

Chapter 11 Solutions

1. The wavelength of the peak is

$$\begin{aligned}\lambda_{pic} &= \frac{2.898 \times 10^{-3} mK}{T} \\ &= \frac{2.898 \times 10^{-3} mK}{3273 K} \\ &= 885 nm\end{aligned}$$

This corresponds to infrared radiation.

2. The temperature is found with

$$\begin{aligned}\lambda_{pic} &= \frac{2.898 \times 10^{-3} mK}{T} \\ 502 \times 10^{-9} m &= \frac{2.898 \times 10^{-3} mK}{T} \\ T &= 5773 K\end{aligned}$$

3. The power is

$$\begin{aligned}P &= \sigma A (T^4 - T_0^4) \\ &= 5.67 \times 10^{-8} \frac{W}{m^2 K^4} 4\pi (3.2 \times 10^{10} m)^2 ((6015 K)^4 - (0 K)^4) \\ &= 9.55 \times 10^{29} W\end{aligned}$$

This is about 2500 times brighter than the Sun.

4. a) The power is

$$\begin{aligned}
 P &= \sigma A (T^4 - T_0^4) \\
 &= 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4} \cdot 1.8 \text{m}^2 \cdot \left((310 \text{K})^4 - (293 \text{K})^4 \right) \\
 &= 190.4 \text{W}
 \end{aligned}$$

(This is the same power as the power required to go biking with a little effort).

a) The power is

$$\begin{aligned}
 P &= \sigma A (T^4 - T_0^4) \\
 &= 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4} \cdot 1.8 \text{m}^2 \cdot \left((310 \text{K})^4 - (243 \text{K})^4 \right) \\
 &= 586.7 \text{W}
 \end{aligned}$$

(This is equivalent to a really strenuous exercising. Go on a stationary bike that shows the power and try to achieve this power...)

5. The filament is a cylinder whose area is

$$\begin{aligned}
 A &= 2\pi r l \\
 &= 2\pi \cdot 0.0005 \text{m} \cdot 0.1 \text{m} \\
 &= 3.1416 \times 10^{-4} \text{m}^2
 \end{aligned}$$

Therefore, the temperature is

$$\begin{aligned}
 P &= \sigma A (T^4 - T_0^4) \\
 60 \text{W} &= 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4} \cdot 3.1416 \times 10^{-4} \text{m}^2 \cdot \left(T^4 - (293 \text{K})^4 \right) \\
 T &= 1355 \text{K} = 1082^\circ \text{C}
 \end{aligned}$$

6. The energy is

$$\begin{aligned}
 E &= \frac{1240 \text{eV} \cdot \text{nm}}{\lambda} \\
 &= \frac{1240 \text{eV} \cdot \text{nm}}{550 \text{nm}} \\
 &= 2.25 \text{eV}
 \end{aligned}$$

7. The energy of a photon is

$$\begin{aligned}
 E &= \frac{1240eV \cdot nm}{\lambda} \\
 &= \frac{1240eV \cdot nm}{632nm} \\
 &= 1.962eV \\
 &= 3.143 \times 10^{-19} J
 \end{aligned}$$

The energy emitted per second is

$$\begin{aligned}
 E &= Pt \\
 &= 0.001W \cdot 1s \\
 &= 0.001J
 \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned}
 N &= \frac{\text{Energy emitted in 1 second}}{\text{Energy of one photon}} \\
 &= \frac{0.001J}{3.143 \times 10^{-19} \frac{J}{\text{photons}}} \\
 &= 3.182 \times 10^{15} \text{ photons}
 \end{aligned}$$

8. The energy of a photon is

$$\begin{aligned}
 E &= \frac{1240eV \cdot nm}{\lambda} \\
 &= \frac{1240eV \cdot nm}{585nm} \\
 &= 2.12eV \\
 &= 3.396 \times 10^{-19} J
 \end{aligned}$$

The energy received in 20 seconds is

$$\begin{aligned}
 E &= IA t \\
 &= 50 \frac{W}{m^2} \cdot 3m^2 \cdot 20s \\
 &= 3000J
 \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned}
 N &= \frac{\text{Energy received in 20 seconds}}{\text{Energy of one photon}} \\
 &= \frac{3000J}{3.396 \times 10^{-19} \frac{J}{\text{photons}}} \\
 &= 8.835 \times 10^{21} \text{ photons}
 \end{aligned}$$

