Chapter 5 Solutions

1. The forces exerted on the 10 kg object are:

1) The weight, 98 N downwards.
2) A normal force F_N, upwards.
3) A friction force F_f towards the left.

The sum of the x-component of the forces is then

$$\sum F_x = ma_x$$
$$-F_f = ma_x$$
$$-\mu_k F_N = ma_x$$

The normal force can be found with the sum of the y-component of the forces.

$$\sum F_y = ma_y$$
$$-mg + F_N = 0$$
$$F_N = mg$$

The sum of the x-component of the forces then becomes

$$-\mu_k F_N = ma_x$$
$$-\mu_k mg = ma_x$$
$$-\mu_k g = a_x$$

To find μ_k, the acceleration must be known. As the block stops in 4 seconds, the acceleration is

$$v_x = v_{0x} + a_x t$$
$$0\tfrac{m}{s} = 10\tfrac{m}{s} + a_x \cdot 4s$$
$$a_x = -2.5\tfrac{m}{s^2}$$

Therefore

$$-\mu_k g = a_x$$
$$-\mu_k g = -2.5\tfrac{m}{s^2}$$
$$\mu_k = 0.2551$$

2. First, let's find the acceleration.

The forces exerted on the Guy are (using an *x*-axis directed downhill):

1) The weight, mg downwards (at -60°).
2) A normal force F_N, perpendicular to the slope.
3) A friction force $F_f = \mu_k F_N$ opposed to the motion, so directed uphill.

The table of force is

Forces	*x*	*y*
Weight	mg cos (-60°)	mg sin (-60°)
Normal force	0	F_N
Friction force	$-\mu_k F_N$	0

The equations of force are then

$$\sum F_x = mg\cos(-60°) - \mu_k F_N = ma_x$$
$$\sum F_y = mg\sin(-60°) + F_N = 0$$

The normal force can be found with the sum of the *y*-component of the forces.

$$mg\sin(-60°) + F_N = 0$$
$$F_N = -mg\sin(-60°)$$
$$F_N = mg\sin(60°)$$

This value is then substituted in the sum of the *x*-component of the forces.

$$mg\cos(-60°) - \mu_k mg\sin(60°) = ma_x$$
$$g\cos(-60°) - \mu_k g\sin(60°) = a_x$$
$$9.8\tfrac{N}{kg}\cos(-60°) - 0.1\cdot 9.8\tfrac{N}{kg}\sin(60°) = a_x$$
$$a_x = 4.051\tfrac{m}{s^2}$$

a) The time required to reach 50 m/s is

$$v_x = v_{0x} + a_x t$$
$$50\tfrac{m}{s} = 10\tfrac{m}{s} + 4.051\tfrac{m}{s^2}\cdot t$$
$$t = 9.873s$$

b) The distance travelled in 5 seconds is

$$\begin{aligned} x &= x_0 + v_{0x} + \tfrac{1}{2} a_x t^2 \\ &= 0m + 10\tfrac{m}{s} \cdot 5s + \tfrac{1}{2} \cdot 4.051\tfrac{m}{s^2} \cdot (5s)^2 \\ &= 100.6m \end{aligned}$$

3. Let's start with the situation on the horizontal surface to find the coefficient of friction

The forces exerted on Manon are:

1) The weight, *mg* downwards.
2) A normal force F_N, upwards.
3) A friction force F_f towards the left.

