Chapter 3 Solutions

1. We have

$$
\begin{aligned}
v &= \lambda f \\
340\tfrac{m}{s} &= \lambda \cdot 50000Hz \\
\lambda &= 0.0068m = 6.8mm
\end{aligned}
$$

2. The wave speed is

$$
\begin{aligned}
v &= 331.3\tfrac{m}{s}\sqrt{\frac{T°}{273.15K}} \\
&= 331.3\tfrac{m}{s}\sqrt{\frac{298.15K}{273.15K}} \\
&= 346.1\tfrac{m}{s}
\end{aligned}
$$

3. a) The wave speed is found with

$$
\begin{aligned}
v &= \frac{\omega}{k} \\
&= \frac{560\,\frac{rad}{s}}{1.6\,\frac{rad}{m}} \\
&= 350\tfrac{m}{s}
\end{aligned}
$$

Thus, the temperature is

$$
\begin{aligned}
v &= 331.3\tfrac{m}{s}\sqrt{\frac{T°}{273.15K}} \\
350\tfrac{m}{s} &= 331.3\tfrac{m}{s}\sqrt{\frac{T°}{273.15K}} \\
T° &= 304.9K = 31.7°C
\end{aligned}
$$

b) The maximum speed of the air molecules is

$$\begin{aligned} v_{\max} &= \omega A \\ &= 560 \tfrac{rad}{s} \cdot 0.000\,01m \\ &= 0.0056 \tfrac{m}{s} \end{aligned}$$

4. a) At this temperature, the wave speed is

$$\begin{aligned} v &= 331.3 \tfrac{m}{s} \sqrt{\frac{T°}{273.15K}} \\ &= 331.3 \tfrac{m}{s} \sqrt{\frac{288.15K}{273.15K}} \\ &= 340.3 \tfrac{m}{s} \end{aligned}$$

The air impedance is therefore

$$\begin{aligned} Z &= \rho v \\ &= 1.3 \tfrac{kg}{m^3} \cdot 340.3 \tfrac{m}{s} \\ &= 442.4 \tfrac{kg}{m^2 \cdot s} \end{aligned}$$

b) The water impedance is

$$\begin{aligned} Z &= \rho v \\ &= 1000 \tfrac{kg}{m^3} \cdot 1520 \tfrac{m}{s} \\ &= 1,520,000 \tfrac{kg}{m^2 \cdot s} \end{aligned}$$

c) No, because the water impedance is too different from the air impedance (3435 times greater).

5. The frequency of the first harmonic is

$$\begin{aligned} f_1 &= \frac{1 \cdot v}{2L} \\ &= \frac{1 \cdot 340 \tfrac{m}{s}}{2 \cdot 0.25m} \\ &= 680Hz \end{aligned}$$

The frequency of the second harmonic is

$$\begin{aligned} f_2 &= \frac{1 \cdot v}{2L} \\ &= \frac{2 \cdot 340 \frac{m}{s}}{2 \cdot 0.25m} \\ &= 1360Hz \end{aligned}$$

The frequency of the third harmonic is

$$\begin{aligned} f_3 &= \frac{1 \cdot v}{2L} \\ &= \frac{3 \cdot 340 \frac{m}{s}}{2 \cdot 0.25m} \\ &= 2040Hz \end{aligned}$$

6. The frequency of the first harmonic is

$$\begin{aligned} f_1 &= \frac{1 \cdot v}{4L} \\ &= \frac{1 \cdot 340 \frac{m}{s}}{4 \cdot 0.40m} \\ &= 212.5Hz \end{aligned}$$

The frequency of the third harmonic is

$$\begin{aligned} f_3 &= \frac{3 \cdot v}{4L} \\ &= \frac{3 \cdot 340 \frac{m}{s}}{4 \cdot 0.40m} \\ &= 637.5Hz \end{aligned}$$

The frequency of the fifth harmonic is

$$\begin{aligned} f_5 &= \frac{5 \cdot v}{4L} \\ &= \frac{5 \cdot 340 \frac{m}{s}}{4 \cdot 0.40m} \\ &= 1062.5Hz \end{aligned}$$

7. At this temperature, the wave speed is

$$\begin{aligned} v &= 331.3\tfrac{m}{s}\sqrt{\frac{T°}{273.15K}} \\ &= 331.3\tfrac{m}{s}\sqrt{\frac{303.15K}{273.15K}} \\ &= 349.0\tfrac{m}{s} \end{aligned}$$

Then, we must have

$$\begin{aligned} f_3 &= \frac{3\cdot v}{4L} \\ 500Hz &= \frac{3\cdot 349.0\tfrac{m}{s}}{4\cdot L} \\ L &= 0.5235m \end{aligned}$$

8. At this temperature, the wave speeds are

$$\begin{aligned} v &= 331.3\tfrac{m}{s}\sqrt{\frac{T°}{273.15K}} \\ &= 331.3\tfrac{m}{s}\sqrt{\frac{298.15K}{273.15K}} \\ &= 346.1\tfrac{m}{s} \end{aligned}$$

$$\begin{aligned} v &= 331.3\tfrac{m}{s}\sqrt{\frac{T°}{273.15K}} \\ &= 331.3\tfrac{m}{s}\sqrt{\frac{273.15K}{273.15K}} \\ &= 331.3\tfrac{m}{s} \end{aligned}$$

At 25 °C, we have

$$\begin{aligned} f_1 &= \frac{1\cdot v}{x\cdot L} \\ 500Hz &= \frac{1\cdot 346.1\tfrac{m}{s}}{x\cdot L} \end{aligned}$$

(x has been used in the denominator since we do not know whether the pipe is open or closed. If it is open, we have $x = 2$, and if it is closed, then $x = 4$.)