9. The energy of a photon is

$$\begin{aligned}
 E &= \frac{1240eV \cdot nm}{\lambda} \\
 &= \frac{1240eV \cdot nm}{470nm} \\
 &= 2.638eV \\
 &= 4.227 \times 10^{-19} J
 \end{aligned}$$

The energy received per second is

$$\begin{aligned}
 E &= IA t \\
 &= 200 \frac{W}{m^2} \cdot \pi (0.0025m)^2 \cdot 1s \\
 &= 0.003927J
 \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned}
 N &= \frac{\text{Energy received in 1 second}}{\text{Energy of one photon}} \\
 &= \frac{0.003927J}{4.227 \times 10^{-19} \frac{J}{\text{photons}}} \\
 &= 9.291 \times 10^{15} \text{ photons}
 \end{aligned}$$

10. The photon energy is

$$\begin{aligned}
 E &= \frac{1240eV \cdot nm}{\lambda} \\
 &= \frac{1240eV \cdot nm}{150nm} \\
 &= 8.267eV
 \end{aligned}$$

The maximum energy of the ejected electrons is, therefore,

$$\begin{aligned}
 E_{k\max} &= hf - \phi \\
 &= 8.267\text{eV} - 4.5\text{eV} \\
 &= 3.767\text{eV}
 \end{aligned}$$

11. The work function of cesium is

$$\begin{aligned}
 \phi &= \frac{1240\text{eV} \cdot \text{nm}}{\lambda_0} \\
 &= \frac{1240\text{eV} \cdot \text{nm}}{686\text{nm}} \\
 &= 1.808\text{eV}
 \end{aligned}$$

a) With a wavelength of 690 nm, the energy of the photons is

$$\begin{aligned}
 E &= \frac{1240\text{eV} \cdot \text{nm}}{\lambda} \\
 &= \frac{1240\text{eV} \cdot \text{nm}}{690\text{nm}} \\
 &= 1.797\text{eV}
 \end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned}
 E_{k\max} &= hf - \phi \\
 &= 1.797\text{eV} - 1.808\text{eV} \\
 &= -0.011\text{eV}
 \end{aligned}$$

This means that there are no electrons ejected since a negative kinetic energy is impossible. Photons don't have enough energy to eject electrons.

b) With a wavelength of 450 nm, the energy of the photons is

$$\begin{aligned}
 E &= \frac{1240\text{eV} \cdot \text{nm}}{\lambda} \\
 &= \frac{1240\text{eV} \cdot \text{nm}}{450\text{nm}} \\
 &= 2.756\text{eV}
 \end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned}
 E_{k\max} &= hf - \phi \\
 &= 2.756eV - 1.808eV \\
 &= 0.948eV
 \end{aligned}$$

12. a) The threshold wavelength is

$$\begin{aligned}
 \phi &= \frac{1240eV \cdot nm}{\lambda_0} \\
 3.2eV &= \frac{1240eV \cdot nm}{\lambda_0} \\
 \lambda_0 &= 387.5nm
 \end{aligned}$$

b) With a wavelength of 250 nm, the energy of the photons is

$$\begin{aligned}
 E &= \frac{1240eV \cdot nm}{\lambda} \\
 &= \frac{1240eV \cdot nm}{250nm} \\
 &= 4.96eV
 \end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned}
 E_{k\max} &= hf - \phi \\
 &= 4.96eV - 3.2eV \\
 &= 1.76eV \\
 &= 2.82 \times 10^{-19} J
 \end{aligned}$$

The speed of the electrons is, therefore,

$$\begin{aligned}
 E_{k\max} &= \frac{1}{2}mv_{\max}^2 \\
 2.82 \times 10^{-19} J &= \frac{1}{2} \cdot 9.11 \times 10^{-31} kg \cdot v_{\max}^2 \\
 v_{\max} &= 7.868 \times 10^5 \frac{m}{s}
 \end{aligned}$$

