

Chapter 6 Solutions

1. a) The magnitude of the centripetal force is

$$\begin{aligned}F_c &= \frac{mv^2}{r} \\&= \frac{200\text{kg} \cdot (34 \frac{\text{m}}{\text{s}})^2}{33\text{m}} \\&= 7006\text{N}\end{aligned}$$

b) The magnitude of the centripetal force is

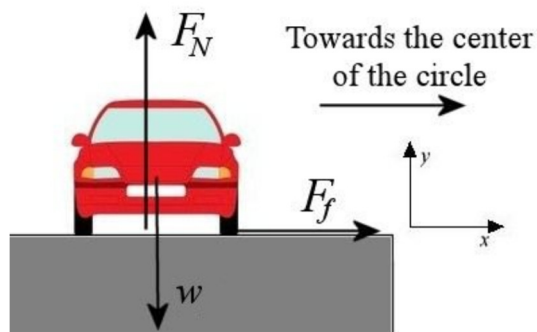
$$\begin{aligned}F_c &= \frac{mv^2}{r} \\&= \frac{200\text{kg} \cdot (34 \frac{\text{m}}{\text{s}})^2}{24\text{m}} \\&= 9633\text{N}\end{aligned}$$

2. In the curve, the following forces act on the car:

- 1) The weight mg directed downwards.
- 2) A normal force F_N directed upwards.
- 3) A friction force F_f directed towards the centre of the circle.

The equations of forces are then

$$\begin{aligned}\sum F_x &= ma_x \\&\rightarrow F_f = m \frac{v^2}{r} \\ \sum F_y &= ma_y \\&\rightarrow -mg + F_N = 0\end{aligned}$$



The second equation gives $F_N = mg$
and the first equation gives $F_f = m \frac{v^2}{r}$.

Using this value in

$$F_f \leq \mu_s F_N$$

the following result is obtained

$$m \frac{v^2}{r} \leq \mu_s F_N$$

$$m \frac{v^2}{r} \leq \mu_s mg$$

$$\frac{v^2}{r} \leq \mu_s g$$

$$\mu_s \geq \frac{v^2}{rg}$$

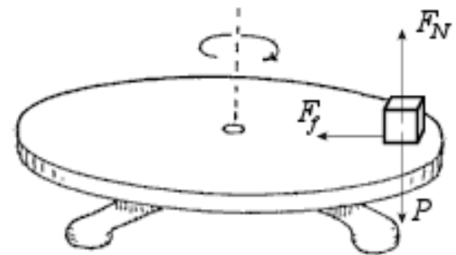
The minimum value of the coefficient of friction is therefore

$$\begin{aligned} \mu_{s\min} &= \frac{v^2}{rg} \\ &= \frac{(33.33 \frac{m}{s})^2}{100m \times 9.8 \frac{N}{kg}} \\ &= 1.134 \end{aligned}$$

3. The forces exerted on the block are:

- 1) The weight mg directed downwards.
- 2) A normal force F_N directed upwards.
- 3) A friction force F_f directed towards the centre of the circular path.

The equations of forces are then (using an axis towards the centre of the circular path)



$$\begin{aligned}\sum F_x &= ma_x \\ &\rightarrow F_f = m \frac{4\pi^2 r}{T^2} \\ \sum F_y &= ma_y \\ &\rightarrow -mg + F_N = 0\end{aligned}$$

As the period of rotation is sought (not the speed), the centripetal force formula with the period of rotation is used.

The second equation gives $F_N = mg$
and the first equation gives $F_f = m \frac{4\pi^2 r}{T^2}$.

Using this value in

$$F_f \leq \mu_s F_N$$

the following result is obtained

$$\begin{aligned}m \frac{4\pi^2 r}{T^2} &\leq \mu_s F_N \\ m \frac{4\pi^2 r}{T^2} &\leq \mu_s mg \\ \frac{4\pi^2 r}{T^2} &\leq \mu_s g \\ T &\geq \sqrt{\frac{4\pi^2 r}{\mu_s g}}\end{aligned}$$

The minimum period is therefore

$$\begin{aligned}T_{\min} &= \sqrt{\frac{4\pi^2 r}{\mu_s g}} \\ &= \sqrt{\frac{4\pi^2 \times 0.1m}{0.6 \times 9.8 \frac{N}{kg}}} \\ &= 0.8194s\end{aligned}$$

- 4.** At the lowest point on the circular path, the forces exerted on the car are (using a y-axis directed upwards):

- 1) The weight (9800 N) directed downwards.
- 2) A normal force F_N directed upwards.

