Chapter 9 Solutions

1. Using a $y = 0$ at the surface of the water, the gravitational energy at points A, B, and C are

$$U_{gA} = mgy = 30kg \cdot 9.8\tfrac{N}{kg} \cdot 2.8m = 823.2J$$
$$U_{gB} = mgy = 30kg \cdot 9.8\tfrac{N}{kg} \cdot 1.5m = 441J$$
$$U_{gC} = mgy = 30kg \cdot 9.8\tfrac{N}{kg} \cdot 0m = 0J$$

a) Going from A to B, the variation of energy is

$$\begin{aligned}\Delta U_g &= U_{gB} - U_{gA} \\ &= 441J - 823,2J \\ &= -382.2J\end{aligned}$$

b) Going from A to C, the variation of energy is

$$\begin{aligned}\Delta U_g &= U_{gC} - U_{gA} \\ &= 0J - 823.2J \\ &= -823.2J\end{aligned}$$

The work is therefore

$$\begin{aligned}W_g &= -\Delta U_g \\ &= --823.2J \\ &= 823.2J\end{aligned}$$

2. a) The formula for the potential energy is

$$\begin{aligned}U &= -\int F_x dx \\ &= -\int \left(2\tfrac{N}{m^3}x^3 + 2N\right)dx \\ &= -\frac{1}{2}\tfrac{N}{m^3}x^4 - 2N \cdot x + Cst\end{aligned}$$

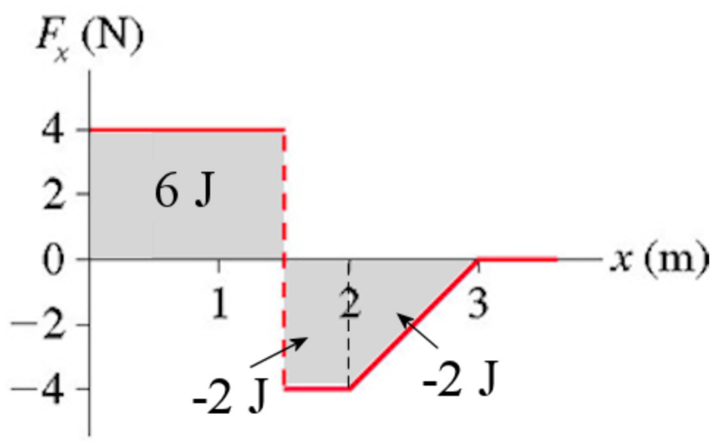
b) The potential energy difference is

$$\begin{aligned}\Delta U &= \left(-\frac{1}{2}\tfrac{N}{m^3}(5m)^4 - 2N\cdot(5m)\right) - \left(-\frac{1}{2}\tfrac{N}{m^3}(-2m)^4 - 2N\cdot(-2m)\right)\\ &= (-322.5J) - (-4J)\\ &= -318.5J\end{aligned}$$

c) The work done by the force is

$$\begin{aligned}W &= -\Delta U\\ &= -(-318.5J)\\ &= 318.5J\end{aligned}$$

3. The change of potential energy is equal to minus the area under the curve. Between $x = 0$ m and $x = 3$ m, the area is



The total area is 2 J. Therefore, the variation of potential energy is -2 J.

4. The force is conservative if

$$\frac{\partial F_y}{\partial x} = \frac{\partial F_x}{\partial y}$$

a) The derivatives are

$$\frac{\partial F_y}{\partial x} = \frac{\partial\left(3\frac{N}{m^2}y^2\right)}{\partial x} = 0$$

$$\frac{\partial F_x}{\partial y} == \frac{\partial\left(3\frac{N}{m^2}x^2\right)}{\partial y} = 0$$

As they are equal, the force is conservative.

b) The derivatives are

$$\frac{\partial F_y}{\partial x} = \frac{\partial\left(3\frac{N}{m^3}xy^2 + 1\frac{N}{m}y\right)}{\partial x} = 3\frac{N}{m^3}y^2$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial\left(3\frac{N}{m^3}x^2y + 1\frac{N}{m}x\right)}{\partial y} = 3\frac{N}{m^3}x^2$$

As they are not equal, the force is not conservative.

c) The derivatives are

$$\frac{\partial F_y}{\partial x} = \frac{\partial\left(3\frac{N}{m^3}x^2y + 1\frac{N}{m}x\right)}{\partial x} = 6\frac{N}{m^3}xy + 1\frac{N}{m}$$

$$\frac{\partial F_x}{\partial y} = \frac{\partial\left(3\frac{N}{m^3}xy^2 + 1\frac{N}{m}y\right)}{\partial y} = 6\frac{N}{m^3}xy + 1\frac{N}{m}$$

As they are equal, the force is conservative.

5. The *x*-component of the force is

$$\begin{aligned} F_x &= -\frac{\partial U}{\partial x} \\ &= -\frac{\partial\left(4\frac{J}{m^2}x^2 + 2\frac{J}{m}(x+y) - 3\frac{J}{m^2}(xy)\right)}{\partial x} \\ &= -\left(8\frac{J}{m^2}x + 2\frac{J}{m} - 3\frac{J}{m^2}(y)\right) \end{aligned}$$

At $x = 1$ m and $y = 2$ m, this component of the force is

$$F_x = -\left(8\tfrac{J}{m^2}(1m) + 2\tfrac{J}{m} - 3\tfrac{J}{m^2}(2m)\right)$$
$$= -4N$$

The y-component of the force is

$$F_y = -\frac{\partial U}{\partial y}$$
$$= -\frac{\partial\left(4\tfrac{J}{m^2}x^2 + 2\tfrac{J}{m}(x+y) - 3\tfrac{J}{m^2}(xy)\right)}{\partial y}$$
$$= -\left(0 + 2\tfrac{J}{m} - 3\tfrac{J}{m^3}(x)\right)$$

At $x = 1$ m and $y = 2$ m, this component of the force is

$$F_y = -\left(2\tfrac{J}{m} - 3\tfrac{J}{m^3}(1m)\right)$$
$$= 1N$$

Therefore, the force is

$$\vec{F} = \left(-4\vec{i} + 1\vec{j}\right)N$$

6. If F_x is integrated with respect to x, the result is

$$U = -\int\left(-4\tfrac{N}{m^2}xy^2 + 2\tfrac{N}{m}x\right)dx$$
$$= 2\tfrac{N}{m^2}x^2y^2 - 1\tfrac{N}{m}x^2 + C_1$$

If F_y is integrated with respect to y, the result is

$$U = -\int\left(-4\tfrac{N}{m^2}x^2y + 4\tfrac{N}{m}y\right)dy$$
$$= 2\tfrac{N}{m^2}x^2y^2 - 2\tfrac{N}{m}y^2 + C_2$$

The second term of the first integral is the part of U that depends only on x. The second term of the second integral is the part of U that depends only on y. The first terms of the two integrals are the part of U that depends on both x and y at the same time. We, therefore, come to the conclusion that U is equal to

$$U = 2\tfrac{N}{m^2}x^2y^2 - 1\tfrac{N}{m}x^2 - 2\tfrac{N}{m}y^2 + Cst$$

b) At the point (2,1), the potential energy is

$$\begin{aligned} U &= 2\tfrac{N}{m^2}(2m)^2(1m)^2 - 1\tfrac{N}{m}(2m)^2 - 2\tfrac{N}{m}(1m)^2 \\ &= 2J \end{aligned}$$

c) At the point (5,2), the potential energy is

$$\begin{aligned} U &= 2\tfrac{N}{m^2}(5m)^2(2m)^2 - 1\tfrac{N}{m}(5m)^2 - 2\tfrac{N}{m}(2m)^2 \\ &= 167J \end{aligned}$$

d) The work is

$$\begin{aligned} W &= -\Delta U \\ &= -(167J - 2J) \\ &= -165J \end{aligned}$$

7. The x-component of the force is

$$\begin{aligned} F_x &= -\frac{\partial U}{\partial x} \\ &= -\frac{\partial\left(4\tfrac{J}{m^2}x^2 + 2\tfrac{J}{m}(x+y+z) - 3\tfrac{J}{m^3}(xyz)\right)}{\partial x} \\ &= -\left(8\tfrac{J}{m^2}x + 2\tfrac{J}{m} - 3\tfrac{J}{m^3}(yz)\right) \end{aligned}$$

At x = 1 m, y = 2 m and z = - 4 m, this component is

$$\begin{aligned} F_x &= -\left(8\tfrac{J}{m^2}(1m) + 2\tfrac{J}{m} - 3\tfrac{J}{m^3}(2m\cdot -4m)\right) \\ &= -34N \end{aligned}$$

The y-component of the force is

$$\begin{aligned} F_y &= -\frac{\partial U}{\partial y} \\ &= -\frac{\partial\left(4\tfrac{J}{m^2}x^2 + 2\tfrac{J}{m}(x+y+z) - 3\tfrac{J}{m^3}(xyz)\right)}{\partial y} \\ &= -\left(0 + 2\tfrac{J}{m} - 3\tfrac{J}{m^3}(xz)\right) \end{aligned}$$

At x = 1 m, y = 2 m and z = - 4 m, this component is

$$\begin{aligned} F_y &= -\left(2\tfrac{J}{m} - 3\tfrac{J}{m^3}(1m \cdot -4m)\right) \\ &= -14N \end{aligned}$$

