Chapter 12 Solutions

1. The wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$
$$= \frac{6.626\times10^{-34}\,Js}{1.6726\times10^{-27}\,kg\cdot10^{4}\,\frac{m}{s}}$$
$$= 3.96\times10^{-11}\,m = 0.0396\,nm$$

2. As the speed is close to the speed of light, the relativistic momentum formula must be used. The wavelength is, therefore,

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mv}$$
$$= \frac{6.626\times10^{-34}\,Js}{\frac{1}{\sqrt{1-\left(\frac{2\times10^{8}\,\frac{m}{s}}{3\times10^{8}\,\frac{m}{s}}\right)^{2}}}1.6726\times10^{-27}\,kg\cdot2\times10^{8}\,\frac{m}{s}}$$
$$= 1.476\times10^{-15}\,m$$

3. With a 10 eV kinetic energy, the speed of the electron is

$$E_k = \frac{1}{2}mv^2$$
$$10\cdot1.602\times10^{-19}\,J = \frac{1}{2}9.1094kg\cdot v^2$$
$$v = 1.875\times10^{6}\,\frac{m}{s}$$

Thus, the wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$
$$= \frac{6.626\times 10^{-34} Js}{9.1094\times 10^{-31} kg \cdot 1.875\times 10^{6} \frac{m}{s}}$$
$$= 3.879\times 10^{-10} m = 0.3879 nm$$

4. With a 6 eV kinetic energy, the speed of the electron is

$$E_k = \frac{1}{2}mv^2$$
$$6\cdot 1.602\times 10^{-19} J = \frac{1}{2} 9,1094\times 10^{-31} kg \cdot v^2$$
$$v = 1.453\times 10^{6} \tfrac{m}{s}$$

Thus, the wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$
$$= \frac{6.626\times 10^{-34} Js}{9.1094\times 10^{-31} kg \cdot 1.875\times 10^{6} \frac{m}{s}}$$
$$= 5.007\times 10^{-10} m = 0.5007 nm$$

When U increases to 2 eV, the kinetic energy decreases to 4 eV. The speed of the electron is then

$$E_k = \frac{1}{2}mv^2$$
$$4\cdot 1.602\times 10^{-19} J = \frac{1}{2} 9.1094 kg \cdot v^2$$
$$v = 1.186\times 10^{6} \tfrac{m}{s}$$

And the wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$
$$= \frac{6.626\times10^{-34}Js}{9.1094\times10^{-31}kg\cdot1.875\times10^{6}\,\tfrac{m}{s}}$$
$$= 6.132\times10^{-10}m = 0.6132nm$$

The change in wavelength is, therefore,

$$\Delta\lambda = \lambda' - \lambda$$
$$= 0.6132nm - 0.5007nm$$
$$= 0.1125nm$$

5. With a 2 eV kinetic energy, the speed of the electron is

$$E_k = \frac{1}{2}mv^2$$
$$2\cdot1.602\times10^{-19}J = \frac{1}{2}9.1094kg\cdot v^2$$
$$v = 8.3877\times10^{5}\,\tfrac{m}{s}$$

The wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$
$$= \frac{6.626\times10^{-34}Js}{9.1094\times10^{-31}kg\cdot8.388\times10^{5}\,\tfrac{m}{s}}$$
$$= 8.672\times10^{-10}m = 0.8672nm$$

According to the figure, we're looking for the distance between the two order 2 maxima. The angle between this maximum and the central maximum is given by

$$d\sin\theta = m\lambda$$
$$0.1\times10^{-6}m\sin\theta = 2\cdot8.672\times10^{-10}m$$
$$\theta = 0.9938°$$

The distance from the central maximum to the order 2 maximum is, therefore,

$$\tan\theta = \frac{y}{L}$$

$$\tan 0.9938° = \frac{y}{300cm}$$

$$y = 5.204cm$$

The distance between the two order 2 maxima is twice as big. Therefore, it is 10.408 cm.

