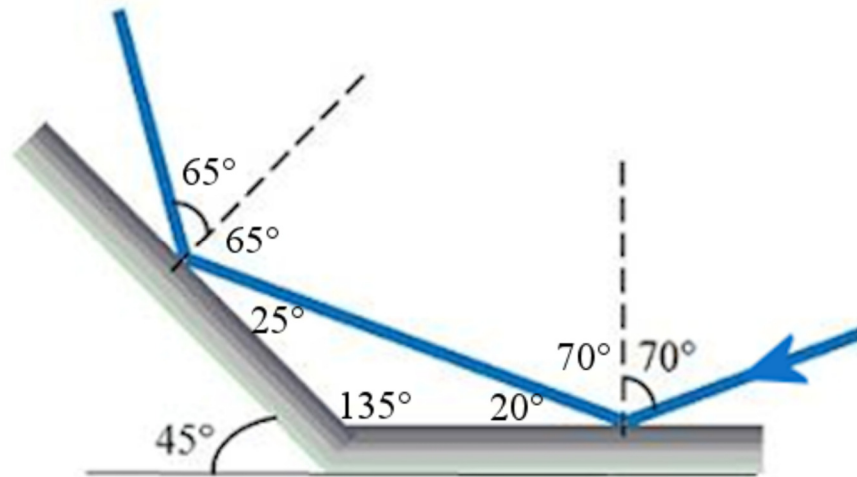


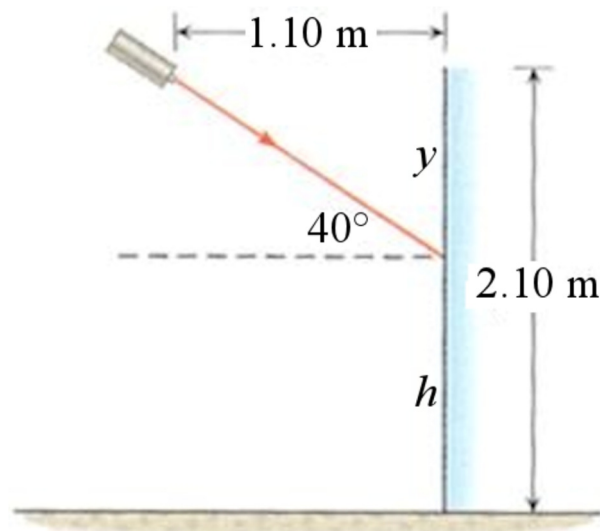
Chapter 5 Solutions

1. We have the following angles.



The 25° angle was obtained from the fact that the sum of the angles of a triangle must be 180° .

2. The height h at which the laser strikes the mirror can be found.

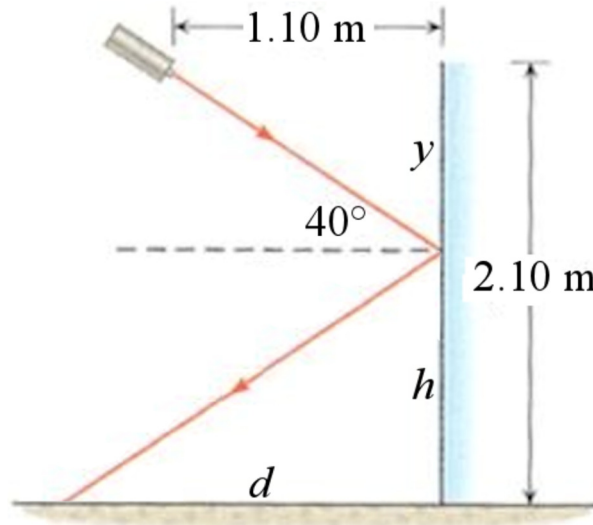


In the figure, we have

$$\tan 50^\circ = \frac{1.10\text{m}}{y}$$

$$y = 0.923\text{m}$$

The height is therefore $h = 2.100\text{ m} - 0.923\text{ m} = 1.177\text{ m}$. Then the distance to the ground is found with

