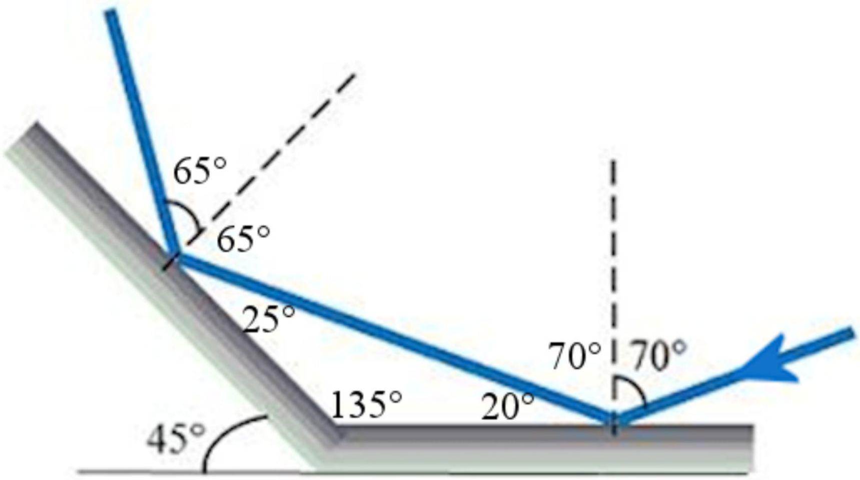
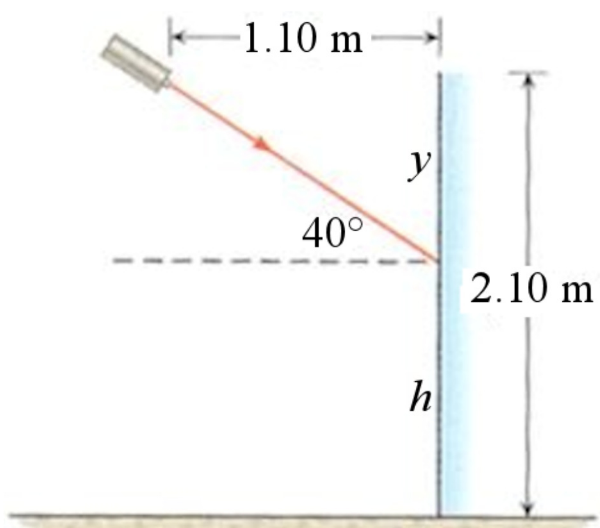
# Chapter 5 Solutions

**1.** We have the following angles.




The 25° angle was obtained from the fact that the sum of the angles of a triangle must be 180°.

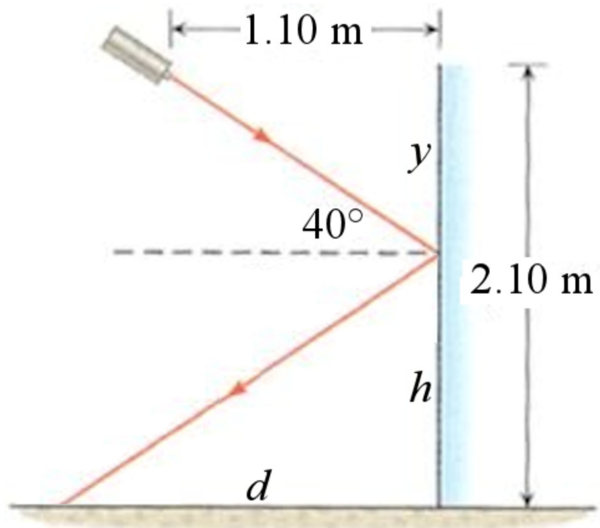
**2.** The height $h$ at which the laser strikes the mirror can be found.




