tfrac{3}{\sqrt{13}}\cos(126.826 rad) + \tfrac{9}{13}\right) \\ &= 0.002353\tfrac{W}{m^2}\end{aligned}$$

21. The phase difference is

$$\begin{aligned}\Delta\phi &= \frac{4\pi e n_f}{\lambda}\\ &= \frac{4\pi\cdot 450nm\cdot 1.3}{\lambda}\\ &= \frac{2340nm\cdot\pi}{\lambda}\end{aligned}$$

With destructive interference, we must have

$$\Delta\phi = (2m+1)\pi$$

Therefore, we must have

$$\begin{aligned}\frac{2340nm\cdot\pi}{\lambda} &= (2m+1)\pi\\ \lambda &= \frac{2340nm}{2m+1}\end{aligned}$$

This equation gives the following values.

m= 0	λ = 2340 nm
m= 1	λ = 780 nm
m= 2	λ = 468 nm
m= 3	λ = 334.3 nm

In the visible spectrum, only the 468 nm wavelength is absent.

22. The phase difference is

$$\begin{aligned}\Delta\phi &= \frac{4\pi e n_f}{\lambda}\\ &= \frac{4\pi\cdot 450nm\cdot 1,3}{\lambda}\\ &= \frac{2340nm\cdot\pi}{\lambda}\end{aligned}$$

With constructive interference, we must have

$$\Delta\phi = 2m\pi$$

Then, we must have

$$\frac{2340nm \cdot \pi}{\lambda} = 2m\pi$$
$$\lambda = \frac{1170nm}{m}$$

This equation gives the following values

m= 1	λ = 1170 nm
m= 2	λ = 585 nm
m= 3	λ = 390 nm

In the visible spectrum, only the 585 nm wavelength is strongly reflected.

23. a) The phase difference is

$$\begin{aligned}\Delta\phi &= \frac{4\pi e n_f}{\lambda} + \pi \\ &= \frac{4\pi \cdot 250nm \cdot 1,6}{450nm} + \pi \\ &= 14.31 rad\end{aligned}$$

b) The intensity is

$$\begin{aligned}I_2 &= 4I_1 \cos^2\left(\frac{\Delta\phi}{2}\right) \\ &= 4I_1 \cos^2\left(\frac{14.31}{2}\right) \\ &= 1.65 I_1\end{aligned}$$

The light is thus 1.65 times brighter than it would be without a thin film.

24. The phase difference is

$$\begin{aligned}\Delta\phi &= \frac{4\pi e n_f}{\lambda}+\pi \\ &= \frac{4\pi\cdot e\cdot 1{,}8}{550nm}+\pi \\ &= \frac{7.2\cdot\pi\cdot e}{550nm}+\pi\end{aligned}$$

With constructive interference, we must have

$$\Delta\phi = 2m\pi$$

Then, we must have

$$\begin{aligned}\frac{7.2\cdot\pi\cdot e}{550nm}+\pi &= 2m\pi \\ \frac{7.2\cdot e}{550nm}+1 &= 2m \\ e &= \frac{550nm(2m-1)}{7.2}\end{aligned}$$

The minimum thickness is found with m = 1. The minimum thickness is

$$e = \frac{550nm}{7.2} = 76.39nm$$

25. The phase difference is

$$\Delta\phi = \frac{4\pi e n_f}{\lambda}+\pi$$

For the wavelength making constructive interference (λ_1), we have

$$\Delta\phi = 2m\pi$$

Therefore

$$\frac{4\pi e n_f}{\lambda_1}+\pi = 2m_1\pi$$

Solving for the thickness, we found

$$e=\frac{(2m_1-1)\lambda_1}{4n_f}=\frac{(2m_1-1)\cdot 638.4nm}{4\cdot 1.33}=(2m_1-1)120nm$$

Giving the values 1, 2, 3, 4, 5…. to m_1, the following thicknesses are obtained: 120 nm, 360 nm, 600 nm, 840 nm, 1080 nm.