13. The maximum kinetic energy of the electrons is

$$\begin{aligned}
 E_{k \max} &= \frac{1}{2} m v_{\max}^2 \\
 &= \frac{1}{2} \cdot 9.11 \times 10^{-31} \text{ kg} \cdot \left(5 \times 10^5 \frac{\text{m}}{\text{s}} \right)^2 \\
 &= 1.139 \times 10^{-19} \text{ J} \\
 &= 0.711 \text{ eV}
 \end{aligned}$$

The energy of the photons is

$$\begin{aligned}
 E &= \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda} \\
 &= \frac{1240 \text{ eV} \cdot \text{nm}}{400 \text{ nm}} \\
 &= 3.1 \text{ eV}
 \end{aligned}$$

The work function is then found with

$$\begin{aligned}
 E_{k \max} &= hf - \phi \\
 0.711 \text{ eV} &= 3.1 \text{ eV} - \phi \\
 \phi &= 2.389 \text{ eV}
 \end{aligned}$$

The threshold wavelength is, therefore,

$$\begin{aligned}
 \phi &= \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda_0} \\
 2,389 \text{ eV} &= \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda_0} \\
 \lambda_0 &= 519 \text{ nm}
 \end{aligned}$$

14. The energy of a photon is

$$\begin{aligned}
 E &= \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda} \\
 &= \frac{1240 \text{ eV} \cdot \text{nm}}{450 \text{ nm}} \\
 &= 2.756 \text{ eV} \\
 &= 4.414 \times 10^{-19} \text{ J}
 \end{aligned}$$

The energy received per second per square centimeter is

$$\begin{aligned}
 E &= IAt \\
 &= 40 \frac{\text{W}}{\text{m}^2} \cdot 0.0001 \text{m}^2 \cdot 1\text{s} \\
 &= 0.004\text{J}
 \end{aligned}$$

Therefore, the number of received photons is

$$\begin{aligned}
 N &= \frac{\text{Energy received in 1 second}}{\text{Energy of one photon}} \\
 &= \frac{0.004\text{J}}{4.414 \times 10^{-19} \frac{\text{J}}{\text{photons}}} \\
 &= 9.091 \times 10^{15} \text{photons}
 \end{aligned}$$

If only 3% of the photons eject an electron, then the number of ejected electrons is

$$3\% \cdot 9.091 \times 10^{15} = 2.718 \times 10^{14}$$

15. a) The wavelength shift is

$$\begin{aligned}
 \Delta\lambda &= 2.4263 \times 10^{-3} \text{nm} (1 - \cos \theta) \\
 &= 2.4263 \times 10^{-3} \text{nm} (1 - \cos 45^\circ) \\
 &= 0.0007106 \text{nm}
 \end{aligned}$$

b) The wavelength of the incident photon is

$$\begin{aligned}
 E &= \frac{1240 \text{eV} \cdot \text{nm}}{\lambda} \\
 62,000 \text{eV} &= \frac{1240 \text{eV} \cdot \text{nm}}{\lambda} \\
 \lambda &= 0.02 \text{nm}
 \end{aligned}$$

The new wavelength is thus

$$\begin{aligned}
 \lambda' &= \lambda + \Delta\lambda \\
 &= 0.02 \text{nm} + 0.0007106 \text{nm} \\
 &= 0.0207106 \text{nm}
 \end{aligned}$$

c) The new energy of the photon is

$$\begin{aligned}
 E &= \frac{1240eV \cdot nm}{\lambda} \\
 &= \frac{1240eV \cdot nm}{0.0207106nm} \\
 &= 59,873eV
 \end{aligned}$$

d) The kinetic energy of the electron is

$$\begin{aligned}
 E_{\gamma} &= E'_{\gamma} + E_{ke} \\
 62,000eV &= 59,873eV + E_{ke} \\
 E_{ke} &= 2127eV
 \end{aligned}$$

e) The angle with the conservation of y-component of the momentum.

$$0 = p'_{\gamma} \sin \theta - p'_e \sin \phi$$

The momentum of the photon is found with

$$\begin{aligned}
 E' &= p'_{\gamma} c \\
 59,873 \cdot 1.602 \times 10^{-19} J &= p'_{\gamma} \cdot 3 \times 10^8 \frac{m}{s} \\
 p'_{\gamma} &= 3.197 \times 10^{-23} \frac{kgm}{s}
 \end{aligned}$$

