Chapter 6 Solutions

1. a) The magnitude of the centripetal force is

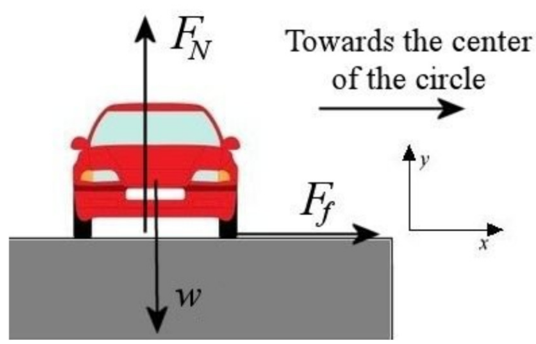
$$\begin{aligned} F_c &= \frac{mv^2}{r} \\ &= \frac{200kg \cdot \left(34\tfrac{m}{s}\right)^2}{33m} \\ &= 7006N \end{aligned}$$

b) The magnitude of the centripetal force is

$$\begin{aligned} F_c &= \frac{mv^2}{r} \\ &= \frac{200kg \cdot \left(34\tfrac{m}{s}\right)^2}{24m} \\ &= 9633N \end{aligned}$$

2. In the curve, the following forces act on the car:

1) The weight mg downwards.
2) A normal force F_N upwards.
3) A friction force F_f towards the centre of the circle.



The equations of forces are then

$$\begin{aligned} \sum F_x &= ma_x \\ &\rightarrow F_f = m\frac{v^2}{r} \\ \sum F_y &= ma_y \\ &\rightarrow -mg + F_N = 0 \end{aligned}$$

The second equation gives us the normal force $F_N = mg$.

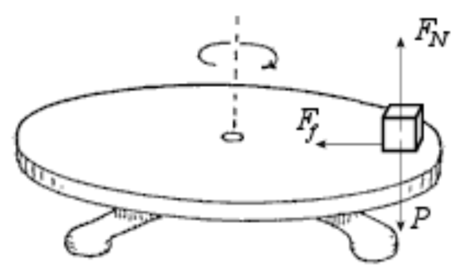
With the sum of the x-component of the forces, we have

$$m\frac{v^2}{r} = F_f \leq \mu_s F_N$$
$$m\frac{v^2}{r} \leq \mu_s F_N$$
$$m\frac{v^2}{r} \leq \mu_s mg$$
$$\frac{v^2}{r} \leq \mu_s g$$
$$\mu_s \geq \frac{v^2}{rg}$$

The minimum value of the coefficient of friction is therefore

$$\begin{aligned}\mu_{s\min} &= \frac{v^2}{rg}\\ &= \frac{\left(33.33\frac{m}{s}\right)^2}{100m\times 9.8\frac{N}{kg}}\\ &= 1.134\end{aligned}$$

3. The forces exerted on the block are:



1) The weight mg downwards.
2) A normal force F_N upwards.
3) A friction force towards the centre of the circular path.

The equations of forces are then (using an axis towards the centre of the circular path)

$$\sum F_x = ma_x$$
$$\rightarrow F_f = m\frac{4\pi^2 r}{T^2}$$
$$\sum F_y = ma_y$$
$$\rightarrow -mg + F_N = 0$$

As the period of rotation is sought (not the speed), the centripetal force formula with the period of rotation is used.

The sum of the y-component of the forces gives us the normal force $F_N = mg$.

The sum of the y-component of the forces then becomes

$$\begin{aligned} m\frac{4\pi^2 r}{T^2} &= F_f \leq \mu_s F_N \\ m\frac{4\pi^2 r}{T^2} &\leq \mu_s F_N \\ m\frac{4\pi^2 r}{T^2} &\leq \mu_s mg \\ \frac{4\pi^2 r}{T^2} &\leq \mu_s g \\ T &\geq \sqrt{\frac{4\pi^2 r}{\mu_s g}} \end{aligned}$$

The minimum period is therefore

$$\begin{aligned} T_{\min} &= \sqrt{\frac{4\pi^2 r}{\mu_s g}} \\ &= \sqrt{\frac{4\pi^2 \times 0.1m}{0.6 \times 9.8 \frac{N}{kg}}} \\ &= 0.8194s \end{aligned}$$

4. At the lowest point on the circular path, the forces exerted on the car are (using a y-axis directed upwards):

1) The weight, 9800 N downwards.
2) A normal force F_N upwards.

