Chapter 2 Solutions

1. a) The period is

$$T = \frac{1}{f} = \frac{1}{400Hz} = 0.0025s$$

b) We have

$$v = \lambda f$$
$$350\tfrac{m}{s} = \lambda \cdot 400Hz$$
$$\lambda = 0.875m$$

2. The wave speed is

$$v = \frac{\Delta x}{\Delta t} = \frac{30m}{4s} = 7.5\tfrac{m}{s}$$

The wave period is

$$T = \frac{4s}{20 \text{ oscillations}} = 0.2s$$

The frequency is therefore

$$f = \frac{1}{T} = \frac{1}{0.2s} = 5Hz$$

The wavelength is found with

$$v = \lambda f$$
$$7.5\tfrac{m}{s} = \lambda \cdot 5Hz$$
$$\lambda = 1.5m$$

3. a) According to the figure, the wavelength is 20 cm. The frequency is then found with

$$\begin{aligned} v &= \lambda f \\ 40\tfrac{m}{s} &= 0.2m \cdot f \\ f &= 200Hz \end{aligned}$$

b) As the rope makes a harmonic oscillation, its maximum speed is

$$v_{\max} = \omega A$$

With the figure, we note that the amplitude is 6 cm. So the maximum speed is

$$\begin{aligned} v_{\max} &= \left(2\pi f\right) A \\ &= \left(2\pi \cdot 200Hz\right) \cdot 0.06m \\ &= 75.4\tfrac{m}{s} \end{aligned}$$

4. a) Since there a + in front of ωt, the wave is travelling towards the negative x-axis.

b) In the equation, we have k = 10 rad/m. the wavelength is thus

$$\begin{aligned} k &= \frac{2\pi}{\lambda} \\ 10\tfrac{rad}{m} &= \frac{2\pi}{\lambda} \\ \lambda &= 0.6283m \end{aligned}$$

c) The speed is

$$\begin{aligned} v &= \frac{\omega}{k} \\ &= \frac{200\tfrac{rad}{s}}{10\tfrac{rad}{m}} \\ &= 20\tfrac{m}{s} \end{aligned}$$

d) The formula for the velocity of the rope is found by deriving the position formula. The formula is then

$$\begin{aligned} v_y &= \frac{\partial y}{\partial t} \\ &= \frac{\partial}{\partial t}\left(0.2m\sin\left(10\tfrac{rad}{m}\cdot x + 200\tfrac{rad}{s}\cdot t + \frac{\pi}{4}\right)\right) \\ &= 0.2m\cdot 200\tfrac{rad}{s}\cos\left(10\tfrac{rad}{m}\cdot x + 200\tfrac{rad}{s}\cdot t + \frac{\pi}{4}\right) \\ &= 40\tfrac{m}{s}\cos\left(10\tfrac{rad}{m}\cdot x + 200\tfrac{rad}{s}\cdot t + \frac{\pi}{4}\right) \end{aligned}$$

At x = 1 m and t = 1 s, the velocity of the rope is then

$$\begin{aligned} v_y &= 40\tfrac{m}{s}\cos\left(10\tfrac{rad}{m}\cdot 1m + 200\tfrac{rad}{s}\cdot 1s + \frac{\pi}{4}\right) \\ &= 40\tfrac{m}{s}\cos\left(210 + \frac{\pi}{4}\right) \\ &= -38.23\tfrac{m}{s} \end{aligned}$$

5. ω is

$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{0.1s} = 20\pi\tfrac{rad}{s}$$

k is

$$\begin{aligned} v &= \frac{\omega}{k} \\ 50\tfrac{m}{s} &= \frac{20\pi\tfrac{rad}{s}}{k} \\ k &= \frac{2\pi}{5}\tfrac{rad}{m} \end{aligned}$$

The amplitude is

$$\begin{aligned} A^2 &= y^2 + \left(\frac{v_y}{\omega}\right)^2 \\ &= (0.02m)^2 + \left(\frac{-1\frac{m}{s}}{20\pi\frac{rad}{s}}\right)^2 \\ &= 6.533\times 10^{-4} m^2 \\ A &= 0.02556m \end{aligned}$$

and the phase constant is

$$\tan(kx - \omega t + \phi) = \frac{\omega y}{-v_y}$$

$$\tan(0+0+\phi) = \frac{20\pi\frac{rad}{s}\cdot 0.02m}{1\frac{m}{s}}$$

$$\phi = 0.8986$$

The equation is therefore

$$y = 0.02556m \cdot \sin\left(\tfrac{2\pi}{5}\tfrac{rad}{m}\cdot x - 20\pi\cdot t + 0.8986\right)$$