The momentum of the electron is found with

$$\begin{aligned}
 E_e &= \frac{p^2}{2m} \\
 2127 \cdot 1.602 \times 10^{-19} J &= \frac{p_e'^2}{2 \cdot 9.11 \times 10^{-31} kg} \\
 p'_e &= 2.491 \times 10^{-23} \frac{kgm}{s}
 \end{aligned}$$

The conservation equation then becomes

$$\begin{aligned}
 0 &= p'_{\gamma} \sin \theta - p'_e \sin \phi \\
 0 &= 3.197 \times 10^{-23} \frac{kgm}{s} \cdot \sin 45^{\circ} - 2.491 \times 10^{-23} \frac{kgm}{s} \cdot \sin \phi \\
 0 &= 3.197 \cdot \sin 45^{\circ} - 2.491 \sin \phi \\
 \phi &= 65.1^{\circ}
 \end{aligned}$$

16. The initial wavelength is

$$E = \frac{1240eV \cdot nm}{\lambda}$$

$$50,000eV = \frac{1240eV \cdot nm}{\lambda}$$

$$\lambda = 0.0248nm$$

The wavelength after the scattering is

$$E' = \frac{1240eV \cdot nm}{\lambda'}$$

$$49,500eV = \frac{1240eV \cdot nm}{\lambda'}$$

$$\lambda' = 0.02505nm$$

So, the wavelength shift is

$$\Delta\lambda = \lambda' - \lambda$$

$$= 0.02505nm - 0.0248nm$$

$$= 0.00025nm$$

The angle is then found with

$$\Delta\lambda = 2.4263 \times 10^{-3} nm (1 - \cos \theta)$$

$$0.00025nm = 2.4263 \times 10^{-3} nm (1 - \cos \theta)$$

$$\theta = 26.3^\circ$$

17. a) The radius is

$$r_n = \frac{n^2}{Z} 0.05292nm$$

$$= \frac{2^2}{3} 0.05292nm$$

$$= 0.07056nm$$

b) The energy of the first level is

$$\begin{aligned}
 E_n &= -\frac{Z^2}{n^2} 13.61 \text{ eV} \\
 &= -\frac{3^2}{1^2} 13.61 \text{ eV} \\
 &= -122.49 \text{ eV}
 \end{aligned}$$

c) To ionize the atom, the energy of the electron must be positive. Therefore, at least 122.49 eV must be given to ionize it.

18. a) The speed is

$$\begin{aligned}
 v_n &= \frac{Z}{n} \cdot \frac{c}{137.04} \\
 &= \frac{3}{2} \cdot \frac{3 \times 10^8 \frac{\text{m}}{\text{s}}}{137.04} \\
 &= 3.2837 \times 10^6 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

b) As the electron is in a circular motion, the acceleration is

$$a = \frac{v^2}{r}$$

Therefore, the radius of the orbit is needed. The radius is

$$\begin{aligned}
 r_n &= \frac{n^2}{Z} 0.05292 \text{ nm} \\
 &= \frac{2^2}{3} 0.05292 \text{ nm} \\
 &= 0.07056 \text{ nm}
 \end{aligned}$$

The acceleration is, therefore,

$$\begin{aligned}
 a &= \frac{v^2}{r} \\
 &= \frac{\left(3.2815 \times 10^6 \frac{\text{m}}{\text{s}}\right)^2}{7.056 \times 10^{-11} \text{ m}} \\
 &= 1.528 \times 10^{23} \frac{\text{m}}{\text{s}^2}
 \end{aligned}$$

19. The energy of level 5 is

$$\begin{aligned} E_5 &= -\frac{Z^2}{n^2}13.61eV \\ &= -\frac{1^2}{5^2}13.61eV \\ &= -0.5444eV \end{aligned}$$

The energy of level 3 is

$$\begin{aligned} E_3 &= -\frac{Z^2}{n^2}13,61eV \\ &= -\frac{1^2}{3^2}13,61eV \\ &= -1,5122eV \end{aligned}$$

The energy lost by the electron is

$$\begin{aligned} \Delta E &= E_3 - E_5 \\ &= -1.5122eV - (-0.5444eV) \\ &= -0.9678eV \end{aligned}$$

If the electron has lost 0.9678 eV, it has emitted a photon whose energy is 0.9678 eV. The wavelength of the photon is, therefore,