The sum of the y -component of the forces is then

$$\begin{aligned}\sum F_y &= ma_y \\ \rightarrow -9800N + F_N &= m \frac{v^2}{r}\end{aligned}$$

The normal force is therefore

$$\begin{aligned}-9800N + F_N &= 1000kg \frac{(12 \frac{m}{s})^2}{5m} \\ -9800N + F_N &= 28,800N \\ F_N &= 38,600N\end{aligned}$$

At the lowest point on the circular path, the forces exerted on the car are (using a y -axis directed upwards):

- 1) The weight (9800 N) directed downwards.
- 2) A normal force F_N directed downwards.

The sum of the y -component of the forces is then

$$\begin{aligned}\sum F_y &= ma_y \\ \rightarrow -9800N - F_N &= -m \frac{v^2}{r}\end{aligned}$$

The normal force is therefore

$$\begin{aligned}-9800N - F_N &= -1000kg \frac{(12 \frac{m}{s})^2}{5m} \\ -9800N - F_N &= -28,800N \\ F_N &= 19,000N\end{aligned}$$

- 5.** At the highest point on the circular path, the forces exerted on the car are (using a y -axis directed upwards):

- 1) The weight (9800 N) directed downwards.

2) A normal force F_N directed downwards.

The sum of the y -component of the forces is then

$$\begin{aligned}\sum F_y &= ma_y \\ \rightarrow -mg - F_N &= -m \frac{v^2}{r}\end{aligned}$$

The normal force is therefore

$$\begin{aligned}-mg - F_N &= -m \frac{v^2}{r} \\ F_N &= m \frac{v^2}{r} - mg\end{aligned}$$

If we want the car to be in contact with the road, the normal force must be positive. This means that

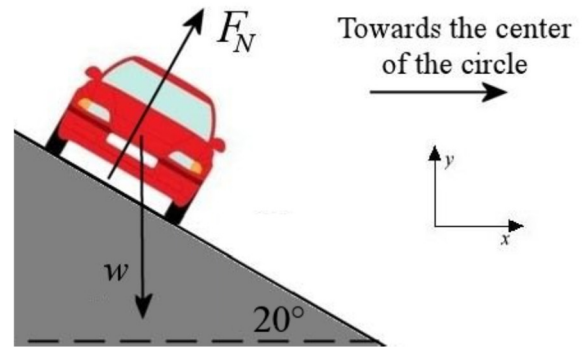
$$\begin{aligned}m \frac{v^2}{r} - mg &\geq 0 \\ m \frac{v^2}{r} &\geq mg \\ \frac{v^2}{r} &\geq g \\ v &\geq \sqrt{rg}\end{aligned}$$

The minimum speed is therefore

$$\begin{aligned}v_{\min} &= \sqrt{rg} \\ &= \sqrt{5m \times 9.8 \frac{N}{kg}} \\ &= 7 \frac{m}{s}\end{aligned}$$

6. The forces exerted on the car are (using an x -axis directed towards the right):

- 1) The weight mg directed downwards.
- 2) A normal force F_N perpendicular to the road.



The equations of forces are then

$$\sum F_x = ma_x$$

$$\rightarrow F_N \cos(70^\circ) = m \frac{v^2}{r}$$

$$\sum F_y = ma_y$$

$$\rightarrow -mg + F_N \sin(70^\circ) = 0$$

The normal force can be found with the sum of the y -component of the forces.

$$-mg + F_N \sin(70^\circ) = 0$$

$$F_N = \frac{mg}{\sin(70^\circ)}$$

The sum of the x -component of the forces then becomes

$$F_N \cos(70^\circ) = m \frac{v^2}{r}$$

$$\frac{mg}{\sin(70^\circ)} \cos(70^\circ) = m \frac{v^2}{r}$$

$$\frac{g \cos(70^\circ)}{\sin(70^\circ)} = \frac{v^2}{r}$$

$$v = \sqrt{\frac{rg \cos(70^\circ)}{\sin(70^\circ)}}$$

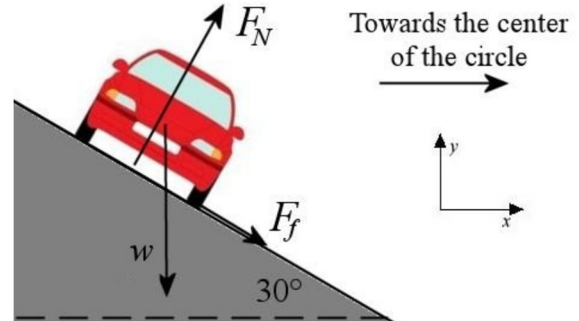
$$v = \sqrt{\frac{rg}{\tan(70^\circ)}}$$

$$v = \sqrt{\frac{80m \times 9.8 \frac{N}{kg}}{\tan(70^\circ)}}$$

$$v = 16.89 \frac{m}{s}$$

7. The forces exerted on the car are (using an x -axis directed towards the right)

- 1) The weight mg directed downwards.
- 2) A normal force F_N perpendicular to the road.
- 3) A friction force F_f parallel to the road.