The sum of the y-component of the forces is then

$$\sum F_y = ma_y$$
$$\to -9800N + F_N = m\frac{v^2}{r}$$

The normal force is therefore

$$-9800N + F_N = 1000kg\,\frac{\left(12\,\frac{m}{s}\right)^2}{5m}$$
$$-9800N + F_N = 28,800N$$
$$F_N = 38,600N$$

At the lowest point on the circular path, the forces exerted on the car are (using a y-axis directed upwards):

1) The weight, 9800 N downwards.
2) A normal force F_N downwards.

The sum of the y-component of the forces is then

$$\sum F_y = ma_y$$
$$\rightarrow -9800N - F_N = -m\frac{v^2}{r}$$

The normal force is therefore

$$-9800N - F_N = -1000kg\,\frac{\left(12\,\frac{m}{s}\right)^2}{5m}$$
$$-9800N - F_N = -28,800N$$
$$F_N = 19,000N$$

5. At the highest point on the circular path, the forces exerted on the car are (using a y-axis directed upwards):

1) The weight, 9800 N downwards.
2) A normal force F_N downwards.

The sum of the y-component of the forces is then

$$\sum F_y = ma_y$$
$$\rightarrow -mg - F_N = -m\frac{v^2}{r}$$

The normal force is therefore

$$-mg - F_N = -m\frac{v^2}{r}$$

$$F_N = m\frac{v^2}{r} - mg$$

If we want the car to be in contact with the road, the normal force must be positive. This means that

$$m\frac{v^2}{r} - mg \geq 0$$

$$m\frac{v^2}{r} \geq mg$$

$$\frac{v^2}{r} \geq g$$

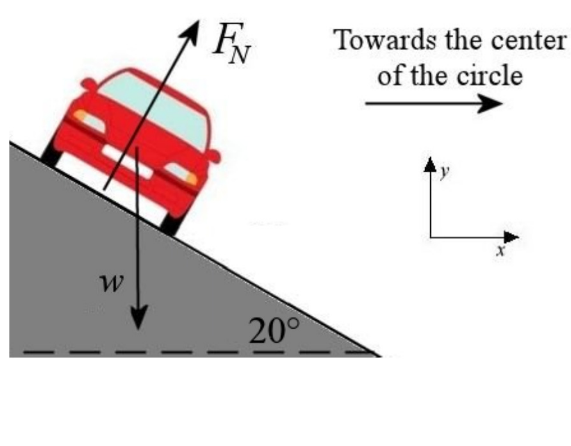
$$v \geq \sqrt{rg}$$

The minimum speed is therefore

$$\begin{aligned} v_{\min} &= \sqrt{rg} \\ &= \sqrt{5m \times 9.8 \tfrac{N}{kg}} \\ &= 7 \tfrac{m}{s} \end{aligned}$$

6. The forces exerted on the car are (using an *x*-axis directed towards the right):

1) The weight *mg* downwards.
2) A normal force F_N perpendicular to the road.



The equations of forces are then

$$\sum F_x = ma_x$$

$$\rightarrow F_N \cos(70°) = m\frac{v^2}{r}$$

$$\sum F_y = ma_y$$

$$\rightarrow -mg + F_N \sin(70°) = 0$$

The normal force can be found with the sum of the *y*-component of the forces.