At 0 °C, we have

$$\begin{aligned} f_1' &= \frac{1\cdot v}{x\cdot L} \\ &= \frac{1\cdot 331.3\tfrac{m}{s}}{x\cdot L} \end{aligned}$$

If we divide this equation by the equation at 25 °C, we obtain

$$\frac{f_1'}{500Hz} = \frac{\left(\frac{1 \cdot 331.3\frac{m}{s}}{x \cdot L}\right)}{\left(\frac{1 \cdot 346.1\frac{m}{s}}{x \cdot L}\right)}$$

$$\frac{f_1'}{500Hz} = \frac{331.3\frac{m}{s}}{346.1\frac{m}{s}}$$

$$f_1' = 478.6Hz$$

9. Let's assume that the pipe is open. Then we would have the following two equations.

$$f_n = \frac{n \cdot v}{2L} \qquad f_{n+1} = \frac{(n+1) \cdot v}{2L}$$

$$630Hz = \frac{n \cdot v}{2L} \qquad 840Hz = \frac{(n+1) \cdot v}{2L}$$

By dividing the equation to the right by the equation to the left, we obtain

$$\frac{840Hz}{630Hz} = \frac{\frac{(n+1) \cdot v}{2L}}{\frac{n \cdot v}{2L}}$$

$$\frac{840}{630} = \frac{n+1}{n}$$

$$840 \cdot n = 630 \cdot (n+1)$$

$$840 \cdot n = 630 \cdot n + 630$$

$$210 \cdot n = 630$$

$$n = 3$$

This is an acceptable solution (because *n* must be an integer).

Let's now assume that the pipe is closed. Then we would have the following two equations.

$$f_n = \frac{n \cdot v}{4L} \qquad f_{n+1} = \frac{(n+2) \cdot v}{4L}$$

$$630Hz = \frac{n \cdot v}{4L} \qquad 840Hz = \frac{(n+2) \cdot v}{4L}$$

By dividing the equation to the right by the equation to the left, we obtain

$$\begin{aligned}\frac{840Hz}{630Hz} &= \frac{\frac{(n+2)\cdot v}{4L}}{\frac{n\cdot v}{4L}}\\ \frac{840}{630} &= \frac{n+2}{n}\\ 840\cdot n &= 630\cdot(n+2)\\ 840\cdot n &= 630\cdot n + 1260\\ 210\cdot n &= 1260\\ n &= 6\end{aligned}$$

This is not an acceptable solution (because n must be an odd integer with a closed pipe).

Therefore, the tube is open.

b) We have

$$\begin{aligned}f_3 &= \frac{3\cdot v}{2L}\\ 630Hz &= \frac{3\cdot 336Hz}{2L}\\ L &= 0.8m\end{aligned}$$

10. a) The sound frequency is

$$\begin{aligned}f_{sound} &= \frac{f_1+f_2}{2}\\ &= \frac{500Hz+508Hz}{2}\\ &= 504Hz\end{aligned}$$

b) The beat frequency is

$$\begin{aligned}f_{beats} &= |f_1-f_2|\\ &= 508Hz-500Hz\\ &= 8Hz\end{aligned}$$

11. We have the equations

$$f_{sound} = \frac{f_1 + f_2}{2}$$
$$350Hz = \frac{f_1 + f_2}{2}$$
$$700Hz = f_1 + f_2$$

and

$$f_{beats} = \left| f_1 - f_2 \right|$$
$$6Hz = f_1 - f_2$$

(The absolute value was removed since we will assume that f_1 is greater than f_2.)

Adding these two equations, we obtain

$$700Hz + 6Hz = (f_1 + f_2) + (f_1 - f_2)$$
$$706Hz = 2f_1$$
$$f_1 = 353Hz$$

The other frequency is then

$$6Hz = f_1 - f_2$$
$$6Hz = 353Hz - f_2$$
$$f_2 = 347Hz$$

12. If there are 4.2 Hz beats, then there is a 4.3 Hz gap between the frequency of the rope and the frequency of the machine. As the machine has a 329.6 Hz frequency, this means that the rope has a frequency of 333.8 Hz or 325.4 Hz. Which of these frequencies is good?

To find out, we'll use the fact that the beat frequency decreases if the tension of the rope is increased. As the frequency of the string is given by

$$f_1 = \frac{1}{2L}\sqrt{\frac{F_T}{\mu}}$$

We can see that frequency of the rope increases if tension is increased.