For the wavelength making destructive interference (λ_2), we have

$$\Delta\phi=(2m+1)\pi$$

Therefore

$$\frac{4\pi e n_f}{\lambda_2}+\pi=(2m_2+1)\pi$$

Simplified, this equation is

$$\frac{4\pi e n_f}{\lambda_2}=2m_2\pi$$

Solving for the thickness, we obtain

$$e=\frac{2m_2\lambda_1}{4n_f}=\frac{2m_2\cdot 478.8nm}{4\cdot 1.33}=m_2\times 180nm$$

Giving the values 1, 2, 3, 4, 5…. to m_2 , the following thicknesses are obtained: 180 nm, 360 nm, 540 nm, 720 nm, 900 nm.

The smallest common value for the thickness is, therefore, 360 nm.

26. a) The maximum value of *m* is

$$m<\frac{d}{\lambda}=\frac{\frac{1}{300}\times 10^{-3}m}{650\times 10^{-9}m}=5.13$$

The maximum value of *m* is thus 5. There are therefore 11 maxima (the *m* = 1 to *m* = 5 maxima on the right, the *m* = 1 to *m* = 5 maxima on the left and the central maximum).

b) The angle of the first-order maximum is

$$d\sin\theta = m\lambda$$
$$\tfrac{1}{300}\times 10^{-3}m\sin\theta = 1\cdot 650\times 10^{-9}m$$
$$\theta = 11.24°$$

Therefore, the position is

$$\tan\theta = \frac{y}{L}$$
$$\tan 11.24° = \frac{y}{240cm}$$
$$y = 47.7cm$$

27. a) The angle of the first-order maximum is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{43.6cm}{100cm}$$
$$\theta = 23.56°$$

Therefore, the wavelength is

$$d\sin\theta = m\lambda$$
$$\tfrac{1}{800}\times 10^{-3}m\cdot\sin 23.55° = 1\cdot\lambda$$
$$\lambda = 499.6nm$$

b) The angle of the second-order maximum is

$$d\sin\theta = m\lambda$$
$$\tfrac{1}{800}\times 10^{-3}m\cdot\sin\theta = 2\cdot 499.6nm$$
$$\theta = 53.07°$$

Thus, the position is

$$\tan\theta = \frac{y}{L}$$
$$\tan 53.07° = \frac{y}{100cm}$$
$$y = 133.02cm$$

The distance between the second-order maximum and the first-order maximum is, therefore,

$$x = 133.02cm - 43.60cm = 89.42cm$$

c) The maximum value of *m* is

$$m < \frac{d}{\lambda} = \frac{\frac{1}{800}\times 10^{-3} m}{499.6\times 10^{-9} m} = 2.50$$

The maximum value of *m* is thus 2. There are therefore 5 maxima (the $m = 1$ and $m = 2$ maxima on the right, the $m = 1$ and $m = 2$ maxima on the left and the central maximum).

28. For the first wavelength, the angle of the first-order maximum is

$$d\sin\theta = m\lambda$$
$$\tfrac{1}{300}\times 10^{-3} m \cdot \sin\theta = 1\cdot 589.0nm$$
$$\theta = 10.1776°$$

The position of this maximum is

$$\tan\theta = \frac{y}{L}$$
$$\tan 10{,}1776° = \frac{y}{200cm}$$
$$y = 35{,}9050cm$$

For the second wavelength, the angle of the first-order maximum is

$$d\sin\theta = m\lambda$$
$$\tfrac{1}{300}\times 10^{-3} m \cdot \sin\theta = 1\cdot 589.6nm$$
$$\theta = 10.1881°$$

The position of this maximum is

$$\tan\theta = \frac{y}{L}$$
$$\tan 10.1881° = \frac{y}{200cm}$$
$$y = 35.9427cm$$

Therefore, the distance between these two maxima is

$$x = 35.9427cm - 35.9050cm = 0.0377cm$$

29. The angle of the first-order maximum is

$$\tan\theta = \frac{y}{L}$$
$$\tan\theta = \frac{35cm}{100cm}$$
$$\theta = 19.29°$$

Therefore,

$$d\sin\theta = m\lambda$$
$$d\sin 19,29° = 1\cdot\lambda$$
$$\frac{\lambda}{d} = 0.3304$$

The angle of the second-order maximum is

$$d\sin\theta = m\lambda$$
$$d\sin\theta = 2\lambda$$
$$\sin\theta = 2\frac{\lambda}{d}$$
$$\sin\theta = 0.6607$$
$$\theta = 41.35°$$