$$-mg + F_N \sin(70°) = 0$$

$$F_N = \frac{mg}{\sin(70°)}$$

The sum of the x-component of the forces then becomes

$$F_N \cos(70°) = m\frac{v^2}{r}$$

$$\frac{mg}{\sin(70°)}\cos(70°) = m\frac{v^2}{r}$$

$$\frac{g\cos(70°)}{\sin(70°)} = \frac{v^2}{r}$$

$$v = \sqrt{\frac{rg\cos(70°)}{\sin(70°)}}$$

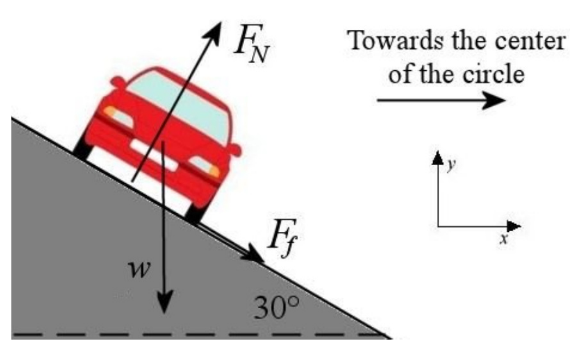
$$v = \sqrt{\frac{rg}{\tan(70°)}}$$

$$v = \sqrt{\frac{80m \times 9.8\frac{N}{kg}}{\tan(70°)}}$$

$$v = 16.89\tfrac{m}{s}$$

7. The forces exerted on the car are (using an x-axis directed towards the right)

1) The weight mg downwards.
2) A normal force F_N perpendicular to the road.
3) A friction force F_f parallel to the road.



Actually, the direction of the frictional force is not known. It may be directed uphill or downhill. Let's assume the direction is as shown in the figure. If, in the end, we have a positive frictional force, we will know that this direction is correct. If the frictional force is negative, the force is directed uphill instead.

The equations of forces are then

$$\sum F_x = ma_x$$
$$\rightarrow F_N \cos(60°) + F_f \cos(-30°) = m\frac{v^2}{r}$$
$$\sum F_y = ma_y$$
$$\rightarrow -mg + F_N \sin(60°) + F_f \sin(-30°) = 0$$

The normal force can be found with the sum of the *y*-component of the forces.

$$-mg + F_N \sin(60°) + F_f \sin(-30°) = 0$$
$$F_N \sin(60°) = mg - F_f \sin(-30°)$$
$$F_N \sin(60°) = mg + F_f \sin(30°)$$
$$F_N = \frac{mg + F_f \sin(30°)}{\sin(60°)}$$

The sum of the *x*-component of the forces then becomes

$$F_N \cos(60°) + F_f \cos(30°) = m\frac{v^2}{r}$$
$$\frac{mg + F_f \sin(30°)}{\sin(60°)}\cos(60°) + F_f \cos(30°) = m\frac{v^2}{r}$$

If we solve the equation for the friction force, we obtain

$$\frac{mg + F_f \sin(30°)}{\sin(60°)}\cos(60°) + F_f \cos(30°) = m\frac{v^2}{r}$$
$$\frac{mg}{\sin(60°)}\cos(60°) + \frac{F_f \sin(30°)}{\sin(60°)}\cos(60°) + F_f \cos(30°) = m\frac{v^2}{r}$$
$$\frac{F_f \sin(30°)}{\sin(60°)}\cos(60°) + F_f \cos(30°) = m\frac{v^2}{r} - \frac{mg}{\sin(60°)}\cos(60°)$$
$$F_f\left(\frac{\sin(30°)}{\sin(60°)}\cos(60°) + \cos(30°)\right) = m\frac{v^2}{r} - \frac{mg}{\sin(60°)}\cos(60°)$$
$$F_f(1.1547) = m\frac{v^2}{r} - \frac{mg}{\sin(60°)}\cos(60°)$$

a) If the speed is 100 m/s, we have

$$F_f\left(1.1547\right)=m\frac{v^2}{r}-\frac{mg}{\sin\left(60°\right)}\cos\left(60°\right)$$

$$F_f\left(1.1547\right)=1200kg\frac{\left(100\frac{m}{s}\right)^2}{80m}-\frac{1200kg\times 9.8\frac{N}{kg}}{\sin\left(60°\right)}\cos\left(60°\right)$$

$$F_f\left(1.1547\right)=143,210N$$

$$F_f=124,023N$$

The friction force is thus directed downhill.

b) If the speed is 10 m/s, we have

$$F_f\left(1.1547\right)=m\frac{v^2}{r}-\frac{mg}{\sin\left(60°\right)}\cos\left(60°\right)$$

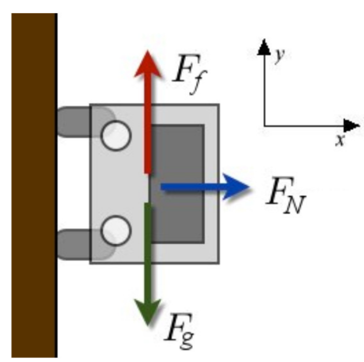
$$F_f\left(1.1547\right)=1200kg\frac{\left(10\frac{m}{s}\right)^2}{80m}-\frac{1200kg\times 9.8\frac{N}{kg}}{\sin\left(60°\right)}\cos\left(60°\right)$$

$$F_f\left(1.1547\right)=-5290N$$

$$F_f=-4581N$$

The friction force is thus directed uphill.