In the figure, we have

$$\tan 50° = \frac{1.10m}{y}$$
$$y = 0.923m$$

The height is therefore $h$ = 2.100 m – 0.923 m = 1.177 m. Then the distance to the ground is found with

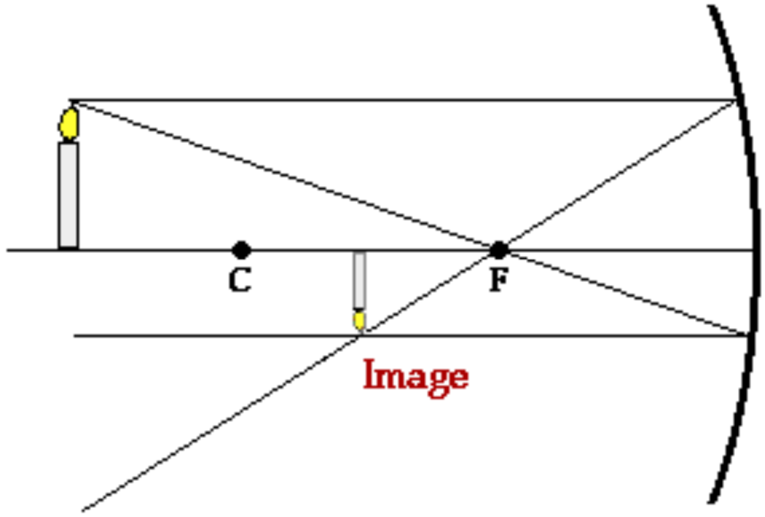



$$\tan 50° = \frac{d}{1.117m}$$
$$d = 1.403m$$

**3.** The object is at the same distance behind the mirror, so 120 cm behind the mirror. The image has the same height as the object (20 cm).

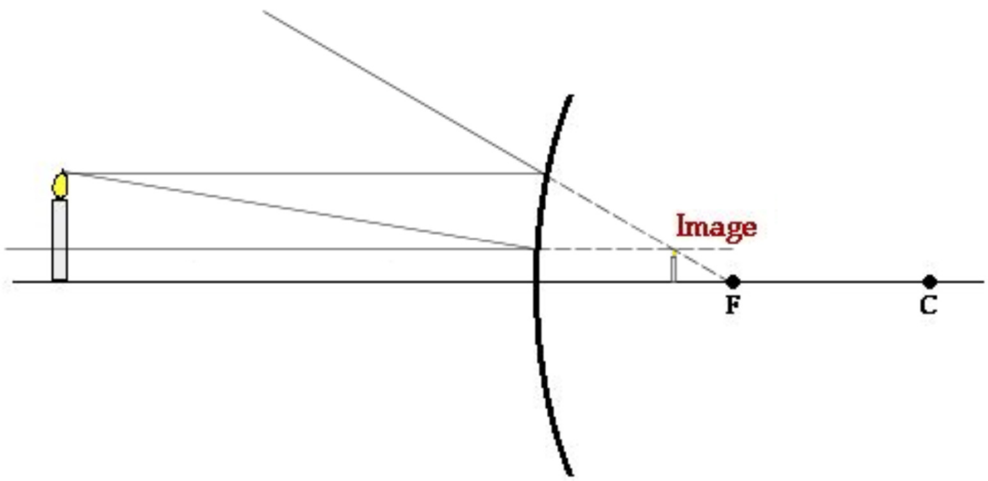
**4.** The image of the candle is 1 m behind the mirror. It is therefore 4 m from Anna.

**5.**




www.physicsclassroom.com/mmedia/optics/rdcma.cfm

**6.**




www.physicsclassroom.com/mmedia/optics/rdcma.cfm

**7.** a) The focal length of the mirror is

$$f = \frac{R}{2} = 20cm$$

The position of the image is therefore

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{10cm}+\frac{1}{q}=\frac{1}{20cm}$$
$$q=-20cm$$

The image is therefore 20 cm behind the mirror.

b) The position of the image is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{50cm}+\frac{1}{q}=\frac{1}{20cm}$$
$$q=33,3cm$$

The image is therefore 33.3 cm before the mirror.