Actually, the direction of the frictional force is not known. It may be directed uphill or downhill. Let's assume the direction is as shown in the figure. If, in the end, we have a positive frictional force, we will know that this direction is correct. If the frictional force is negative, the force is directed uphill instead.

The equations of forces are then

$$\sum F_x = ma_x$$

$$\rightarrow F_N \cos(60^\circ) + F_f \cos(-30^\circ) = m \frac{v^2}{r}$$

$$\sum F_y = ma_y$$

$$\rightarrow -mg + F_N \sin(60^\circ) + F_f \sin(-30^\circ) = 0$$

The normal force can be found with the sum of the y -component of the forces.

$$-mg + F_N \sin(60^\circ) + F_f \sin(-30^\circ) = 0$$

$$F_N \sin(60^\circ) = mg - F_f \sin(-30^\circ)$$

$$F_N \sin(60^\circ) = mg + F_f \sin(30^\circ)$$

$$F_N = \frac{mg + F_f \sin(30^\circ)}{\sin(60^\circ)}$$

The sum of the x -component of the forces then becomes

$$F_N \cos(60^\circ) + F_f \cos(30^\circ) = m \frac{v^2}{r}$$

$$\frac{mg + F_f \sin(30^\circ)}{\sin(60^\circ)} \cos(60^\circ) + F_f \cos(30^\circ) = m \frac{v^2}{r}$$

If we solve the equation for the friction force, we obtain

$$\frac{mg + F_f \sin(30^\circ)}{\sin(60^\circ)} \cos(60^\circ) + F_f \cos(30^\circ) = m \frac{v^2}{r}$$

$$\frac{mg}{\sin(60^\circ)} \cos(60^\circ) + \frac{F_f \sin(30^\circ)}{\sin(60^\circ)} \cos(60^\circ) + F_f \cos(30^\circ) = m \frac{v^2}{r}$$

$$\frac{F_f \sin(30^\circ)}{\sin(60^\circ)} \cos(60^\circ) + F_f \cos(30^\circ) = m \frac{v^2}{r} - \frac{mg}{\sin(60^\circ)} \cos(60^\circ)$$

$$F_f \left(\frac{\sin(30^\circ)}{\sin(60^\circ)} \cos(60^\circ) + \cos(30^\circ) \right) = m \frac{v^2}{r} - \frac{mg}{\sin(60^\circ)} \cos(60^\circ)$$

$$F_f (1.1547) = m \frac{v^2}{r} - \frac{mg}{\sin(60^\circ)} \cos(60^\circ)$$

a) If the speed is 100 m/s, we have

$$F_f (1.1547) = m \frac{v^2}{r} - \frac{mg}{\sin(60^\circ)} \cos(60^\circ)$$

$$F_f (1.1547) = 1200 \text{kg} \frac{(100 \frac{\text{m}}{\text{s}})^2}{80 \text{m}} - \frac{1200 \text{kg} \times 9.8 \frac{\text{N}}{\text{kg}}}{\sin(60^\circ)} \cos(60^\circ)$$

$$F_f (1.1547) = 143,210 \text{N}$$

$$F_f = 124,023 \text{N}$$

The friction force is thus directed downhill.

b) If the speed is 10 m/s, we have

$$F_f (1.1547) = m \frac{v^2}{r} - \frac{mg}{\sin(60^\circ)} \cos(60^\circ)$$

$$F_f (1.1547) = 1200 \text{kg} \frac{(10 \frac{\text{m}}{\text{s}})^2}{80 \text{m}} - \frac{1200 \text{kg} \times 9.8 \frac{\text{N}}{\text{kg}}}{\sin(60^\circ)} \cos(60^\circ)$$

$$F_f (1.1547) = -5290 \text{N}$$

$$F_f = -4581 \text{N}$$

The friction force is thus directed uphill.

8. The forces exerted on the car are:

- 1) The weight mg directed downwards.
- 2) A normal force F_N directed towards the right.
- 3) A friction force F_f directed upwards.