The z-component of the force is

$$\begin{aligned} F_z &= -\frac{\partial U}{\partial z} \\ &= -\frac{\partial\left(4\tfrac{J}{m^2}x^2 + 2\tfrac{J}{m}(x+y+z) - 3\tfrac{J}{m^3}(xyz)\right)}{\partial z} \\ &= -\left(0 + 2\tfrac{J}{m} - 3\tfrac{J}{m^3}(xy)\right) \end{aligned}$$

At x = 1 m, y = 2 m and z = - 4 m, this component is

$$\begin{aligned} F_z &= -\left(2\tfrac{J}{m} - 3\tfrac{J}{m^3}(1m \cdot 2m)\right) \\ &= 4N \end{aligned}$$

The force is therefore

$$\vec{F} = \left(-34\vec{i} - 14\vec{j} + 4\vec{k}\right)N$$

8. a) At equilibrium, the force exerted by the spring is equal to the force of gravity.

$$kx = mg$$

The stretching of the spring is therefore

$$\begin{aligned} x &= \frac{mg}{k} \\ &= \frac{0.05kg \cdot 9.8\frac{N}{kg}}{50\frac{N}{m}} \\ &= 0.0098m \end{aligned}$$

The spring energy is therefore

$$\begin{aligned} U_{sp} &= \frac{1}{2}kx^2 \\ &= \frac{1}{2}50\tfrac{N}{m} \cdot (0.0098m)^2 \\ &= 0.002401J \end{aligned}$$

b) At equilibrium, the force exerted by the spring is equal to the force of gravity.

$$kx' = mg$$

The stretching of the spring is therefore

$$\begin{aligned} x' &= \frac{mg}{k} \\ &= \frac{0.55kg \cdot 9.8\frac{N}{kg}}{50\frac{N}{m}} \\ &= 0.1078m \end{aligned}$$

The spring energy is therefore

$$\begin{aligned} U_{sp}' &= \frac{1}{2}kx'^2 \\ &= \frac{1}{2}50\tfrac{N}{m} \cdot (0.1078m)^2 \\ &= 0.290521J \end{aligned}$$

c) The work done by the spring is

$$\begin{aligned} W_{sp} &= -\Delta U_{sp} \\ &= -(0.290521J - 0.002401J) \\ &= -0.28812J \end{aligned}$$

9. a) As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

Originally (car at point A), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy \\ &= \frac{1}{2}mv^2 + 2000kg \times 9.8\tfrac{N}{kg} \times 25m \\ &= \frac{1}{2}mv^2 + 490,000J \end{aligned}$$

We have chosen to set the $y = 0$ at point B.

The mechanical energy to point B is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= \frac{1}{2}2000kg \cdot \left(25\tfrac{m}{s}\right)^2 + 0J \\ &= 625,000J \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ \frac{1}{2}mv^2 + 490,000J &= 625,000J \\ \frac{1}{2}mv^2 &= 135,000J \\ \frac{1}{2}2000kg \cdot v^2 &= 135,000J \\ v &= 11.62\tfrac{m}{s} \end{aligned}$$

b) The mechanical energy at point C is

$$\begin{aligned}E'' &= \frac{1}{2}mv''^2 + mgy'' \\ &= \frac{1}{2}mv''^2 + 2000kg \cdot 9.8\tfrac{N}{kg} \cdot 5m \\ &= \frac{1}{2}mv''^2 + 98{,}000J\end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned}E' &= E'' \\ 625{,}000J &= \frac{1}{2}mv''^2 + 98{,}000J \\ 527{,}000J &= \frac{1}{2}2000kg \times v''^2 \\ v &= 22.96\tfrac{m}{s}\end{aligned}$$

10. As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

Initially (spring compressed), the mechanical energy is

$$\begin{aligned}E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0J + 0J + \frac{1}{2}2000\tfrac{N}{m} \cdot (0.2m)^2 \\ &= 40J\end{aligned}$$

We have chosen to set the $y = 0$ at the ground level.

a) When the spring is compressed 5 cm, the mechanical energy is

$$\begin{aligned}E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}mv'^2 + 0J + \frac{1}{2}2000\tfrac{N}{m} \cdot (0.05m)^2 \\ &= \frac{1}{2}mv'^2 + 2.5J\end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$
\begin{aligned}
E &= E' \\
40J &= \frac{1}{2}mv'^2 + 2.5J \\
37.5J &= \frac{1}{2}10kg \times v'^2 \\
v' &= 2.739\tfrac{m}{s}
\end{aligned}
$$

b) When the spring is no longer compressed, the mechanical energy is

$$
\begin{aligned}
E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\
&= \frac{1}{2}mv'^2 + 0J + 0J \\
&= \frac{1}{2}mv'^2
\end{aligned}
$$

According to the law of conservation of mechanical energy, we have

$$
\begin{aligned}
E &= E' \\
40J &= \frac{1}{2}mv'^2 \\
40J &= \frac{1}{2}10kg \times v'^2 \\
v' &= 2.828\tfrac{m}{s}
\end{aligned}
$$

11. As the system is composed of an object and a spring, the mechanical energy of the system is

$$
E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2
$$

Initially (spring compressed), the mechanical energy is

$$
\begin{aligned}
E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\
&= 0J + 0J + \frac{1}{2}500\tfrac{N}{m} \cdot (0.2m)^2 \\
&= 10J
\end{aligned}
$$

We have chosen to set the $y = 0$ at the ground level.

When the mass is at its maximum height on the slope, the mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= 0J + mgy' + 0J \\ &= mgy' \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ 10J &= mgy' \\ 10J &= 2kg \cdot 9.8\tfrac{N}{kg} \cdot y' \\ y' &= 0.5102m \end{aligned}$$

The displacement is therefore

$$\begin{aligned} \sin 45° &= \frac{y'}{D} \\ D &= \frac{y'}{\sin 45°} \\ D &= \frac{0.5102m}{\sin 45°} \\ D &= 0.7215m = 72.15cm \end{aligned}$$

12. As the system is composed of an object and two springs, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2$$

Initially (instant 1), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 \\ &= 0J + 0J + \frac{1}{2}100\tfrac{N}{m} \cdot (0.5m)^2 + \frac{1}{2}200\tfrac{N}{m} \cdot (0.1m)^2 \\ &= 12.5J + 1J \\ &= 13.5J \end{aligned}$$

We have chosen to set the $y = 0$ at the ground level.

The mechanical energy at instant 2 is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}k_1x_1'^2 + \frac{1}{2}k_2x_2'^2 \\ &= \frac{1}{2}mv'^2 + 0J + \frac{1}{2}100\tfrac{N}{m}\cdot(0.25m)^2 + \frac{1}{2}200\tfrac{N}{m}\cdot(0.15m)^2 \\ &= \frac{1}{2}mv'^2 + 3.125J + 2.25J \\ &= \frac{1}{2}mv^2 + 5.375J \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ 13.5J &= \frac{1}{2}mv'^2 + 5.375J \\ 8.125J &= \frac{1}{2}5kg \cdot v'^2 \\ v' &= 1.803\tfrac{m}{s} \end{aligned}$$

13. As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

Initially (instant 1), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0J + 2kg \cdot 9.8\tfrac{N}{kg} \cdot 0.5m + 0J \\ &= 9.8J \end{aligned}$$

We have chosen to set the $y = 0$ at the end of the tube.

The mechanical energy at instant 2 is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= 0J + 2kg \cdot 9.8\tfrac{N}{kg} \cdot (-d) + \frac{1}{2}100\tfrac{N}{m} \cdot d^2 \\ &= 19.6N \cdot (-d) + 50\tfrac{N}{m} d^2 \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ 9.8J &= 19.6N \cdot (-d) + 50\tfrac{N}{m} d^2 \\ 50\tfrac{N}{m} d^2 &- 19.6N \cdot d - 9.8J = 0 \end{aligned}$$

The solution of this equation is d = 0.6801 m.

(The other solution d = -0.2881 m must be rejected since it corresponds to a stretching of the spring, which does not make sense here.)

14. As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

Initially (spring compressed), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0J + 0J + \frac{1}{2}2000\tfrac{N}{m} \cdot (0.2m)^2 \\ &= 40J \end{aligned}$$

We have chosen to set the y = 0 at the ground level.