6. Since the waves are all going at the same speed on a rope, the speed is also 30 m/s.

7. If the wave goes from one end to the other in 0.05 s, its speed is

$$v = \frac{\Delta x}{\Delta t} = \frac{2m}{0.05s} = 40\tfrac{m}{s}$$

Then

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$40\tfrac{m}{s} = \sqrt{\frac{200N}{\mu}}$$

$$\mu = 0.125\tfrac{kg}{m}$$

The mass is therefore

$$\mu = \frac{mass}{length}$$
$$0.125\tfrac{kg}{m} = \frac{mass}{2m}$$
$$mass = 0.25kg$$

8. The linear density is found with

$$v = \sqrt{\frac{F_T}{\mu}}$$
$$40\tfrac{m}{s} = \sqrt{\frac{50N}{\mu}}$$
$$\mu = 0,03125\tfrac{kg}{m}$$

If the tension is 80 N, then the speed is

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ &= \sqrt{\frac{80N}{0.03125\tfrac{kg}{m}}} \\ &= 50.6\tfrac{m}{s} \end{aligned}$$

9. a) The velocity of the wave is

$$v = \frac{\omega}{k} = \frac{200\tfrac{rad}{s}}{10\tfrac{rad}{m}} = 20\tfrac{m}{s}$$

Therefore

$$v = \sqrt{\frac{F_T}{\mu}}$$
$$20\tfrac{m}{s} = \sqrt{\frac{50N}{\mu}}$$
$$\mu = 0.125\tfrac{kg}{m}$$

b) The maximum speed of the rope is

$$\begin{aligned} v_{\max} &= \omega A \\ &= 200\tfrac{rad}{s}\cdot 0.2m \\ &= 40\tfrac{m}{s} \end{aligned}$$

10. a) The wave frequency is

$$\begin{aligned} v &= \lambda f \\ 50\tfrac{m}{s} &= 0.4m\cdot f \\ f &= 125Hz \end{aligned}$$

The energy is therefore

$$\begin{aligned} E &= \frac{1}{2}\mu D\omega^2 A^2 \\ &= \frac{1}{2}0.025\tfrac{kg}{m}\cdot 10m\cdot\left(2\pi\cdot 125Hz\right)^2\left(0.002m\right)^2 \\ &= 0.3084J \end{aligned}$$

b) The power is

$$\begin{aligned} P &= \frac{1}{2}\mu v\omega^2 A^2 \\ &= \frac{1}{2}0.025\tfrac{kg}{m}\cdot 50\tfrac{m}{s}\cdot\left(2\pi\cdot 125Hz\right)^2\left(0.002m\right)^2 \\ &= 1.542W \end{aligned}$$

11. The speed of the wave is

$$\begin{aligned} v &= \lambda f \\ &= 0.125m\cdot 200Hz \\ &= 25\tfrac{m}{s} \end{aligned}$$

The linear density of the rope is

$$v = \sqrt{\frac{F_T}{\mu}}$$
$$25\tfrac{m}{s} = \sqrt{\frac{80N}{\mu}}$$
$$\mu = 0.128\tfrac{kg}{m}$$

The amplitude is then found with the power formula.

$$P = \frac{1}{2}\mu v\omega^2 A^2$$
$$20W = \frac{1}{2}0.128\tfrac{kg}{m}\cdot 25\tfrac{m}{s}\cdot\left(2\pi\cdot 200Hz\right)^2 A^2$$
$$A = 0.002813m$$