Therefore, the position of this maximum is

$$\tan\theta = \frac{y}{L}$$

$$\tan 41.35° = \frac{y}{100cm}$$

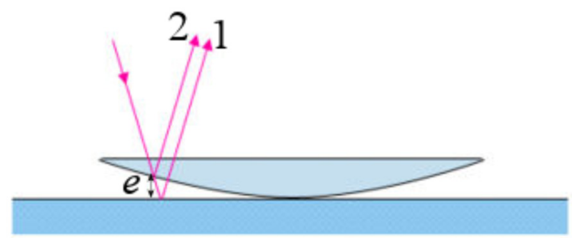
$$y = 88.02cm$$

30. At a dark ring, there must be destructive interference. Therefore, the phase difference is

$$\Delta\phi = (2m+1)\pi$$

As this is a thin film, the phase difference is

$$\Delta\phi = \frac{4\pi e n_f}{\lambda} + \Delta\phi_R$$



$\Delta\phi_R$ must then be found. As wave 1 (the one with the longest path) is travelling in air and is reflected on a medium having a larger index (glass), the wave is inverted and undergoes a phase shift ($\phi_{R1} = \pi$). As wave 2 (the one with the shortest path) is travelling in glass and is reflected on a medium having a smaller index of refraction (air), the wave is not inverted and does not undergo a phase shift ($\phi_{R2} = 0$). The difference between these two phase shifts is

$$\begin{aligned}\Delta\phi_R &= \phi_{R2} - \phi_{R1} \\ &= 0 - \pi \\ &= -\pi\end{aligned}$$

Therefore, the total phase difference is

$$\Delta\phi = \frac{4\pi e n_f}{\lambda} - \pi$$

With destructive interference, this means that

$$\frac{4\pi e n_f}{\lambda} - \pi = (2m+1)\pi$$

This gives

$$\frac{4en_f}{\lambda} - 1 = 2m + 1$$
$$\frac{4en_f}{\lambda} = 2m + 2$$
$$e = \frac{(2m+2)\lambda}{4n_f}$$

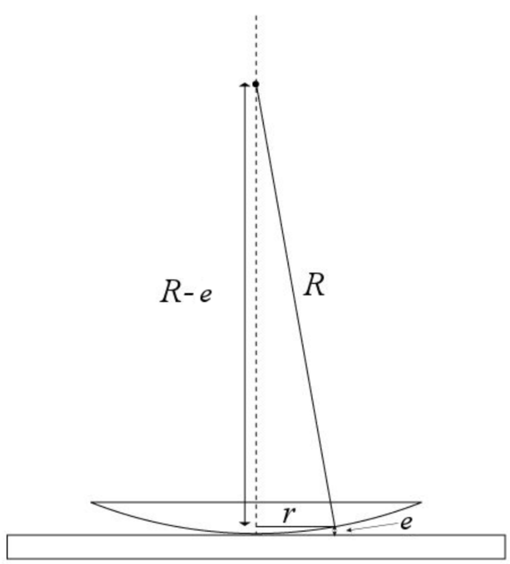
As the index of the film is 1, the equation becomes

$$e = \frac{(2m+2)\lambda}{4}$$
$$= \frac{(m+1)\lambda}{2}$$

Values of *m* smaller than -2 are not possible here. *m* = -1 corresponds to the central dark spot, *m* = 0 corresponds to the first ring, *m* = 1 to the second ring and *m* = 2 to the third ring. Therefore, *e* for the third ring is

$$e = \frac{(2+1)\lambda}{2}$$
$$= \frac{3\lambda}{2}$$
$$= \frac{3 \cdot 600nm}{2}$$
$$= 900nm$$

It only remains to find the radius of the ring. This radius is



$$(R-e)^2 + r^2 = R^2$$
$$(0.60m - 900\times 10^{-9} m)^2 + r^2 = (0.60m)^2$$
$$r^2 = (0.60m)^2 - (0.60m - 900\times 10^{-9} m)^2$$
$$r = 1.039mm$$

tr.wikipedia.org/wiki/Newton_halkalar%C4%B1

31. a) The light intensity with N slits is

$$I_N = I_1 \frac{\sin^2\left(\frac{N\Delta\phi}{2}\right)}{\sin^2\left(\frac{\Delta\phi}{2}\right)}$$

The maximum begins and ends when the intensity is zero. The intensity is zero when the sine function in the numerator is zero (but without the denominator being zero, because the fraction is then 0/0, which is not 0). Thus the intensity vanishes when

$$\sin\left(\frac{N\Delta\phi}{2}\right) = 0$$

$$\frac{N\Delta\phi}{2} = M\pi$$

$$\frac{\Delta\phi}{2} = \frac{M}{N}\pi$$

where M is an integer (but it cannot be a whole number of N, because then the denominator is zero and this corresponds to the phase difference of the large maxima).