Now assume that the frequency of the rope is 333.8 Hz. If the tension is increased, the frequency increases, and the gap between the frequency of the string and the 329.6 Hz sound will become larger, and the beat frequency will increase.

Now assume that the frequency of the rope is 325.4 Hz. If the tension is increased, the frequency increases, and the gap between the frequency of the string and the 329.6 Hz sound will become smaller, and the beat frequency will decrease.

As it is said that the beat frequency decreases if the tension is increased, then the frequency of the rope must be 325.4 Hz.

With a 1300 N tension, we have

$$325.4Hz = \frac{1}{2L}\sqrt{\frac{1300N}{\mu}}$$

If a 329,6 Hz frequency is needed, then

$$329.6Hz = \frac{1}{2L}\sqrt{\frac{F_T'}{\mu}}$$

By dividing this last equation by the previous one, we obtain

$$\frac{329.6Hz}{325.4Hz} = \frac{\left(\frac{1}{2L}\sqrt{\frac{F_T'}{\mu}}\right)}{\left(\frac{1}{2L}\sqrt{\frac{1300N}{\mu}}\right)}$$

$$\frac{329.6}{325.4} = \sqrt{\frac{F_T'}{1300N}}$$

$$F_T' = 1333.8N$$

13. a) The frequency is

$$\begin{aligned} f' &= f\frac{v-v_0}{v-v_s} \\ &= 350Hz\frac{340\frac{m}{s}-0\frac{m}{s}}{340\frac{m}{x}-35\frac{m}{s}} \\ &= 390.1Hz \end{aligned}$$

b) The wavelength is

$$\begin{aligned} \lambda' &= \lambda\left(1-\frac{v_s}{v}\right) \\ &= \frac{v}{f}\left(1-\frac{v_s}{v}\right) \\ &= \frac{340\frac{m}{s}}{350Hz}\left(1-\frac{35\frac{m}{s}}{340Hz}\right) \\ &= 0.8714m \end{aligned}$$

c) The frequency is

$$\begin{aligned} f' &= f\frac{v-v_0}{v-v_s} \\ &= 350Hz\frac{340\frac{m}{s}-0\frac{m}{s}}{340\frac{m}{x}-\left(-35\frac{m}{s}\right)} \\ &= 317.3Hz \end{aligned}$$

d) The wavelength is

$$\begin{aligned} \lambda' &= \lambda\left(1-\frac{v_s}{v}\right) \\ &= \frac{v}{f}\left(1-\frac{v_s}{v}\right) \\ &= \frac{340\frac{m}{s}}{350Hz}\left(1-\frac{-35\frac{m}{s}}{340Hz}\right) \\ &= 1.0714m \end{aligned}$$

14. a) The frequency is

$$\begin{aligned} f' &= f\frac{v-v_0}{v-v_s} \\ &= 400Hz\frac{340\frac{m}{s}-10\frac{m}{s}}{340\frac{m}{x}-0\frac{m}{s}} \\ &= 388.2Hz \end{aligned}$$

b) The wavelength is

$$\begin{aligned} \lambda' &= \lambda\left(1-\frac{v_s}{v}\right) \\ &= \frac{v}{f}\left(1-\frac{v_s}{v}\right) \\ &= \frac{340\frac{m}{s}}{400Hz}\left(1-\frac{0\frac{m}{s}}{340Hz}\right) \\ &= 0.85m \end{aligned}$$

c) The frequency is

$$\begin{aligned} f' &= f\frac{v-v_0}{v-v_s} \\ &= 400Hz\frac{340\frac{m}{s}-\left(-12\frac{m}{s}\right)}{340\frac{m}{x}-0\frac{m}{s}} \\ &= 414.1Hz \end{aligned}$$

d) The wavelength is

$$\begin{aligned} \lambda' &= \lambda\left(1-\frac{v_s}{v}\right) \\ &= \frac{v}{f}\left(1-\frac{v_s}{v}\right) \\ &= \frac{340\frac{m}{s}}{400Hz}\left(1-\frac{0\frac{m}{s}}{340Hz}\right) \\ &= 0.85m \end{aligned}$$

15. The frequency is

$$\begin{aligned} f' &= f\frac{v-v_0}{v-v_s} \\ &= 400Hz\frac{340\frac{m}{s}-\left(-41.67\frac{m}{s}\right)}{340\frac{m}{x}-22.22\frac{m}{s}} \\ &= 480.4Hz \end{aligned}$$

16. By removing the wind speed (by adding 15 km/h towards the left to all velocities), the speed of the red car is 135 km/h and the speed of the police car is 95 km/h. Therefore, the frequency is

$$\begin{aligned} f' &= f\frac{v-v_0}{v-v_s} \\ &= 400Hz\frac{340\frac{m}{s}-\left(-37.5\frac{m}{s}\right)}{340\frac{m}{x}-26.39\frac{m}{s}} \\ &= 481.4Hz \end{aligned}$$