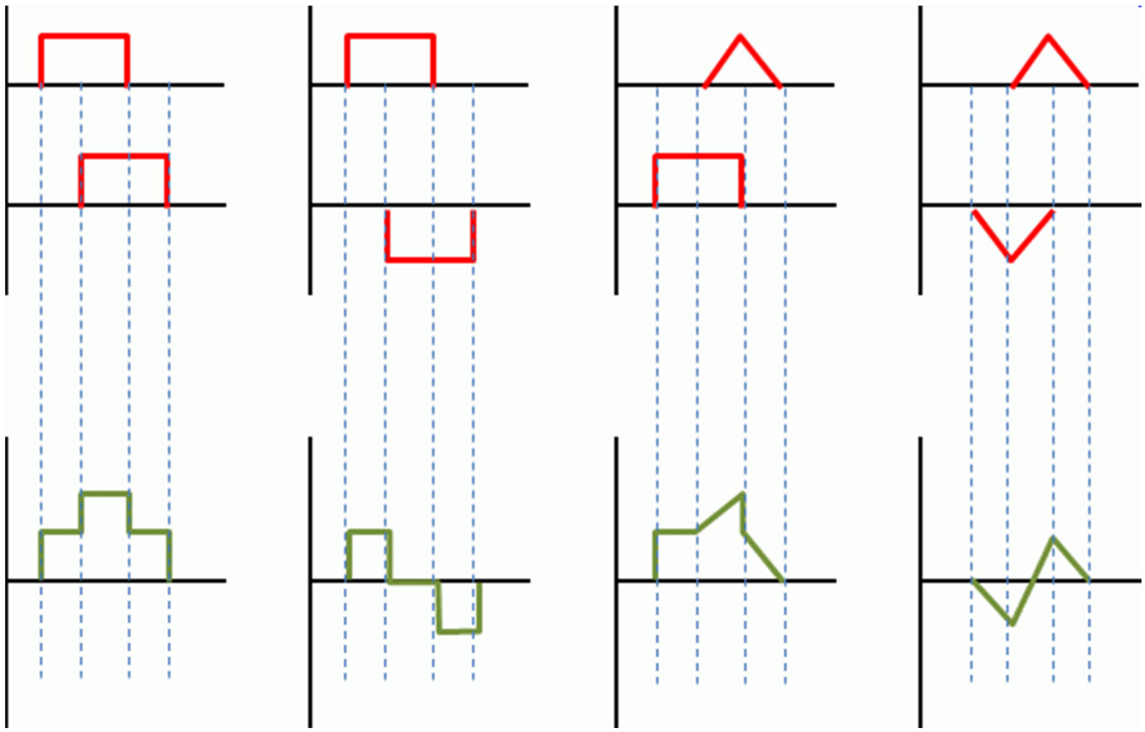
12. The wave speed is

$$\begin{aligned} v &= \frac{\omega}{k} \\ &= \frac{50\pi\tfrac{rad}{s}}{10\pi\tfrac{rad}{m}} \\ &= 5\tfrac{m}{s} \end{aligned}$$

The power is thus

$$\begin{aligned} P &= \frac{1}{2}\mu v\omega^2 A^2 \\ &= \frac{1}{2}0.05\tfrac{kg}{m}\cdot 5\tfrac{m}{s}\cdot\left(50\pi\tfrac{rad}{s}\right)^2\left(0.02m\right)^2 \\ &= 1.234W \end{aligned}$$

13.



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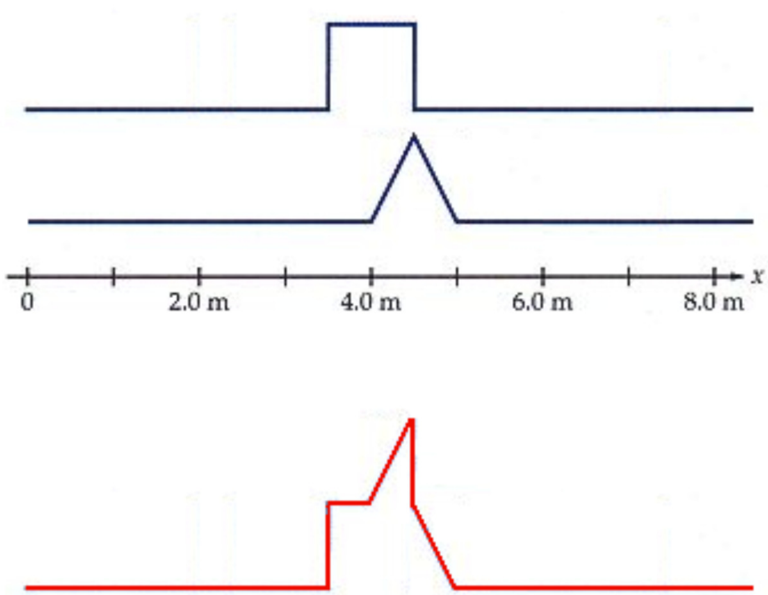

