Chapter 6 Solutions

1. a) According to the law of refraction, the index is

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$n_X \sin(25°) = 1.33 \sin(48°)$$
$$n_X = 2.34$$

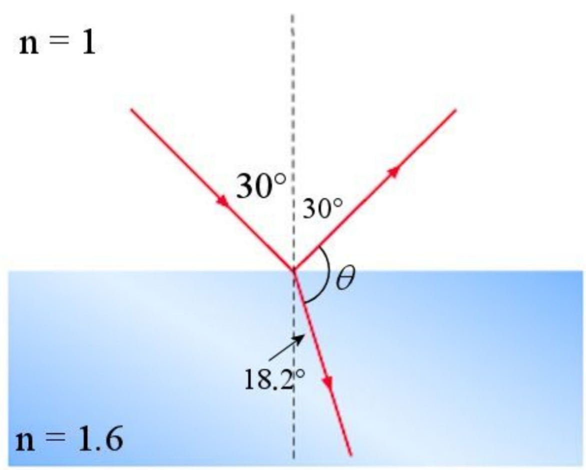
b) The speed of light in the unknown substance is

$$v = \frac{c}{n} = \frac{3\times10^8 \ \frac{m}{s}}{2.34} = 1.28\times10^8 \ \frac{m}{s}$$

2. The angle of refraction is

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1\sin(30°) = 1.6\sin\theta_2$$
$$\theta_2 = 18.21°$$

Therefore, we have



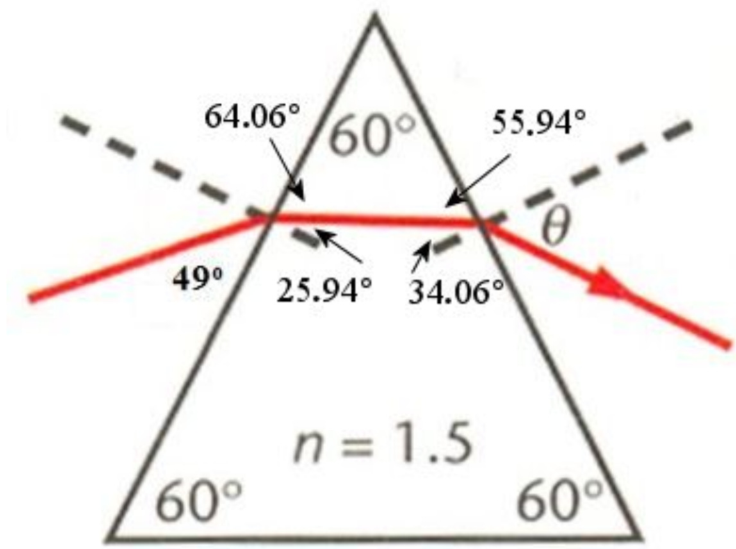
The angle is thus

$$30° + \theta + 18.21° = 180°$$
$$\theta = 131.79°$$

3. The angle θ_2 is found with the law of refraction.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1\sin(41°) = 1.5\sin\theta_2$$
$$\theta_2 = 25.94°$$

Therefore, we have



This is how these angles were found.

64.06°: We have this angle because 25.94° and 64.06° must be equal to 90° when added.

55.94°: We have this angle because the sum of the angles of a triangle is 180°. Thus, we must have 64.06°+60°+55.94° = 180°.

34.06°: We have this angle because 55.94° and 34.06° must be equal to 90° when added.

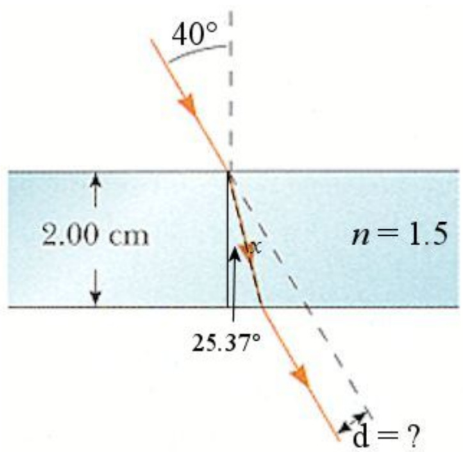
Finally, the angle θ is found with the law of refraction.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1.5\sin(34.06°) = 1\sin\theta$$
$$\theta = 57.16°$$

4. First, the angle of refraction in the glass is found with the law of refraction.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1\sin(40°) = 1.5\sin\theta_2$$
$$\theta = 25.37°$$