17. a) When the train is moving towards the stationary observer, the frequency is

$$\begin{aligned} f' &= f\frac{v-v_0}{v-v_s} \\ 150Hz &= f\frac{335\frac{m}{s}-0\frac{m}{s}}{335\frac{m}{x}-v} \\ 150Hz &= f\frac{335\frac{m}{s}}{335\frac{m}{x}-v} \end{aligned}$$

When the train is moving away from the stationary observer, the frequency is

$$\begin{aligned} f' &= f\frac{v-v_0}{v-v_s} \\ 125Hz &= f\frac{335\frac{m}{s}-0\frac{m}{s}}{335\frac{m}{x}--v} \\ 125Hz &= f\frac{335\frac{m}{s}}{335\frac{m}{x}+v} \end{aligned}$$

We then have 2 equations and 2 unknowns. By dividing the equations one by the other, we obtain

$$\frac{150Hz}{125Hz}=\frac{f\,\frac{335\frac{m}{s}}{335\frac{m}{x}-v}}{f\,\frac{335\frac{m}{s}}{335\frac{m}{x}+v}}$$

$$1.2=\frac{335\frac{m}{x}+v}{335\frac{m}{x}-v}$$

$$1.2\cdot\left(335\tfrac{m}{x}-v\right)=335\tfrac{m}{x}+v$$

$$402\tfrac{m}{x}-1.2\cdot v=335\tfrac{m}{x}+v$$

$$67\tfrac{m}{x}=2.2\cdot v$$

$$v=30.45\tfrac{m}{s}$$

b) Using the speed, the frequency is easily calculated

$$125Hz=f\,\frac{335\frac{m}{s}}{335\frac{m}{x}+v}$$

$$125Hz=f\,\frac{335\frac{m}{s}}{335\frac{m}{x}+30.45\frac{m}{s}}$$

$$f=136.4Hz$$

18. We have

$$f\,'=f\,\frac{v-v_0}{v-v_s}$$

$$420Hz=400Hz\,\frac{345\frac{m}{s}-v}{345\frac{m}{x}-\left(v+15\frac{m}{s}\right)}$$

The solution is

$$\frac{420Hz}{400Hz}=\frac{345\frac{m}{s}-v}{345\frac{m}{s}-\left(v+15\frac{m}{s}\right)}$$

$$1{,}05=\frac{345\frac{m}{s}-v}{330\frac{m}{s}-v}$$

$$1{,}05\cdot\left(330\tfrac{m}{s}-v\right)=345\tfrac{m}{s}-v$$

$$346{,}5\tfrac{m}{s}-1{,}05\cdot v=345\tfrac{m}{s}-v$$

$$1{,}5\tfrac{m}{s}=0{,}05\cdot v$$

$$v=30\tfrac{m}{s}$$

The speed of the red car is therefore 30 m/s (108 km/h) and the speed of the police car is 45 m/s (162 km/h).

19. At this temperature, the speed of sound is

$$\begin{aligned} v &= 331.3\tfrac{m}{s}\sqrt{\frac{T^{\circ}}{273.15K}} \\ &= 331.3\tfrac{m}{s}\sqrt{\frac{298.15K}{273.15K}} \\ &= 346.13\tfrac{m}{s} \end{aligned}$$

The frequency of the sound arriving directly to the car is

$$\begin{aligned} f' &= f\frac{v-v_0}{v-v_s} \\ &= 400Hz\frac{346.13\frac{m}{s}-5\frac{m}{s}}{346.13\frac{m}{x}-25\frac{m}{s}} \\ &= 424.91Hz \end{aligned}$$

Now let's find the frequency of the sound arriving at the wall. This frequency is

$$\begin{aligned} f' &= f\frac{v-v_0}{v-v_s} \\ &= 400Hz\frac{346.13\frac{m}{s}-0\frac{m}{s}}{346.13\frac{m}{x}-25\frac{m}{s}} \\ &= 431.14Hz \end{aligned}$$

The wall now becomes a source emitting at this frequency. The frequency of this sound received by the person in the car is

$$\begin{aligned} f' &= f\frac{v-v_0}{v-v_s} \\ &= 431.14Hz\frac{346.13\frac{m}{s}-\left(-5\frac{m}{s}\right)}{346.13\frac{m}{x}-0\frac{m}{s}} \\ &= 437.37Hz \end{aligned}$$

The person in the car therefore receives a sound at 424.91 Hz and sound at 437.37 Hz. The beat frequency is therefore

$$\begin{aligned} f_{beats} &= 437.37Hz - 424.91Hz \\ &= 12.46Hz \end{aligned}$$

20. At this temperature, the speed of sound is

$$\begin{aligned} v &= 331.3\tfrac{m}{s}\sqrt{\frac{T°}{273.15K}} \\ &= 331.3\tfrac{m}{s}\sqrt{\frac{293.15K}{273.15K}} \\ &= 343.21\tfrac{m}{s} \end{aligned}$$

The frequency of the sound arriving directly to the car is 400 Hz.