The mechanical energy at instant 2 (spring partially compressed) is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}mv'^2 + 0J + \frac{1}{2}kx'^2 \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$E = E'$$

$$40J = \frac{1}{2}mv'^2 + \frac{1}{2}kx'^2$$

Since the kinetic energy is equal to the spring energy

$$\frac{1}{2}mv'^2 = \frac{1}{2}kx'^2$$

Our equation becomes

$$\begin{aligned}
40J &= \frac{1}{2}mv'^2 + \frac{1}{2}kx'^2 \\
40J &= \frac{1}{2}kx'^2 + \frac{1}{2}kx'^2 \\
40J &= kx'^2 \\
40J &= 2000\tfrac{N}{m}x'^2 \\
x' &= 0.1414m
\end{aligned}$$

15. As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

Initially (car at point 1), the mechanical energy is

$$\begin{aligned}
E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\
&= \frac{1}{2}2000kg \cdot \left(10\tfrac{m}{s}\right)^2 + 2000kg \cdot 9.8\tfrac{N}{kg} \cdot 30m + 0J \\
&= 100,000J + 588,000J \\
&= 688,000J
\end{aligned}$$

We have chosen to set the $y = 0$ at the point 2.

a) The maximum speed will be reached at the lowest point, so at point 2. At this point, the mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}mv'^2 + 0J + 0J \\ &= \frac{1}{2}mv'^2 \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ 688{,}000J &= \frac{1}{2}mv'^2 \\ 688{,}000J &= \frac{1}{2}2000kg \cdot v'^2 \\ v' &= 26.23\tfrac{m}{s} \end{aligned}$$

b) When the spring compression is maximum, the mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= 0J + 2000kg \cdot 9{,}8\tfrac{N}{m} \cdot 15m + \frac{1}{2}kx'^2 \\ &= 294{,}000J + \frac{1}{2}kx'^2 \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ 688{,}000J &= 294{,}000J + \frac{1}{2}kx'^2 \\ 394{,}000J &= \frac{1}{2}800\tfrac{N}{m} \cdot x'^2 \\ x' &= 31.38m \end{aligned}$$

c) At this instant, the mechanical energy is

$$\begin{aligned}E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}mv'^2 + 2000kg \cdot 9.8\tfrac{N}{m} \cdot 15m + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}mv'^2 + 294{,}000J + \frac{1}{2}kx'^2\end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned}E &= E' \\ 688{,}000J &= \frac{1}{2}mv'^2 + 294{,}000J + \frac{1}{2}kx'^2 \\ 394{,}000J &= \frac{1}{2}mv'^2 + \frac{1}{2}kx'^2\end{aligned}$$

Since the kinetic energy is equal to twice the spring energy, we have

$$\begin{aligned}\frac{1}{2}mv'^2 &= 2 \cdot \frac{1}{2}kx'^2 \\ \frac{1}{4}mv'^2 &= \frac{1}{2}kx'^2\end{aligned}$$

Our equation becomes

$$\begin{aligned}394{,}000J &= \frac{1}{2}mv'^2 + \frac{1}{2}kx'^2 \\ 394{,}000J &= \frac{1}{2}mv'^2 + \frac{1}{4}mv'^2 \\ 394{,}000J &= \frac{3}{4}mv'^2 \\ 394{,}000J &= \frac{3}{4}2000kg \cdot v'^2 \\ v' &= 16.21\tfrac{m}{s}\end{aligned}$$

16. As the system is composed of two objects and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}m_1v_1^2 + m_1gy_1 + \frac{1}{2}m_2v_2^2 + m_2gy_2 + \frac{1}{2}kx^2$$

Initially (instant 1), the mechanical energy is

$$\begin{aligned}E &= \frac{1}{2}m_1v_1^2 + m_1gy_1 + \frac{1}{2}m_2v_2^2 + m_2gy_2 + \frac{1}{2}kx^2 \\ &= 0J + 0J + 0J + 0J + \frac{1}{2}195\tfrac{N}{m}\cdot(0.15m)^2 \\ &= 2.19375J\end{aligned}$$

We have chosen to set the $y = 0$ of each block at its position at instant 1.

At instant 2, the 30 kg block is 10 cm lower while the 25 kg block has moved 10 cm towards the top of the slope. The height variation in height of this block is given by

$$\begin{aligned}\sin 40° &= \frac{y}{10cm} \\ y &= 6.4279m\end{aligned}$$

The mechanical energy at instant 2 is then

$$\begin{aligned}E' &= \frac{1}{2}m_1v'^2 + m_1gy_1' + \frac{1}{2}m_2v'^2 + m_2gy_2' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}30kg\cdot v'^2 + 30kg\cdot 9.8\tfrac{N}{kg}\cdot(-0.1m) + \frac{1}{2}25kg\cdot v'^2 \\ &\qquad + 25kg\cdot 9.8\tfrac{N}{kg}\cdot 0.064279m + \frac{1}{2}195\tfrac{N}{m}\cdot(0.05m)^2 \\ &= \frac{1}{2}30kg\cdot v'^2 + -29.4J + \frac{1}{2}25kg\cdot v'^2 + 15.748J + 0.24375J \\ &= \frac{1}{2}55kg\cdot v'^2 + -13.408J\end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned}E &= E' \\ 2.19375J &= \frac{1}{2}55kg\cdot v'^2 + -13.408J \\ 15.602J &= \frac{1}{2}55kg\cdot v'^2 \\ v' &= 0.7532\tfrac{m}{s}\end{aligned}$$

17. As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

Initially (pendulum at point A), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy \\ &= 0J + mgy \end{aligned}$$

We have chosen to set the $y = 0$ at the lowest point of the path of the pendulum.

The height of the pendulum at point A is

$$\begin{aligned} y &= L(1-\cos\theta) \\ &= 1.2m(1-\cos 35°) \\ &= 0.217m \end{aligned}$$

The mechanical energy at point A is therefore

$$\begin{aligned} E &= mgy \\ &= 4kg \times 9.8\tfrac{N}{m} \times 0.217m \\ &= 8.507J \end{aligned}$$

The mechanical energy at point B is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= \frac{1}{2}mv'^2 + 0J \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ 8.507J &= \frac{1}{2}mv'^2 \\ 8.507J &= \frac{1}{2}4kg \cdot v'^2 \\ v' &= 2.062\tfrac{m}{s} \end{aligned}$$

18. As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

Initially (Radu end of a vertical rope with a speed v) the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy \\ &= \frac{1}{2}mv^2 \end{aligned}$$

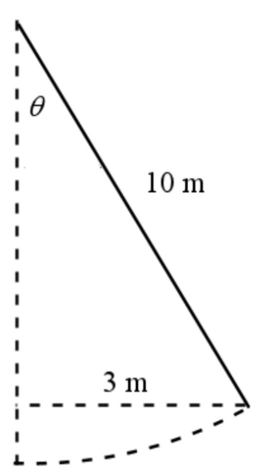
We have chosen to set the $y = 0$ at the lowest point on the path of the pendulum.

The mechanical energy at instant 2 (Radu at the highest point of the pendulum's motion) is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= mgy' \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ \frac{1}{2}mv^2 &= mgy' \\ \frac{1}{2}v^2 &= gy' \end{aligned}$$



The value of y_{min} when Radu crosses the ravine must be found. First, the minimum angle is found with

$$\begin{aligned} \sin\theta &= \frac{3m}{10m} \\ \theta &= 17.46° \end{aligned}$$

This corresponds to a height of

$$\begin{aligned} y_{\min} &= L(1-\cos\theta_{\min}) \\ &= 10m\cdot(1-\cos 17.46°) \\ &= 0.4606m \end{aligned}$$

The energy equation then becomes

$$\frac{1}{2}v_{\min}^2 = gy_{\min}$$
$$\frac{1}{2}v_{\min}^2 = 9.8\tfrac{N}{kg}\cdot 0.4606m$$
$$v_{\min} = 3.005\tfrac{m}{s}$$

19. As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

Initially (configuration shown in the figure), the height of the pendulum is

$$\begin{aligned} y &= L(1-\cos\theta) \\ &= 4m\cdot(1-\cos 25°) \\ &= 0.3748m \end{aligned}$$

The mechanical energy is then

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy \\ &= \frac{1}{2}2kg\cdot\left(2\tfrac{m}{s}\right)^2 + 2kg\cdot 9.8\tfrac{N}{kg}\cdot 0.3748m \\ &= 11.345J \end{aligned}$$

We have chosen to set the $y = 0$ at the lowest point on the path of the pendulum.

a) The mechanical energy at instant 2 (vertical pendulum) is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= \frac{1}{2}mv'^2 \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$
\begin{aligned}
E &= E' \\
11.345J &= \frac{1}{2}mv'^2 \\
11.345J &= \frac{1}{2}2kg \cdot v'^2 \\
v' &= 3.368\tfrac{m}{s}
\end{aligned}
$$

b) The mechanical energy at instant 3 (pendulum at its highest point) is

$$
\begin{aligned}
E' &= \frac{1}{2}mv'^2 + mgy' \\
&= mgy'
\end{aligned}
$$

According to the law of conservation of mechanical energy, we have

$$
\begin{aligned}
E &= E' \\
11.345J &= mgy' \\
11.345J &= 2kg \cdot 9.8\tfrac{N}{kg} \cdot y' \\
y' &= 0.57885m
\end{aligned}
$$

This corresponds to the angle

$$
\begin{aligned}
\cos\theta &= \frac{L - y'}{L} \\
\cos\theta &= \frac{4m - 0.57885m}{4m} \\
\theta &= 31.2^\circ
\end{aligned}
$$

20. As the system is composed of only one object, the mechanical energy of the system is

$$
E_{mec} = \frac{1}{2}mv^2 + mgy
$$

Initially (instant 1), the height of the pendulum is

$$
y = L(1 - \cos\theta)
$$

The mechanical energy is then

$$\begin{aligned}E &= \frac{1}{2}mv^2 + mgy \\ &= 0J + mgy \\ &= mgL(1-\cos 50°)\end{aligned}$$

We have chosen to set the $y = 0$ at the lowest point on the path of the pendulum.