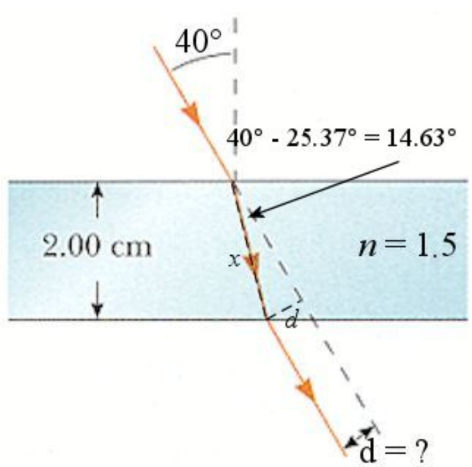
Therefore, we have



Then, the length of the ray of light in the glass (identified by the dotted line *x*) is found. It is found with

$$\cos 25.37 = \frac{2cm}{x}$$
$$x = 2.214cm$$

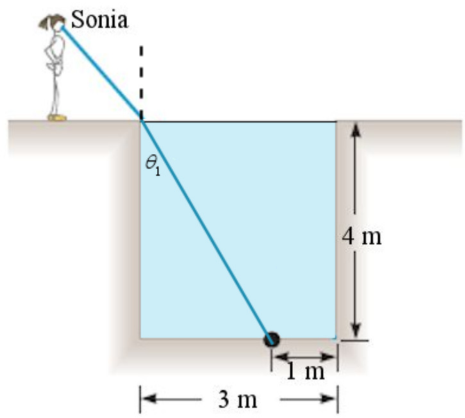
Finally, we have another right triangle with the sides *x* and *d*.



With this triangle, *d* is found

$$\sin(14.63°) = \frac{d}{x}$$
$$\sin(14.63°) = \frac{d}{2.21cm}$$
$$d = 0.559cm$$

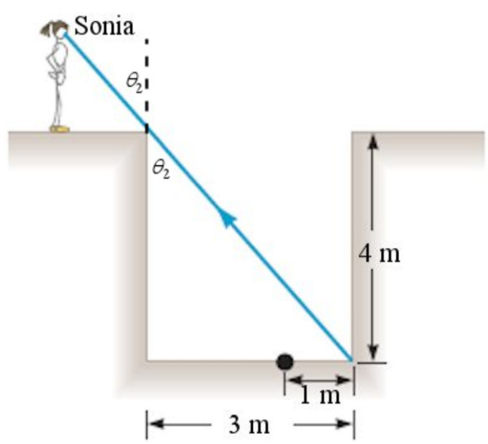
5. To see the point, we have, at worst, the following situation.



The angle in this figure is

$$\tan\theta_1 = \frac{2m}{4m}$$
$$\theta_1 = 26.57°$$

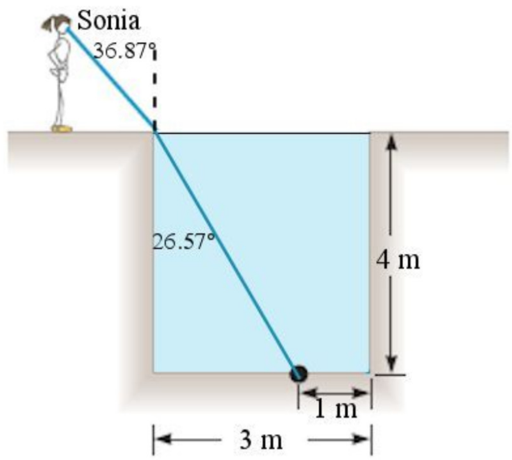
Only the angle on the outside the liquid is missing. This angle is found with the initial situation.



The angle in this figure is

$$\tan\theta_2 = \frac{3m}{4m}$$
$$\theta_1 = 36.87°$$

We then have the following situation.



The index of refraction must then be

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$n\sin(26.57°) = 1\sin 36.87°$$
$$n = 1.342$$

6. If the critical angle is 60°, then the index of refraction is

$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\sin 60° = \frac{n}{1.33}$$
$$n = 1.152$$

Therefore, the speed of light is

$$v = \frac{c}{n} = \frac{3\times10^8 \frac{m}{s}}{1.152} = 2.60\times10^8 \frac{m}{s}$$

7. The critical angle is

$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\sin\theta_c = \frac{1}{1.5}$$
$$\theta_c = 41.81^\circ$$