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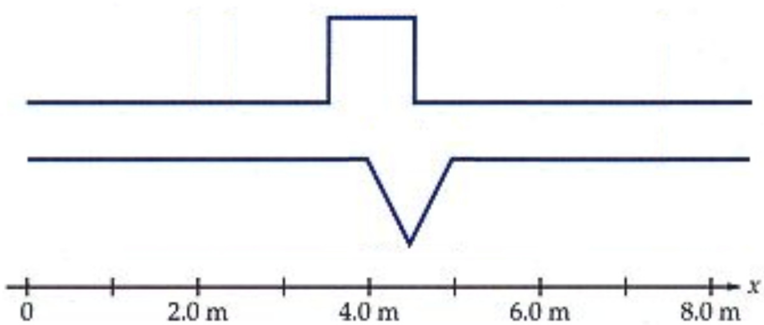


16. a) The tension in the rope is equal to the weight of the block. This weight is

$$F_T = mg = 2kg \cdot 9.8\tfrac{N}{kg} = 19.6N$$

The wave speed in the aluminum wire is therefore

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ &= \sqrt{\frac{19.6N}{0.01\frac{kg}{m}}} \\ &= 44.27\tfrac{m}{s} \end{aligned}$$

b) The frequency is

$$\begin{aligned} v &= \lambda f \\ 44.27\tfrac{m}{s} &= 0.2m \cdot f \\ f &= 221.4Hz \end{aligned}$$

c) The wave speed in the steel wire is

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ &= \sqrt{\frac{19.6N}{0.025\frac{kg}{m}}} \\ &= 28\tfrac{m}{s} \end{aligned}$$

d) As the frequency does not change from one medium to another, it is still 221.4 Hz.

e) The wavelength is

$$\begin{aligned} v &= \lambda f \\ 28\tfrac{m}{s} &= \lambda \cdot 221.4Hz \\ \lambda &= 0.1265m \end{aligned}$$

f) The impedance is

$$\begin{aligned} Z_1 &= \sqrt{F_T \mu} \\ &= \sqrt{19.6N \cdot 0.01\tfrac{kg}{m}} \\ &= 0.4427\tfrac{kg}{s} \end{aligned}$$

g) The impedance is

$$\begin{aligned} Z_2 &= \sqrt{F_T \mu} \\ &= \sqrt{19.6N \cdot 0.025\tfrac{kg}{m}} \\ &= 0.7\tfrac{kg}{s} \end{aligned}$$

h) The amplitude of the reflected wave is

$$\begin{aligned} A_R &= \frac{Z_1 - Z_2}{Z_1 + Z_2} A \\ &= \frac{0.4427\frac{kg}{s} - 0.7\frac{kg}{s}}{0.4427\frac{kg}{s} + 0.7\frac{kg}{s}} 5mm \\ &= -1.126mm \end{aligned}$$

It is therefore an inverted wave with an amplitude of 1.126 mm.

i) The amplitude of the transmitted wave is

$$\begin{aligned}A_T &= \frac{2Z_1}{Z_1+Z_2}A\\ &= \frac{2\cdot 0.4427\frac{kg}{s}}{0.4427\frac{kg}{s}+0.7\frac{kg}{s}}5mm\\ &= 3.874mm\end{aligned}$$

j) The power of the incident wave is

$$\begin{aligned}P &= \frac{1}{2}\mu v\omega^2 A^2\\ &= \frac{1}{2}Z_1\omega^2 A^2\end{aligned}$$

The power of the transmitted wave is

$$\begin{aligned}P_T &= \frac{1}{2}\mu v\omega^2 A^2\\ &= \frac{1}{2}Z_2\omega^2 A^2\end{aligned}$$

The ration of the power transmitted and the initial power is

$$\begin{aligned}\frac{P_T}{P} &= \frac{\frac{1}{2}Z_1\omega^2 A_T^2}{\frac{1}{2}Z_2\omega^2 A^2}\\ &= \frac{Z_2 A_T^2}{Z_1 A^2}\\ &= \frac{0.7\frac{kg}{s}(0.003874m)^2}{0.4427\frac{kg}{s}(0.005m)^2}\\ &= 0.949\end{aligned}$$

94,9% of the power is transmitted.