The equations of forces are then

$$\begin{aligned}\sum F_x &= ma_x \\ \rightarrow F_N &= m \frac{v^2}{r} \\ \sum F_y &= ma_y \\ \rightarrow -mg + F_f &= 0\end{aligned}$$

(There is no y -component for the acceleration since the car is not sliding down the wall.)

The first equation gives $F_N = m \frac{v^2}{r}$
and the second equation gives $F_f = mg$.

Using this value in

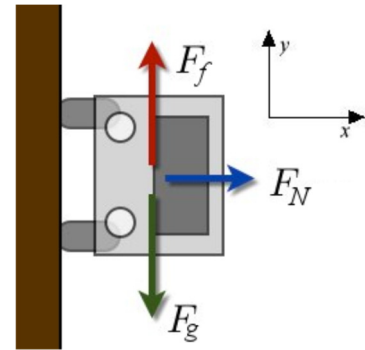
$$F_f \leq \mu_s F_N$$

the following result is obtained.

$$mg \leq \mu_s F_N$$

Since the normal force is $F_N = mv^2/r$ (from the sum of the x -component of the forces) this equation becomes

$$\begin{aligned}mg &\leq \mu_s m \frac{v^2}{r} \\ g &\leq \mu_s \frac{v^2}{r} \\ v &\geq \sqrt{\frac{rg}{\mu_s}}\end{aligned}$$



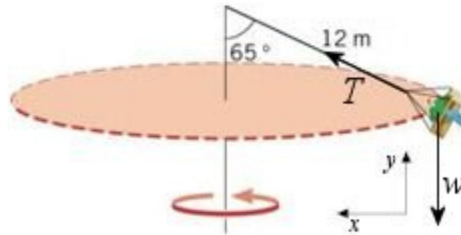
Therefore, the minimum speed is

$$\begin{aligned} v_{\min} &= \sqrt{\frac{5\text{m} \times 9.8 \frac{\text{N}}{\text{kg}}}{0.8}} \\ &= 7.826 \frac{\text{m}}{\text{s}} = 28.2 \frac{\text{km}}{\text{h}} \end{aligned}$$

and the normal force is

$$\begin{aligned} F_N &= m \frac{v^2}{r} \\ &= 1200\text{kg} \frac{(7.826 \frac{\text{m}}{\text{s}})^2}{5\text{m}} \\ &= 14,700\text{N} \end{aligned}$$

9. The forces exerted on the person are:



- 1) The weight mg directed downwards.
- 2) A tension force T directed at 25° .

The equations of forces are then

$$\begin{aligned} \sum F_x &= ma_x \\ &\rightarrow T \cos(25^\circ) = m \frac{4\pi^2 r}{T^2} \\ \sum F_y &= ma_y \\ &\rightarrow mg + T \sin(25^\circ) = 0 \end{aligned}$$

As the period of rotation is sought (not the speed), the centripetal force formula with the period of rotation is used.

The tension can be found with the sum of the y-component of the forces.

$$mg + T \sin(25^\circ) = 0$$

$$T = \frac{mg}{\sin(25^\circ)}$$

$$T = \frac{60\text{kg} \times 9.8 \frac{\text{N}}{\text{kg}}}{\sin(25^\circ)}$$

$$T = 1391.3\text{N}$$

The sum of the y-component of the forces then becomes

$$T \cos(25^\circ) = m \frac{4\pi^2 r}{T^2}$$

$$1391.3\text{N} \cos(25^\circ) = m \frac{4\pi^2 r}{T^2}$$

Since the radius of the circular path is

$$\frac{r}{12\text{m}} = \sin(65^\circ)$$

$$r = 12\text{m} \sin(65^\circ)$$

the period of rotation is

$$1391.3\text{N} \cos(25^\circ) = 60\text{kg} \times \frac{4\pi^2 12\text{m} \cdot \sin(65^\circ)}{T^2}$$

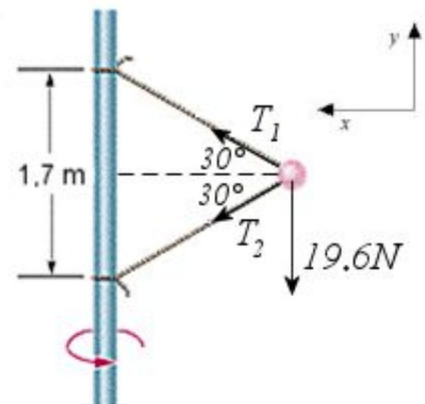
$$T = 4.52\text{s}$$

10. The forces exerted on the 2 kg object are

- 1) The weight (19.6 N) directed downwards.
- 2) A tension force T_1 directed at 30° .
- 3) A tension force T_2 directed at -30° .

