Chapter 4 Solutions

1. The time is

$$\begin{aligned}\Delta t &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{1.496\times10^{11}\,m}{299{,}792{,}458\,\frac{m}{s}} \\ &= 499s \\ &= 8\min 19s\end{aligned}$$

2. The distance is

$$\begin{aligned}\Delta x &= v\Delta t \\ &= 299{,}792{,}458\,\tfrac{m}{s}\cdot\left(365.25\cdot24\cdot60\cdot60s\right) \\ &= 9.46\times10^{15}\,m\end{aligned}$$

3. The wavelength is

$$\begin{aligned}\lambda_{\text{substance}} &= \frac{\lambda_{\text{vacuum}}}{n} \\ &= \frac{500nm}{1.33} \\ &= 375.9nm\end{aligned}$$

4. The refraction index is found with

$$\begin{aligned}\lambda_{\text{substance}} &= \frac{\lambda_{\text{vacuum}}}{n} \\ 480nm &= \frac{600nm}{n} \\ n &= 1.25\end{aligned}$$

The speed of light is therefore

$$\begin{aligned} v &= \frac{c}{n} \\ &= \frac{299{,}792{,}458 \frac{m}{s}}{1.25} \\ &= 2.4 \times 10^8 \tfrac{m}{s} \end{aligned}$$

5. The amplitude of the incident wave is

$$\begin{aligned} I &= \frac{cn\varepsilon_0 E_0^2}{2} \\ 5 \tfrac{W}{m^2} &= \frac{3\times 10^8 \frac{m}{s} \cdot 1 \cdot 8.854\times 10^{-12} \frac{C^2}{Nm^2} E_0^2}{2} \\ E_0 &= 61.36 \tfrac{N}{C} \end{aligned}$$

Thus, the amplitude of the reflected and transmitted waves are

$$\begin{aligned} E_{0R} &= \frac{n_1 - n_2}{n_1 + n_2} E_0 \\ &= \frac{1 - 1.33}{1 + 1.33} \cdot 61.36 \tfrac{N}{C} \\ &= -8.690 \tfrac{N}{C} \end{aligned}$$

$$\begin{aligned} E_{0T} &= \frac{2n_1}{n_1 + n_2} E_0 \\ &= \frac{2 \cdot 1}{1 + 1.33} \cdot 61.36 \tfrac{N}{C} \\ &= 52.67 \tfrac{N}{C} \end{aligned}$$

This means that the intensity of the transmitted and reflected waves are

$$I_R = \frac{cn\varepsilon_0 E_{0R}^2}{2}$$
$$= \frac{3\times10^8 \frac{m}{s}\cdot 1\cdot 8.854\times10^{-12}\frac{C^2}{Nm^2}\cdot\left(-8.690\frac{N}{C}\right)^2}{2}$$
$$= 0.100\frac{W}{m^2}$$

$$I_T = \frac{cn\varepsilon_0 E_{0T}^2}{2}$$
$$= \frac{3\times10^8 \frac{m}{s}\cdot 1.33\cdot 8,854\times10^{-12}\frac{C^2}{Nm^2}\cdot\left(52.67\frac{N}{C}\right)^2}{2}$$
$$= 4.900\frac{W}{m^2}$$

Therefore, 98% of the energy is transmitted.

6. The wavelength is

$$c = \lambda f$$
$$3\times10^8 \frac{m}{s} = \lambda\cdot 10^{15} Hz$$
$$\lambda = 3\times10^{-7} m = 300nm$$

This is a wavelength corresponding to ultraviolet light.

7. a) The frequency is

$$c = \lambda f$$
$$3\times10^8 \frac{m}{s} = \left(585\times10^{-9} m\right) f$$
$$f = 5.128\times10^{14} Hz$$

The frequency received by the observer is

$$f' = f\,\frac{c - v_0}{c - v_s}$$
$$= \left(5.128\times10^{14} Hz\right)\cdot\frac{3\times10^8 \frac{m}{s} - 0\frac{m}{s}}{3\times10^8 \frac{m}{s} - 3\times10^7 \frac{m}{s}}$$
$$= 5.698\times10^{14} Hz$$

This frequency corresponds to the wavelength

$$\begin{aligned} c &= \lambda f \\ 3\times10^8 \tfrac{m}{s} &= \lambda\cdot\left(5.698\times10^{14} Hz\right) \\ \lambda &= 526.5nm \end{aligned}$$

Which corresponds to bluish-green light.

b) the frequency received by the observer is

$$\begin{aligned} f' &= f\frac{c-v_0}{c-v_s} \\ &= \left(5.128\times10^{14} Hz\right)\cdot\frac{3\times10^8 \tfrac{m}{s} - 0\tfrac{m}{s}}{3\times10^8 \tfrac{m}{s} + 3\times10^7 \tfrac{m}{s}} \\ &= 4.662\times10^{14} Hz \end{aligned}$$

This frequency corresponds to the wavelength

$$\begin{aligned} c &= \lambda f \\ 3\times10^8 \tfrac{m}{s} &= \lambda\cdot\left(4.662\times10^{14} Hz\right) \\ \lambda &= 643.5nm \end{aligned}$$

Which corresponds to red light.