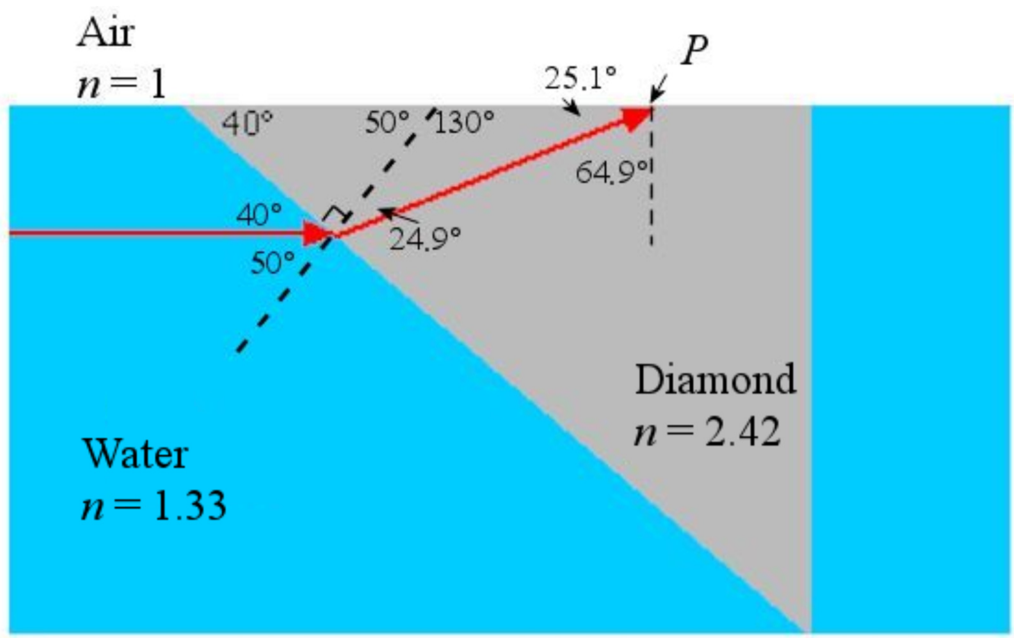
As the angle of incidence is 52° (90° – 38°), there is a total reflection since the angle of incidence is greater than the critical angle.

8. The critical angle is

$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\sin\theta_c = \frac{1.5}{1}$$
$$\theta_c \text{ does not exist}$$

As there is no critical angle, a total reflection is impossible.

9. We have the following angles.



This is how those angles were found.

40° between the red beam and the interface between water and diamond. Alternate internal of the 40° at the end of the piece of diamond

50° angle incidence of the ray at the water-diamond interface.
40° and 50° must be equal to 90° when added.

24,9° angle of refraction
Comes from the law of refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1.33\sin(50°) = 2.42\sin\theta_2$$
$$\theta_2 = 24.9°$$

50° angle (at the end of the dotted line, near the interface between air and diamond).
The sum of the angles of a triangle must be 180°.
We must then have 40° + 90° + 50° = 180°.

130° angle
The sum of the 50° angle and the 130° angle must give 180° (supplementary angles).

25.1° angle
The sum of the angles of a triangle must be 180°.
We must then have 130° + 24.9° + 25.1° = 180°.

64.9° angle
25.1° and 64.9° must be equal to 90° when added.

The angle of incidence of the ray is 64.9°. Is it greater than the critical angle? The critical angle is

$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\sin\theta_c = \frac{1}{2.42}$$
$$\theta_c = 24.4°$$

As the angle of incidence is greater than the critical angle, there is a total reflection at point P.

10. The angle of refraction for the red light is

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1\sin(80°) = 1.62\sin\theta_2$$
$$\theta_2 = 37.44°$$

The angle of refraction for the violet light is

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1\sin(80°) = 1.66\sin\theta_2$$
$$\theta_2 = 36.39°$$

The difference between those two angles is 37.44° - 36.39° = 1.05°.

11. The position of the image is found with

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$
$$\frac{1.33}{10cm} + \frac{1}{q} = \frac{1 - 1.33}{-20cm}$$
$$q = -8.58cm$$

Therefore, the image of the fish is 8.58 cm behind the wall of the aquarium.

12. The position of the image is found with

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$
$$\frac{1.33}{10cm} + \frac{1}{q} = \frac{1 - 1.33}{40cm}$$
$$q = -7.08cm$$

Therefore, the image of the fish is 7.08 cm behind the wall of the aquarium.