(We have 30° angles because the strings and the vertical post form an equilateral triangle, whose angle at each vertex is 60° .)

The equations of forces are then



$$\begin{aligned}\sum F_x &= ma_x \\ &\rightarrow T_1 \cos(30^\circ) + T_2 \cos(-30^\circ) = m \frac{4\pi^2 r}{T^2} \\ \sum F_y &= ma_y \\ &\rightarrow -19.6N + T_1 \sin(30^\circ) + T_2 \sin(-30^\circ) = 0\end{aligned}$$

As the period of rotation is known (0.1 s) instead of the speed, the centripetal force formula with the period of rotation is used.

If we solve for T_1 in the sum of the y -component of the forces, we obtain

$$\begin{aligned}-19.6N + T_1 \sin(30^\circ) + T_2 \sin(-30^\circ) &= 0 \\ T_1 &= \frac{19.6N - T_2 \sin(-30^\circ)}{\sin(30^\circ)} \\ T_1 &= \frac{19.6N + T_2 \sin(30^\circ)}{\sin(30^\circ)} \\ T_1 &= \frac{19.6N}{\sin(30^\circ)} + T_2 \\ T_1 &= 39.2N + T_2\end{aligned}$$

We then substitute this value in the sum of the x -component of the forces to obtain

$$\begin{aligned}T_1 \cos(30^\circ) + T_2 \cos(30^\circ) &= m \frac{4\pi^2 r}{T^2} \\ T_1 + T_2 &= m \frac{4\pi^2 r}{T^2 \cos(30^\circ)} \\ (39.2N + T_2) + T_2 &= m \frac{4\pi^2 r}{T^2 \cos(30^\circ)} \\ 39.2N + 2T_2 &= m \frac{4\pi^2 r}{T^2 \cos(30^\circ)} \\ 2T_2 &= m \frac{4\pi^2 r}{T^2 \cos(30^\circ)} - 39.2N \\ T_2 &= m \frac{2\pi^2 r}{T^2 \cos(30^\circ)} - 19.6N\end{aligned}$$

Since the radius of the circular path is (dotted line in the figure)

$$\frac{r}{1.7m} = \cos(30^\circ)$$

$$r = 1.7m \cos(30^\circ)$$

we have

$$T_2 = m \frac{2\pi^2 r}{T^2 \cos(30^\circ)} - 19.6N$$

$$T_2 = m \frac{2\pi^2 \cdot 1.7m \cos(30^\circ)}{T^2 \cos(30^\circ)} - 19.6N$$

$$T_2 = m \frac{2\pi^2 \cdot 1.7m}{T^2} - 19.6N$$

$$T_2 = 2kg \frac{2\pi^2 \cdot 1.7m}{(1s)^2} - 19.6N$$

$$T_2 = 47.51N$$

Thus, the tension T_1 is

$$T_1 = 39.2N + T_2$$

$$T_1 = 86.71N$$

11. The forces acting on the small block are

- 1) The weight, mg directed downwards
- 2) A normal force directed at 30°

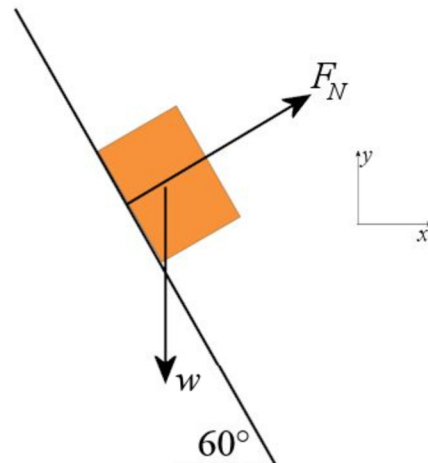
The equations of forces are then

$$\sum F_x = ma_x$$

$$\rightarrow F_N \cos(30^\circ) = m \frac{4\pi^2 r}{T^2}$$

$$\sum F_y = ma_y$$

$$\rightarrow -mg + F_N \sin(30^\circ) = 0$$



If we solve for F_N in the sum of the y-component of the forces, we obtain

$$F_N = \frac{mg}{\sin(30^\circ)}$$

We then substitute this value in the sum of the x-component of the forces to obtain

$$\frac{mg}{\sin(30^\circ)} \cos(30^\circ) = m \frac{4\pi^2 r}{T^2}$$

It only remains to solve for r .

$$\begin{aligned} \frac{g}{\sin(30^\circ)} \cos(30^\circ) &= \frac{4\pi^2 r}{T^2} \\ r &= \frac{gT^2}{4\pi^2 \tan(30^\circ)} \end{aligned}$$