6. a) The energy of the first level is

$$E_n = \frac{n^2h^2}{8mL^2}$$

$$E_1 = \frac{1^2h^2}{8mL^2}$$

$$= \frac{\left(6.626\times10^{-34}Js\right)^2}{8\cdot1.6749\times10^{-27}kg\cdot\left(10^{-14}m\right)^2}$$

$$= 3.276\times10^{-13}J$$

$$= 2.045MeV$$

The energy of the second level is

$$E_n = \frac{n^2h^2}{8mL^2}$$

$$E_2 = \frac{2^2h^2}{8mL^2}$$

$$= 4\cdot\frac{h^2}{8mL^2}$$

$$= 4\cdot E_1$$

$$= 4\cdot2.045MeV$$

$$= 8.181MeV$$

b) When it goes from level 2 to level 1, the energy of the neutron decreases by

$$\Delta E = E_1 - E_2$$

$$= 2.045MeV - 8.181MeV$$

$$= -6.136MeV$$

Since the neutron loses 6.136 MeV, the energy of the emitted photon must be 6.136 MeV. The wavelength of this photon is

$$E = \frac{1240 eV \cdot nm}{\lambda}$$
$$6.136 \times 10^6 eV = \frac{1240 eV \cdot nm}{\lambda}$$
$$\lambda = 2.021 \times 10^{-4} nm$$

c) At level 1, the wavelength of the neutron is

$$\lambda_n = \frac{2L}{n}$$
$$\lambda_1 = \frac{2L}{1}$$
$$= \frac{2 \cdot 10^{-14} m}{1}$$
$$= 2 \times 10^{-14} m$$

7. The width is found with the formula for the energy of the 4th level.

$$E_n = \frac{n^2 h^2}{8mL^2}$$
$$E_4 = \frac{4^2 h^2}{8mL^2}$$
$$10 \cdot 1.602 \times 10^{-19} J = \frac{16 \cdot \left(6.626 \times 10^{-34} Js\right)^2}{8 \cdot 9.1094 \times 10^{-31} kg \cdot L^2}$$
$$L = 7.757 \times 10^{-10} m = 0.7757 nm$$

8. a) The energy of the first level is

$$\begin{aligned} E_n &= \frac{n^2h^2}{8mL^2} \\ E_1 &= \frac{1^2h^2}{8mL^2} \\ &= \frac{\left(6.626\times10^{-34}Js\right)^2}{8\cdot 9.1094\times10^{-31}kg\cdot\left(2\times10^{-9}m\right)^2} \\ &= 1.506\times10^{-20}J \end{aligned}$$

The energy of the fourth level is

$$\begin{aligned} E_n &= \frac{n^2h^2}{8mL^2} \\ E_4 &= \frac{4^2h^2}{8mL^2} \\ &= 16\cdot\frac{h^2}{8mL^2} \\ &= 16\cdot E_1 \\ &= 16\cdot 1.506\times10^{-20}J \\ &= 2.410\times10^{-19}J \end{aligned}$$

To go from the first level to the fourth level, the electron must gain the following energy.

$$\begin{aligned} \Delta E &= E_4 - E_1 \\ &= 2.410\times10^{-19}J - 1.506\times10^{-20}J \\ &= 2.259\times10^{-19}J \\ &= 1.41eV \end{aligned}$$

Therefore, the photon must have this energy. The wavelength of the photon has to be

$$\begin{aligned} E &= \frac{1240eV\cdot nm}{\lambda} \\ 1.41eV &= \frac{1240eV\cdot nm}{\lambda} \\ \lambda &= 879nm \end{aligned}$$

9. The electron has its lowest speed when it has the smallest energy, so when it is on the first level. The energy of the first level is

$$\begin{aligned} E_n &= \frac{n^2 h^2}{8mL^2} \\ E_1 &= \frac{1^2 h^2}{8mL^2} \\ &= \frac{\left(6.626\times10^{-34} Js\right)^2}{8\cdot 9.1094\times10^{-31} kg\cdot\left(6\times10^{-9} m\right)^2} \\ &= 1.674\times10^{-21} J \end{aligned}$$

