Chapter 11 Solutions

1. The wavelength of the peak is

$$\begin{aligned}\lambda_{pic} &= \frac{2.898\times10^{-3}mK}{T}\\ &= \frac{2.898\times10^{-3}mK}{3273K}\\ &= 885nm\end{aligned}$$

This corresponds to infrared radiation.

2. The temperature is found with

$$\begin{aligned}\lambda_{pic} &= \frac{2.898\times10^{-3}mK}{T}\\ 502\times10^{-9}m &= \frac{2.898\times10^{-3}mK}{T}\\ T &= 5773K\end{aligned}$$

3. The power is

$$\begin{aligned}P &= \sigma A\left(T^4 - T_0^4\right)\\ &= 5.67\times10^{-8}\tfrac{W}{m^2K^4}4\pi\left(3.2\times10^{10}m\right)^2\left(\left(6015K\right)^4 - \left(0K\right)^4\right)\\ &= 9.55\times10^{29}W\end{aligned}$$

This is about 2500 times brighter than the Sun.

4. a) The power is

$$\begin{aligned} P &= \sigma A\left(T^4 - T_0^4\right) \\ &= 5.67\times10^{-8}\,\tfrac{W}{m^2K^4}\cdot 1.8m^2\cdot\left((310K)^4-(293K)^4\right) \\ &= 190.4W \end{aligned}$$

(This is the same power as the power required to go biking with a little effort).

a) The power is

$$\begin{aligned} P &= \sigma A\left(T^4 - T_0^4\right) \\ &= 5.67\times10^{-8}\,\tfrac{W}{m^2K^4}\cdot 1.8m^2\cdot\left((310K)^4-(243K)^4\right) \\ &= 586.7W \end{aligned}$$

(This is equivalent to a really strenuous exercising. Go on a stationary bike that shows the power and try to achieve this power...)

5. The filament is a cylinder whose area is

$$\begin{aligned} A &= 2\pi rl \\ &= 2\pi\cdot 0.0005m\cdot 0.1m \\ &= 3.1416\times10^{-4}m^2 \end{aligned}$$

Therefore, the temperature is

$$\begin{aligned} P &= \sigma A\left(T^4 - T_0^4\right) \\ 60W &= 5.67\times10^{-8}\,\tfrac{W}{m^2K^4}\cdot 3.1416\times10^{-4}m^2\cdot\left(T^4-(293K)^4\right) \\ T &= 1355K = 1082°C \end{aligned}$$

6. The energy is

$$\begin{aligned} E &= \frac{1240eV\cdot nm}{\lambda} \\ &= \frac{1240eV\cdot nm}{550nm} \\ &= 2.25eV \end{aligned}$$

7. The energy of a photon is

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ &= \frac{1240eV \cdot nm}{632nm} \\ &= 1.962eV \\ &= 3.143 \times 10^{-19} J \end{aligned}$$

The energy emitted per second is

$$\begin{aligned} E &= Pt \\ &= 0.001W \cdot 1s \\ &= 0.001J \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned} N &= \frac{\text{Energy emitted in 1 second}}{\text{Energy of one photon}} \\ &= \frac{0.001J}{3.143 \times 10^{-19} \frac{J}{photons}} \\ &= 3.182 \times 10^{15}\ photons \end{aligned}$$

8. The energy of a photon is

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ &= \frac{1240eV \cdot nm}{585nm} \\ &= 2.12eV \\ &= 3.396 \times 10^{-19} J \end{aligned}$$

The energy received in 20 seconds is

$$\begin{aligned} E &= IAt \\ &= 50\tfrac{W}{m^2} \cdot 3m^2 \cdot 20s \\ &= 3000J \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned} N &= \frac{\text{Energy received in 20 seconds}}{\text{Energy of one photon}} \\ &= \frac{3000J}{3.396\times 10^{-19}\frac{J}{photons}} \\ &= 8.835\times 10^{21}\ photons \end{aligned}$$