8. The forces exerted on the car are:



1) The weight mg downwards.
2) A normal force F_N towards the right.
3) A friction force F_f upwards.

The equations of forces are then

$$\sum F_x=ma_x$$

$$\to F_N=m\frac{v^2}{r}$$

$$\sum F_y=ma_y$$

$$\to -mg+F_f=0$$

The sum of the y-component of the forces gives

$$F_f = mg$$

If the car does not slip, we must have

$$mg = F_f \leq \mu_s F_N$$
$$mg \leq \mu_s F_N$$

The normal force $F_N = mv^2/r$ can be found with the sum of the x-component of the forces. Therefore

$$mg \leq \mu_s F_N$$
$$mg \leq \mu_s m \frac{v^2}{r}$$
$$g \leq \mu_s \frac{v^2}{r}$$
$$v \geq \sqrt{\frac{rg}{\mu_s}}$$

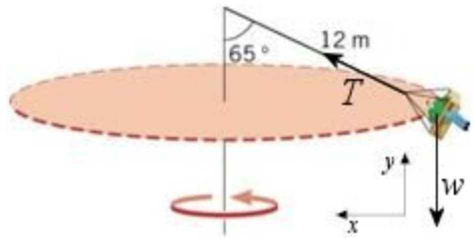
The minimum speed is therefore

$$v_{\min} = \sqrt{\frac{5m \times 9.8 \frac{N}{kg}}{0.8}}$$
$$= 7.826 \tfrac{m}{s} = 28.2 \tfrac{km}{h}$$

and the normal force is

$$F_N = m\frac{v^2}{r}$$
$$= 1200kg \frac{\left(7.826 \frac{m}{s}\right)^2}{5m}$$
$$= 14,700N$$

9. The forces exerted on the person are:



1) The weight mg downwards.
2) A tension force at 25°.

The equations of forces are then

$$\sum F_x = ma_x$$
$$\rightarrow T\cos(25°) = m\frac{4\pi^2 r}{T^2}$$
$$\sum F_y = ma_y$$
$$\rightarrow mg + T\sin(25°) = 0$$

As the period of rotation is sought (not the speed), the centripetal force formula with the period of rotation is used.

The tension can be found with the sum of the y-component of the forces.

$$mg + T\sin(25°) = 0$$
$$T = \frac{mg}{\sin(25°)}$$
$$T = \frac{60kg \times 9.8\frac{N}{kg}}{\sin(25°)}$$
$$T = 1391.3N$$

The sum of the y-component of the forces then becomes

$$T\cos(25°) = m\frac{4\pi^2 r}{T^2}$$
$$1391.3N\cos(25°) = m\frac{4\pi^2 r}{T^2}$$

Since the radius of the circular path is

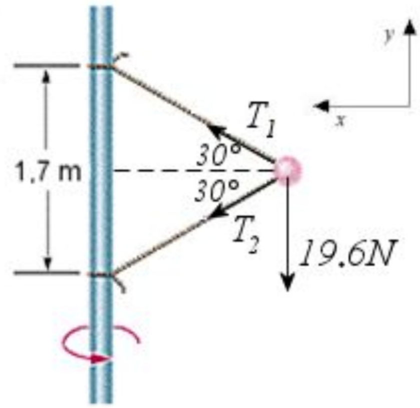
$$\frac{r}{12m} = \sin(65°)$$
$$r = 12m\sin(65°)$$

the period of rotation is

$$1391.3N\cos(25°) = 60kg \times \frac{4\pi^2 12m \cdot \sin(65°)}{T^2}$$
$$T = 4.52s$$

10. The forces exerted on the 2 kg object are

1) The weight, 19.6 N downwards.
2) A tension force T_1 at 30°.
3) A tension force T_2 at -30°.



(We have 30° angles because the strings and the vertical post form an equilateral triangle, whose angle at each vertex is 60°.)

The equations of forces are then

$$\sum F_x = ma_x$$
$$\rightarrow T_1\cos(30°) + T_2\cos(-30°) = m\frac{4\pi^2 r}{T^2}$$
$$\sum F_y = ma_y$$
$$\rightarrow -19,6N + T_1\sin(30°) + T_2\sin(-30°) = 0$$

As the period of rotation is known (0.1 s) instead of the speed, the centripetal force formula with the period of rotation is used.