Thus, the speed of the electron is

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ 1.674\times10^{-21} J &= \frac{1}{2}9.1094\times10^{-31} kg\cdot v^2 \\ v &= 6.062\times10^4 \tfrac{m}{s} \end{aligned}$$

10. From the energy of the fourth level, the energy of the first level can be found

$$\begin{aligned} E_n &= \frac{n^2 h^2}{8mL^2} \\ E_4 &= \frac{4^2 h^2}{8mL^2} \\ &= 16\cdot\frac{h^2}{8mL^2} \\ &= 16\cdot E_1 \end{aligned}$$

Therefore

$$\begin{aligned} E_4 &= 16\cdot E_1 \\ 24eV &= 16\cdot E_1 \\ E_1 &= 1.5eV \end{aligned}$$

Then, the energy of the third level can be found from the energy of the first level.

$$E_n = \frac{n^2 h^2}{8mL^2}$$
$$E_3 = \frac{4^2 h^2}{8mL^2}$$
$$= 9 \cdot \frac{h^2}{8mL^2}$$
$$= 9 \cdot E_1$$
$$= 9 \cdot 1.5eV$$
$$= 13.5eV$$

11. With a period of 4 x 10^{-15} s, the frequency is

$$f = \frac{1}{T} = \frac{1}{4 \times 10^{-15} s} = 2.5 \times 10^{14} Hz$$

a) The smallest energy is at the level $n = 0$.

$$E_n = \left(n + \tfrac{1}{2}\right) hf$$
$$E_0 = \left(0 + \tfrac{1}{2}\right) hf$$
$$= \frac{1}{2} \cdot 6.626 \times 10^{-34} Js \cdot 2.5 \times 10^{14} Hz$$
$$= 8.283 \times 100 J$$
$$= 0.517 eV$$

b) The energy of the level n =3 is

$$E_n = \left(n + \tfrac{1}{2}\right) hf$$
$$E_3 = \left(3 + \tfrac{1}{2}\right) hf$$
$$= \frac{7}{2} hf$$
$$= 7 \cdot \frac{1}{2} hf$$
$$= 7 \cdot E_0$$
$$= 7 \cdot 0.517 eV$$
$$= 3.619 eV$$

The energy of the level n = 1 is

$$\begin{aligned}E_n &= \left(n+\tfrac{1}{2}\right)hf\\ E_1 &= \left(1+\tfrac{1}{2}\right)hf\\ &= \frac{3}{2}hf\\ &= 3\cdot\frac{1}{2}hf\\ &= 3\cdot E_0\\ &= 3\cdot 0.517eV\\ &= 1.551eV\end{aligned}$$

Therefore, the energy lost when the electron goes from the level n =3 to the level n = 1 is

$$\begin{aligned}\Delta E &= E_1 - E_3\\ &= 1.551eV - 3.619eV\\ &= -2.068eV\end{aligned}$$

A photon with an energy of 2.068 eV is, therefore, emitted. The wavelength of this photon is

$$\begin{aligned}E &= \frac{1240eV\cdot nm}{\lambda}\\ 2.068eV &= \frac{1240eV\cdot nm}{\lambda}\\ \lambda &= 599.6nm\end{aligned}$$

12. The photon energy is

$$\begin{aligned}E &= \frac{1240eV\cdot nm}{\lambda}\\ &= \frac{1240eV\cdot nm}{496nm}\\ &= 2.5eV\end{aligned}$$

This corresponds to the difference in energy between levels 5 and 2. Thus

$$\begin{aligned}\Delta E &= E_5 - E_2 \\ &= \left(5+\tfrac{1}{2}\right)hf - \left(2+\tfrac{1}{2}\right)hf \\ &= 5hf + \tfrac{1}{2}hf - 2hf - \tfrac{1}{2}hf \\ &= 3hf\end{aligned}$$