9. The energy of a photon is

$$\begin{aligned} E &= \frac{1240eV\cdot nm}{\lambda} \\ &= \frac{1240eV\cdot nm}{470nm} \\ &= 2.638eV \\ &= 4.227\times 10^{-19}J \end{aligned}$$

The energy received per second is

$$\begin{aligned} E &= IAt \\ &= 200\tfrac{W}{m^2}\cdot \pi\left(0.0025m\right)^2\cdot 1s \\ &= 0.003927J \end{aligned}$$

Therefore, the number of photons is

$$\begin{aligned} N &= \frac{\text{Energy received in 1 second}}{\text{Energy of one photon}} \\ &= \frac{0.003927J}{4.227\times 10^{-19}\frac{J}{photons}} \\ &= 9.291\times 10^{15}\ photons \end{aligned}$$

10. The photon energy is

$$\begin{aligned} E &= \frac{1240eV\cdot nm}{\lambda} \\ &= \frac{1240eV\cdot nm}{150nm} \\ &= 8.267eV \end{aligned}$$

The maximum energy of the ejected electrons is, therefore,

$$\begin{aligned} E_{k\max} &= hf - \phi \\ &= 8.267eV - 4.5eV \\ &= 3.767eV \end{aligned}$$

11. The work function of cesium is

$$\begin{aligned} \phi &= \frac{1240eV \cdot nm}{\lambda_0} \\ &= \frac{1240eV \cdot nm}{686nm} \\ &= 1.808eV \end{aligned}$$

a) With a wavelength of 690 nm, the energy of the photons is

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ &= \frac{1240eV \cdot nm}{690nm} \\ &= 1.797eV \end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned} E_{k\max} &= hf - \phi \\ &= 1.797eV - 1.808eV \\ &= -0.011eV \end{aligned}$$

This means that there are no electrons ejected since a negative kinetic energy is impossible. Photons don't have enough energy to eject electrons.

b) With a wavelength of 450 nm, the energy of the photons is

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ &= \frac{1240eV \cdot nm}{450nm} \\ &= 2.756eV \end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned} E_{k\max} &= hf - \phi \\ &= 2.756eV - 1.808eV \\ &= 0.948eV \end{aligned}$$

12. a) The threshold wavelength is

$$\phi = \frac{1240eV \cdot nm}{\lambda_0}$$
$$3.2eV = \frac{1240eV \cdot nm}{\lambda_0}$$
$$\lambda_0 = 387.5nm$$

b) With a wavelength of 250 nm, the energy of the photons is

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ &= \frac{1240eV \cdot nm}{250nm} \\ &= 4.96eV \end{aligned}$$

The energy of the ejected electrons is then

$$\begin{aligned} E_{k\max} &= hf - \phi \\ &= 4.96eV - 3.2eV \\ &= 1.76eV \\ &= 2.82\times10^{-19} J \end{aligned}$$

The speed of the electrons is, therefore,

$$E_{k\max} = \frac{1}{2} m v_{\max}^2$$
$$2.82\times10^{-19} J = \frac{1}{2}\cdot 9.11\times10^{-31} kg \cdot v_{\max}^2$$
$$v_{\max} = 7.868\times10^5 \tfrac{m}{s}$$

13. The maximum kinetic energy of the electrons is

$$\begin{aligned}E_{k\max} &= \frac{1}{2}mv_{\max}^2\\ &= \frac{1}{2}\cdot 9.11\times10^{-31}kg\cdot\left(5\times10^5\tfrac{m}{s}\right)^2\\ &= 1.139\times10^{-19}J\\ &= 0.711eV\end{aligned}$$

The energy of the photons is

$$\begin{aligned}E &= \frac{1240eV\cdot nm}{\lambda}\\ &= \frac{1240eV\cdot nm}{400nm}\\ &= 3.1eV\end{aligned}$$