If we solve for T_1 in the sum of the *y*-component of the forces, we obtain

$$-19.6N + T_1\sin(30°) + T_2\sin(-30°) = 0$$
$$T_1 = \frac{19.6N - T_2\sin(-30°)}{\sin(30°)}$$
$$T_1 = \frac{19.6N + T_2\sin(30°)}{\sin(30°)}$$
$$T_1 = \frac{19.6N}{\sin(30°)} + T_2$$
$$T_1 = 39.2N + T_2$$

We then substitute this value in the sum of the *x*-component of the forces to obtain

$$T_1\cos(30°)+T_2\cos(30°)=m\frac{4\pi^2 r}{T^2}$$

$$T_1+T_2=m\frac{4\pi^2 r}{T^2\cos(30°)}$$

$$(39.2N+T_2)+T_2=m\frac{4\pi^2 r}{T^2\cos(30°)}$$

$$39.2N+2T_2=m\frac{4\pi^2 r}{T^2\cos(30°)}$$

$$2T_2=m\frac{4\pi^2 r}{T^2\cos(30°)}-39.2N$$

$$T_2=m\frac{2\pi^2 r}{T^2\cos(30°)}-19.6N$$

Since the radius of the circular path is (dotted line in the figure)

$$\frac{r}{1.7m}=\cos(30°)$$
$$r=1.7m\cos(30°)$$

we have

$$T_2=m\frac{2\pi^2 r}{T^2\cos(30°)}-19.6N$$

$$T_2=m\frac{2\pi^2\cdot 1.7m\cos(30°)}{T^2\cos(30°)}-19.6N$$

$$T_2=m\frac{2\pi^2\cdot 1,7m}{T^2}-19.6N$$

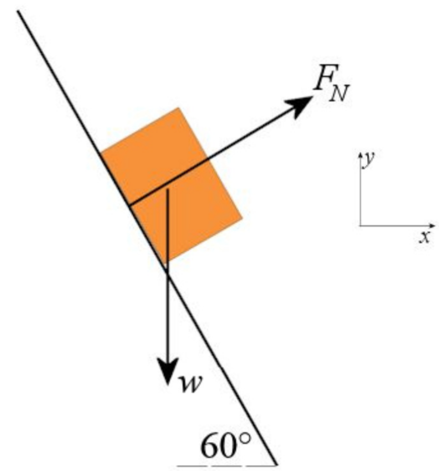
$$T_2=2kg\,\frac{2\pi^2\cdot 1.7m}{(1s)^2}-19,6N$$

$$T_2=47.51N$$

Thus, the tension T_1 is

$$T_1=39.2N+T_2$$
$$T_1=86.71N$$

11. The forces acting on the small block are



1) The weight, mg downwards
2) A normal force at 30°

The equations of forces are then

$$\sum F_x = ma_x$$

$$\rightarrow F_N \cos(30°) = m\frac{4\pi^2 r}{T^2}$$

$$\sum F_y = ma_y$$

$$\rightarrow -mg + F_N \sin(30°) = 0$$

If we solve for F_N in the sum of the y-component of the forces, we obtain

$$F_N = \frac{mg}{\sin(30°)}$$

We then substitute this value in the sum of the x-component of the forces to obtain

$$\frac{mg}{\sin(30°)}\cos(30°) = m\frac{4\pi^2 r}{T^2}$$

It only remains to solve for r.

$$\frac{g}{\sin(30°)}\cos(30°) = \frac{4\pi^2 r}{T^2}$$

$$r = \frac{gT^2}{4\pi^2 \tan(30°)}$$

Thus, the value of r is

$$r = \frac{9.8\frac{N}{kg}\cdot(0.5s)^2}{4\pi^2 \tan(30°)}$$

$$= 0.1075m$$

Therefore, the value of x, which is the radius of the circular path, is 10.75 cm

12. a) We'll first find the tangential acceleration. It is calculated with

$$\begin{aligned} F_t &= ma_t \\ 1.5N &= 3kg \cdot a_t \\ a_t &= 0.5 \tfrac{m}{s^2} \end{aligned}$$