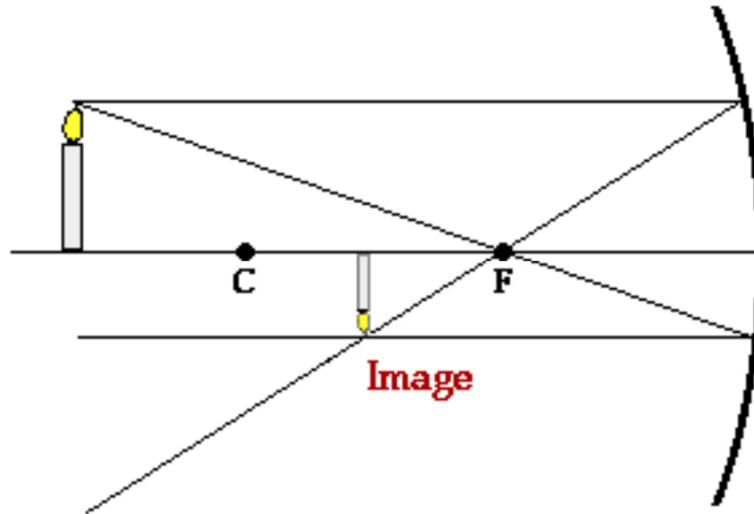


$$\tan 50^\circ = \frac{d}{1.117\text{m}}$$

$$d = 1.403\text{m}$$

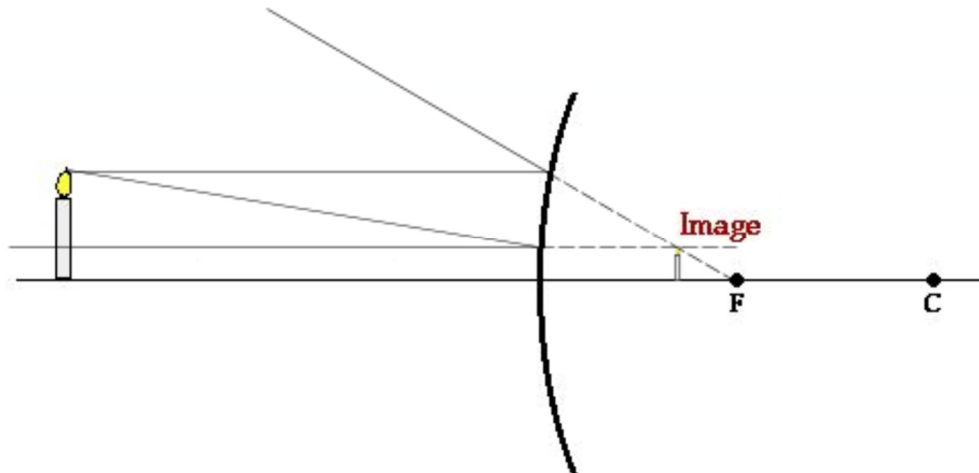
3. The object is at the same distance behind the mirror, so 120 cm behind the mirror. The image has the same height as the object (20 cm).
4. The image of the candle is 1 m behind the mirror. It is therefore 4 m from Anna.

5.



www.physicsclassroom.com/mmedia/optics/rdcma.cfm

6.



www.physicsclassroom.com/mmedia/optics/rdcma.cfm

7. a) The focal length of the mirror is

$$f = \frac{R}{2} = 20\text{cm}$$

The position of the image is therefore

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{10\text{cm}} + \frac{1}{q} = \frac{1}{20\text{cm}}$$

$$q = -20\text{cm}$$

The image is therefore 20 cm behind the mirror.

b) The position of the image is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{50\text{cm}} + \frac{1}{q} = \frac{1}{20\text{cm}}$$

$$q = 33,3\text{cm}$$

The image is therefore 33.3 cm before the mirror.