Therefore

$$\Delta E = 3hf$$
$$2.5 \cdot 1.602\times10^{-19} J = 3 \cdot 6.626\times10^{-34} Js \cdot f$$
$$f = 2.015\times10^{14} Hz$$

The period of oscillation is therefore

$$\begin{aligned}T &= \frac{1}{f} \\ &= \frac{1}{2{,}015\times10^{14} Hz} \\ &= 4.963\times10^{-15} s\end{aligned}$$

13. The uncertainty of the momentum is

$$\begin{aligned}\Delta p &= p_{\max} - p_{\min} \\ &= 2.05\times10^{-23} \tfrac{kgm}{s} - 2\times10^{-23} \tfrac{kgm}{s} \\ &= 5\times10^{-25} \tfrac{kgm}{s}\end{aligned}$$

Therefore, the uncertainty of the position is

$$\Delta x \Delta p = h$$
$$\Delta x \cdot 5\times10^{-25} \tfrac{kgm}{s} = 6.626\times10^{-34} Js$$
$$\Delta x = 1.325\times10^{-9} m = 1.325 nm$$

14. The uncertainty of the energy is

$$\Delta E \Delta t = h$$
$$\Delta E \cdot 10^{-8} \tfrac{kgm}{s} = 6.626\times10^{-34} Js$$
$$\Delta E = 6.626\times10^{=26} J = 4.136\times10^{-7} eV$$

15. In the box, there is a standing wave which is identical to a standing wave on a string. Thus, the amplitude of the wave will be identical to the amplitude standing wave on a string.

$$\psi = 2A\sin kx$$

The wavelength of the first level is

$$\begin{aligned}\lambda_1 &= \frac{2L}{1} \\ &= 2L \\ &= 2\cdot 10nm \\ &= 20nm\end{aligned}$$

Thus, the amplitude of the wave is

$$\psi = 2A\sin kx$$

$$\psi = 2A\sin\left(\frac{2\pi}{\lambda}x\right)$$

$$\psi = 2A\sin\left(\frac{2\pi}{20nm}x\right)$$

First, this wave function must comply with

$$\int_{\substack{\text{all possible}\\ \text{locations}}} \psi^2 dx = 1$$

As the only possible locations are between 0 nm and 10 nm, this equation becomes

$$\int_{0nm}^{10nm} \left(2A\sin\left(\frac{2\pi}{20nm}x\right)\right)^2 dx = 1$$

The value of *A* can then be found with this equation.

$$\int_{0nm}^{10nm} 4A^2 \sin^2\left(\frac{2\pi}{20nm}x\right)dx = 1$$

$$4A^2\left[\frac{x}{2} - \frac{20nm}{8\pi}\sin\left(\frac{4\pi}{20nm}x\right)\right]_{0nm}^{10nm} = 1$$

$$4A^2\left[\frac{10nm}{2} - \frac{20nm}{8\pi}\sin\left(\frac{4\pi}{20nm}10nm\right)\right] - A^2\left[\frac{0nm}{2} - \frac{10nm}{8\pi}\sin\left(\frac{4\pi}{20nm}0nm\right)\right] = 1$$

$$4A^2\left[\frac{10nm}{2} - \frac{20nm}{8\pi}\sin(2\pi)\right] - A^2\left[0 - \frac{10nm}{8\pi}\sin(0)\right] = 1$$

$$4A^2 \cdot 5nm = 1$$

$$A = \frac{1}{2\sqrt{5nm}}$$

Thus, the amplitude of the wave is

$$\psi = 2A\sin\left(\frac{2\pi}{20nm}x\right)$$

$$\psi = \frac{1}{\sqrt{5nm}}\sin\left(\frac{2\pi}{20nm}x\right)$$

Therefore, the probability of finding the particle between x = 0 nm and x = 3 nm is

$$P = \int_{0nm}^{3nm} \frac{1}{5nm}\sin^2\left(\frac{2\pi}{20nm}x\right)dx$$

$$= \frac{1}{5nm}\left[\frac{x}{2} - \frac{20nm}{8\pi}\sin\left(\frac{4\pi}{20nm}x\right)\right]_{0nm}^{3nm}$$