To find centripetal acceleration, the speed of the object is required. This speed is

$$\begin{aligned} v &= v_0 + a_t t \\ &= 0 \tfrac{m}{s} + 0.5 \tfrac{m}{s^2} \cdot 2s \\ &= 1 \tfrac{m}{s} \end{aligned}$$

The centripetal acceleration is therefore

$$a_c = \frac{v^2}{r} = \frac{\left(1 \tfrac{m}{s}\right)^2}{1.2m} = 0.8333 \tfrac{m}{s^2}$$

The total acceleration is therefore

$$\begin{aligned} a &= \sqrt{a_c^2 + a_t^2} \\ &= \sqrt{\left(0.8333 \tfrac{m}{s^2}\right)^2 + \left(0.5 \tfrac{m}{s^2}\right)^2} \\ &= 0.9718 \tfrac{m}{s^2} \end{aligned}$$

b) The tube is the only thing that exerts a centripetal force. Its tension must therefore be equal to the centripetal force. This force is

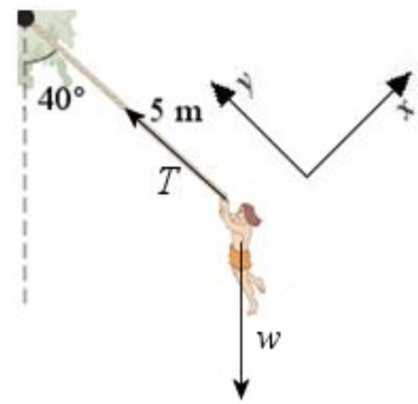
$$\begin{aligned} T &= ma_c \\ &= 3kg \times 0.8333 \tfrac{m}{s^2} \\ &= 2.5N \end{aligned}$$

13. a) The centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{\left(10 \tfrac{m}{s}\right)^2}{5m} = 20 \tfrac{m}{s^2}$$

b) The forces exerted on Gontran are:

1) The weight, 637 N downwards.
2) The tension force *T*.



The equations of forces are then

$$\sum F_x = ma_x$$
$$\rightarrow mg\cos(-130°) = ma_t$$
$$\sum F_y = ma_y$$
$$\rightarrow T + mg\sin(-130°) = m\frac{v^2}{r}$$

The tangential acceleration can be found with the sum of the *x*-component of the forces.

$$mg\cos(-130°) = ma_t$$
$$a_t = g\cos(-130°)$$
$$a_t = -6.299\tfrac{m}{s^2}$$

c) The acceleration is therefore

$$a = \sqrt{a_c^2 + a_t^2}$$
$$= \sqrt{\left(20\tfrac{m}{s^2}\right)^2 + \left(6.299\tfrac{m}{s^2}\right)^2}$$
$$= 20.969\tfrac{m}{s^2}$$

d) The tension can be found with the sum of the *x*-component of the forces.

$$T + mg\sin(-130°) = m\frac{v^2}{r}$$
$$T = m\frac{v^2}{r} - mg\sin(-130°)$$
$$T = m\frac{v^2}{r} + mg\sin(130°)$$
$$T = 65kg\frac{\left(10\tfrac{m}{s}\right)^2}{5m} + 65kg \times 9.8\tfrac{N}{kg}\sin(130°)$$
$$T = 1300N + 488N$$
$$T = 1788N$$

14. We have

$$T = 2\pi\sqrt{\frac{r^3}{GM_c}}$$

$$27.32\times 24\times 60\times 60s = 2\pi\sqrt{\frac{\left(3.844\times 10^8 m\right)^3}{6.674\times 10^{-11}\frac{Nm^2}{kg^2}M_{Earth}}}$$

$$M_{Earth} = 6.03\times 10^{24}kg$$

15. With what is known about Io, the mass of Jupiter can be found.

$$T = 2\pi\sqrt{\frac{r^3}{GM_c}}$$

$$1.796\times 24\times 60\times 60s = 2\pi\sqrt{\frac{\left(4.217\times 10^8 m\right)^3}{6.674\times 10^{-11}\frac{Nm^2}{kg^2}M_{Jupiter}}}$$

$$M_{Jupiter} = 1.8422\times 10^{27}kg$$

This information is then used to find the period of revolution of Ganymede.