The work function is then found with

$$\begin{aligned}E_{k\max} &= hf-\phi\\ 0.711eV &= 3.1eV-\phi\\ \phi &= 2.389eV\end{aligned}$$

The threshold wavelength is, therefore,

$$\begin{aligned}\phi &= \frac{1240eV\cdot nm}{\lambda_0}\\ 2,389eV &= \frac{1240eV\cdot nm}{\lambda_0}\\ \lambda_0 &= 519nm\end{aligned}$$

14. The energy of a photon is

$$\begin{aligned}E &= \frac{1240eV\cdot nm}{\lambda}\\ &= \frac{1240eV\cdot nm}{450nm}\\ &= 2.756eV\\ &= 4.414\times10^{-19}J\end{aligned}$$

The energy received per second per square centimetre is

$$\begin{aligned} E &= IAt \\ &= 40\tfrac{W}{m^2}\cdot 0.0001m^2\cdot 1s \\ &= 0.004J \end{aligned}$$

Therefore, the number of photons received is

$$\begin{aligned} N &= \frac{\text{Energy received in 1 second}}{\text{Energy of one photon}} \\ &= \frac{0.004J}{4.414\times10^{-19}\,\frac{J}{photons}} \\ &= 9.091\times10^{15}\ photons \end{aligned}$$

If only 3% of the photons eject an electron, then the number of ejected electrons is

$$3\%\cdot 9.061\times10^{15} = 2.718\times10^{14}$$

15. a) The wavelength shift is

$$\begin{aligned} \Delta\lambda &= 2.4263\times10^{-3}nm\left(1-\cos\theta\right) \\ &= 2.4263\times10^{-3}nm\left(1-\cos 45°\right) \\ &= 0.0007106nm \end{aligned}$$

b) The wavelength of the incident photon is

$$\begin{aligned} E &= \frac{1240eV\cdot nm}{\lambda} \\ 62{,}000eV &= \frac{1240eV\cdot nm}{\lambda} \\ \lambda &= 0.02nm \end{aligned}$$

The new wavelength is thus

$$\begin{aligned} \lambda' &= \lambda+\Delta\lambda \\ &= 0.02nm+0.0007106nm \\ &= 0.0207106nm \end{aligned}$$

c) The new energy of the photon is

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ &= \frac{1240eV \cdot nm}{0.0207106nm} \\ &= 59{,}873eV \end{aligned}$$

d) The kinetic energy of the electron is

$$\begin{aligned} E_\gamma &= E'_\gamma + E_{k\,e} \\ 62{,}000eV &= 59{,}873eV + E_{k\,e} \\ E_{k\,e} &= 2127eV \end{aligned}$$

e) The angle with the conservation of *y*-component of the momentum.

$$0 = p'_\gamma \sin\theta - p'_e \sin\phi$$

The momentum of the photon is found with

$$\begin{aligned} E' &= p'_\gamma c \\ 59{,}873 \cdot 1.602\times10^{-19} J &= p'_\gamma \cdot 3\times10^8 \tfrac{m}{s} \\ p'_\gamma &= 3.197\times10^{-23} \tfrac{kgm}{s} \end{aligned}$$

The momentum of the electron is found with

$$\begin{aligned} E_e &= \frac{p^2}{2m} \\ 2127 \cdot 1.602\times10^{-19} J &= \frac{p_e'^2}{2 \cdot 9.11\times10^{-31} kg} \\ p'_e &= 2.491\times10^{-23} \tfrac{kgm}{s} \end{aligned}$$

The conservation equation then becomes

$$\begin{aligned} 0 &= p'_\gamma \sin\theta - p'_e \sin\phi \\ 0 &= 3.197\times10^{-23} \tfrac{kgm}{s} \cdot \sin 45° - 2.491\times10^{-23} \tfrac{kgm}{s} \cdot \sin\phi \\ 0 &= 3.197 \cdot \sin 45° - 2.491 \sin\phi \\ \phi &= 65.1° \end{aligned}$$