Now let's find the frequency of the sound arriving at the wall. In this case, the observer (the wall) is stationary and the car has a positive velocity. The received frequency is, therefore,

$$\begin{aligned} f' &= 400Hz\frac{v - v_0}{v - v_s} \\ &= 400Hz\frac{v}{v - v_{car}} \end{aligned}$$

Then, the wall becomes a source emitting that frequency. We then have a stationary source (the wall) and an observer who has a negative velocity (the car). Thus, the sound received by the person in the car has the following frequency

$$\begin{aligned} f'' &= f'\frac{v - v_0}{v - v_s} \\ &= f'\frac{v - -v_{car}}{v} \\ &= f'\frac{v + v_{car}}{v} \end{aligned}$$

Substituting f' by the value found earlier, the equation becomes

$$\begin{aligned} f'' &= 400Hz\frac{v}{v - v_{car}}\frac{v + v_{car}}{v} \\ &= 400Hz\frac{v + v_{car}}{v - v_{car}} \end{aligned}$$

The person in the car then hears a sound with a 400 Hz frequency, and another sound with a frequency f''. The beat frequency is, therefore,

$$f_{beats} = 400Hz \frac{v + v_{car}}{v - v_{car}} - 400Hz$$

As this beat frequency is equal to 15 Hz, we have

$$\begin{aligned} 15Hz &= 400Hz \frac{v + v_{car}}{v - v_{car}} - 400Hz \\ 415Hz &= 400Hz \frac{v + v_{car}}{v - v_{car}} \\ 1.0375 &= \frac{v + v_{car}}{v - v_{car}} \\ 1.0375v - 1.0375v_{car} &= v + v_{car} \\ 0.0375v &= 2.0375v_{car} \\ v_{car} &= \frac{0.0375}{2.0375} v \end{aligned}$$

Since the speed of sound is 343.21 m/s, the speed is

$$\begin{aligned} v_{car} &= \frac{0.0375}{2.0375} 343.21 \tfrac{m}{s} \\ &= 6.317 \tfrac{m}{s} \end{aligned}$$

21. The intensity of the sound at this distance is

$$\begin{aligned} I &= \frac{P}{4\pi r^2} \\ &= \frac{50W}{4\pi (30m)^2} \\ &= 4.42 \times 10^{-3} \tfrac{W}{m^2} \end{aligned}$$

In decibels, the intensity is

$$\begin{aligned}\beta &= 10dB\log\left(\frac{I}{10^{-12}\frac{W}{m^2}}\right)\\ &= 10dB\log\left(\frac{4.42\times10^{-3}\frac{W}{m^2}}{10^{-12}\frac{W}{m^2}}\right)\\ &= 96.5dB\end{aligned}$$

22. The intensity is

$$\begin{aligned}\beta &= 10dB\log\left(\frac{I}{10^{-12}\frac{W}{m^2}}\right)\\ 110dB &= 10dB\log\left(\frac{I}{10^{-12}\frac{W}{m^2}}\right)\\ I &= 0.1\tfrac{W}{m^2}\end{aligned}$$

The power received is therefore

$$\begin{aligned}P_{received} &= IA_{receiver}\\ &= 0.1\tfrac{W}{m^2}\cdot 0.001m^2\\ &= 0.0001W\end{aligned}$$

The energy received in 2 minutes is

$$\begin{aligned}E &= Pt\\ &= 0.0001\tfrac{W}{m^2}\cdot 120s = 0.012J\end{aligned}$$

23. At 90 dB, the intensity is

$$\begin{aligned}\beta &= 10dB\log\left(\frac{I}{10^{-12}\frac{W}{m^2}}\right)\\ 90dB &= 10dB\log\left(\frac{I}{10^{-12}\frac{W}{m^2}}\right)\\ I &= 0.001\tfrac{W}{m^2}\end{aligned}$$

The power of the source is therefore

$$I = \frac{P}{4\pi r^2}$$
$$0.001\tfrac{W}{m^2} = \frac{P}{4\pi(10m)^2}$$
$$P = 1.2566W$$

To have 70 dB, the intensity must be

$$\beta = 10dB\log\left(\frac{I}{10^{-12}\tfrac{W}{m^2}}\right)$$
$$70dB = 10dB\log\left(\frac{I}{10^{-12}\tfrac{W}{m^2}}\right)$$
$$I = 10^{-5}\tfrac{W}{m^2}$$

The distance is then found with

$$I = \frac{P}{4\pi r^2}$$
$$10^{-5}\tfrac{W}{m^2} = \frac{1.2566W}{4\pi r^2}$$
$$r = 100m$$

24. At 40 dB, the intensity is

$$\beta = 10dB\log\left(\frac{I}{10^{-12}\tfrac{W}{m^2}}\right)$$
$$40dB = 10dB\log\left(\frac{I}{10^{-12}\tfrac{W}{m^2}}\right)$$
$$I = 10^{-8}\tfrac{W}{m^2}$$

The power of 1 firecracker is therefore

$$I = \frac{P}{4\pi r^2}$$

$$10^{-8}\,\tfrac{W}{m^2} = \frac{P}{4\pi(50m)^2}$$

$$P = 3.142\times 10^{-4} W$$

The power of 1000 firecrackers is then

$$P' = 1000\cdot 3.142\times 10^{-4} W$$
$$= 0.3142W$$

The intensity at a distance of 200 m is then

$$I = \frac{P}{4\pi r^2}$$
$$= \frac{0.3142W}{4\pi(200m)^2}$$
$$= 6.25\times 10^{-7} W$$