13. a) The position of the image is found with

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$
$$\frac{1.5}{25cm}+\frac{1}{q}=\frac{1-1.5}{-40cm}$$
$$q=-21.05cm$$

Therefore, the image of the spot is 21.05 cm underneath the top of the glass dome. Thus, the distance between the image and the observer is 45cm+21.05cm=66.05 cm.

b) The magnification is

$$\begin{aligned}m&=-\frac{n_1 q}{n_2 p}\\&=-\frac{1.5\cdot(-21.05cm)}{1\cdot 25cm}\\&=1.263\end{aligned}$$

The radius of the image is 1.263 times larger than the radius of the object. The radius of the image is thus

$$\begin{aligned}r&=1.263\cdot 2cm\\&=2.53cm\end{aligned}$$

14. There are two surfaces

First surface (curved surface)

The position of the image is found with

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$
$$\frac{1}{5cm}+\frac{1.5}{q}=\frac{1.5-1}{4cm}$$
$$q=-20cm$$

The magnification is

$$
\begin{aligned}
m &= -\frac{n_1 q}{n_2 p} \\
&= -\frac{1 \cdot (-20cm)}{1.5 \cdot 5cm} \\
&= 2.667
\end{aligned}
$$

<u>*Second surface (flat surface)*</u>

The image of the first surface is used as the object for the second surface. As the image is 20 cm to the left of the curved surface, it is 36 cm from the flat surface. So, we have p = 36 cm. Therefore, the position of the final image is

$$
\begin{aligned}
\frac{n_1}{p} + \frac{n_2}{q} &= \frac{n_2 - n_1}{R} \\
\frac{1.5}{36cm} + \frac{1}{q} &= \frac{1-1.5}{\infty} \\
q &= -24cm
\end{aligned}
$$

Therefore, the image is 24 cm to the left of the flat surface. For the observer, the image is 24 cm behind the flat surface.

The magnification is

$$
\begin{aligned}
m &= -\frac{n_1 q}{n_2 p} \\
&= -\frac{1.5 \cdot (-24cm)}{1 \cdot 36cm} \\
&= 1
\end{aligned}
$$

The total magnification is

$$m_{tot} = m_1 \cdot m_2 = 2.667 \cdot 1 = 2.667$$

The final image is 2.667 times larger than the object. Its diameter is thus

$$y_i = 2.667 \text{ x } 1 \text{ cm} = 2.667 \text{ cm}$$

15. There are two surfaces

First surface (water-glass interface)

The position of the image is found with

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$
$$\frac{1.33}{4m}+\frac{1.5}{q}=\frac{1.5-1.33}{\infty}$$
$$q=-4.511m$$

Second surface (glass-air interface)

The image of the first surface is used as the object for the second surface. As the image is 4.511 m under the water-glass interface, it is 7.511 m under the glass-air interface. So, we have p = 7.511 cm. Therefore, the position of the final image is

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$$
$$\frac{1.5}{7.511m}+\frac{1}{q}=\frac{1-1.5}{\infty}$$
$$q=-5.01m$$

The image is therefore to 5.01 m under the glass-air interface. For the observer, the image is therefore 5.01 m below the top of the glass surface.

16. a) The position of the image is given by

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{4m}+\frac{1}{q}=\frac{1}{0.5m}$$
$$q=57.1cm$$

b) The magnification is

$$m = \frac{-q}{p}$$
$$= -\frac{0.571m}{4m}$$
$$= -0.143$$

Therefore, the height of the image is

$$y_i = m \cdot y_0$$
$$= -0.143 \cdot 1cm$$
$$= -0.143cm$$

A 0.143 cm high inverted image is then obtained.

17. a) The magnification is

$$m = \frac{y_i}{y_o}$$
$$= \frac{-0.5cm}{1cm}$$
$$= -0.5$$

Therefore,

$$m = \frac{-q}{p}$$
$$-0.5 = -\frac{q}{2m}$$
$$q = 1m$$

b) The focal length is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{2m} + \frac{1}{1m} = \frac{1}{f}$$
$$f = 66.6cm$$

18. a) Firstly, it should be noted that we don't know if the image is virtual or real. Let's try both possibilities. Since the magnification is + 4, we have (with a real image)

$$m = \frac{-q}{p}$$
$$4 = -\frac{20cm}{p}$$
$$p = -5cm$$

This answer is not possible since *p* cannot be negative in this situation.