16. The initial wavelength is

$$E = \frac{1240eV \cdot nm}{\lambda}$$
$$50{,}000eV = \frac{1240eV \cdot nm}{\lambda}$$
$$\lambda = 0.0248nm$$

The wavelength after the scattering is

$$E' = \frac{1240eV \cdot nm}{\lambda'}$$
$$49{,}500eV = \frac{1240eV \cdot nm}{\lambda'}$$
$$\lambda' = 0.02505nm$$

So, the wavelength shift is

$$\begin{aligned}\Delta\lambda &= \lambda' - \lambda \\ &= 0.02505nm - 0.0248nm \\ &= 0.00025nm\end{aligned}$$

The angle is then found with

$$\Delta\lambda = 2.4263\times 10^{-3}\,nm\left(1-\cos\theta\right)$$
$$0.00025nm = 2.4263\times 10^{-3}\,nm\left(1-\cos\theta\right)$$
$$\theta = 26.3°$$

17. a) The radius is

$$\begin{aligned}r_n &= \frac{n^2}{Z}0.05292nm \\ &= \frac{2^2}{3}0.05292nm \\ &= 0.07056nm\end{aligned}$$

b) The energy of the first level is

$$\begin{aligned} E_n &= -\frac{Z^2}{n^2} 13.61 eV \\ &= -\frac{3^2}{1^2} 13.61 eV \\ &= -122.49 eV \end{aligned}$$

c) To ionize the atom, the energy of the electron must be positive. Therefore, at least 122.49 eV must be given to ionize it.

18. a) The speed is

$$\begin{aligned} v_n &= \frac{Z}{n} \cdot \frac{c}{137.04} \\ &= \frac{3}{2} \cdot \frac{3\times10^8 \frac{m}{s}}{137.04} \\ &= 3.2837\times10^6 \frac{m}{s} \end{aligned}$$

b) As the electron is in a circular motion, the acceleration is

$$a = \frac{v^2}{r}$$

Therefore, the radius of the orbit is needed. The radius is

$$\begin{aligned} r_n &= \frac{n^2}{Z} 0.05292 nm \\ &= \frac{2^2}{3} 0.05292 nm \\ &= 0.07056 nm \end{aligned}$$

The acceleration is, therefore,

$$\begin{aligned} a &= \frac{v^2}{r} \\ &= \frac{\left(3.2815\times10^6 \frac{m}{s}\right)^2}{7.056\times10^{-11} m} \\ &= 1.528\times10^{23} \frac{m}{s^2} \end{aligned}$$

19. The energy of level 5 is

$$\begin{aligned} E_5 &= -\frac{Z^2}{n^2} 13.61 eV \\ &= -\frac{1^2}{5^2} 13.61 eV \\ &= -0.5444 eV \end{aligned}$$

The energy of level 3 is

$$\begin{aligned} E_3 &= -\frac{Z^2}{n^2} 13,61 eV \\ &= -\frac{1^2}{3^2} 13,61 eV \\ &= -1,5122 eV \end{aligned}$$

The energy lost by the electron is

$$\begin{aligned} \Delta E &= E_3 - E_5 \\ &= -1.5122 eV - -0.5444 eV \\ &= -0.9678 eV \end{aligned}$$

If the electron has lost 0.9678 eV, it has emitted a photon whose energy is 0.9678 eV. The wavelength of the photon is, therefore,

$$\begin{aligned} E &= \frac{1240 eV \cdot nm}{\lambda} \\ 0.9678 eV &= \frac{1240 eV \cdot nm}{\lambda} \\ \lambda &= 1281 nm \end{aligned}$$

20. The energy of level 6 is

$$\begin{aligned} E_6 &= -\frac{Z^2}{n^2} 13.61 eV \\ &= -\frac{1^2}{6^2} 13.61 eV \\ &= -0.378 eV \end{aligned}$$