The mechanical energy at instant 2 (the length of the rope is now *L'*) is

$$\begin{aligned}E' &= \frac{1}{2}mv'^2 + mgy' \\ &= 0J + mgy' \\ &= mgL'(1-\cos\theta)\end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned}E &= E' \\ mgL(1-\cos 50°) &= mgL'(1-\cos\theta) \\ L(1-\cos 50°) &= L'(1-\cos\theta) \\ 1.5m(1-\cos 50°) &= 0.5m(1-\cos\theta) \\ \cos\theta &= -0.0716 \\ \theta &= 94.1°\end{aligned}$$

21. As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}Mv^2 + Mgy$$

(*M* is used instead of *m* to avoid the confusion between the mass and the units (metre) later.)

Initially (car at position A), the height of the car is

$$\begin{aligned}y &= L(1-\cos\theta) \\ &= 5m(1-\cos 80°) \\ &= 4.132m\end{aligned}$$

The mechanical energy of the car is then

$$\begin{aligned} E &= \frac{1}{2}Mv^2 + Mgy \\ &= 0J + Mgy \\ &= Mg \cdot 4.132m \end{aligned}$$

We have chosen to set the $y = 0$ at the lowest point on the path of the car.

The mechanical energy at instant 2 (car at position B) is

$$\begin{aligned} E' &= \frac{1}{2}Mv'^2 + Mgy' \\ &= \frac{1}{2}Mv'^2 + 0J \\ &= \frac{1}{2}Mv'^2 \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ Mg \cdot 4.132m &= \frac{1}{2}Mv'^2 \\ g \cdot 4.132m &= \frac{1}{2}v'^2 \\ v' &= 8.999\tfrac{m}{s} \end{aligned}$$

The centripetal acceleration of a person in the car is then

$$a_c = \frac{v^2}{r} = \frac{\left(8.999\tfrac{m}{s}\right)^2}{5m} = 16.1965\tfrac{m}{s^2}$$

upwards.

The components of the apparent weight of a person in the car are therefore

$$\begin{aligned} w_{app\ x} &= -Ma_x \\ &\rightarrow w_{app\ x} = 0 \\ w_{app\ y} &= -Mg - Ma_y \\ &\rightarrow w_{app\ y} = -M \cdot 9.8\tfrac{N}{kg} - M \cdot 16.1965\tfrac{m}{s^2} \\ &\rightarrow w_{app\ y} = -M \cdot 25.9965\tfrac{N}{kg} \end{aligned}$$

The number of g is therefore

$$n_g = \frac{|w_{app}|}{w_{\text{actual on Earth}}} = \frac{M \cdot 25.9965 \frac{N}{kg}}{M \cdot 9.8 \frac{N}{kg}} = 2.653$$

22. The system being formed of one object (the ball) and two springs, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2$$

At instant 1 (stretched springs), the ball has no speed. The $y = 0$ is set between the two springs (where the ball is) so that $mgy = 0$. However, the springs are stretched. The length of the spring is equal to the hypotenuse of a triangle whose sides adjacent to the right angle are 50 cm and 30 cm long. The length of the springs is, therefore,

$$L = \sqrt{(50cm)^2 + (30cm)^2} = 58.31cm$$

As the springs were neither stretched nor compressed when their length was 50 cm, then they are stretched 8.31 cm. Therefore, the energy at the instant 1 is

$$\begin{aligned} E_{mec} &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2 x_2^2 \\ &= 0 + 0 + \frac{1}{2}500\tfrac{N}{m} \cdot (0.0831m)^2 + \frac{1}{2}500\tfrac{N}{m} \cdot (0.0831m)^2 \\ &= 3.452J \end{aligned}$$

At instant 2, the ball has some speed, the springs are no more stretched and the ball is still at $y = 0$. The energy at instant 2 is, therefore,

$$\begin{aligned} E'_{mec} &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}k_1 x_1'^2 + \frac{1}{2}k_2 x_2'^2 \\ &= \frac{1}{2}mv'^2 + 0 + 0 + 0 \end{aligned}$$

The mechanical energy conservation then gives

$$E = E'$$
$$3.452J = \frac{1}{2}mv'^2$$
$$3.452J = \frac{1}{2}\cdot 0.1kg \cdot v'^2$$
$$v' = 8.31\tfrac{m}{s}$$

23. As the system is composed of two objects, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}m_1v_1^2 + m_1gy_1 + \frac{1}{2}m_2v_2^2 + m_2gy_2$$

Initially (configuration shown in the figure), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}m_1v_1^2 + m_1gy_1 + \frac{1}{2}m_2v_2^2 + m_2gy_2 \\ &= 0J + 0J + 0J + 0J \\ &= 0J \end{aligned}$$

We have chosen to set the $y = 0$ each block at its initial position.

We know that the blocks moved 2 m, but we do not know in which direction. Therefore, there must be two solutions.

1st solution: the 6 kg block moves 2 m upwards

The mechanical energy after a 2 m upwards displacement is

$$\begin{aligned} E' &= \frac{1}{2}m_1v_1'^2 + m_1gy_1' + \frac{1}{2}m_2v_2'^2 + m_2gy_2' \\ &= \frac{1}{2}m_1v'^2 + 6kg\cdot 9.8\tfrac{N}{kg}\cdot 2m + \frac{1}{2}m_2v'^2 + 10kg\cdot 9.8\tfrac{N}{kg}\cdot 0m \\ &= \frac{1}{2}6kg\cdot v'^2 + \frac{1}{2}10kg\cdot v'^2 + 117.6J \\ &= 8kg\cdot v'^2 + 117.6J \end{aligned}$$

As there are external forces, the work done by these forces must be calculated.

$$W_{10N} = 10N\cdot 2m\cdot\cos(0°) = 20J$$
$$W_{20N} = 20N\cdot 2m\cdot\cos(180°) = -40J$$

The net work done by the external forces is thus -20 J.

We then have

$$E + W_{ext} = E'$$
$$0J + -20J = 8kg \cdot v'^2 + 117.6J$$
$$v'^2 = -17.2 \tfrac{m^2}{s^2}$$

This has no solution. It is, therefore, impossible for the 6 kg block to move 2 m upwards.

2[nd] solution: the 6 kg block moves 2 m downwards

The mechanical energy after a 2 m downwards displacement is

$$\begin{aligned}E' &= \frac{1}{2}m_1v_1'^2 + m_1gy_1' + \frac{1}{2}m_2v_2'^2 + m_2gy_2' \\ &= \frac{1}{2}m_1v'^2 + 6kg \cdot 9.8\tfrac{N}{kg} \cdot (-2m) + \frac{1}{2}m_2v'^2 + 10kg \cdot 9.8\tfrac{N}{kg} \cdot 0m \\ &= \frac{1}{2}6kg \cdot v'^2 + \frac{1}{2}10kg \cdot v'^2 - 117.6J \\ &= 8kg \cdot v'^2 - 117.6J\end{aligned}$$

As there are external forces, the work done by these forces must be calculated.

$$W_{10N} = 10N \cdot 2m \cdot \cos(180°) = -20J$$
$$W_{20N} = 20N \cdot 2m \cdot \cos(0°) = 40J$$

The net work done by the external forces is thus 20 J.

We thus have

$$E + W_{ext} = E'$$
$$0J + 20J = 8kg \cdot v'^2 - 117.6J$$
$$v'^2 = 17.2 \tfrac{m^2}{s^2}$$
$$v' = 4.147 \tfrac{m}{s}$$

24. As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

Initially (configuration shown in the figure), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0J + 0J + \frac{1}{2}20{,}000\tfrac{N}{m} \cdot (3m)^2 \\ &= 90{,}000J \end{aligned}$$

We have chosen to set the $y = 0$ at ground level.

The mechanical energy after a 50 m displacement is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}mv'^2 + 0J + 0J \\ &= \frac{1}{2}mv'^2 \end{aligned}$$

As there is a non-conservative force, the work done by this force must be calculated.

$$W_{non-cons} = 2000N \cdot 50m \cdot \cos(0°) = 100{,}000J$$

We thus have

$$\begin{aligned} E + W_{nc} &= E' \\ 90{,}000J + 100{,}000J &= \frac{1}{2}mv'^2 \\ 190{,}000J &= \frac{1}{2}500kg \cdot v'^2 \\ v' &= 27.568\tfrac{m}{s} \end{aligned}$$

25. As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

Initially (configuration shown in the figure), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy \\ &= 0J + mg\cdot(62m) \\ &= mg\cdot(62m) \end{aligned}$$

We have chosen to set the $y = 0$ at the base of the hill.

The mechanical energy after the descent and the motion on the flat surface is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' \\ &= 0J + 0J + 0J \\ &= 0J \end{aligned}$$

As there is a non-conservative force (the friction force), the work done by this force must be calculated.

$$\begin{aligned} W_{non-cons} &= F_f\Delta s\cdot\cos(180°) \\ &= \mu_k F_N \Delta s\cdot\cos(180°) \\ &= \mu_k mg\Delta s\cdot\cos(180°) \\ &= -\mu_k mg\Delta s \end{aligned}$$

We thus have

$$\begin{gathered} E + W_{nc} = E' \\ mg\cdot(62m) + -\mu_k mg\Delta s = 0J \\ (62m) - \mu_k\Delta s = 0m \\ 62m - 0.2\cdot\Delta s = 0m \\ \Delta s = 310m \end{gathered}$$

26. As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

Initially (configuration shown in the figure), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0J + 0J + \frac{1}{2}2000\tfrac{N}{m}\cdot(0.5m)^2 \\ &= 250J \end{aligned}$$

We have chosen to set the $y = 0$ at ground level.