In decibels, this intensity is

$$\beta = 10dB\log\left(\frac{I}{10^{-12}\,\tfrac{W}{m^2}}\right)$$
$$= 10dB\log\left(\frac{6.25\times 10^{-7}\,\tfrac{W}{m^2}}{10^{-12}\,\tfrac{W}{m^2}}\right)$$
$$= 57.96dB$$

25. At 90 dB, the intensity is

$$\beta = 10dB\log\left(\frac{I}{10^{-12}\,\tfrac{W}{m^2}}\right)$$

$$90dB = 10dB\log\left(\frac{I}{10^{-12}\,\tfrac{W}{m^2}}\right)$$

$$I = 10^{-3}\,\tfrac{W}{m^2}$$

At 95 dB, the intensity is

$$\beta = 10dB \log\left(\frac{I}{10^{-12}\,\frac{W}{m^2}}\right)$$
$$95dB = 10dB \log\left(\frac{I}{10^{-12}\,\frac{W}{m^2}}\right)$$
$$I = 3.162\times 10^{-3}\,\tfrac{W}{m^2}$$

The total intensity is thus

$$\begin{aligned} I_{tot} &= 10^{-3}\,\tfrac{W}{m^2} + 3.162\times 10^{-3}\,\tfrac{W}{m^2} \\ &= 4.162\times 10^{-3}\,\tfrac{W}{m^2} \end{aligned}$$

In decibels, this intensity is

$$\begin{aligned} \beta &= 10dB \log\left(\frac{I}{10^{-12}\,\frac{W}{m^2}}\right) \\ &= 10dB \log\left(\frac{4.162\times 10^{-3}\,\frac{W}{m^2}}{10^{-12}\,\frac{W}{m^2}}\right) \\ &= 96.2dB \end{aligned}$$

26. At 25 m, the intensity is

$$\begin{aligned} I &= \frac{P}{4\pi r^2} \\ &= \frac{50W}{4\pi (25m)^2} \\ &= 6.366\times 10^{-3} W \end{aligned}$$

The amplitude is then found with

$$I = \frac{1}{2}\rho v \omega^2 A^2$$
$$6.366\times 10^{-3}\,\tfrac{W}{m^2} = \frac{1}{2}\cdot 1.3\tfrac{kg}{m^3}\cdot 340\tfrac{m}{s}\cdot (2\pi\cdot 200Hz)^2 A^2$$
$$A = 4.271\mu m$$

27. a) 5 km from the source, the intensity is

$$\begin{aligned} I &= \frac{P}{4\pi r^2} \\ &= \frac{20,000W}{4\pi\left(5000m\right)^2} \\ &= 6.366\times 10^{-5}W \end{aligned}$$

Thus, the intensity is

$$\begin{aligned} \beta &= 10dB\log\left(\frac{I}{10^{-12}\frac{W}{m^2}}\right) \\ &= 10dB\log\left(\frac{6.366\times 10^{-5}\frac{W}{m^2}}{10^{-12}\frac{W}{m^2}}\right) \\ &= 78.04dB \end{aligned}$$

b) If we lose 7 dB per km, the intensity will be 35 dB lower if the absorption by the air is taken into account absorption air. The intensity is, therefore, 43,04 dB.

28. At 60 dB, the intensity is

$$\begin{aligned} \beta &= 10dB\log\frac{I}{10^{-12}\frac{W}{m^2}} \\ 60dB &= 10dB\log\frac{I}{10^{-12}\frac{W}{m^2}} \\ I &= 10^{-6}\frac{W}{m^2} \end{aligned}$$

The wave amplitude is then

$$\begin{aligned} I &= \frac{1}{2}\rho v\omega^2 A^2 \\ 10^{-6}\frac{W}{m^2} &= \frac{1}{2}\left(1.3\frac{kg}{m^3}\right)\left(330\frac{m}{s}\right)\left(2\pi\times 200Hz\right)^2 A^2 \\ A &= 5.433\times 10^{-8}m \end{aligned}$$

To find the amplitude of the wave transmitted in water, the impedances of water and air are needed. These impedances are

$$Z_1 = \rho v = 1.3\tfrac{kg}{m^3}\cdot 330\tfrac{m}{s} = 429\tfrac{kg}{m^2 s}$$
$$Z_2 = \rho v = 1000\tfrac{kg}{m^3}\cdot 1450\tfrac{m}{s} = 1,450,000\tfrac{kg}{m^2 s}$$

The amplitude of the transmitted wave is therefore

$$\begin{aligned} A_T &= \frac{2Z_1}{Z_1+Z_2}A \\ &= \frac{2\cdot 429\tfrac{kg}{m^2 s}}{429\tfrac{kg}{m^2 s}+1,450,000\tfrac{kg}{m^2 s}}5.4335\times 10^{-8}m \\ &= 3.214\times 10^{-11}m \end{aligned}$$