With a virtual image, we have

$$m = \frac{-q}{p}$$
$$4 = -\frac{-20cm}{p}$$
$$p = 5cm$$

This is an acceptable answer. The object is 5 cm from the lens.

b) The focal length is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{5cm} + \frac{1}{-20cm} = \frac{1}{f}$$
$$f = 6.66cm$$

19. a) With a magnification of -3, we have

$$m = \frac{-q}{p}$$
$$-3 = -\frac{q}{p}$$
$$q = 3p$$

Therefore,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{p}+\frac{1}{3p}=\frac{1}{25cm}$$
$$\frac{1}{p}\left(1+\frac{1}{3}\right)=\frac{1}{25cm}$$
$$p=33.3cm$$

b) With a magnification of +3, we have

$$m=\frac{-q}{p}$$
$$3=-\frac{q}{p}$$
$$q=-3p$$

Therefore,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{p}+\frac{1}{-3p}=\frac{1}{25cm}$$
$$\frac{1}{p}\left(1-\frac{1}{3}\right)=\frac{1}{25cm}$$
$$p=16.7cm$$

20. a) With a magnification of –0.4, we have

$$m=\frac{-q}{p}$$
$$-0.4=-\frac{q}{p}$$
$$q=0.4p$$

Therefore,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{p}+\frac{1}{0.4p}=\frac{1}{-25cm}$$
$$\frac{1}{p}\left(1+\frac{1}{0.4}\right)=\frac{1}{-25cm}$$
$$p=-87.5cm$$

Since *p* cannot be negative here, this answer is impossible.

b) With a magnification of +0.4, we have

$$m=\frac{-q}{p}$$
$$0.4=-\frac{q}{p}$$
$$q=-0.4p$$

Therefore,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{p}+\frac{1}{-0.4p}=\frac{1}{-25cm}$$
$$\frac{1}{p}\left(1-\frac{1}{0.4}\right)=\frac{1}{-25cm}$$
$$p=37.5cm$$

21. The thin lens equation gives

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{x}+\frac{1}{2m-x}=\frac{1}{0.4m}$$

The solution of this equation is found with

$$\frac{1}{x}+\frac{1}{2m-x}=\frac{1}{0.4m}$$
$$\frac{2m-x}{x(2m-x)}+\frac{x}{x(2m-x)}=\frac{1}{0.4m}$$
$$\frac{2m-x+x}{x(2m-x)}=\frac{1}{0.4m}$$
$$\frac{2m}{x(2m-x)}=\frac{1}{0.4m}$$
$$2m\cdot 0.4m=x(2m-x)$$
$$0.8m^2=2m\cdot x-x^2$$
$$x^2-2m\cdot x+0.8m^2=0$$

The solutions of this quadratic equation are x = 1.4472 m and x = 0.5528 m.

22. We will deal with one lens at the same time.

First Lens

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{10cm}+\frac{1}{q}=\frac{1}{12cm}$$
$$q=-60cm$$

Magnification

$$\begin{aligned}m&=-\frac{q}{p}\\&=-\frac{-60cm}{10cm}\\&=6\end{aligned}$$

Second Lens

The image of the first lens is used as an object for the second lens. With an image 60 cm to the left of the first lens, the distance between this image and the second lens is 90 cm. So, we have p = 90 cm.

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{90cm}+\frac{1}{q}=\frac{1}{20cm}$$
$$q=25.7cm$$

The final image is 25.7 cm to the right of the lens to the right.

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{-25.7cm}{90cm} \\ &= -0.286 \end{aligned}$$

The total magnification is thus

$$\begin{aligned} m_{tot} &= m_1 \cdot m_2 \\ &= 6 \cdot -0.286 \\ &= -1.714 \end{aligned}$$

The height of the image is

$$\begin{aligned} y_i &= my_0 \\ &= -1.714 \cdot 1cm \\ &= -1.714cm \end{aligned}$$

Therefore, the final image is a 1.714 high inverted image.

23. We will deal with one lens at the same time.

First Lens

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{24cm}+\frac{1}{q}=\frac{1}{14cm}$$
$$q=33.6cm$$

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{-33.6cm}{24cm} \\ &= -1.4 \end{aligned}$$

Second Lens

The image of the first lens is used as an object for the second lens. With an image 33.6 cm to the left of the first lens, the distance between this image and the second lens is 8.6 cm. As the object is on the side where the light goes, the value of p is negative. So, we have p = -8.6 cm.

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{-8.6cm}+\frac{1}{q}=\frac{1}{-7cm}$$
$$q=-37.625cm$$

Therefore, the final image is 37.625 cm to the left of the lens to the right.

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{-37.625cm}{-8.6cm} \\ &= -4.375 \end{aligned}$$

Therefore, the total magnification is

$$\begin{aligned} m_{tot} &= m_1 \cdot m_2 \\ &= -1.4 \cdot -4.375 \\ &= 6.125 \end{aligned}$$

The height of the image is thus

$$\begin{aligned} y_i &= m y_0 \\ &= 6.125 \cdot 2cm \\ &= 12.25cm \end{aligned}$$

Therefore, the final image is erect and is 12.25 cm high.