17. a) The transmitted wave is always in the same direction as the original wave. It will be upwards.

b) Since the density of the rope to the right is greater, the reflected wave will be reversed, so downwards.

c) Since the speed is 20 m/s on the rope to the left, the tension is

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$20\tfrac{m}{s} = \sqrt{\frac{F_T}{0.02\tfrac{kg}{m}}}$$

$$F_T = 8N$$

The speed of the wave on the rope to the right is then

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{8N}{0.05\tfrac{kg}{m}}} = 12.65\tfrac{m}{s}$$

d) The reflected wave being on the rope to the left, its speed is the same as the initial wave, so 20 m/s.

18. The power of a wave is given by

$$P = \frac{1}{2}\mu v \omega^2 A^2$$

We know that the power of the reflected wave is equal to 50% of the power of the initial wave. This means that

$$\begin{aligned} 0.5 &= \frac{P_R}{P} \\ &= \frac{\frac{1}{2}\mu v \omega^2 A_R^2}{\frac{1}{2}\mu v \omega^2 A^2} \\ &= \left(\frac{A_R}{A}\right)^2 \end{aligned}$$

Since the amplitude of the reflected wave is

$$A_R = \frac{Z_1 - Z_2}{Z_1 + Z_2} A$$

the equation becomes

$$\begin{aligned}0.5 &= \left(\frac{A_R}{A}\right)^2 \\ &= \left(\frac{\frac{Z_1-Z_2}{Z_1+Z_2}A}{A}\right)^2 \\ &= \left(\frac{Z_1-Z_2}{Z_1+Z_2}\right)^2 \\ &= \left(\frac{1-\frac{Z_2}{Z_1}}{1+\frac{Z_2}{Z_1}}\right)^2\end{aligned}$$

Setting $Z_2/Z_1 = x$ (which is what we're looking for), we arrive at

$$0.5 = \left(\frac{1-x}{1+x}\right)^2$$

It only remains to solve for x.

$$\pm\sqrt{0.5} = \frac{1-x}{1+x}$$

To simplify, let's set $C = \pm\sqrt{0.5}$. Therefore,

$$\begin{aligned}C &= \frac{1-x}{1+x} \\ C(1+x) &= 1-x \\ C+Cx &= 1-x \\ x+Cx &= 1-C \\ x(1+C) &= 1-C \\ x &= \frac{1-C}{1+C}\end{aligned}$$

If $C = \sqrt{0,5}$, the result is

$$x = \frac{1-\sqrt{0.5}}{1+\sqrt{0.5}} = 0.1716$$

This cannot be the right answer because the impedance of the second string is greater than the impedance of the first rope (this means that $Z_2/Z_1 > 1$)

If $C = -\sqrt{0.5}$, the result is

$$x = \frac{1 - -\sqrt{0.5}}{1 + -\sqrt{0.5}} = 5.828$$

This is the right answer.

19. a) Since $\omega = 200\pi$ rad/s, the frequency is

$$f = \frac{\omega}{2\pi} = \frac{200\pi \frac{rad}{s}}{2\pi} = 100Hz$$

b) Since $k = 40\pi$ rad/m, the wavelength is

$$\begin{aligned} k &= \frac{2\pi}{\lambda} \\ 40\pi \tfrac{rad}{m} &= \frac{2\pi}{\lambda} \\ \lambda &= 0.05m \end{aligned}$$

c) The speed is

$$v = \lambda f = 0.05m \cdot 100Hz = 5 \tfrac{m}{s}$$

d) The formula for the velocity of the rope is obtained by deriving the formula for the position with respect to time

$$\begin{aligned} v_y &= \frac{\partial y}{\partial t} \\ &= \frac{\partial}{\partial t}\left(0.06m \cdot \sin\left(40\pi \tfrac{rad}{m} \cdot x\right)\cos\left(200\pi \tfrac{rad}{s} \cdot t\right)\right) \\ &= -0.06m \cdot 200\pi \tfrac{rad}{s} \sin\left(40\pi \tfrac{rad}{m} \cdot x\right)\sin\left(200\pi \tfrac{rad}{s} \cdot t\right) \\ &= -12\pi \tfrac{m}{s} \cdot \sin\left(40\pi \tfrac{rad}{m} \cdot x\right)\sin\left(200\pi \tfrac{rad}{s} \cdot t\right) \end{aligned}$$