When the block has stopped, the mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= 0J + 0J + 0J \\ &= 0J \end{aligned}$$

As there is a non-conservative force (the friction force), the work done by this force must be calculated.

$$\begin{aligned} W_{non-cons} &= F_f \Delta s \cdot \cos(180°) \\ &= \mu_k F_N \Delta s \cdot \cos(180°) \\ &= \mu_k mg \Delta s \cdot \cos(180°) \\ &= -\mu_k mg \Delta s \end{aligned}$$

We thus have

$$\begin{gathered} E + W_{non-cons} = E' \\ 250J + -\mu_k mg\Delta s = 0J \\ 250J - \mu_k \cdot 10kg \cdot 9.8\tfrac{N}{kg} \cdot 42m = 0m \\ \mu_k = 0.06074 \end{gathered}$$

27. a) As the system is composed of two objects and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}m_1v_1^2 + m_1gy_1 + \frac{1}{2}m_2v_2^2 + m_2gy_2 + \frac{1}{2}kx^2$$

Initially (configuration shown in the figure), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}m_1v_1^2 + m_1gy_1 + \frac{1}{2}m_2v_2^2 + m_2gy_2 + \frac{1}{2}kx^2 \\ &= 0J + 0J + 0J + 0J + 0J \\ &= 0J \end{aligned}$$

We have chosen to set the $y = 0$ of each block at its initial position.

We know that the 36 kg block moved downwards 1 m (as it is impossible for this block to move upwards).

After the 1 m displacement, the mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2}m_1v_1'^2 + m_1gy_1' + \frac{1}{2}m_2v_2'^2 + m_2gy_2' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}36kg \cdot v'^2 + 36kg \cdot 9.8\tfrac{N}{kg} \cdot (-1m) + \frac{1}{2}12kg \cdot v'^2 + 10kg \cdot 9.8\tfrac{N}{kg} \cdot 0m + \frac{1}{2}200\tfrac{N}{m}(1m)^2 \\ &= 18kg \cdot v'^2 - 352.8J + 6kg \cdot v'^2 + 0J + 100J \\ &= 24kg \cdot v'^2 - 252.8J \end{aligned}$$

As there is a non-conservative force (the friction force on the 12 kg block), the work done by this force must be calculated.

$$\begin{aligned} W_{non-cons} &= F_f \Delta s \cdot \cos(180°) \\ &= \mu_k F_N \Delta s \cdot \cos(180°) \\ &= \mu_k mg \Delta s \cos(180°) \\ &= 0.4 \cdot 12kg \cdot 9.8\tfrac{N}{kg} \cdot 1m \cdot \cos 180° \\ &= -47.04J \end{aligned}$$

We thus have

$$E + W_{non-cons} = E'$$
$$0J + -47.04J = 24kg \cdot v'^2 - 252.8J$$
$$205.76J = 24kg \cdot v'^2$$
$$v' = 2.928 \tfrac{m}{s}$$

b) After the 3.2 m displacement, the mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2}m_1 v_1'^2 + m_1 g y_1' + \frac{1}{2}m_2 v_2'^2 + m_2 g y_2' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}36kg \cdot v'^2 + 36kg \cdot 9.8\tfrac{N}{kg} \cdot (-3.2m) + \frac{1}{2}12kg \cdot v'^2 + 10kg \cdot 9.8\tfrac{N}{kg} \cdot 0m + \frac{1}{2}200\tfrac{N}{m}(3.2m)^2 \\ &= 18kg \cdot v'^2 - 1128.96J + 6kg \cdot v'^2 + 0J + 1024J \\ &= 24kg \cdot v'^2 - 104.96J \end{aligned}$$

As there is a non-conservative force (the friction force on the 12 kg block), the work done by this force must be calculated.

$$\begin{aligned} W_{non-cons} &= F_f \Delta s \cdot \cos(180°) \\ &= \mu_k F_N \Delta s \cdot \cos(180°) \\ &= \mu_k mg \Delta s \cos(180°) \\ &= 0.4 \cdot 12kg \cdot 9.8\tfrac{N}{kg} \cdot 3.2m \cdot \cos 180° \\ &= -150.528J \end{aligned}$$

We thus have

$$E + W_{non-cons} = E'$$
$$0J + -150.528J = 24kg \cdot v'^2 - 104.96J$$
$$-45.568J = 24kg \cdot v'^2$$

This has no solution. This means that the blocks cannot have a 3.2 m displacement.

28. As the system is composed of an object and a spring, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2$$

Initially (configuration shown in the figure), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + mgy + \frac{1}{2}kx^2 \\ &= 0J + 0J + 0J \\ &= 0J \end{aligned}$$

We have chosen to set the $y = 0$ at the initial position of the block.

When the block is at rest after hitting the spring, it has descended 3m + d. The height of the block is then

$$\begin{aligned} \sin 30° &= \frac{y}{3m + d} \\ y &= (3m + d)\sin 30° \\ y &= \tfrac{1}{2}(3m + d) \end{aligned}$$

The mechanical energy is now

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + mgy' + \frac{1}{2}kx'^2 \\ &= 0J + mg \cdot -\frac{1}{2}(3m + d) + \frac{1}{2}kd^2 \\ &= -\frac{1}{2}mg(3m + d) + \frac{1}{2}kd^2 \end{aligned}$$

As there is a non-conservative force (the friction force), the work done by this force must be calculated.

$$\begin{aligned} W_{non-cons} &= F_f \Delta s \cdot \cos(180°) \\ &= \mu_k F_N \Delta s \cdot \cos(180°) \\ &= \mu_k mg \sin(60°) \cdot (3m + d)\cos(180°) \\ &= -\mu_k mg \sin(60°)(3m + d) \end{aligned}$$

We thus have

$$E + W_{non-cons} = E'$$
$$0J + -\mu_k mg \sin(60°)(3m + d) = -\frac{1}{2} mg(3m + d) + \frac{1}{2} kd^2$$
$$-\mu_k mg \sin(60°)(3m + d) = -\frac{1}{2} mg(3m + d) + \frac{1}{2} kd^2$$
$$-0.2 \cdot 3kg \cdot 9.8\tfrac{N}{kg} \sin(60°)(3m + d) = -\frac{1}{2} 3kg \cdot 9.8\tfrac{N}{kg}(3m + d) + \frac{1}{2} 400\tfrac{N}{m} d^2$$
$$-5.09223N(3m + d) = -14.7N(3m + d) + 200\tfrac{N}{m} d^2$$
$$-15.2767J - 5.09223N \cdot d = -44.1J - 14.7N \cdot d + 200\tfrac{N}{m} d^2$$
$$-28.8233J - 9.60777N \cdot d + 200\tfrac{N}{m} d^2 = 0$$

The solution of this equation is d = 0.4044 m.

(There is another negative solution corresponding to a stretching of the spring, which makes no sense here.)

29. As the system is composed of two objects, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2} m_1 v_1^2 + m_1 g y_1 + \frac{1}{2} m_2 v_2^2 + m_2 g y_2$$

Initially (configuration shown in the figure), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2} m_1 v_1^2 + m_1 g y_1 + \frac{1}{2} m_2 v_2^2 + m_2 g y_2 \\ &= 0J + 0J + 0J + 0J \\ &= 0J \end{aligned}$$

We have chosen to set the y = 0 each block at its initial position.

The mechanical energy after a 2 m downwards displacement is

$$\begin{aligned} E' &= \frac{1}{2} m_1 v_1'^2 + m_1 g y_1' + \frac{1}{2} m_2 v_2'^2 + m_2 g y_2' \\ &= \frac{1}{2} m_1 v'^2 + 6kg \cdot 9.8\tfrac{N}{kg} \cdot (-2m) + \frac{1}{2} m_2 v'^2 + 10kg \cdot 9.8\tfrac{N}{kg} \cdot 0m \\ &= \frac{1}{2} 6kg \cdot v'^2 + \frac{1}{2} 10kg \cdot v'^2 - 117.6J \\ &= 8kg \cdot v'^2 - 117.6J \end{aligned}$$

As there is an external force, the work done by this force must be calculated.

$$W_{20N} = 20N \cdot 2m \cdot \cos(0°) = 40J$$

As there is a non-conservative force (the friction force), the work done by this force must be calculated.

$$\begin{aligned}W_{non-cons} &= F_f \Delta s \cdot \cos(180°) \\ &= \mu_k F_N \Delta s \cdot \cos(180°) \\ &= \mu_k F_N \Delta s \cdot \cos(180°) \\ &= 0.4 \cdot 10kg \cdot 9.8\tfrac{N}{kg} \cdot 2m \cos(180°) \\ &= -78,4J\end{aligned}$$

We thus have

$$\begin{aligned}E + W_{ext} + W_{non-cons} &= E' \\ 0J + 40J - 78,4J &= 8kg \cdot v'^2 - 117.6J \\ v'^2 &= 9,9\tfrac{m^2}{s^2} \\ v' &= 3.146\tfrac{m}{s}\end{aligned}$$

30. a) It cannot be at the places where U is larger than E, therefore at

$$x < 4 \text{ m} \quad \text{and} \quad x > 26 \text{ m}$$

b) At the minimum of U, thus at $x = 10$ m.

c) at $x = 20$, we have $U = 2.6$ J. Since the mechanical energy is 4 J, the kinetic energy is

$$\begin{aligned}E_{mec} &= E_k + U \\ 4J &= E_k + 2.6J \\ E_k &= 1.4J\end{aligned}$$

Then

$$E_K = 1.4J$$
$$\frac{1}{2}mv^2 = 1.4J$$
$$\frac{1}{2}2kg \cdot v^2 = 1.4J$$
$$v = 1.1832\tfrac{m}{s}$$

As this is a rough estimate, let's say 1.2 m/s.

d) Stable equilibrium at x = 10 m. The mechanical energy must be 0.2 J.
Stable equilibrium at x = 22 m. The mechanical energy must be 2 J.

e) Unstable equilibrium at x = 17 m. The mechanical energy must be 3.4 J.