24. We will deal with one item (lens or mirror) at the same time.

First passage through the lens (the light is travelling towards the right)

Position of the image

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{20cm} + \frac{1}{q} = \frac{1}{15cm}$$

$$q = 60cm$$

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{60cm}{20cm} \\ &= -3 \end{aligned}$$

Mirror

The image of the first lens is used as an object for the mirror. With an image 60 cm to the right of the first lens, the distance between this image and the mirror is 15 cm. As the object behind the mirror, the value of p is negative. So, we have p = -15 cm.

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{-15cm}+\frac{1}{q}=\frac{1}{10cm}$$
$$q=6cm$$

Therefore, the image is 6 cm in front of the mirror.

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{6cm}{-15cm} \\ &= 0.4 \end{aligned}$$

<u>Second passage through the lens (the light is travelling towards the left)</u>

The image of the mirror is used as an object for the lens. With an image 6 cm in front of the mirror, the distance between this image and the lens is 39 cm. So, we have p = 39 cm.

Position of the image

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{39cm}+\frac{1}{q}=\frac{1}{15cm}$$
$$q=24.375cm$$

Therefore, the final image is 24.375 cm to the left of the lens.

Magnification

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{24.375cm}{39cm} \\ &= -0.625 \end{aligned}$$

Therefore, the total magnification is

$$\begin{aligned} m_{tot} &= m_1 \cdot m_2 \cdot m_3 \\ &= -3 \cdot 0.4 \cdot -0.625 \\ &= 0.75 \end{aligned}$$

The height of the image is then

$$\begin{aligned} y_i &= m y_0 \\ &= 0.75 \cdot 2cm \\ &= 1.5cm \end{aligned}$$

The final image is erect and is 1.5 cm high.

25. a) The focal length is (assuming that light passes from left to right through the lens)

$$\begin{aligned} \frac{1}{f} &= \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\ \frac{1}{f} &= \frac{1.6-1}{1}\left(\frac{1}{0.1m} - \frac{1}{-0.15m}\right) \\ f &= 10cm \end{aligned}$$

This is a converging lens whose focal length is 10 cm.

b) The focal length is (assuming that light passes from left to right through the lens)

$$\begin{aligned} \frac{1}{f} &= \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \\ \frac{1}{f} &= \frac{1.6-1}{1}\left(\frac{1}{0.1m} - \frac{1}{\infty}\right) \\ f &= 16.7cm \end{aligned}$$

This is a converging lens whose focal length is 16.7 cm.

c) The focal length is (assuming that light passes from left to right through the lens)

$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
$$\frac{1}{f}=\frac{1.6-1}{1}\left(\frac{1}{-0.1m}-\frac{1}{0.15m}\right)$$
$$f=-10cm$$

This is a diverging lens whose focal length is 10 cm.

d) The focal length is (assuming that light passes from left to right through the lens)

$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
$$\frac{1}{f}=\frac{1.6-1}{1}\left(\frac{1}{-0.1m}-\frac{1}{-0.15m}\right)$$
$$f=-50cm$$

This is a diverging lens whose focal length is 50 cm.

26. We have (assuming that light passes from left to right through the lens)

$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
$$\frac{1}{-30cm}=\frac{1.62-1.33}{1.33}\left(\frac{1}{-10cm}-\frac{1}{R}\right)$$
$$R=18.9cm$$

27. For red light, the focal length is (assuming that light passes from left to right through the lens)

$$\frac{1}{f}=\frac{n_l-n_m}{n_m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$
$$\frac{1}{f}=\frac{1.62-1}{1}\left(\frac{1}{30cm}-\frac{1}{-20cm}\right)$$
$$f=19.35cm$$

For violet light, the focal length is (assuming that light passes from left to right through the lens)

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{f} = \frac{1.67 - 1}{1}\left(\frac{1}{30cm} - \frac{1}{-20cm}\right)$$
$$f = 17.91cm$$

The distance between the focuses is then 1.44 cm.