At $x = 0.02$ m and $t = 0.022$ s the velocity is

$$\begin{aligned} v_y &= -12\pi \tfrac{m}{s} \cdot \sin\left(40\pi \tfrac{rad}{m} \cdot 0,02m\right) \sin\left(200\pi \tfrac{rad}{s} \cdot 0,022s\right) \\ &= -12\pi \tfrac{m}{s} \cdot \sin\left(\tfrac{4\pi}{5}\right) \sin\left(\tfrac{22\pi}{5}\right) \\ &= -21.07 \tfrac{m}{s} \end{aligned}$$

e) The amplitude is

$$\begin{aligned} A_{tot} &= \left|2A \sin kx\right| \\ &= \left|6cm \cdot \sin\left(40\pi \tfrac{rad}{m} \cdot x\right)\right| \end{aligned}$$

At x = 5 cm, we have

$$\begin{aligned} A_{tot} &= \left|6cm \cdot \sin\left(40\pi \tfrac{rad}{m} \cdot 0.005m\right)\right| \\ &= 3.527cm \end{aligned}$$

20. The equation is

$$y_{tot} = 4cm \cdot \sin\left(10\pi \tfrac{rad}{m} \cdot x\right) \cos\left(50\pi \tfrac{rad}{s} \cdot t\right)$$

21. a) With a 20 cm wavelength, k is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.2m} = 10\pi \tfrac{rad}{m}$$

With a 0.05 s period, ω is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.05s} = 40\pi \tfrac{rad}{s}$$

Therefore, the equation is

$$y_{tot} = 0.1m \cdot \sin\left(10\pi \tfrac{rad}{m} \cdot x\right) \cos\left(40\pi \tfrac{rad}{s} \cdot t\right)$$

b) The distance between the nodes is equal to half of the wavelength. It is thus 10 cm.

c) Since there is a node at x = 0, the amplitude 1 cm from the nodes is the amplitude at x = 1 cm. This amplitude is

$$\begin{aligned} A_{tot} &= \left|2A\sin kx\right| \\ &= \left|0.1m\cdot\sin\left(10\pi\tfrac{rad}{m}\cdot x\right)\right| \\ &= \left|0.1m\cdot\sin\left(10\pi\tfrac{rad}{m}\cdot 0.01m\right)\right| \\ &= 0.03090m \end{aligned}$$

22. Since the distance between the nodes is 20 cm, the wavelength is 40 cm. With a 40 cm wavelength, *k* is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.4m} = 5\pi\tfrac{rad}{m}$$

The wave speed is

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{12N}{0.03\frac{kg}{m}}} = 20\tfrac{m}{s}$$

Therefore, ω is

$$\begin{aligned} v &= \frac{\omega}{k} \\ 20\tfrac{m}{s} &= \frac{\omega}{5\pi\frac{rad}{m}} \\ \omega &= 100\pi\tfrac{rad}{s} \end{aligned}$$

This means that

$$y_{tot} = 0.005m\cdot\sin\left(5\pi\tfrac{rad}{m}\cdot x\right)\cos\left(100\pi\tfrac{rad}{s}\cdot t\right)$$

23. We have

$$\begin{aligned} f_n &= \frac{n}{2L}\sqrt{\frac{F_T}{\mu}} \\ 400Hz &= \frac{4}{2\cdot 0.6m}\sqrt{\frac{F_T}{0.02\frac{kg}{m}}} \\ F_T &= 288N \end{aligned}$$

24. We have

$$f_n = \frac{nv}{2L}$$
$$400Hz = \frac{2v}{2\cdot 2m}$$
$$v = 800\tfrac{m}{s}$$

25. Since $k = 20\pi$ rad/m, the wavelength is

$$k = \frac{2\pi}{\lambda}$$
$$20\pi\tfrac{rad}{m} = \frac{2\pi}{\lambda}$$
$$\lambda = 0.1m$$