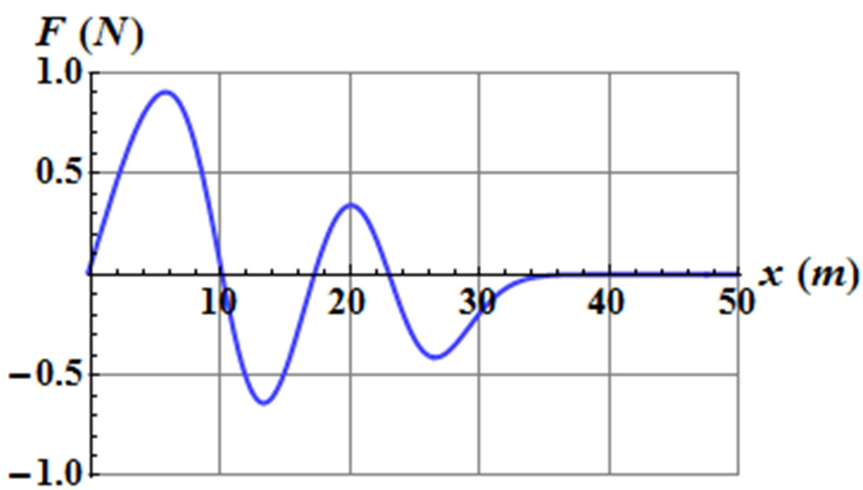
f) At x = 10 m, we have U = 0.2 J and at x = 20 m, we have U = 2.6 J. Therefore

$$\begin{aligned}\Delta U &= U_2 - U_1 \\ &= 2.6J - 0.2J \\ &= 2.4J\end{aligned}$$

The work is then

$$\begin{aligned}W &= -\Delta U \\ &= -2.4J\end{aligned}$$

g)



31. As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + \frac{-GM_T m}{r}$$

Initially (Neil at 400,000 km from the surface of the Earth), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + \frac{-GM_T m}{r} \\ &= 0J + \frac{-6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.972\times10^{24}\,kg\cdot 100kg}{406,371,000m} \\ &= -9.80806\times10^{7}\,J \end{aligned}$$

a) When the speed is 5000 m/s, the mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2}mv^2 + \frac{-GM_T m}{r} \\ &= \frac{1}{2}100kg\cdot\left(5000\,\tfrac{m}{s}\right)^2 + \frac{-6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.972\times10^{24}\,kg\cdot 100kg}{r'} \\ &= 1.25\times10^{9}\,J - \frac{3.991\times10^{16}\,Jm}{r'} \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ -9.80806\times10^{7}\,J &= 1.25\times10^{9}\,J - \frac{3.991\times10^{16}\,Jm}{r'} \\ -1.3481\times10^{9}\,J &= -\frac{3.991\times10^{16}\,Jm}{r'} \\ r' &= 2.9566\times10^{7}\,m \\ r' &= 29,566km \end{aligned}$$

Therefore, the distance from the Earth's surface is

29,566 km – 6371 km = 23,195 km

b) At the arrival at the surface of the Earth, the mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + \frac{-GM_T m}{r} \\ &= \frac{1}{2}mv'^2 + \frac{-6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.972\times10^{24}kg\cdot 100kg}{6,371,000m} \\ &= \frac{1}{2}mv'^2 - 6,2560\times10^9 J \end{aligned}$$

We then have

$$\begin{aligned} E &= E' \\ -9.80806\times10^7 J &= \frac{1}{2}mv'^2 - 6.2560\times10^9 J \\ 6.1579\times10^9 J &= \frac{1}{2}mv'^2 \\ 6.1579\times10^9 J &= \frac{1}{2}100kg\cdot v'^2 \\ v' &= 11,098\,\tfrac{m}{s} = 11.098\,\tfrac{km}{s} \end{aligned}$$

32. The escape velocity is

$$\begin{aligned} v_{lib} &= \sqrt{\frac{2GM_c}{R_c}} \\ &= \sqrt{\frac{2\cdot 6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 7.34\times10^{22}kg}{1.737\times10^6 m}} \\ &= 2375\,\tfrac{m}{s} \end{aligned}$$

33. As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + \frac{-GM_M m}{r}$$

Initially (object at the surface of the Moon), the mechanical energy is

$$\begin{aligned} E &= \frac{1}{2}mv^2 + \frac{-GM_M m}{r} \\ &= \frac{1}{2}mv^2 + \frac{-6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot 7.34\times10^{22}kg\cdot m}{1.737\times10^6 m} \\ &= \frac{1}{2}mv^2 - 2.8202\times10^6\frac{J}{kg}\cdot m \end{aligned}$$

When the object reaches its maximum height of 3000 km, its velocity is zero. The mechanical energy is then

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + \frac{-GM_M m}{r'} \\ &= 0J + \frac{-6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot 7.34\times10^{22}kg\cdot m}{4.737\times10^6 m} \\ &= -1.0341\times10^6\frac{J}{kg}\cdot m \end{aligned}$$

According to the law of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ \frac{1}{2}mv^2 - 2.8202\times10^6\frac{J}{kg}\cdot m &= -1.0341\times10^6\frac{J}{kg}\cdot m \\ \frac{1}{2}v^2 - 2.8202\times10^6\frac{J}{kg} &= -1.0341\times10^6\frac{J}{kg} \\ \frac{1}{2}v^2 &= 1.786\times10^6\frac{J}{kg} \\ v &= 1890\frac{m}{s} \end{aligned}$$

34. As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + \frac{-GM_E m}{r}$$

Initially (satellite at rest on the surface of the Earth), the mechanical energy is

$$\begin{aligned}
E &= \frac{1}{2}mv^2 + \frac{-GM_E m}{r} \\
&= 0J + \frac{-6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.972\times10^{24}\,kg\cdot 350kg}{6.371\times10^6\,m} \\
&= -2.1896\times10^{10}\,J
\end{aligned}$$

When the satellite is in orbit, its mechanical energy is

$$\begin{aligned}
E' &= \frac{1}{2}mv'^2 + \frac{-GM_E m}{r'} \\
&= \frac{-GM_T m}{2r'} \\
&= \frac{-6.674\times10^{-11}\,\frac{Nm^2}{kg^2}\cdot 5.972\times10^{24}\,kg\cdot 350kg}{2\cdot 6.871\times10^6\,m} \\
&= -1.0151\times10^{10}\,J
\end{aligned}$$

We thus have

$$\begin{aligned}
E + W_{non-cons} &= E' \\
-2.1896\times10^{10}\,J + W_{non-con} &= -1.0151\times10^{10}\,J \\
W_{non-cons} &= 1.174\times10^{10}\,J
\end{aligned}$$

35. As the system is composed of only one object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + \frac{-GM_E m}{r}$$

Initially (satellite at rest on the surface of the Earth), the mechanical energy is

$$\begin{aligned}
E &= \frac{1}{2}mv^2 + \frac{-GM_T m}{R_T} \\
&= 0J + \frac{-GM_T m}{R_T}
\end{aligned}$$

When the satellite is in orbit, its mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2}mv'^2 + \frac{-GM_T m}{r'} \\ &= \frac{-GM_T m}{2r'} \end{aligned}$$

The energy given to the satellite is, therefore,

$$\begin{aligned} E + W_{non-cons} &= E' \\ \frac{-GM_T m}{R_T} + W_{non-con} &= \frac{-GM_T m}{2r'} \\ W_{non-cons} &= \frac{-GM_T m}{2r'} + \frac{GM_T m}{R_T} \\ W_{non-cons} &= GM_T m\left(\frac{1}{R_T} - \frac{1}{2r'}\right) \end{aligned}$$

The kinetic energy of the satellite is ½mv^2. However, the speed of the orbiting satellite is

$$v^2 = \sqrt{\frac{GM_T}{r}}$$

Which means that the kinetic energy on an orbit of radius *r'* is

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{GmM_T}{r'}$$

Thus, the proportion of kinetic energy given relative to the total energy given is

$$\begin{aligned} \frac{mv^2}{W_{non-cons}} &= \frac{\frac{1}{2}\frac{GM_T m}{r'}}{GM_T m\left(\frac{1}{R_T} - \frac{1}{2r'}\right)} \\ &= \frac{\frac{1}{2r'}}{\left(\frac{1}{R_T} - \frac{1}{2r'}\right)} \\ &= \frac{R_T}{2r' - R_T} \\ &= \frac{6371km}{2(6371km + 150km) - 6371km} \\ &= 0.9550 \end{aligned}$$