28. In air, we have

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{15cm} = \frac{1.6 - 1}{1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{9}cm^{-1}$$

In water, we then have

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{f} = \frac{1.6 - 1.33}{1.33} \cdot \frac{1}{9}cm^{-1}$$
$$f = 44.33cm$$

29. The focal length of the lens is

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{f} = \frac{1.5 - 1}{1}\left(\frac{1}{30cm} - \frac{1}{-25cm}\right)$$
$$f = 27.3cm$$

The position of the image is then found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{40cm}+\frac{1}{q}=\frac{1}{27.3cm}$$
$$q=85.7cm$$

30. The equivalent focal length of the two lenses is

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$
$$\frac{1}{f_{eq}}=\frac{1}{10cm}+\frac{1}{15cm}$$
$$f_{eq}=6cm$$

The position of the image is then found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{40cm}+\frac{1}{q}=\frac{1}{6cm}$$
$$q=7.06cm$$

The magnification is

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{7.06cm}{40cm} \\ &= -0.176 \end{aligned}$$

The height of the image is thus

$$\begin{aligned} y_i &= my_0 \\ &= -0.176\cdot 3cm \\ &= -0.529cm \end{aligned}$$

Therefore, the image is inverted and is 0.529 cm high.

31. a) The distance is

$$\begin{aligned} p &= f \\ &= 3cm \end{aligned}$$

b) The maximum angular magnification is

$$\begin{aligned} G_{\min} &= \frac{d_{pp}}{f} \\ &= \frac{20cm}{3cm} \\ &= 6.67 \end{aligned}$$

c) The distance is

$$\begin{aligned} p &= \frac{d_{pp} f}{d_{pp} + f} \\ &= \frac{20cm \cdot 3cm}{20cm + 3cm} \\ &= 2.61cm \end{aligned}$$

d) The maximum angular magnification is

$$\begin{aligned} G_{\max} &= 1 + \frac{d_{pp}}{f} \\ &= 1 + \frac{20cm}{3cm} \\ &= 7.67 \end{aligned}$$

32. Let's find the position and the height of the image if the object is at p = 1.9 cm. The position is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{1.9cm} + \frac{1}{q} = \frac{1}{2cm}$$
$$q = -38cm$$

The height of the image is

$$m = -\frac{q}{p}$$

$$m = -\frac{-38cm}{1.9cm}$$

$$m = 20$$

Therefore, Subrahmanyan sees an image 20 times bigger, 38 cm from his eye.

Without lens, the angle is

$$\alpha = \frac{y_0}{d_{pp}}$$

$$\alpha = \frac{y_0}{20cm}$$

With a magnifying glass, the angle is

$$\beta = \frac{y_i}{38cm}$$

Therefore, the angular magnification is

$$G = \frac{\beta}{\alpha}$$

$$= \frac{\frac{y_i}{38cm}}{\frac{y_0}{20cm}}$$

$$= \frac{y_i}{y_o}\frac{20cm}{38cm}$$

As y_i / y_0 is the magnification, the angular magnification is

$$G = m\frac{20}{38}$$

$$= 20\frac{20}{38}$$

$$= 10.53$$

33. The power of the glasses is

$$\begin{aligned} P_{gla} &= -\frac{1}{d_{pr}} \\ &= -\frac{1}{5m} \\ &= -0.2D \end{aligned}$$

34. The power of the glasses is

$$\begin{aligned} P_{gla} &= \frac{1}{d'_{pp}} - \frac{1}{d_{pp}} \\ &= \frac{1}{0.2m} - \frac{1}{0.45m} \\ &= 2.78D \end{aligned}$$

35. a) As this person is myopic, the power of the glasses is

$$\begin{aligned} P_{gla} &= -\frac{1}{d_{pr}} \\ &= -\frac{1}{2,4m} \\ &= -0.417D \end{aligned}$$

b) Without glasses, the power of accommodation is

$$\begin{aligned} P_{acc} &= \frac{1}{d_{pp}} - \frac{1}{d_{pr}} \\ &= \frac{1}{0.18m} - \frac{1}{2.4m} \\ &= 5.139D \end{aligned}$$

This power remains the same with glasses. So we have

$$P_{acc} = \frac{1}{d'_{pp}} - \frac{1}{d'_{pr}}$$

$$5.139D = \frac{1}{d'_{pp}} - \frac{1}{\infty}$$

$$d'_{pp} = 0.1946m = 19.46cm$$

With his glasses, this person sees clearly from 19.46 cm to infinity.

36. a) As this person is farsighted, the power of the glasses is

$$P_{gla} = \frac{1}{d'_{pp}} - \frac{1}{d_{pp}}$$

$$= \frac{1}{0.25m} - \frac{1}{0.5m}$$

$$= 2D$$

b) As the power of accommodation is 3 D, we have

$$P_{acc} = \frac{1}{d'_{pp}} - \frac{1}{d'_{pr}}$$

$$3D = \frac{1}{0.25m} - \frac{1}{d'_{pr}}$$

$$d'_{pr} = 1m$$

With his glasses, this person sees clearly from 25 cm to 1 m.