The energy of the first level is

$$\begin{aligned} E_1 &= -\frac{Z^2}{n^2} 13.61eV \\ &= -\frac{1^2}{1^2} 13.61eV \\ &= -13.61eV \end{aligned}$$

If the electron goes to level 6, it must gain the following energy.

$$\begin{aligned} \Delta E &= E_f - E_i \\ &= -0.378eV - -13.61eV \\ &= 13.232eV \end{aligned}$$

This must be the energy of the photons. The corresponding wavelength is

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ 13.232eV &= \frac{1240eV \cdot nm}{\lambda} \\ \lambda &= 93.71nm \end{aligned}$$

21. The energy of level 1 is

$$\begin{aligned} E_1 &= -\frac{Z^2}{n^2} 13.61eV \\ &= -\frac{1^2}{1^2} 13.61eV \\ &= -13.61eV \end{aligned}$$

If 12 eV is added, the energy of the electron is -1.61 eV. The energy level corresponding to the energy is

$$\begin{aligned} E_n &= -\frac{Z^2}{n^2} 13.61eV \\ -1.61eV &= -\frac{1^2}{n^2} 13.61eV \\ n &= 2.9 \end{aligned}$$

This is not an integer. This means that there is no level of energy with an energy of -1.61 eV. Photons cannot be absorbed and the electrons remain at the level n = 1.

22. The energy of the absorbed photon is

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ &= \frac{1240eV \cdot nm}{250nm} \\ &= 4.96eV \end{aligned}$$

The energy of the electron has increased by 4.96 eV.

By emitting the first photon, the electron loses the energy of this photon. This energy is

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ &= \frac{1240eV \cdot nm}{800nm} \\ &= 1.55eV \end{aligned}$$

As the electron had gained 4.96 eV and had just lost 1.55 eV, he still has 3.41 eV to lose to return to its initial energy level. It had to emit a 3.41 eV photon, which has the wavelength given by this formula.

$$\begin{aligned} E &= \frac{1240eV \cdot nm}{\lambda} \\ 3.41eV &= \frac{1240eV \cdot nm}{\lambda} \\ \lambda &= 363.6nm \end{aligned}$$

23. The equation for centripetal force

$$\frac{m_e v^2}{r} = \frac{kZe^2}{r^2}$$

allows us to write the kinetic energy in the form

$$\frac{1}{2} m_e v^2 = \frac{kZe^2}{2r}$$

As the mechanical energy of the electron is given by

$$E = -\frac{kZe^2}{2r}$$

We have

$$\frac{E_k}{E} = \frac{\frac{kZe^2}{2r}}{-\frac{kZe^2}{2r}} = -1$$

This means that the kinetic energy of the electron is the absolute value of the energy of the electron on the level. For example, at the first level of hydrogen, the energy of the electron is -13.61 eV. This means that its kinetic energy is 13.61 eV.

24. a) On the orbit, the centripetal force is equal to the force between the nucleus and the electron

$$\frac{m_e v^2}{r} = kr$$

As the quantization condition is $m_e vr = n\hbar$, the equation becomes

$$\frac{m_e\left(\frac{n\hbar}{m_e r}\right)^2}{r} = kr$$

$$\frac{n^2\hbar^2}{m_e r^3} = kr$$

$$n^2\hbar^2 = km_e r^4$$

$$r = \sqrt[4]{\frac{n^2\hbar^2}{km_e}}$$

b) The energy levels are given by

$$E = \frac{1}{2}m_e v^2 + \frac{1}{2}kr^2$$

To find this energy, the speed of the electron is needed. However, the equation for centripetal force

$$\frac{m_e v^2}{r} = kr$$

can be used to arrive at

$$m_e v^2 = kr^2$$

Thus, the energy equation can be written as

$$\begin{aligned} E &= \frac{1}{2} m_e v^2 + \frac{1}{2} kr^2 \\ &= \frac{1}{2} kr^2 + \frac{1}{2} kr^2 \\ &= kr^2 \end{aligned}$$