The intensity of this wave in water is

$$\begin{aligned} I &= \frac{1}{2}\rho v\omega^2 A^2 \\ &= \frac{1}{2}\left(1000\tfrac{kg}{m^3}\right)\left(1450\tfrac{m}{s}\right)\left(2\pi\times 200Hz\right)^2\left(3.214\times 10^{-11}m\right)^2 \\ &= 1.183\times 10^{-9}\tfrac{W}{m^2} \end{aligned}$$

In decibels, this intensity is

$$\begin{aligned} \beta &= 10dB\log\frac{I}{10^{-12}\tfrac{W}{m^2}} \\ &= 10dB\log\frac{1.183\times 10^{-9}\tfrac{W}{m^2}}{10^{-12}\tfrac{W}{m^2}} \\ &= 30.7dB \end{aligned}$$

The amplitude of the reflected wave is

$$\begin{aligned} A_R &= \frac{Z_1-Z_2}{Z_1+Z_2}A \\ &= \frac{429\tfrac{kg}{m^2 s}-1,450,000\tfrac{kg}{m^2 s}}{429\tfrac{kg}{m^2 s}+1,450,000\tfrac{kg}{m^2 s}}\cdot 5.4335\times 10^{-8}m \\ &= -5.4302\times 10^{-6}m \end{aligned}$$

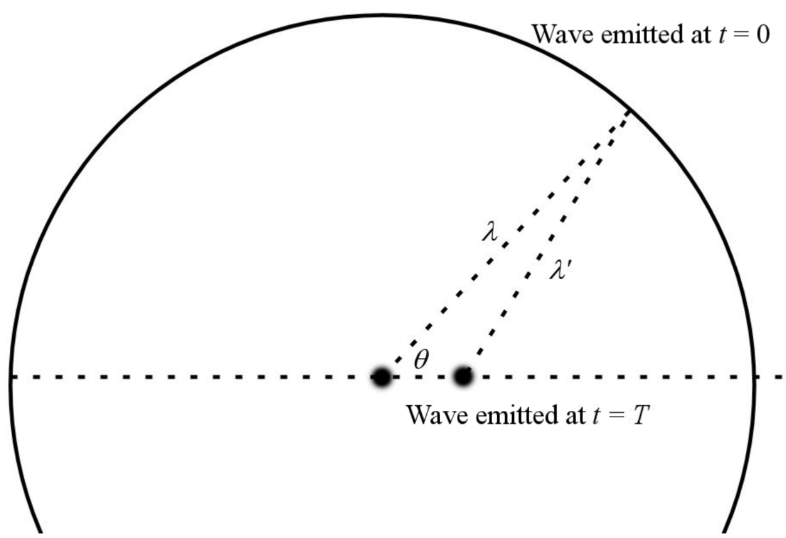
The intensity of this wave in air is

$$\begin{aligned} I &= \frac{1}{2}\rho v\omega^2 A^2 \\ &= \frac{1}{2}\left(1.3\tfrac{kg}{m^3}\right)\left(330\tfrac{m}{s}\right)\left(2\pi\times 200Hz\right)^2\left(5.4302\times 10^{-8}m\right)^2 \\ &= 9.988\times 10^{-7}\tfrac{W}{m^2} \end{aligned}$$

In decibels, this intensity is

$$\begin{aligned} \beta &= 10dB\log\frac{I}{10^{-12}\tfrac{W}{m^2}} \\ &= 10dB\log\frac{9.988\times 10^{-7}\tfrac{W}{m^2}}{10^{-12}\tfrac{W}{m^2}} \\ &= 59.99dB \end{aligned}$$

29. To find the frequency, the wavelength must be found. Let's take the following situation: the source has emitted a crest at $t = 0$. At time T later, the source is again emitting a crest because it emits a crest at every period. To know the wavelength λ', the distance between the crest must be found. This distance is the distance between the crest emitted at $t = 0$ and the source when it is emitting the next crest to time T.

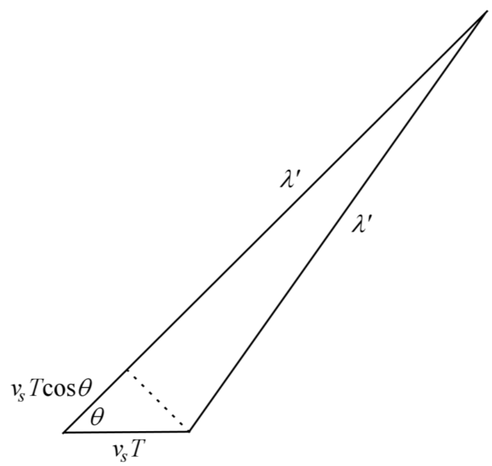


The wavelength without Doppler effect corresponds to the radius of the circle, which is the distance travelled by the wave from the position of the source at $t = 0$. This distance is

$$\lambda = vT$$

Between $t = 0$ and $t = T$, the source, which is travelling at speed v_s, has moved by a distance equal to