8. The emitted frequency is

$$\begin{aligned} c &= \lambda f \\ 3\times10^8 \tfrac{m}{s} &= \left(600\times10^{-9} m\right) f \\ f &= 5\times10^{14} Hz \end{aligned}$$

The frequency received is

$$\begin{aligned} c &= \lambda f' \\ 3\times10^8 \tfrac{m}{s} &= \left(470\times10^{-9} m\right) f' \\ f' &= 6.383\times10^{14} Hz \end{aligned}$$

Therefore, the speed is found with

$$f' = f\frac{c - v_0}{c - v_s}$$

$$6.383\times 10^{14} Hx = \left(5\times 10^{14} Hz\right)\cdot\frac{3\times 10^{8}\,\frac{m}{s} - v_0}{3\times 10^{8}\,\frac{m}{s} - 0\,\frac{m}{s}}$$

$$v_0 = -8.3\times 10^{7}\,\frac{m}{s}$$

The negative sign indicates that the observer must move towards the source. The speed is 8.3 x 10^7 m/s.

9. The speed is found with

$$\begin{aligned} v_{car} &= \frac{c\Delta f}{2f} \\ &= \frac{3\times 10^{8}\,\frac{m}{s}\times 6000Hz}{2\times 25\times 10^{9}\,Hz} \\ &= 36\,\frac{m}{s} \\ &= 129.6\,\frac{km}{h} \end{aligned}$$

10. Let's start by finding the percentages of the reflected and transmitted wave at each surface.

When the light comes from air to glass, the transmitted and reflected amplitudes are

$$\begin{aligned} E_{0R} &= \frac{n_1 - n_2}{n_1 + n_2} E_0 \\ &= \frac{1-1,5}{1+1,5}\cdot E_0 \\ &= -0.2E_0 \end{aligned}$$

$$\begin{aligned} E_{0T} &= \frac{2n_1}{n_1 + n_2} E_0 \\ &= \frac{2\cdot 1}{1+1,5}\cdot E_0 \\ &= 0.8E_0 \end{aligned}$$

This means that the ratio of the intensities of the transmitted and reflected wave to the intensity of the initial wave are

$$\begin{aligned}\frac{I_R}{I_0} &= \frac{\left(\frac{cn_1\varepsilon_0 E_{0R}^2}{2}\right)}{\left(\frac{cn_1\varepsilon_0 E_0^2}{2}\right)} \\ &= \frac{E_{0R}^2}{E_0^2} \\ &= \frac{\left(0.2E_0\right)^2}{E_0^2} \\ &= \left(0.2\right)^2 \\ &= 0.04\end{aligned}$$

$$\begin{aligned}I_T &= \frac{\left(\frac{cn_2\varepsilon_0 E_{0T}^2}{2}\right)}{\left(\frac{cn_1\varepsilon_0 E_0^2}{2}\right)} \\ &= \frac{n_2 E_{0T}^2}{n_1 E_0^2} \\ &= \frac{n_2\left(0.8E_0\right)^2}{n_1 E_0^2} \\ &= \frac{n_2\left(0.8\right)^2}{n_1} \\ &= \frac{1.5\left(0.8\right)^2}{1} \\ &= 0.96\end{aligned}$$

Thus, 96% of the energy enters the glass.

Once inside the glass, the light reaches the glass air interface. In this interface, the amplitudes of reflected and transmitted waves are

$$\begin{aligned} E_{0R} &= \frac{n_1 - n_2}{n_1 + n_2} E_0 \\ &= \frac{1.5 - 1}{1 + 1.5} \cdot E_0 \\ &= 0.2 E_0 \end{aligned}$$

$$\begin{aligned} E_{0T} &= \frac{2n_1}{n_1 + n_2} E_0 \\ &= \frac{2 \cdot 1.5}{1 + 1.5} \cdot E_0 \\ &= 1.2 E_0 \end{aligned}$$