**8.** a) The focal length of the mirror is

$$f=\frac{R}{2}=\frac{-40cm}{2}=-20cm$$

The position of the image is therefore

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{10cm}+\frac{1}{q}=\frac{1}{-20cm}$$
$$q=-6.67cm$$

The image is therefore 6.67 cm behind the mirror.

b) The position of the image is

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{50cm}+\frac{1}{q}=\frac{1}{-20cm}$$
$$q=-14.29cm$$

The image is therefore 14.29 cm behind the mirror.

**9.** The focal length of the mirror is

$$f=\frac{R}{2}=\frac{28cm}{2}=14cm$$

The position of the image is therefore

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{16cm}+\frac{1}{q}=\frac{1}{14cm}$$
$$q=112cm$$

The image is therefore to 112 cm in front of the mirror.

The magnification is

$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\frac{112cm}{16cm} \\ &= -7 \end{aligned}$$

The height of the image is therefore

$$m=\frac{y_i}{y_o}$$
$$-7=\frac{y_i}{3cm}$$
$$y_i=-21cm$$

The image is therefore inverted and has a height of 21 cm.

**10.** With the magnification formula we find that

$$\frac{y_i}{y_o} = -\frac{q}{p}$$
$$-0.3 = -\frac{q}{p}$$
$$q = 0.3p$$
$$q = 0.3 \cdot 30cm$$
$$q = 9cm$$

Then

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{30cm} + \frac{1}{9cm} = \frac{1}{f}$$
$$f = 6.923cm$$

As the value is positive, this is a concave mirror. Its radius of curvature is

$$f = \frac{R}{2}$$
$$6,923cm = \frac{R}{2}$$
$$R = 13,85cm$$

**11.** If the object is 60 cm from the mirror, and the image is 20 cm in front of the mirror, the focal length of the mirror is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$
$$\frac{1}{60cm} + \frac{1}{20cm} = \frac{1}{f}$$
$$f = 15cm$$

Now, if the object is 10 cm from the mirror, the position of the image is found with

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{10cm}+\frac{1}{q}=\frac{1}{15cm}$$
$$q=-30cm$$

The image is therefore 30 cm behind the mirror.

**12.** a) With the magnification formula, we find that

$$\frac{y_i}{y_o}=-\frac{q}{p}$$
$$0.25=-\frac{q}{p}$$
$$q=-0.25p$$

The focal length is

$$\begin{aligned} f &= \frac{R}{2} \\ &= \frac{-40cm}{2} \\ &= -20cm \end{aligned}$$

Therefore

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$\frac{1}{p}+\frac{1}{-0,25p}=\frac{1}{-20cm}$$
$$\frac{1}{p}\left(1+\frac{1}{-0,25}\right)=\frac{1}{-20cm}$$
$$\frac{1}{p}(-3)=\frac{1}{-20cm}$$
$$p=60cm$$

b) The position of the image is therefore

$$\begin{aligned} q &= -0.25p \\ &= -0.25 \cdot 60cm \\ &= -15cm \end{aligned}$$

The image is therefore 15 cm behind the mirror.

**13.** We have $p = x$ and $q = x + 1$ m. The lens equation then gives

$$\frac{1}{x} + \frac{1}{x+1m} = \frac{1}{1,2m}$$

This equation gives

$$\begin{gathered} \frac{x+1m}{x(x+1m)} + \frac{x}{x(x+1m)} = \frac{1}{1,2m} \\ \frac{2x+1m}{x(x+1m)} = \frac{1}{1,2m} \\ 1,2m \cdot (2x+1m) = x(x+1m) \\ 2,4m \cdot x + 1,2m^2 = x^2 + 1m \cdot x \\ x^2 - 1,4m \cdot x - 1,2m^2 = 0 \end{gathered}$$

The solutions to this equation are

$$\begin{aligned} x &= \frac{1,4m \pm \sqrt{(1,4m)^2 + 4 \cdot 1,2m^2}}{2} \\ &= \frac{1,4m \pm 2,6m}{2} \end{aligned}$$

The only positive answer is $x = 2$ m.