$$D_{source} = v_s T$$



To find the new wavelength, the line λ will be separated into two parts with a dotted line arriving perpendicularly, as shown in the figure to the left. Since the speed of the source is much smaller than the velocity of the wave, the distance v_sT is much smaller than λ. The part between the dotted line and the observer is thus almost equal to λ'. For the other part, its length can be found to be

$$v_s T\cos\theta$$

With a little trigonometry. Therefore,

$$\lambda' = \lambda - v_s T\cos\theta$$

$$\lambda' = vt - v_s T\cos\theta$$

$$\lambda' = vT\left(1 - \frac{v_s\cos\theta}{v}\right)$$

$$\lambda' = \lambda\left(1 - \frac{v_s\cos\theta}{v}\right)$$

Therefore, the received frequency is

$$f' = \frac{v}{\lambda'}$$

$$f' = \frac{v}{vT\left(1 - \frac{v_s \cos\theta}{v}\right)}$$

$$f' = \frac{1}{T\left(1 - \frac{v_s \cos\theta}{v}\right)}$$

$$f' = \frac{f}{1 - \frac{v_s \cos\theta}{v}}$$

$$f' = f\frac{v}{v - v_s \cos\theta}$$

Thus the frequency is

$$\begin{aligned} f' &= 500Hz\frac{340\tfrac{m}{s}}{340\tfrac{m}{s} - 10\tfrac{m}{s}\cos 50°} \\ &= 509,6Hz \end{aligned}$$

30. a) With a 10 dB intensity, the intensity *I* is

$$\beta = 10dB\log\frac{I}{10^{-12}\tfrac{W}{m^2}}$$

$$10dB = 10dB\log\frac{I}{10^{-12}\tfrac{W}{m^2}}$$

$$I = 10^{-11}\tfrac{W}{m^2}$$

Therefore, the distance is

$$I = \frac{P}{4\pi r^2}$$

$$10^{-11}\tfrac{W}{m^2} = \frac{20,000W}{4\pi r^2}$$

$$r = 12,616km$$

This answer seems much too high...This noise would be heard on almost half of the surface of the Earth. Yet, this is sound source 10 times less powerful than a jet taking off. It’s normal to get such a high distance since the fact that air absorbs sound was ignored.

b) If the air absorbs the sound at the rate of 7 dB/km, then the number of decibels will be

$$\beta' = \beta - 0,007 \tfrac{dB}{m} r$$

where β is the intensity that we would have without absorption. With a source at a distance *r*, this intensity without absorption is

$$\beta = 10dB \log \frac{\frac{P}{4\pi r^2}}{10^{-12} \frac{W}{m^2}}$$

$$\beta = 10dB \log \frac{P}{4\pi \times 10^{-12} \frac{W}{m^2} r^2}$$

$$\beta = 10dB \log \frac{P}{4\pi \times 10^{-12} \frac{W}{m^2} (1m)^2} - 10dB \log \left(\frac{r}{1m}\right)^2$$

$$\beta = 10dB \log \frac{20,000W}{4\pi \times 10^{-12} \frac{W}{m^2}} - 20dB \log r$$

$$\beta = 152.02dB - 20dB \log \left(\frac{r}{1m}\right)$$

Thus, the intensity with absorption is

$$\beta' = 152.02dB - 20dB \log \left(\frac{r}{1m}\right) - 0.007 \tfrac{dB}{m} \cdot r$$

To hear a 10 dB sound, the equation gives

$$10dB = 152.02dB - 20dB \log \left(\frac{r}{1m}\right) - 0.007 \tfrac{dB}{m} \cdot r$$

$$1 = 15.202 - 2\log \left(\frac{r}{1m}\right) - 0.0007m^{-1} \cdot r$$

$$0.0007m^{-1} \cdot r = 15.202 - 1 - 2\log \left(\frac{r}{1m}\right)$$

$$0.0007m^{-1} \cdot r = 14.202 - 2\log \left(\frac{r}{1m}\right)$$

$$r = 20,288m - \tfrac{2m}{0.0007} \log \left(\frac{r}{1m}\right)$$

It only remains to solve for r, but it's not easy. Of course, Maple can be used to solve. An iterative method can also be used. In this method, a value of r is taken at random and r is calculated with the formula. The value obtained is then used to calculate r again with the formula. This new value is then used again to determine r and so on until the value does not change anymore. If this happens, the answer is obtained. Let's try it here by using 20 km as the starting value (the equation indicates that r is smaller than 20,288 m). Then, the results are

1st iteration

$$\begin{aligned} r &= 20,288m - \tfrac{2m}{0.0007}\log(20,000) \\ &= 8000m \end{aligned}$$

2nd iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(8000) \\ &= 9137m \end{aligned}$$

3rd iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(9137) \\ &= 8972m \end{aligned}$$

4th iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(8972) \\ &= 8994m \end{aligned}$$

5th iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(8994) \\ &= 8991m \end{aligned}$$

6th iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(8991) \\ &= 8992m \end{aligned}$$

7th iteration

$$\begin{aligned} r &= 20\,288m - \tfrac{2m}{0.0007}\log(8992) \\ &= 8992m \end{aligned}$$

Et voilà! The distance is 8992 m.