Thus, the value of r is

$$\begin{aligned} r &= \frac{9.8 \frac{N}{kg} \cdot (0.5s)^2}{4\pi^2 \tan(30^\circ)} \\ &= 0.1075m \end{aligned}$$

Therefore, the value of x , which is the radius of the circular path, is 10.75 cm

12. a) We'll first find the tangential acceleration. It is calculated with

$$\begin{aligned} F_t &= ma_t \\ 1.5N &= 3kg \cdot a_t \\ a_t &= 0.5 \frac{m}{s^2} \end{aligned}$$

To find centripetal acceleration, the speed of the object is required. This speed is

$$\begin{aligned} v &= v_0 + a_t t \\ &= 0 \frac{m}{s} + 0.5 \frac{m}{s^2} \cdot 2s \\ &= 1 \frac{m}{s} \end{aligned}$$

Therefore, the centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(1 \frac{m}{s})^2}{1.2m} = 0.8333 \frac{m}{s^2}$$

Therefore, the total acceleration is

$$\begin{aligned} a &= \sqrt{a_c^2 + a_t^2} \\ &= \sqrt{(0.8333 \frac{m}{s^2})^2 + (0.5 \frac{m}{s^2})^2} \\ &= 0.9718 \frac{m}{s^2} \end{aligned}$$

b) The tube is the only thing that exerts a centripetal force. Its tension must therefore be equal to the centripetal force. This force is

$$\begin{aligned} T &= ma_c \\ &= 3kg \times 0.8333 \frac{m}{s^2} \\ &= 2.5N \end{aligned}$$

13. a) The centripetal acceleration is

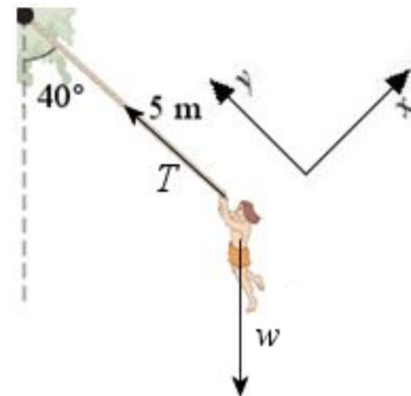
$$a_c = \frac{v^2}{r} = \frac{(10 \frac{m}{s})^2}{5m} = 20 \frac{m}{s^2}$$

b) The forces exerted on Gontran are:

- 1) The weight (637 N) directed downwards.
- 2) The tension force T .

The equations of forces are then

$$\begin{aligned} \sum F_x &= ma_x \\ &\rightarrow mg \cos(-130^\circ) = ma_t \\ \sum F_y &= ma_y \\ &\rightarrow T + mg \sin(-130^\circ) = m \frac{v^2}{r} \end{aligned}$$



The tangential acceleration can be found with the sum of the x -component of the forces.

$$mg \cos(-130^\circ) = ma_t$$

$$a_t = g \cos(-130^\circ)$$

$$a_t = -6.299 \frac{m}{s^2}$$

c) The acceleration is therefore

$$\begin{aligned} a &= \sqrt{a_c^2 + a_t^2} \\ &= \sqrt{\left(20 \frac{m}{s^2}\right)^2 + \left(6.299 \frac{m}{s^2}\right)^2} \\ &= 20.969 \frac{m}{s^2} \end{aligned}$$

d) The tension can be found with the sum of the x -component of the forces.

$$T + mg \sin(-130^\circ) = m \frac{v^2}{r}$$

$$T = m \frac{v^2}{r} - mg \sin(-130^\circ)$$

$$T = m \frac{v^2}{r} + mg \sin(130^\circ)$$

$$T = 65kg \frac{\left(10 \frac{m}{s}\right)^2}{5m} + 65kg \times 9.8 \frac{N}{kg} \sin(130^\circ)$$

$$T = 1300N + 488N$$

$$T = 1788N$$

14. We have

$$T = 2\pi \sqrt{\frac{r^3}{GM_c}}$$

$$27.32 \times 24 \times 60 \times 60 s = 2\pi \sqrt{\frac{\left(3.844 \times 10^8 m\right)^3}{6.674 \times 10^{-11} \frac{Nm^2}{kg^2} M_{Earth}}}$$

$$M_{Earth} = 6.03 \times 10^{24} kg$$

15. With what is known about Io, the mass of Jupiter can be found.

$$T = 2\pi \sqrt{\frac{r^3}{GM_c}}$$

$$1.796 \times 24 \times 60 \times 60 \text{ s} = 2\pi \sqrt{\frac{(4.217 \times 10^8 \text{ m})^3}{6.674 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} M_{\text{Jupiter}}}}$$

$$M_{\text{Jupiter}} = 1.8422 \times 10^{27} \text{ kg}$$

This information is then used to find the period of revolution of Ganymede.