Thus, 95.5% of the given energy given to this satellite in low orbit is in the form of kinetic energy.

b) If 50% of the given energy is in the form of kinetic energy, then

$$\begin{aligned} 0.5 &= \frac{R_T}{2r' - R_T} \\ 0.5 &= \frac{6371km}{2r' - 6371km} \\ r' &= 9556.5km \end{aligned}$$

36. a) As the system is composed of only one object (the Moon), the mechanical energy of the system is

$$E_{mec} = \frac{1}{2} M_M v^2 + \frac{-GM_E M_M}{r}$$

Initially (Moon orbiting the Earth), the mechanical energy is

$$\begin{aligned} E &= \frac{-GM_E M_M}{2r} \\ &= \frac{-6.674 \times 10^{-11} \frac{Nm^2}{kg^2} \cdot 5.972 \times 10^{24} kg \cdot 7.34 \times 10^{22} kg}{2 \cdot 3.844 \times 10^8 m} \\ &= -3{,}8053 \times 10^{28} J \end{aligned}$$

When the Moon is far from the Earth, the mechanical energy is

$$\begin{aligned} E' &= \frac{1}{2} M_M v'^2 + \frac{-GM_E M_M}{r'} \\ &= \frac{1}{2} M_M v'^2 + 0J \end{aligned}$$

We thus have

$$\begin{aligned} E + W_{ext} &= E' \\ -3.8053 \times 10^{28} J + W_{ext} &= \frac{1}{2} M_M v'^2 \\ W_{ext} &= 3.8053 \times 10^{28} J + \frac{1}{2} M_M v'^2 \end{aligned}$$

The minimum energy is therefore 3.8053 x 10^{28} J.

b) This energy corresponds to

$$\frac{3.8053\times10^{28}J}{6.3\times10^{13}J}=6.04\times10^{14}$$

times the Hiroshima atomic bomb. This is 604,000 billion times the Hiroshima bomb. Nasty explosion!

37. As the system is composed of only one object (the probe), the mechanical energy of the system is

$$E_{mec}=\frac{1}{2}mv^2+\frac{-GM_Em}{r_E}+\frac{-GM_Mm}{r_M}$$

There are two gravitational energies because here we must take into account the gravitational energy made by the Earth and the Moon.

Initially (probe at rest on the surface of the Earth), the mechanical energy is

$$\begin{aligned}E&=\frac{1}{2}mv^2+\frac{-GM_Em}{r_E}+\frac{-GM_Mm}{r_M}\\&=0J-\frac{6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot5.972\times10^{24}kg\cdot100kg}{6.371\times10^6m}-\frac{6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot7.34\times10^{22}kg\cdot100kg}{3.78029\times10^8m}\\&=-6.2560\times10^9J-1.2959\times10^6J\\&=-6.2573\times10^9J\end{aligned}$$

When the probe is at rest on the surface of the Moon, the mechanical energy is

$$\begin{aligned}E'&=\frac{1}{2}mv'^2+\frac{-GM_Em}{r_E'}+\frac{-GM_Mm}{r_M'}\\&=0J-\frac{6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot5.972\times10^{24}kg\cdot100kg}{3.82663\times10^8m}-\frac{6.674\times10^{-11}\frac{Nm^2}{kg^2}\cdot7.34\times10^{22}kg\cdot100kg}{1.737\times10^6m}\\&=-1.0416\times10^8J-2.8202\times10^8J\\&=-3.8618\times10^8J\end{aligned}$$

We thus have

$$\begin{aligned} E + W_{non-cons} &= E' \\ -6.2573\times10^{9} J + W_{non-cons} &= -3.8618\times10^{8} J \\ W_{non-cons} &= 5.871\times10^{9} J \end{aligned}$$

38. a) The mechanical energy is

$$\begin{aligned} E_{mec} &= \frac{1}{2}mv^2 + \frac{-GM_E m}{r} \\ &= \frac{-GM_E m}{2r} \\ &= \frac{-6.674\times10^{-11} \frac{Nm^2}{kg^2} \cdot 5.972\times10^{24} kg \cdot 500kg}{2\cdot 7.371\times10^{6} m} \\ &= -1.3518\times10^{10} J \end{aligned}$$

b) The gravitational energy is

$$\begin{aligned} U_g &= \frac{-GM_E m}{r} \\ &= \frac{-6.674\times10^{-11} \frac{Nm^2}{kg^2} \cdot 5.972\times10^{24} kg \cdot 500kg}{7.371\times10^{6} m} \\ &= -2.7036\times10^{10} J \end{aligned}$$

c) The kinetic energy is

$$\begin{aligned} E_{mec} &= E_k + U_g \\ -1,3518\times10^{10} J &= E_k + -2,7036\times10^{10} J \\ E_k &= 1,3518\times10^{10} J \end{aligned}$$

Alternate solution

The speed of the satellite is

$$\begin{aligned} v &= \sqrt{\frac{GM_E}{r}} \\ &= \sqrt{\frac{6.674\times 10^{-11} \frac{Nm^2}{kg^2} \cdot 5.972\times 10^{24} kg}{7.371\times 10^6 m}} \\ &= 7353 \tfrac{m}{s} \end{aligned}$$

The kinetic energy is then

$$\begin{aligned} E_k &= \frac{1}{2} mv^2 \\ &= \frac{1}{2} 500kg \times \left(7353 \tfrac{m}{s}\right)^2 \\ &= 1.3518\times 10^{10} J \end{aligned}$$

d) For the satellite to escape the Earth, its energy must be positive. At a minimum, it vanishes. Therefore

$$\begin{aligned} E + W_{ext} &= E' \\ -1.3518\times 10^{10} J + W_{ext} &= 0J \\ W_{ext} &= 1.3518\times 10^{10} J \end{aligned}$$

(We assumed that this work is done by an external force, but we might also have assumed that this work is done by a non-conservative force.)

39. a) Let's check if the force respects all 3 conditions. The first condition is

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

The derivatives are

$$\frac{\partial F_x}{\partial y} = \frac{\partial \left(5 \tfrac{N}{m^2} yz - 2 \tfrac{N}{m} x + 2 \tfrac{N}{m} y\right)}{\partial y} = 5 \tfrac{N}{m^2} z + 2 \tfrac{N}{m}$$

$$\frac{\partial F_y}{\partial x} = \frac{\partial \left(5 \tfrac{N}{m^2} xz - 4 \tfrac{N}{m} y + 2 \tfrac{N}{m} x\right)}{\partial x} = 5 \tfrac{N}{m^2} z + 2 \tfrac{N}{m}$$

Since the derivatives are equal, the first condition is met.

The second condition is

$$\frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x}$$

The derivatives are

$$\frac{\partial F_x}{\partial z} = \frac{\partial\left(5\frac{N}{m^2}yz - 2\frac{N}{m}x + 2\frac{N}{m}y\right)}{\partial z} = 5\frac{N}{m^2}y$$

$$\frac{\partial F_z}{\partial x} = \frac{\partial\left(5\frac{N}{m^2}xy + 6\frac{N}{m}z\right)}{\partial x} = 5\frac{N}{m^2}y$$

Since the derivatives are equal, the second condition is met.

The third condition is

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}$$

The derivatives are

$$\frac{\partial F_y}{\partial z} = \frac{\partial\left(5\frac{N}{m^2}xz - 4\frac{N}{m}y + 2\frac{N}{m}x\right)}{\partial z} = 5\frac{N}{m^2}x$$

$$\frac{\partial F_z}{\partial y} = \frac{\partial\left(5\frac{N}{m^2}xy + 6\frac{N}{m}z\right)}{\partial y} = 5\frac{N}{m^2}x$$

Since the derivatives are equal, the third condition is met.

Since all three conditions are met, the force is conservative.

b) To find U, the partial integrals must be calculated.

The partial integral with respect to x will give us all the terms that depend on x only, on x and y at the same time, on x and z at the same time and on x, y and z at the same time.

$$U = -\int F_x dx = \int\left(5\frac{N}{m^2}yx - 2\frac{N}{m}x + 2\frac{N}{m}y\right)dx = 5\frac{N}{m^2}xyx - 1\frac{N}{m}x^2 + 2\frac{N}{m}xy + C_1$$

The partial integral with respect to *x* will give us all the terms that depend on *y* only, on *x* and *y* at the same time, on *y* and *z* at the same time and on *x*, *y* and *z* at the same time.

$$U=-\int F_y dy=\int\left(5\tfrac{N}{m^2}yx-4\tfrac{N}{m}y+2\tfrac{N}{m}x\right)dx=5\tfrac{N}{m^2}xyx-2\tfrac{N}{m}y^2+2\tfrac{N}{m}xy+C_2$$

The partial integral with respect to *z* will give us all the terms that depend on *z* only, on *x* and *z* at the same time, on *y* and *z* at the same time and on *x*, *y* and *z* at the same time.

$$U=-\int F_z dz=\int\left(5\tfrac{N}{m^2}xy+6\tfrac{N}{m}z\right)dx=5\tfrac{N}{m^2}xyx-3\tfrac{N}{m}z^2+C_3$$

Combining these results, it can be inferred that U is

$$U=5\tfrac{N}{m^2}xyx-1\tfrac{N}{m}x^2-2\tfrac{N}{m}y^2-3\tfrac{N}{m}z^2+2\tfrac{N}{m}xy+Cst$$

Here, we'll choose $cst = 0$.