**14.** Let's assume that the point of the image closer to the mirror is at a distance $p_1$ from the mirror. This side of the image is at a distance $q_1$ from the mirror. This means that

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$$

The side of the object farther away from the mirror is at a distance $p_1 + l$ from the mirror. As this is further away from the mirror than the other side, the image of this point is closer to the mirror. Therefore, it is at a distance $q_1 - l'$ from the mirror. This means that

$$\frac{1}{p_1+l}+\frac{1}{q_1-l'}=\frac{1}{f}$$

Combining these two equations, we obtain

$$\frac{1}{p_1+l}+\frac{1}{q_1-l'}=\frac{1}{p_1}+\frac{1}{q_1}$$

Since $l$ is much smaller than $p_1$ and $l'$ is much smaller than $q_1$, Taylor series can be used. In addition, as $l$ and $l'$ are small, we can write $p_1 = p$ and $q_1 = q$. Thus

$$\frac{1}{p+l}=\frac{1}{p}\left(\frac{1}{1+\frac{l}{p}}\right)=\frac{1}{p}\left(1-\frac{l}{p}+\ldots\right)$$

$$\frac{1}{q-l'}=\frac{1}{q}\left(\frac{1}{1-\frac{l'}{q}}\right)=\frac{1}{q}\left(1+\frac{l'}{q}+\ldots\right)$$

The equation thus becomes

$$\frac{1}{p+l}+\frac{1}{q-l'}=\frac{1}{p}+\frac{1}{q}$$

$$\frac{1}{p}\left(1-\frac{l}{p}\right)+\frac{1}{q}\left(1+\frac{l'}{q}\right)=\frac{1}{p}+\frac{1}{q}$$

$$\frac{1}{p}-\frac{l}{p^2}+\frac{1}{q}+\frac{l'}{q^2}=\frac{1}{p}+\frac{1}{q}$$

$$-\frac{l}{p^2}+\frac{l'}{q^2}=0$$

$$\frac{l'}{q^2}=\frac{l}{p^2}$$

$$\frac{l'}{l}=\frac{q^2}{p^2}$$

This is the desired result

**15.** The first distance of the object that gives a real image is $p_1$ and the second distance of the object that gives a virtual image is $p_2$. We know that

$$p_1 - p_2 = 1.2m$$

If the object forms a 3 times larger real image, we have

$$\begin{aligned} -3 &= -\frac{q_1}{p_1} \\ q_1 &= 3p_1 \end{aligned}$$

(The magnification is negative for a real image.) The lens equation then gives

$$\frac{1}{p_1} + \frac{1}{3p_1} = \frac{1}{f}$$

Solving this equation for $p_1$, we obtain

$$\begin{aligned} \frac{1}{p_1}\left(1+\frac{1}{3}\right) &= \frac{1}{f} \\ \frac{1}{p_1}\left(\frac{4}{3}\right) &= \frac{1}{f} \\ p_1 &= \frac{4f}{3} \end{aligned}$$

If the object forms a 3 times larger virtual image, we have

$$\begin{aligned} 3 &= -\frac{q_2}{p_2} \\ q_2 &= -3p_2 \end{aligned}$$

(The magnification is positive for a virtual image.) The lens equation then gives

$$\frac{1}{p_2} + \frac{1}{-3p_2} = \frac{1}{f}$$

Solving this equation for $p_2$, we obtain

$$\frac{1}{p_2}\left(1-\frac{1}{3}\right)=\frac{1}{f}$$
$$\frac{1}{p_1}\left(\frac{2}{3}\right)=\frac{1}{f}$$
$$p_1=\frac{2f}{3}$$

Therefore,

$$p_1-p_2=1.2m$$
$$\frac{4f}{3}-\frac{2f}{3}=1.2m$$
$$\frac{2f}{3}=1.2m$$
$$f=1.8m$$