$$T = 2\pi \sqrt{\frac{r^3}{GM_c}}$$

$$= 2\pi \sqrt{\frac{(1.0704 \times 10^9 \text{ m})^3}{6.674 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \times 1.8422 \times 10^{27} \text{ kg}}}$$

$$= 627,529 \text{ s} = 7.263 \text{ j}$$

b) The speed of Ganymede is

$$v = \sqrt{\frac{GM_c}{r}}$$

$$= \sqrt{\frac{6.674 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \cdot 1.8422 \times 10^{27} \text{ kg}}{1.0704 \times 10^9 \text{ m}}}$$

$$= 10,717 \frac{\text{m}}{\text{s}} = 10.717 \frac{\text{km}}{\text{s}}$$

16. The period of revolution is

$$T = 2\pi \sqrt{\frac{r^3}{GM_c}}$$

$$= 2\pi \sqrt{\frac{(1.837 \times 10^6 \text{ m})^3}{6.674 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \times 7.35 \times 10^{22} \text{ kg}}}$$

$$= 7063 \text{ s} = 1.962 \text{ h}$$

17. The distance is found with

$$T = 2\pi \sqrt{\frac{r^3}{GM_c}}$$

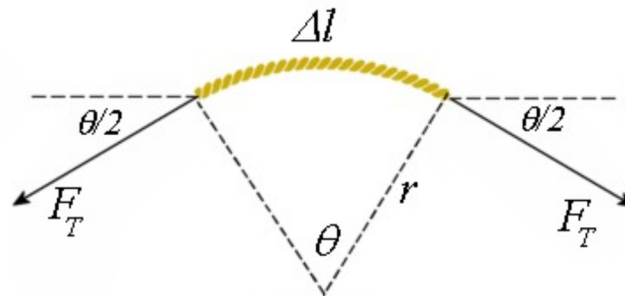
$$2 \times 24 \times 60 \times 60 \text{ s} = 2\pi \sqrt{\frac{r^3}{6.674 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \times 5.97 \times 10^{24} \text{ kg}}}$$

$$r = 6.7044 \times 10^7 \text{ m} = 67,044 \text{ km}$$

This is the distance from the centre of the Earth. Therefore, the distance from the surface is

$$\text{dist} = 67,044 \text{ km} - 6378 \text{ km} = 60,666 \text{ km}$$

- 18.** Let's take a small piece of rope and examine the forces on this piece. (The angle θ is small on the figure.)



Since this piece makes a circular motion, the sum of y-components of the force acting on this piece is (using a y-axis directed upwards)

$$\sum F_y = ma_y$$

$$\rightarrow F_T \sin\left(-\frac{\theta}{2}\right) + F_T \sin\left(180^\circ + \frac{\theta}{2}\right) = -m \frac{4\pi^2 r}{T^2}$$

Since $\sin -x = -\sin x$ and $\sin (180^\circ + x) = -\sin x$, it becomes

$$-F_T \sin\left(\frac{\theta}{2}\right) - F_T \sin\left(\frac{\theta}{2}\right) = -m \frac{4\pi^2 r}{T^2}$$

$$2F_T \sin\left(\frac{\theta}{2}\right) = m \frac{4\pi^2 r}{T^2}$$

Since the angle is small, $\sin x = x$. (This means that we are now working with angles in radians.)

$$2F_T \frac{\theta}{2} = m \frac{4\pi^2 r}{T^2}$$

$$F_T \theta = m \frac{4\pi^2 r}{T^2}$$

The mass of the piece depends on the angle. The proportion of the mass of the rope in the small piece is the same as that of the angle compared to 2π radians.