37. With his 2 D glasses, the punctum proximum is at 45 cm. Let's find where the punctum proximum without glasses is.

$$P_{gla} = \frac{1}{d'_{pp}} - \frac{1}{d_{pp}}$$

$$2D = \frac{1}{0.45m} - \frac{1}{d_{pp}}$$

$$d_{pp} = 4.5m$$

To bring back the d_{pp} at 25 cm, he needs glasses with a power of

$$\begin{aligned} P_{gla} &= \frac{1}{d'_{pp}} - \frac{1}{d_{pp}} \\ &= \frac{1}{0.25m} - \frac{1}{4.5m} \\ &= 3.78D \end{aligned}$$

38. If x is the distance between the object and the focus, then the distance between the object and the lens is

$$p = f + x$$

(If the object is between the focus and the lens, x is negative.)

If x' is the distance between the image and the other focus, then the distance between the lens and the image is

$$q = f + x'$$

(If the image is between the focus and the lens, x' is negative.)

Therefore,

$$\begin{gathered} \frac{1}{p} + \frac{1}{q} = \frac{1}{f} \\ \frac{1}{f+x} + \frac{1}{f+x'} = \frac{1}{f} \\ \frac{f+x'}{(f+x)(f+x')} + \frac{f+x}{(f+x)(f+x')} = \frac{1}{f} \\ \frac{2f+x+x'}{(f+x)(f+x')} = \frac{1}{f} \\ f(2f+x+x') = (f+x)(f+x') \\ 2f^2 + fx + fx' = f^2 + fx + fx' + x \cdot x' \\ 2f^2 = f^2 + x \cdot x' \\ f^2 = x \cdot x' \end{gathered}$$

which is the desired result.

39. If x is the distance between the object and the lens, then the distance between the lens and the image is

$$q = L - x$$

The lens equation is, therefore,

$$\frac{1}{x} + \frac{1}{L-x} = \frac{1}{f}$$

Let's solve this equation for x.

$$\frac{L-x}{x(L-x)} + \frac{x}{x(L-x)} = \frac{1}{f}$$
$$\frac{L}{x(L-x)} = \frac{1}{f}$$
$$Lf = x(L-x)$$
$$Lf = xL - x^2$$
$$x^2 - xL + Lf = 0$$

The solution of this equation is

$$x = \frac{L \pm \sqrt{L^2 - 4Lf}}{2}$$

To have a solution, the expression inside the square root must be positive. Thus,

$$L^2 - 4Lf \geq 0$$
$$L^2 \geq 4Lf$$
$$L \geq 4f$$

which is the desired result.

40. When p = 36.8 cm, the distance of the image is q. Thus

$$\frac{1}{36.8cm} + \frac{1}{q} = \frac{1}{f}$$

When p = 36 cm, the distance of the image is q + 3 cm. thus

$$\frac{1}{36cm}+\frac{1}{q+3cm}=\frac{1}{f}$$

We have two equations and two unknowns. Then, q is found with

$$\frac{1}{36.8cm}+\frac{1}{q}=\frac{1}{36cm}+\frac{1}{q+3cm}$$

If this equation is solved for q, we obtain

$$\frac{1}{q}-\frac{1}{q+3cm}=\frac{1}{36cm}-\frac{1}{36.8cm}$$

$$\frac{1}{q}-\frac{1}{q+3cm}=\frac{1}{1656cm}$$

$$\frac{q+3cm}{q(q+3cm)}-\frac{q}{q(q+3cm)}=\frac{1}{1656cm}$$

$$\frac{3cm}{q(q+3cm)}=\frac{1}{1656cm}$$

$$4968cm^2=q(q+3cm)$$

$$4968cm^2=q^2+q\cdot 3cm$$

$$q^2+q\cdot 3cm-4968cm^2=0$$

The solutions of this equation is

$$q=\frac{-3cm\pm\sqrt{(3cm)^2+4\cdot 4968cm^2}}{2}$$

$$q=\frac{-3cm\pm\sqrt{19\,881cm^2}}{2}$$

$$q=\frac{-3cm\pm 141cm}{2}$$

Since the image is real, q must be positive. The only positive solution is

$$q=\frac{-3cm+141cm}{2}$$
$$=69cm$$

Thus, the focal distance is

$$\frac{1}{36.8cm} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{36.8cm} + \frac{1}{69cm} = \frac{1}{f}$$

$$f = 24cm$$