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ 0.9678eV &= \frac{1240eV \cdot nm}{\lambda} \\ \lambda &= 1281nm \end{aligned}$$

20. The energy of level 6 is

$$\begin{aligned} E_6 &= -\frac{Z^2}{n^2}13.61eV \\ &= -\frac{1^2}{6^2}13.61eV \\ &= -0.378eV \end{aligned}$$

The energy of the first level is

$$\begin{aligned}
 E_1 &= -\frac{Z^2}{n^2} 13.61 \text{ eV} \\
 &= -\frac{1^2}{1^2} 13.61 \text{ eV} \\
 &= -13.61 \text{ eV}
 \end{aligned}$$

If the electron goes to level 6, it must gain the following energy.

$$\begin{aligned}
 \Delta E &= E_f - E_i \\
 &= -0.378 \text{ eV} - (-13.61 \text{ eV}) \\
 &= 13.232 \text{ eV}
 \end{aligned}$$

This must be the energy of the photons. The corresponding wavelength is

$$\begin{aligned}
 E &= \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda} \\
 13.232 \text{ eV} &= \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda} \\
 \lambda &= 93.71 \text{ nm}
 \end{aligned}$$

21. The energy of level 1 is

$$\begin{aligned}
 E_1 &= -\frac{Z^2}{n^2} 13.61 \text{ eV} \\
 &= -\frac{1^2}{1^2} 13.61 \text{ eV} \\
 &= -13.61 \text{ eV}
 \end{aligned}$$

If 12 eV is added, the energy of the electron is -1.61 eV. The energy level corresponding to the energy is

$$\begin{aligned}
 E_n &= -\frac{Z^2}{n^2} 13.61 \text{ eV} \\
 -1.61 \text{ eV} &= -\frac{1^2}{n^2} 13.61 \text{ eV} \\
 n &= 2.9
 \end{aligned}$$

This is not an integer. This means that there is no level of energy with an energy of -1.61 eV. Photons cannot be absorbed and the electrons remain at the level $n = 1$.

22. The energy of the absorbed photon is

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ &= \frac{1240eV \cdot nm}{250nm} \\ &= 4.96eV \end{aligned}$$

The energy of the electron has increased by 4.96 eV.

By emitting the first photon, the electron loses the energy of this photon. This energy is

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ &= \frac{1240eV \cdot nm}{800nm} \\ &= 1.55eV \end{aligned}$$

As the electron had gained 4.96 eV and had just lost 1.55 eV, he still has 3.41 eV to lose to return to its initial energy level. It had to emit a 3.41 eV photon, which has the wavelength given by this formula.

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ 3.41eV &= \frac{1240eV \cdot nm}{\lambda} \\ \lambda &= 363.6nm \end{aligned}$$

23. The equation for centripetal force

$$\frac{m_e v^2}{r} = \frac{kZe^2}{r^2}$$

allows us to write the kinetic energy in the form

$$\frac{1}{2} m_e v^2 = \frac{kZe^2}{2r}$$

As the mechanical energy of the electron is given by

$$E = -\frac{kZe^2}{2r}$$

We have

$$\frac{E_k}{E} = \frac{\frac{kZe^2}{2r}}{-\frac{kZe^2}{2r}} = -1$$

This means that the kinetic energy of the electron is the absolute value of the energy of the electron on the level. For example, at the first level of hydrogen, the energy of the electron is -13.61 eV. This means that its kinetic energy is 13.61 eV.

- 24.** a) On the orbit, the centripetal force is equal to the force between the nucleus and the electron

$$\frac{m_e v^2}{r} = kr$$

As the quantization condition is $m_e v r = n\hbar$, the equation becomes

$$\begin{aligned} \frac{m_e \left(\frac{n\hbar}{m_e r} \right)^2}{r} &= kr \\ \frac{n^2 \hbar^2}{m_e r^3} &= kr \\ n^2 \hbar^2 &= km_e r^4 \\ r &= \sqrt[4]{\frac{n^2 \hbar^2}{km_e}} \end{aligned}$$