$$\frac{m}{6kg} = \frac{\theta}{2\pi}$$

$$m = \frac{\theta}{2\pi} 6kg$$

The force equation then becomes

$$F_T \theta = m \frac{4\pi^2 r}{T^2}$$

$$F_T \theta = \frac{\theta}{2\pi} 6kg \frac{4\pi^2 r}{T^2}$$

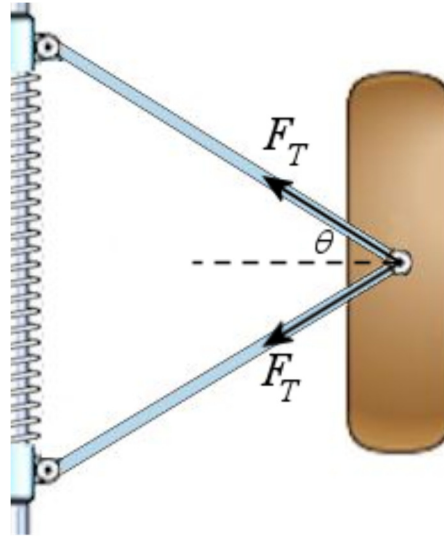
$$F_T = 6kg \frac{2\pi r}{T^2}$$

With the values of r and T , the tension is

$$F_T = 6kg \frac{2\pi \cdot 0.5m}{(0.25s)^2}$$

$$= 301.6N$$

19. Let's with considering the forces on one of the masses.



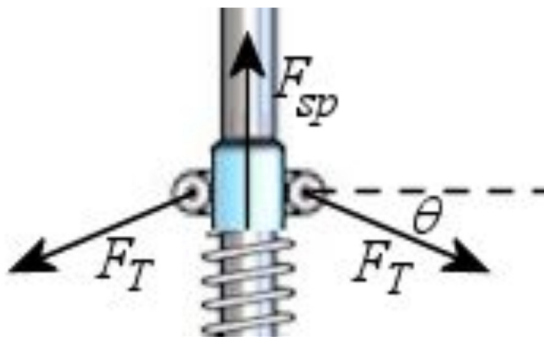
The sum of the x -components of the forces is

$$\begin{aligned}\sum F_x &= ma_x \\ \rightarrow -F_T \cos \theta - F_T \cos \theta &= -m \frac{4\pi^2 r}{T^2}\end{aligned}$$

Therefore, the tension force exerted by the rods is

$$F_T = \frac{2m\pi^2 r}{T^2 \cos \theta}$$

Now, let's consider the force acting at the end of the spring.



The sum of the y -components of the forces is

$$\begin{aligned}\sum F_y &= ma_y \\ \rightarrow F_T \sin(-\theta) + F_T \sin(-\theta) + F_{sp} &= 0\end{aligned}$$

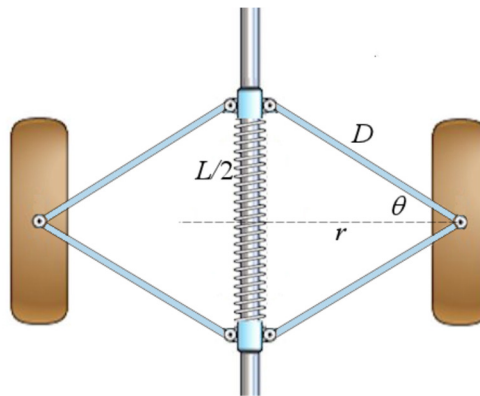
Thus, the force exerted by the spring is

$$F_{sp} = 2F_T \sin \theta$$

Using the value of the tension found previously, this force is

$$\begin{aligned} F_{sp} &= 2F_T \sin \theta \\ &= \frac{4m\pi^2 r}{T^2 \cos \theta} \sin \theta \end{aligned}$$

Now, let's make the connection between a few variables.



Firstly,

$$D \cos \theta = r$$

so that

$$\begin{aligned} F_{sp} &= \frac{4m\pi^2 r}{T^2 \cos \theta} \sin \theta \\ &= \frac{4m\pi^2 D}{T^2} \sin \theta \end{aligned}$$

Secondly,

$$D \sin \theta = \frac{L}{2}$$

so that

$$F_{sp} = \frac{4m\pi^2 D}{T^2} \sin \theta$$

$$= \frac{2m\pi^2 L}{T^2}$$

Since the force exerted by the spring is

$$F_{sp} = k(x_0 - L)$$

where x_0 is the length of the spring when it is neither stretched nor compressed, the equation becomes

$$k(x_0 - L) = \frac{2m\pi^2 L}{T^2}$$

It only remains to solve this equation for L .

$$kx_0 - kL = \frac{2m\pi^2 L}{T^2}$$

$$kx_0 = \left(\frac{2m\pi^2}{T^2} + k \right) L$$

$$L = \frac{x_0}{1 + \frac{2m\pi^2}{kT^2}}$$

With the values given, the length is

$$L = \frac{x_0}{1 + \frac{2m\pi^2}{kT^2}}$$

$$= \frac{80cm}{1 + \frac{2 \cdot 1kg \cdot \pi^2}{2000 \frac{kg}{m} \cdot (0.1s)^2}}$$

$$= 40.263cm$$