$$= \frac{1}{5nm}\left[\frac{3nm}{2} - \frac{20nm}{8\pi}\sin\left(\frac{4\pi}{20nm}3nm\right)\right] - \frac{1}{5nm}\left[\frac{0nm}{2} - \frac{20nm}{8\pi}\sin\left(\frac{4\pi}{20nm}0nm\right)\right]$$

$$= \frac{1}{5nm}\left[\frac{3nm}{2} - \frac{20nm}{8\pi}\sin(0.6\pi)\right] - \frac{1}{5nm}\left[0 - \frac{20nm}{8\pi}\sin(0)\right]$$

$$= \frac{1}{5nm}\left[\frac{3nm}{2} - \frac{20nm}{8\pi}\sin(1.2\pi)\right]$$

$$= \frac{3}{10} - \frac{1}{2\pi}\sin(0.6\pi)$$

$$= 0.1486$$

Therefore, the probability is 14.86 %

16. The Schrödinger equation with the potential $U=½kx^2$ is

$$\frac{d^2\psi}{dx^2}+\frac{2m}{\hbar^2}\left(E-\frac{1}{2}kx^2\right)\psi=0$$

Let's check whether

$$\psi=Ae^{-Bx^2}$$

is a solution. To check this, the second derivative of this function is needed.

$$\begin{aligned}\frac{d\psi}{dx}&=Ae^{-Bx^2}\left(-2Bx\right)\\&=2ABxe^{-Bx^2}\end{aligned}$$

$$\begin{aligned}\frac{d^2\psi}{dx^2}&=\frac{d}{dx}\left(-2ABxe^{-Bx^2}\right)\\&=-2ABxe^{-Bx^2}\left(-2Bx\right)+-2ABe^{-Bx^2}\\&=4AB^2x^2e^{-Bx^2}-2ABe^{-Bx^2}\end{aligned}$$

Therefore

$$\frac{d^2\psi}{dx^2}+\frac{2m}{\hbar^2}\left(E-\frac{1}{2}kx^2\right)\psi=0$$

$$4AB^2x^2e^{-Bx^2}-2ABe^{-Bx^2}+\frac{2m}{\hbar^2}\left(E-\frac{1}{2}kx^2\right)Ae^{-Bx^2}=0$$

$$4B^2x^2-2B+\frac{2m}{\hbar^2}\left(E-\frac{1}{2}kx^2\right)=0$$

$$\frac{2m}{\hbar^2}\left(E-\frac{1}{2}kx^2\right)=2B-4B^2x^2$$

$$\frac{2m}{\hbar^2}E-\frac{mk}{\hbar^2}x^2=2B-4B^2x^2$$

The function is a solution if both sides are equal. To be equal, the constant terms must be equal and that the x^2 terms must also be equal. Thus

$$\frac{2m}{\hbar^2}E=2B \qquad \text{and} \qquad \frac{mk}{\hbar^2}x^2=4B^2x^2$$

The second equation gives

$$\frac{mk}{\hbar^2}x^2 = 4B^2x^2$$
$$\frac{mk}{4\hbar^2} = B^2$$

Using this value in the first equation, the result is

$$\frac{2m}{\hbar^2}E = 2B$$
$$\frac{2m}{\hbar^2}E = 2\sqrt{\frac{mk}{4\hbar^2}}$$
$$\frac{m}{\hbar^2}E = \frac{\sqrt{mk}}{2\hbar}$$
$$E = \frac{\sqrt{mk}}{2m}\hbar$$
$$E = \frac{1}{2}\sqrt{\frac{k}{m}}\hbar$$

For a harmonic oscillation, we have

$$\omega = \sqrt{\frac{k}{m}}$$

Thus

$$\begin{aligned} E &= \frac{1}{2}\hbar\omega \\ &= \frac{1}{2}\frac{h}{2\pi}2\pi f \\ &= \frac{1}{2}hf \end{aligned}$$