The angle of the large order 1 maximum is

$$\frac{\Delta\phi}{2} = \pi$$

This means that the first order maximum occurs when $M = N$. The minimum preceding it is thus at $M = N - 1$ and the minimum following it is at $M = N + 1$. Therefore, the change of phase difference between the two minima (which is the width of the central maximum) is given by

$$\frac{\Delta\phi_{\text{min after}}}{2} - \frac{\Delta\phi_{\text{min before}}}{2} = \frac{N+1}{N}\pi - \frac{N-1}{N}\pi$$

$$\frac{\Delta\phi_{\text{min after}}}{2} - \frac{\Delta\phi_{\text{min before}}}{2} = \frac{2\pi}{N}$$

$$\Delta\phi_{\text{min after}} - \Delta\phi_{\text{min before}} = \frac{4\pi}{N}$$

But the phase difference is

$$\Delta\phi = \frac{d\sin\theta}{\lambda}2\pi$$

so that

$$\frac{d\sin\theta_{\text{min after}}}{\lambda}2\pi - \frac{d\sin\theta_{\text{min before}}}{\lambda}2\pi = \frac{4\pi}{N}$$
$$\sin\theta_{\text{min after}} - \sin\theta_{\text{min before}} = \frac{2\lambda}{Nd}$$
$$\Delta(\sin\theta) = \frac{2\lambda}{Nd}$$

Since the angle are close to each other, the following relation holds.

$$\Delta(\sin\theta) = \frac{d\sin\theta}{d\theta}\Delta\theta$$
$$= \cos\theta\Delta\theta$$

Thus, the angle is

$$\cos\theta\Delta\theta = \frac{2\lambda}{Nd}$$
$$\Delta\theta = \frac{2\lambda}{Nd\cos\theta}$$

It is then obvious that the width of the maxima gets smaller as N gets larger.

Note: This can be simplified further since

$$\sin\theta = \frac{\lambda}{d}$$

Thus, we could have written

$$\Delta\theta = \frac{2\lambda}{Nd\cos\theta}$$
$$\Delta\theta = \frac{2\sin\theta}{N\cos\theta}$$
$$\Delta\theta = \frac{2\tan\theta}{N}$$

b) Let's find by how much the angle of the first order maximum changes when the wavelength is changed a little. This means that we're looking for $\Delta\theta$ when $\Delta\lambda$ is small. It is found with

$$\Delta\theta = \frac{d\theta}{d\lambda}\Delta\lambda$$

Since the angle of the first order maximum is given by

$$d\sin\theta = \lambda$$

we have

$$\sin\theta = \frac{\lambda}{d}$$
$$\frac{d\sin\theta}{d\theta} = \frac{1}{d}\frac{d\lambda}{d\theta}$$
$$\cos\theta = \frac{1}{d}\frac{d\lambda}{d\theta}$$
$$\frac{d\theta}{d\lambda} = \frac{1}{d\cos\theta}$$

Thus

$$\Delta\theta = \frac{d\theta}{d\lambda}\Delta\lambda$$
$$\Delta\theta = \frac{1}{d\cos\theta}\Delta\lambda$$

However, this angle must be (approximately) greater than or equal to half the width of the central maximum. This means that

$$\frac{1}{d\cos\theta}\Delta\lambda \geq \frac{\lambda}{Nd\cos\theta}$$

Simplifying, the result is

$$\Delta\lambda \geq \frac{\lambda}{N}$$

Thus, the number of slits needed is

$$0.59nm \geq \frac{589.00nm}{N}$$
$$N \geq 998.3\text{slits}$$

Approximately, it takes 1000 slits to see the two maxima separately.