At the starting point (1 m, 1 m, 0 m), the potential energy is

$$\begin{aligned}U&=5\tfrac{N}{m^2}(1m)\cdot(1m)\cdot(0m)-1\tfrac{N}{m}(1m)^2-2\tfrac{N}{m}(1m)^2-3\tfrac{N}{m}(0m)^2+2\tfrac{N}{m}(1m)\cdot(1m)\\&=0J-1J-2J+0J+2J\\&=-1J\end{aligned}$$

At the ending point (4 m, -2 m, 3 m), the potential energy is

$$\begin{aligned}U&=5\tfrac{N}{m^2}(4m)\cdot(-2m)\cdot(3m)-1\tfrac{N}{m}(4m)^2-2\tfrac{N}{m}(-2m)^2-3\tfrac{N}{m}(3m)^2+2\tfrac{N}{m}(4m)\cdot(-2m)\\&=-120J-16J-8J-27J-16J\\&=-187J\end{aligned}$$

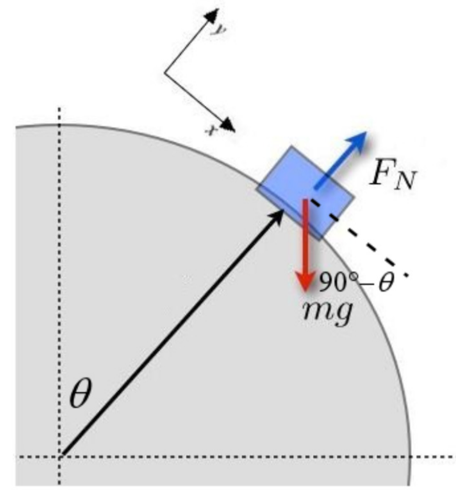
Thus, the work done is

$$\begin{aligned}W&=-\Delta U\\&=-(-187J--1J)\\&=186J\end{aligned}$$

40. a) When the person is in contact with the sphere, there is a normal force. Therefore, the angle when the normal force becomes zero must be found.

There are two forces exerted on the person:

1) The weight
2) A normal force



Using the axes shown in the figure, the equations of forces are:

$$\sum F_x = mg\cos-\left(90°-\theta\right) = ma_t$$

$$\sum F_x = mg\sin-\left(90°-\theta\right) + N = -\frac{mv^2}{R}$$

The second equation gives

$$mg\sin-\left(90°-\theta\right) + N = -\frac{mv^2}{R}$$

$$-mg\cos\theta + N = -\frac{mv^2}{R}$$

$$N = mg\cos\theta - \frac{mv^2}{R}$$

Initially (small angle), the first term is greater than the second, and there is a normal force. As the person slides, the angle increases (and the first term decreases) and the speed increases (and the second term increases). At a certain angle, the normal force becomes zero and the contact is lost. When normal is zero, the equation becomes

$$0 = mg\cos\theta - \frac{mv^2}{r}$$

$$g\cos\theta = \frac{v^2}{r}$$

To find the angle, the speed as a function of the angle must be found. This speed can be found with the conservation of mechanical energy.

As there is only a single object (the person who slides), the formula of the mechanical energy is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

At instant 1, (the person is at the top of the dome), the mechanical energy is

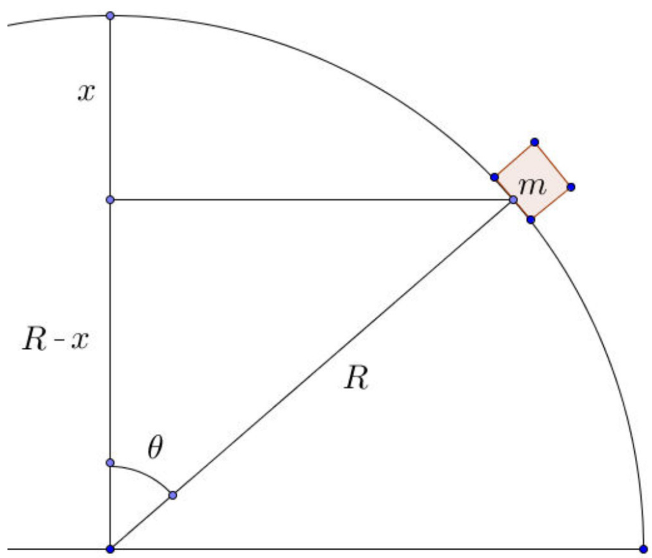
$$\begin{aligned} E_{mec} &= \frac{1}{2}mv^2 + mgy \\ &= 0 + 0 \\ &= 0 \end{aligned}$$

(The $y = 0$ was put at the top of the dome.)

At instant 2 (the person is at the angle θ), the mechanical energy is

$$E'_{mec} = \frac{1}{2}mv'^2 + mgy'$$

There is a link between height and angle. This link can be found with the following figure.



The height is $-x$. On the figure, it is easy to see that

$$\cos\theta = \frac{R - x}{R}$$

Therefore,

$$\begin{aligned} R\cos\theta &= R - x \\ x &= R - R\cos\theta \\ x &= R(1 - \cos\theta) \end{aligned}$$

Thus, the height is

$$y' = -R\left(1-\cos\theta\right)$$

Therefore, the mechanical energy at the instant 2 is

$$E'_{mec} = \frac{1}{2}mv'^2 - mgR\left(1-\cos\theta\right)$$

The conservation of mechanical energy then gives

$$\begin{aligned} E_{mec} &= E'_{mec} \\ 0 &= \frac{1}{2}mv'^2 - mgR\left(1-\cos\theta\right) \\ \frac{1}{2}mv'^2 &= mgR\left(1-\cos\theta\right) \\ v'^2 &= 2gR\left(1-\cos\theta\right) \end{aligned}$$

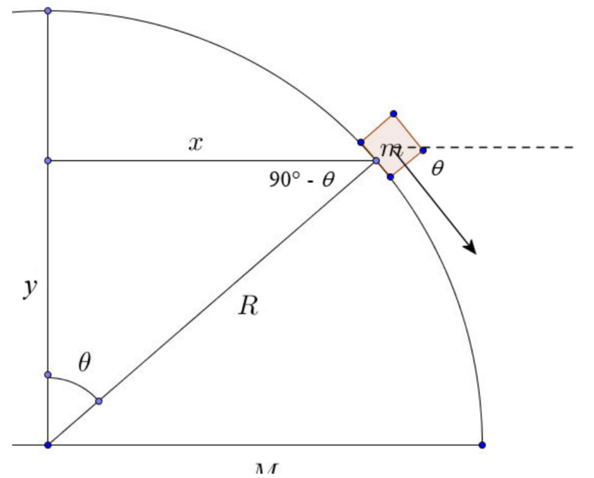
The condition for having a vanishing normal force then becomes

$$\begin{aligned} g\cos\theta &= \frac{v^2}{R} \\ g\cos\theta &= \frac{2gR\left(1-\cos\theta\right)}{R} \end{aligned}$$

It only remains to solve this equation for θ.

$$\begin{aligned} g\cos\theta &= \frac{2gR\left(1-\cos\theta\right)}{R} \\ g\cos\theta &= 2g\left(1-\cos\theta\right) \\ \cos\theta &= 2\left(1-\cos\theta\right) \\ \cos\theta &= 2-2\cos\theta \\ 3\cos\theta &= 2 \\ \cos\theta &= 2/3 \\ \theta &= 48.19° \end{aligned}$$

b) When the person leaves the surface, the direction of the velocity is tangent to the circle. From there on, it's a projectile that follows a parabolic trajectory.



Using an origin at the centre of the hemisphere, the initial position of the object is

$$x = R\sin(48.19°) = 6m\sin(48.19°) = 4.472m$$
$$y = R\cos(48.19°) = 6m\cos(48.19°) = 4m$$

The initial speed of the projectile motion is

$$\begin{aligned} v'^2 &= 2gR(1-\cos\theta) \\ &= 2\cdot 9.8\tfrac{m}{s^2}\cdot 6m\left(1-\tfrac{2}{3}\right) \\ &= 39.2\tfrac{m^2}{s^3} \\ v' &= 6.261\tfrac{m}{s} \end{aligned}$$

The components of this velocity (with an *x*-axis towards the right and a *y*-axis upwards) are

$$v_x = 6.261\tfrac{m}{s}\cos(-48.19°) = 4.174\tfrac{m}{s}$$
$$v_y = 6.261\tfrac{m}{s}\sin(-48.19°) = -4.667\tfrac{m}{s}$$

Therefore, the time of flight is

$$y = y_0 + v_{0y}t - 4.9\tfrac{m}{s^2}t^2$$
$$0 = 4m - 4.667\tfrac{m}{s}t - 4.9\tfrac{m}{s^2}t^2$$
$$t = 1.09025s$$

The *x*-component of the position at the end of the free-fall is

$$\begin{aligned} x &= x_0 + v_{0x}t \\ &= 4.472m + 4.174\tfrac{m}{s} \cdot 1.09025s \\ &= 9.023m \end{aligned}$$

As the edge of the hemisphere is at x = 6 m, the person hits the ground 3.023 m away from the edge of the sphere.