- b) The energy levels are given by

$$E = \frac{1}{2} m_e v^2 + \frac{1}{2} kr^2$$

To find this energy, the speed of electron is needed. However, the equation for centripetal force

$$\frac{m_e v^2}{r} = kr$$

can be used to arrive at

$$m_e v^2 = kr^2$$

Thus, the energy equation can be written as

$$\begin{aligned} E &= \frac{1}{2} m_e v^2 + \frac{1}{2} kr^2 \\ &= \frac{1}{2} kr^2 + \frac{1}{2} kr^2 \\ &= kr^2 \end{aligned}$$

Using the formula for r , the equation becomes

$$\begin{aligned} E &= kr^2 \\ &= k \sqrt{\frac{n^2 \hbar^2}{km_e}} \\ &= \sqrt{\frac{kn^2 \hbar^2}{m_e}} \\ &= n\hbar \sqrt{\frac{k}{m_e}} \end{aligned}$$

It remains to find the frequency of revolution. The period of revolution is

$$T = \frac{2\pi r}{v}$$

However, the speed can be found with

$$\begin{aligned} \frac{m_e v^2}{r} &= kr \\ v &= \sqrt{\frac{kr^2}{m_e}} \end{aligned}$$

Thus, the period is

$$\begin{aligned} T &= \frac{2\pi r}{v} \\ &= \frac{2\pi r}{\sqrt{\frac{kr^2}{m_e}}} \\ &= 2\pi \sqrt{\frac{m_e}{k}} \end{aligned}$$

Therefore, the frequency is

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{2\pi} \sqrt{\frac{k}{m_e}} \end{aligned}$$

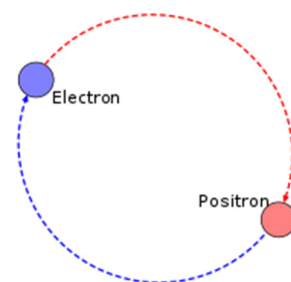
Finally, the following result is found for the energy.

$$\begin{aligned} E &= n\hbar \sqrt{\frac{k}{m_e}} \\ &= n\hbar 2\pi f \\ &= n \frac{h}{2\pi} 2\pi f \\ &= nhf \end{aligned}$$

- 25.** a) In the positronium, the centripetal force is equal to the force of attraction between the electron and the positron. As the radius of the orbit is r is that the distance between the two particles is $2r$, we have

$$\begin{aligned} \frac{m_e v^2}{r} &= \frac{ke^2}{(2r)^2} \\ m_e v^2 &= \frac{ke^2}{4r} \end{aligned}$$

Since the angular momentum is quantized, we have



$$\begin{aligned}
 L_{tot} &= n\hbar \\
 m_e v r + m_e v r &= n\hbar \\
 2m_e v r &= n\hbar \\
 v &= \frac{n\hbar}{2m_e r}
 \end{aligned}$$

Therefore, the formula for the radius is

$$\begin{aligned}
 m_e v^2 &= \frac{ke^2}{4r} \\
 m_e \left(\frac{n\hbar}{2m_e r} \right)^2 &= \frac{ke^2}{4r} \\
 m_e \frac{n^2 \hbar^2}{4m_e^2 r^2} &= \frac{ke^2}{4r} \\
 \frac{n^2 \hbar^2}{m_e r} &= ke^2 \\
 r_n &= \frac{n^2 \hbar^2}{m_e ke^2}
 \end{aligned}$$

b) The energy is

$$\begin{aligned}
 E_n &= E_k + U \\
 &= \frac{1}{2} m v_e^2 + \frac{1}{2} m v_e^2 - \frac{ke^2}{2r} \\
 &= m v_e^2 - \frac{ke^2}{2r}
 \end{aligned}$$

Since

$$m_e v^2 = \frac{ke^2}{4r}$$

the energy becomes

$$\begin{aligned}
 E_n &= \frac{ke^2}{4r} - \frac{ke^2}{2r} \\
 &= -\frac{ke^2}{4r}
 \end{aligned}$$

Using the formula for the radius of the orbit, the equation becomes

$$\begin{aligned} E_n &= -\frac{ke^2}{4r} \\ &= -\frac{ke^2}{4} \frac{m_e ke^2}{n^2 \hbar^2} \\ &= -\frac{m_e k^2 e^4}{4n^2 \hbar^2} \end{aligned}$$

c) The energy of the first level is

$$\begin{aligned} E_1 &= -\frac{m_e k^2 e^4}{4\hbar^2} \\ &= -6,803eV \end{aligned}$$