At the third harmonic, we have

$$\lambda = \frac{2L}{n}$$
$$0{,}1m = \frac{2L}{3}$$
$$L = 0{,}15m$$

26. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$50Hz = \frac{1}{2\cdot 0.5m}\sqrt{\frac{350N}{\mu}}$$
$$\mu = 0.14\tfrac{kg}{m}$$

27. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$200Hz = \frac{1}{2L}\sqrt{\frac{100N}{\mu}}$$

and

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$500Hz = \frac{5}{2L}\sqrt{\frac{F_T'}{\mu}}$$

By dividing the second equation by the first equation, we obtain

$$\frac{500Hz}{200Hz} = \frac{\frac{5}{2L}\sqrt{\frac{F_T'}{\mu}}}{\frac{1}{2L}\sqrt{\frac{100N}{\mu}}}$$
$$2.5 = \frac{5\sqrt{F_T'}}{\sqrt{100N}}$$
$$F_T' = 25N$$

28. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$160Hz = \frac{4}{2\cdot 1.2m}\sqrt{\frac{F_T}{0.036\frac{kg}{m}}}$$
$$F_T = 331.8N$$

The mass is then

$$F_T = mg$$
$$331.8N = m\cdot 9.8\tfrac{N}{kg}$$
$$m = 33.85kg$$

29. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$50Hz = \frac{1}{2\cdot 1m}\sqrt{\frac{F_T}{\mu}}$$

and

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$400Hz = \frac{2}{2L'}\sqrt{\frac{F_T}{\mu}}$$

By dividing the second equation by the first equation, we obtain

$$\frac{400Hz}{50Hz} = \frac{\frac{2}{2L'}\sqrt{\frac{F_T}{\mu}}}{\frac{1}{2\cdot 1m}\sqrt{\frac{F_T}{\mu}}}$$
$$8 = \frac{\frac{2}{L'}}{\frac{1}{1m}}$$
$$8 = \frac{2}{L'}\cdot\frac{1m}{1}$$
$$L' = 0.25m$$

30. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$
$$f_{1A} = \frac{1}{2\cdot 1m}\sqrt{\frac{100N}{\mu}}$$

and

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$

$$f_{1B} = \frac{1}{2\cdot 0.25m}\sqrt{\frac{25N}{\mu}}$$

The ratio of the frequencies is

$$\frac{f_{1A}}{f_{1B}} = \frac{\frac{1}{2\cdot 1m}\sqrt{\frac{100N}{\mu}}}{\frac{1}{2\cdot 0.25m}\sqrt{\frac{25N}{\mu}}}$$

$$\frac{f_{1A}}{f_{1B}} = \frac{\frac{1}{1m}\sqrt{100N}}{\frac{1}{0.25m}\sqrt{25N}}$$

$$\frac{f_{1A}}{f_{1B}} = \frac{0.25m\cdot\sqrt{100N}}{1m\cdot\sqrt{25N}}$$

$$\frac{f_{1A}}{f_{1B}} = 0.5$$

31. We have

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$

$$360Hz = \frac{1}{2\cdot L}\sqrt{\frac{F_T}{\mu}}$$

and

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}}$$

$$f_{1B} = \frac{1}{2\cdot(0.4L)}\sqrt{\frac{F_T}{\mu}}$$

By dividing the second equation by the first equation, we obtain

$$\frac{f_{1B}}{360Hz}=\frac{\frac{1}{2\cdot(0.4L)}\sqrt{\frac{F_T}{\mu}}}{\frac{1}{2\cdot L}\sqrt{\frac{F_T}{\mu}}}$$

$$\frac{f_{1B}}{360Hz}=\frac{1}{0.4}$$

$$f_{1B}=900Hz$$

32. We have

$$f_n=520Hz$$

$$nf_1=520Hz$$

At the following harmonic, the integer in front of f_1 is $n + 1$. Then

$$(n+1)f_1=650Hz$$

Using the first equation, we obtain

$$(n+1)f_1=650Hz$$

$$nf_1+f_1=650Hz$$

$$520Hz+f_1=650Hz$$

$$f_1=130Hz$$

33. The speed of the wave depends on the tension. Let's find the tension as a function of the position on the string. To ensure that there is an equilibrium of force, this tension must be equal to the weight of the rope under the point considered. A y-axis pointing downwards with an origin $y = 0$ at the top of the rope will be used. At the position y, the length of the rope under this position is