8. a) The focal length of the mirror is

$$f = \frac{R}{2} = \frac{-40\text{cm}}{2} = -20\text{cm}$$

The position of the image is therefore

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{10\text{cm}} + \frac{1}{q} = \frac{1}{-20\text{cm}}$$

$$q = -6.67\text{cm}$$

The image is therefore 6.67 cm behind the mirror.

b) The position of the image is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{50\text{cm}} + \frac{1}{q} = \frac{1}{-20\text{cm}}$$

$$q = -14.29\text{cm}$$

The image is therefore 14.29 cm behind the mirror.

9. The focal length of the mirror is

$$f = \frac{R}{2} = \frac{28\text{cm}}{2} = 14\text{cm}$$

The position of the image is therefore

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{16\text{cm}} + \frac{1}{q} = \frac{1}{14\text{cm}}$$

$$q = 112\text{cm}$$

The image is therefore to 112 cm in front of the mirror.

The magnification is

$$m = -\frac{q}{p}$$

$$= -\frac{112\text{cm}}{16\text{cm}}$$

$$= -7$$

The height of the image is therefore

$$m = \frac{y_i}{y_o}$$

$$-7 = \frac{y_i}{3\text{cm}}$$

$$y_i = -21\text{cm}$$

The image is therefore inverted and has a height of 21 cm.

10. With the magnification formula we find that

$$\frac{y_i}{y_o} = -\frac{q}{p}$$

$$-0.3 = -\frac{q}{p}$$

$$q = 0.3p$$

$$q = 0.3 \cdot 30\text{cm}$$

$$q = 9\text{cm}$$

Then

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{30\text{cm}} + \frac{1}{9\text{cm}} = \frac{1}{f}$$

$$f = 6.923\text{cm}$$

As the value is positive, this is a concave mirror. Its radius of curvature is

$$f = \frac{R}{2}$$

$$6.923\text{cm} = \frac{R}{2}$$

$$R = 13.85\text{cm}$$

11. If the object is 60 cm from the mirror, and the image is 20 cm in front of the mirror, the focal length of the mirror is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{60\text{cm}} + \frac{1}{20\text{cm}} = \frac{1}{f}$$

$$f = 15\text{cm}$$

Now, if the object is 10 cm from the mirror, the position of the image is found with

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{10\text{cm}} + \frac{1}{q} = \frac{1}{15\text{cm}}$$

$$q = -30\text{cm}$$

The image is therefore 30 cm behind the mirror.

12. a) With the magnification formula, we find that

$$\frac{y_i}{y_o} = -\frac{q}{p}$$

$$0.25 = -\frac{q}{p}$$

$$q = -0.25p$$

The focal length is

$$f = \frac{R}{2}$$

$$= \frac{-40\text{cm}}{2}$$

$$= -20\text{cm}$$

Therefore

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{-0,25p} = \frac{1}{-20\text{cm}}$$

$$\frac{1}{p} \left(1 + \frac{1}{-0,25} \right) = \frac{1}{-20\text{cm}}$$

$$\frac{1}{p} (-3) = \frac{1}{-20\text{cm}}$$

$$p = 60\text{cm}$$

b) The position of the image is therefore

$$\begin{aligned}
 q &= -0.25p \\
 &= -0.25 \cdot 60\text{cm} \\
 &= -15\text{cm}
 \end{aligned}$$

The image is therefore 15 cm behind the mirror.

13. We have $p = x$ and $q = x + 1$ m. The lens equation then gives

$$\frac{1}{x} + \frac{1}{x+1m} = \frac{1}{1,2m}$$

This equation gives

$$\begin{aligned}
 \frac{x+1m}{x(x+1m)} + \frac{x}{x(x+1m)} &= \frac{1}{1,2m} \\
 \frac{2x+1m}{x(x+1m)} &= \frac{1}{1,2m} \\
 1,2m \cdot (2x+1m) &= x(x+1m) \\
 2,4m \cdot x + 1,2m^2 &= x^2 + 1m \cdot x \\
 x^2 - 1,4m \cdot x - 1,2m^2 &= 0
 \end{aligned}$$

The solutions to this equation are

$$\begin{aligned}
 x &= \frac{1,4m \pm \sqrt{(1,4m)^2 + 4 \cdot 1,2m^2}}{2} \\
 &= \frac{1,4m \pm 2,6m}{2}
 \end{aligned}$$

The only positive answer is $x = 2$ m.