This means that the ratio of the intensities of the transmitted and reflected wave to the intensity of the initial wave are

$$\begin{aligned} \frac{I_R}{I_0} &= \frac{\left(\frac{cn_2\varepsilon_0 E_{0R}^2}{2}\right)}{\left(\frac{cn_2\varepsilon_0 E_0^2}{2}\right)} \\ &= \frac{E_{0R}^2}{E_0^2} \\ &= \frac{(0.2E_0)^2}{E_0^2} \\ &= (0.2)^2 \\ &= 0.04 \end{aligned}$$

$$
\begin{aligned}
I_T &= \frac{\left(\frac{cn_1\varepsilon_0 E_{0T}^2}{2}\right)}{\left(\frac{cn_2\varepsilon_0 E_0^2}{2}\right)} \\
&= \frac{n_1 E_{0T}^2}{n_2 E_0^2} \\
&= \frac{n_1 \left(1.2E_0\right)^2}{n_2 E_0^2} \\
&= \frac{n_1 \left(1.2\right)^2}{n_2} \\
&= \frac{1\left(1.2\right)^2}{1.5} \\
&= 0.96
\end{aligned}
$$

The intensity of the transmitted light can now be found.

No reflection

In this case, the light passes directly through both surfaces. The percentage of light doing this is

$$0.96 \times 0.96 = 0.9216 = 92.16\%$$

1 round trip

In this case, the light enters glass, reflects 2 times and comes out of the glass. The percentage of light doing this is

$$0.96 \times 0.04 \times 0.04 \times 0.96 = 0.96^2 \times 0.04^2$$

2 round trips

In this case, the light enters glass, reflects 4 times and comes out of the glass. The percentage of light doing this is

$$0.96 \times 0.04 \times 0.04 \times 0.04 \times 0.04 \times 0.96 = 0.96^2 \times 0.04^4$$

3 round trips

In this case, the light enters glass, reflects 6 times and comes out of the glass. The percentage of light doing this is

$$0.96^2 \times 0.04^6$$

And so on...

Summing all these intensities, the result is

$$\begin{aligned} I_{tot} &= (0.96)^2 + (0.96)^2 (0.04)^2 + (0.96)^2 (0.04)^4 + (0.96)^2 (0.04)^6 + ... \\ &= (0.96)^2 \left(1 + 0.0016 + 0.0016^2 + 0.0016^3 + ...\right) \end{aligned}$$

Such a sum is a geometric series. You can calculate it directly or you can take a formula that gives the sum. Let's find this formula. If you have a geometric series

$$S = a + ar + ar^2 + ar^3 + ...$$

Then

$$\begin{aligned} Sr &= ar + ar^2 + ar^3 + ar^4 + ... \\ Sr &= S - a \\ S - Sr &= a \\ S(1-r) &= a \\ S &= \frac{a}{1-r} \end{aligned}$$

Therefore, the sum here is

$$\begin{aligned} I_{tot} &= (0.96)^2 \frac{1}{1-0.0016} \\ &= 0.9231 \end{aligned}$$

Thus, 92.31% of the light is transmitted. This means that 7,69% of the light is reflected. Let's see if a calculation gives the same result.

Light directly reflected

In this case, the light is reflected off the first surface 1. The percentage reflected

$$0,04$$

1 round trip

In this case, the light enters glass, reflects 1 times and comes out of the glass. The percentage of light doing this is

$$0.96 \times 0.04 \times 0.96 = 0.96^2 \times 0.04$$

2 round trips

In this case, the light enters glass, reflects 3 times and comes out of the glass. The percentage of light doing this is

$$0.96 \times 0.04 \times 0.04 \times 0.04 \times 0.96 = 0.96^2 \times 0.04^3$$

3 round trips

In this case, the light enters glass, reflects 5 times and comes out of the glass. The percentage of light doing this is

$$0.96^2 \times 0.04^5$$

And so on…

Summing all these intensities, the result is

$$\begin{aligned} I_{tot} &= 0.04 + (0.96)^2 (0.04)^1 + (0.96)^2 (0.04)^3 + (0.96)^2 (0.04)^5 + \ldots \\ &= 0.04 + (0.96)^2 (0.04)\left(1 + 0.0016 + 0.0016^2 + 0.0016^3 + \ldots\right) \end{aligned}$$

The sum of the geometric series is

$$\begin{aligned} S &= (0.96)^2 \cdot 0.04 \frac{1}{1 - 0.0016} \\ &= 0.03692 \end{aligned}$$

Therefore, the total intensity is

$$\begin{aligned} I_{tot} &= 0.04 + 0.0369 \\ &= 0.07692 \end{aligned}$$

which is the predicted result.