$$l=L-y$$

where L is the total length of the rope. The mass of this part of rope is

$$m=\mu(L-y)$$

This means that the tension of the rope at y is

$$F_T=\mu g(L-y)$$

Then, the speed of the wave at y is

$$v=\sqrt{\frac{F_T}{\mu}}=\sqrt{g\left(L-y\right)}$$

To travel the small distance dy, the time is

$$dt=\frac{dy}{v}=\frac{dy}{\sqrt{g\left(L-y\right)}}$$

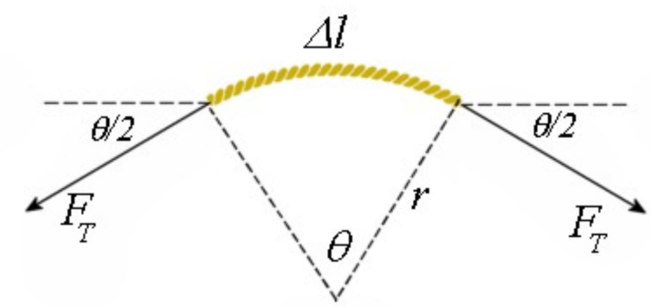
If all these times are added now, we obtain

$$\begin{aligned} t&=\int_0^L\frac{dy}{\sqrt{g\left(L-y\right)}}\\ &=\frac{1}{\sqrt{g}}\int_0^L\frac{dy}{\sqrt{\left(L-y\right)}}\end{aligned}$$

Setting $u = L - y$, the integral becomes

$$\begin{aligned} t&=\frac{1}{\sqrt{g}}\int_L^0\frac{-du}{\sqrt{u}}\\ &=\frac{-1}{\sqrt{g}}\int_L^0 u^{-1/2}du\\ &=\frac{-1}{\sqrt{g}}\left[\frac{u^{1/2}}{1/2}\right]_L^0\\ &=\frac{-2}{\sqrt{g}}\left[0^{1/2}-L^{1/2}\right]\\ &=2\sqrt{\frac{L}{g}}\\ &=2\sqrt{\frac{2m}{9.8\frac{m}{s^2}}}\\ &=0.9035s\end{aligned}$$

34. The speed of the wave depends on the tension. Therefore, the tension in this ring must be found. Let's take a small piece of rope and examine the forces on this piece. (The angle θ is small on the figure.)



Since this piece makes a circular motion, the sum of y-components of the force acting on this piece is (using a y-axis directed upwards)

$$\sum F_y = ma_y$$
$$\rightarrow F_T \sin\left(-\frac{\theta}{2}\right) + F_T \sin\left(180° + \frac{\theta}{2}\right) = -m\omega^2 r$$

Since sin -x = -sin x and sin (180°+x) = -sin x, it becomes

$$-F_T \sin\left(\frac{\theta}{2}\right) - F_T \sin\left(\frac{\theta}{2}\right) = -m\omega^2 r$$
$$2F_T \sin\left(\frac{\theta}{2}\right) = m\omega^2 r$$

Since the angle is small, sin $x = x$. (This means that we are now working with angles in radians.)

$$2F_T \frac{\theta}{2} = m\omega^2 r$$
$$F_T \theta = m\omega^2 r$$

The mass of the piece depends on the angle. Since the density is μ, the mass of the piece is

$$m = \mu \Delta l$$
$$= \mu r \theta$$

The force equation then becomes

$$F_T\theta = m\omega^2 r$$
$$F_T\theta = \mu r\theta\omega^2 r$$
$$F_T = \mu r^2\omega^2$$

Thus, the speed of the wave is

$$\begin{aligned} v &= \sqrt{\frac{F_T}{\mu}} \\ &= \sqrt{\frac{\mu r^2\omega^2}{\mu}} \\ &= r\omega \end{aligned}$$

With the values, we arrive at

$$\begin{aligned} v &= 0.25m \cdot 2\tfrac{rad}{s} \\ &= 0.5\tfrac{m}{s} \end{aligned}$$

Note that this speed ($r\omega$) is also the speed of the rope. Thus, the wave, if it goes in the opposite direction to the rope, always remains at the same place!