$$\begin{aligned} T &= 2\pi\sqrt{\frac{r^3}{GM_c}} \\ &= 2\pi\sqrt{\frac{\left(1.0704\times 10^9 m\right)^3}{6.674\times 10^{-11}\frac{Nm^2}{kg^2}\times 1.8422\times 10^{27}kg}} \\ &= 627,529s = 7.263j \end{aligned}$$

b) The speed of Ganymede is

$$\begin{aligned} v &= \sqrt{\frac{GM_c}{r}} \\ &= \sqrt{\frac{6.674\times 10^{-11}\frac{Nm^2}{kg^2}\cdot 1.8422\times 10^{27}kg}{1.0704\times 10^9 m}} \\ &= 10,717\tfrac{m}{s} = 10.717\tfrac{km}{s} \end{aligned}$$

16. The period of revolution is

$$T = 2\pi\sqrt{\frac{r^3}{GM_c}}$$
$$= 2\pi\sqrt{\frac{\left(1.837\times10^6 m\right)^3}{6.674\times10^{-11}\frac{Nm^2}{kg^2}\times7.35\times10^{22}kg}}$$
$$= 7063s = 1.962h$$

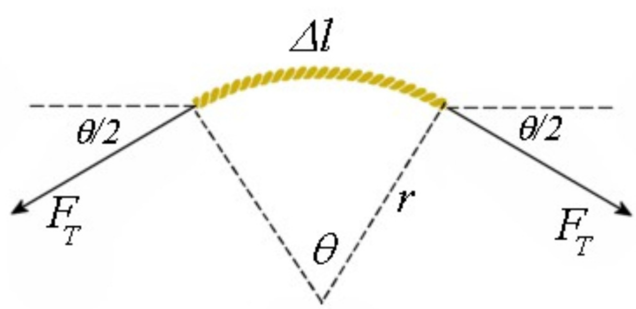
17. The distance is found with

$$T = 2\pi\sqrt{\frac{r^3}{GM_c}}$$
$$2\times24\times60\times60s = 2\pi\sqrt{\frac{r^3}{6.674\times10^{-11}\frac{Nm^2}{kg^2}\times5.97\times10^{24}kg}}$$
$$r = 6.7044\times10^7 m = 67,044km$$

This is the distance from the centre of the Earth. Therefore, the distance from the surface is

$$dist = 67,044km - 6378km = 60,666km$$

18. Let's take a small piece of rope and examine the forces on this piece. (The angle θ is small on the figure.)



Since this piece makes a circular motion, the sum of y-components of the force acting on this piece is (using a y-axis directed upwards)

$$\sum F_y = ma_y$$

$$\rightarrow F_T \sin\left(-\frac{\theta}{2}\right) + F_T \sin\left(180° + \frac{\theta}{2}\right) = -m\frac{4\pi^2 r}{T^2}$$

Since sin -x = -sin x and sin (180°+x) = -sin x, it becomes

$$-F_T \sin\left(\frac{\theta}{2}\right) - F_T \sin\left(\frac{\theta}{2}\right) = -m\frac{4\pi^2 r}{T^2}$$

$$2F_T \sin\left(\frac{\theta}{2}\right) = m\frac{4\pi^2 r}{T^2}$$

Since the angle is small, sin $x = x$. (This means that we are now working with angles in radians.)

$$2F_T \frac{\theta}{2} = m\frac{4\pi^2 r}{T^2}$$

$$F_T\theta = m\frac{4\pi^2 r}{T^2}$$

The mass of the piece depends on the angle. The proportion of the mass of the rope in the small piece is the same as that of the angle compared to 2π radians.