14. Let's assume that the point of the image closer to the mirror is at a distance p_1 from the mirror. This side of the image is at a distance q_1 from the mirror. This means that

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$$

The side of the object farther away from the mirror is at a distance $p_1 + l$ from the mirror. As this is further away from the mirror than the other side, the image of this point is closer to the mirror. Therefore, it is at a distance $q_1 - l'$ from the mirror. This means that

$$\frac{1}{p_1 + l} + \frac{1}{q_1 - l'} = \frac{1}{f}$$

Combining these two equations, we obtain

$$\frac{1}{p_1 + l} + \frac{1}{q_1 - l'} = \frac{1}{p_1} + \frac{1}{q_1}$$

Since l is much smaller than p_1 and l' is much smaller than q_1 , Taylor series can be used. In addition, as l and l' are small, we can write $p_1 = p$ and $q_1 = q$. Thus

$$\begin{aligned}\frac{1}{p+l} &= \frac{1}{p} \left(\frac{1}{1+\frac{l}{p}} \right) = \frac{1}{p} \left(1 - \frac{l}{p} + \dots \right) \\ \frac{1}{q-l'} &= \frac{1}{q} \left(\frac{1}{1-\frac{l'}{q}} \right) = \frac{1}{q} \left(1 + \frac{l'}{q} + \dots \right)\end{aligned}$$

The equation thus becomes

$$\begin{aligned}\frac{1}{p+l} + \frac{1}{q-l'} &= \frac{1}{p} + \frac{1}{q} \\ \frac{1}{p} \left(1 - \frac{l}{p} \right) + \frac{1}{q} \left(1 + \frac{l'}{q} \right) &= \frac{1}{p} + \frac{1}{q} \\ \frac{1}{p} - \frac{l}{p^2} + \frac{1}{q} + \frac{l'}{q^2} &= \frac{1}{p} + \frac{1}{q} \\ -\frac{l}{p^2} + \frac{l'}{q^2} &= 0 \\ \frac{l'}{q^2} &= \frac{l}{p^2} \\ \frac{l'}{l} &= \frac{q^2}{p^2}\end{aligned}$$

This is the desired result

- 15.** The first distance of the object that gives a real image is p_1 and the second distance of the object that gives a virtual image is p_2 . We know that

$$p_1 - p_2 = 1.2m$$

If the object forms a 3 times larger real image, we have

$$-3 = -\frac{q_1}{p_1}$$

$$q_1 = 3p_1$$

(The magnification is negative for a real image.) The lens equation then gives

$$\frac{1}{p_1} + \frac{1}{3p_1} = \frac{1}{f}$$

Solving this equation for p_1 , we obtain

$$\frac{1}{p_1} \left(1 + \frac{1}{3} \right) = \frac{1}{f}$$

$$\frac{1}{p_1} \left(\frac{4}{3} \right) = \frac{1}{f}$$

$$p_1 = \frac{4f}{3}$$

If the object forms a 3 times larger virtual image, we have

$$3 = -\frac{q_2}{p_2}$$

$$q_2 = -3p_2$$

(The magnification is positive for a virtual image.) The lens equation then gives

$$\frac{1}{p_2} + \frac{1}{-3p_2} = \frac{1}{f}$$

Solving this equation for p_2 , we obtain

$$\frac{1}{p_2} \left(1 - \frac{1}{3} \right) = \frac{1}{f}$$
$$\frac{1}{p_1} \left(\frac{2}{3} \right) = \frac{1}{f}$$
$$p_1 = \frac{2f}{3}$$

Therefore,

$$p_1 - p_2 = 1.2m$$
$$\frac{4f}{3} - \frac{2f}{3} = 1.2m$$
$$\frac{2f}{3} = 1.2m$$
$$f = 1.8m$$