Using the formula for *r,* the equation becomes

$$\begin{aligned} E &= kr^2 \\ &= k\sqrt{\frac{n^2\hbar^2}{km_e}} \\ &= \sqrt{\frac{kn^2\hbar^2}{m_e}} \\ &= n\hbar\sqrt{\frac{k}{m_e}} \end{aligned}$$

It remains to find the frequency of revolution. The period of revolution is

$$T = \frac{2\pi r}{v}$$

However, the speed can be found with

$$\begin{aligned} \frac{m_e v^2}{r} &= kr \\ v &= \sqrt{\frac{kr^2}{m_e}} \end{aligned}$$

Thus, the period is

$$\begin{aligned} T &= \frac{2\pi r}{v} \\ &= \frac{2\pi r}{\sqrt{\frac{kr^2}{m_e}}} \\ &= 2\pi\sqrt{\frac{m_e}{k}} \end{aligned}$$

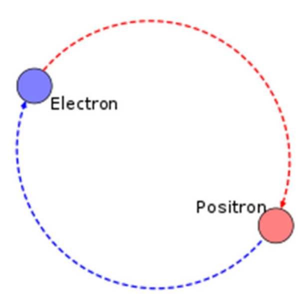
Therefore, the frequency is

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{2\pi}\sqrt{\frac{k}{m_e}} \end{aligned}$$

Finally, the following result is found for the energy.

$$\begin{aligned} E &= n\hbar\sqrt{\frac{k}{m_e}} \\ &= n\hbar 2\pi f \\ &= n\frac{h}{2\pi}2\pi f \\ &= nhf \end{aligned}$$

25. a) In the positronium, the centripetal force is equal to the force of attraction between the electron and the positron. As the radius of the orbit is r is that the distance between the two particles is $2r$, we have



$$\begin{aligned} \frac{m_e v^2}{r} &= \frac{ke^2}{(2r)^2} \\ m_e v^2 &= \frac{ke^2}{4r} \end{aligned}$$

Since the angular momentum is quantized, we have

$$L_{tot} = n\hbar$$
$$m_e vr + m_e vr = n\hbar$$
$$2m_e vr = n\hbar$$
$$v = \frac{n\hbar}{2m_e r}$$

Therefore, the formula for the radius is

$$m_e v^2 = \frac{ke^2}{4r}$$
$$m_e \left(\frac{n\hbar}{2m_e r}\right)^2 = \frac{ke^2}{4r}$$
$$m_e \frac{n^2\hbar^2}{4m_e^2 r^2} = \frac{ke^2}{4r}$$
$$\frac{n^2\hbar^2}{m_e r} = ke^2$$
$$r_n = \frac{n^2\hbar^2}{m_e ke^2}$$

b) The energy is

$$\begin{aligned} E_n &= E_k + U \\ &= \frac{1}{2}mv_e^2 + \frac{1}{2}mv_e^2 - \frac{ke^2}{2r} \\ &= mv_e^2 - \frac{ke^2}{2r} \end{aligned}$$

Since

$$m_e v^2 = \frac{ke^2}{4r}$$

the energy becomes

$$\begin{aligned} E_n &= \frac{ke^2}{4r} - \frac{ke^2}{2r} \\ &= -\frac{ke^2}{4r} \end{aligned}$$

Using the formula for the radius of the orbit, the equation becomes

$$\begin{aligned} E_n &= -\frac{ke^2}{4r} \\ &= -\frac{ke^2}{4}\frac{m_e ke^2}{n^2\hbar^2} \\ &= -\frac{m_e k^2 e^4}{4n^2\hbar^2} \end{aligned}$$

c) The energy of the first level is

$$\begin{aligned} E_1 &= -\frac{m_e k^2 e^4}{4\hbar^2} \\ &= -6,803 eV \end{aligned}$$