$$\frac{m}{6kg} = \frac{\theta}{2\pi}$$

$$m = \frac{\theta}{2\pi}6kg$$

The force equation then becomes

$$F_T\theta = m\frac{4\pi^2 r}{T^2}$$

$$F_T\theta = \frac{\theta}{2\pi}6kg\frac{4\pi^2 r}{T^2}$$

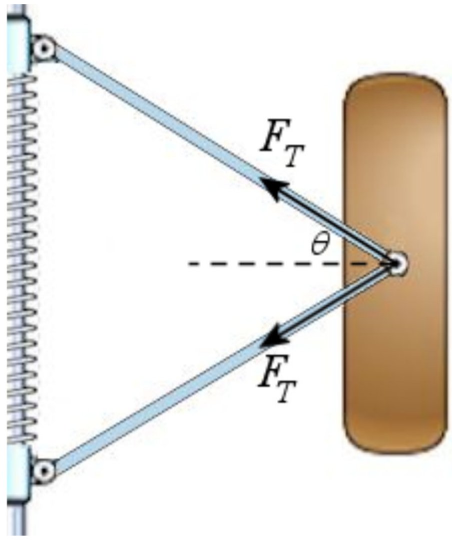
$$F_T = 6kg\frac{2\pi r}{T^2}$$

With the values of r and T, the tension is

$$F_T = 6kg\frac{2\pi \cdot 0.5m}{(0.25s)^2}$$

$$= 301.6N$$

19. Let's with considering the forces on one of the masses.



The sum of the x-components of the forces is

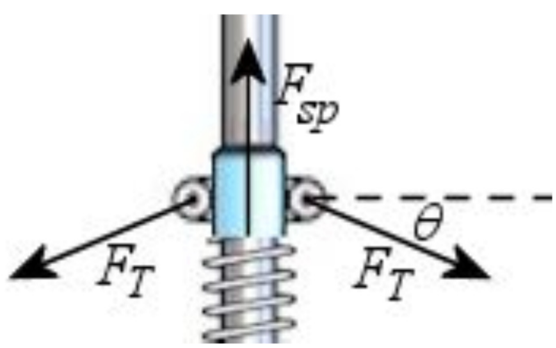
$$\sum F_x = ma_x$$

$$\rightarrow -F_T \cos\theta - F_T \cos\theta = -m\frac{4\pi^2 r}{T^2}$$

Therefore, the tension force exerted by the rods is

$$F_T = \frac{2m\pi^2 r}{T^2 \cos\theta}$$

Now, let's consider the force acting at the end of the spring.



The sum of the y-components of the forces is

$$\sum F_y = ma_y$$
$$\rightarrow F_T \sin(-\theta) + F_T \sin(-\theta) + F_{sp} = 0$$

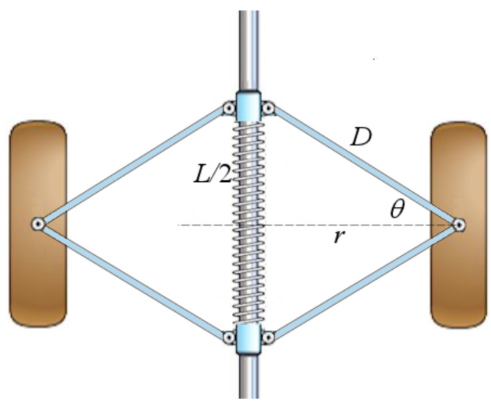
Thus, the force exerted by the spring is

$$F_{sp} = 2F_T \sin\theta$$

Using the value of the tension found previously, this force is

$$\begin{aligned} F_{sp} &= 2F_T \sin\theta \\ &= \frac{4m\pi^2 r}{T^2 \cos\theta}\sin\theta \end{aligned}$$

Now, let's make the connection between a few variables.



Firstly,

$$D\cos\theta = r$$

so that

$$\begin{aligned} F_{sp} &= \frac{4m\pi^2 r}{T^2 \cos\theta}\sin\theta \\ &= \frac{4m\pi^2 D}{T^2}\sin\theta \end{aligned}$$

Secondly,

$$D\sin\theta = \frac{L}{2}$$

so that

$$F_{sp} = \frac{4m\pi^2 D}{T^2}\sin\theta$$
$$= \frac{2m\pi^2 L}{T^2}$$

Since the force exerted by the spring is

$$F_{sp} = k\left(x_0 - L\right)$$

where x_0 is the length of the spring when it is neither stretched nor compressed, the equation becomes

$$k\left(x_0 - L\right) = \frac{2m\pi^2 L}{T^2}$$

It only remains to solve this equation for *L*.

$$kx_0 - kL = \frac{2m\pi^2 L}{T^2}$$
$$kx_0 = \left(\frac{2m\pi^2}{T^2} + k\right)L$$
$$L = \frac{x_0}{1 + \frac{2m\pi^2}{kT^2}}$$

With the values given, the length is

$$L = \frac{x_0}{1 + \frac{2m\pi^2}{kT^2}}$$
$$= \frac{80cm}{1 + \frac{2\cdot 1kg\cdot\pi^2}{2000\frac{N}{m}\cdot(0.1s)^2}}$$
$$= 40.263cm$$