The sum of the *x*-component of the forces is then

$$\begin{aligned} \sum F_x &= ma_x \\ -F_f &= ma_x \\ -\mu_k F_N &= ma_x \end{aligned}$$

The normal force can be found with the sum of the *y*-component of the forces.

$$\begin{aligned} \sum F_y &= ma_y \\ -mg + F_N &= 0 \\ F_N &= mg \end{aligned}$$

The sum of the *x*-component of the forces then becomes

$$\begin{aligned} -\mu_k F_N &= ma_x \\ -\mu_k mg &= ma_x \\ -\mu_k g &= a_x \end{aligned}$$

To find μ_k, the acceleration must be known. As Manon stops over a distance of 10 m, the acceleration is

$$\begin{aligned} 2a_x (x - x_0) &= v_x^2 - v_{0x}^2 \\ 2a_x (40m - 0m) &= \left(0\tfrac{m}{s}\right)^2 - \left(10\tfrac{m}{s}\right)^2 \\ a_x &= -1.25\tfrac{m}{s^2} \end{aligned}$$

Then

$$-\mu_k g = a_x$$
$$-\mu_k g = -1.25\tfrac{m}{s^2}$$
$$\mu_k = 0.12755$$

Let us now examine the situation on the slope.

The forces exerted on Manon are (using an *x*-axis directed uphill):

1) The weight, mg downwards (at -120°).
2) A normal force F_N, perpendicular to the slope.
3) A friction force $F_f = \mu_k F_N$ opposed to the motion, so directed downhill.

The table of force is

Forces	*x*	*y*
Weight	mg cos (-120°)	mg sin (-120°)
Normal force	0	F_N
Friction force	$-\mu_k F_N$	0

The equations of force are then

$$\sum F_x = mg\cos(-120°) - \mu_k F_N = ma_x$$
$$\sum F_y = mg\sin(-120°) + F_N = 0$$

The normal force can be found with the sum of the *y*-component of the forces.

$$mg\sin(-120°) + F_N = 0$$
$$F_N = -mg\sin(-120°)$$
$$F_N = mg\sin(120°)$$

This value is then substituted in the sum of the *x*-component of the forces.

$$mg\cos(-120°) - \mu_k mg\sin(120°) = ma_x$$
$$g\cos(120°) - \mu_k g\sin(120°) = a_x$$
$$9.8\tfrac{N}{kg}\cos(120°) - 0.12755\cdot 9.8\tfrac{N}{kg}\sin(120°) = a_x$$
$$a_x = -5.9825\tfrac{m}{s^2}$$

The stopping distance is therefore

$$2a_x\left(x-x_0\right)=v_x^2-v_{0x}^2$$
$$2\cdot -5.9825\tfrac{m}{s^2}\cdot\left(x-0m\right)=\left(0\tfrac{m}{s}\right)^2-\left(10\tfrac{m}{s}\right)^2$$
$$x=8.358m$$

4. The forces exerted on the 24 kg block on the horizontal surface are (m_1 = 24 kg) (using an x-axis directed to the right):

1) The weight, 235.2 N downwards.
2) A normal force F_{N1}, upwards.
3) A tension force, towards the right.
4) A friction force $F_{f1}=\mu_k F_{N1}$ towards the right.

The equations of force are then

$$\sum F_x = ma_x$$
$$\rightarrow T+\mu_k F_{N1}=m_1 a$$
$$\sum F_y = ma_y$$
$$\rightarrow -235.2N+F_{N1}=0$$

The sum of the y-component of the forces allows us to find that F_{N1} = 235.2 N. The sum of the x-component of the forces then becomes

$$T+\mu_k F_{N1}=m_1 a$$
$$T+0.4\cdot 235.2N=m_1 a$$
$$T+94.08N=m_1 a$$

The forces exerted on the 18 kg block on the slope are (m_2 = 18 kg) (using an x-axis directed downhill):

1) The weight, 176.4 N downwards, therefore at -30°.
2) A normal force F_{N2}, perpendicular to the surface, so towards the positive y-axis
3) A tension force directed uphill.
4) A friction force $F_{f2}=\mu_k F_{N2}$ directed downhill.

The table of force is

Forces	*x*	*y*
Weight	176,4 N cos (-30°)	176,4 N sin (-30°)
Normal	0	F_{N2}
Friction	$\mu_k F_{N2}$	0
Tension	-*T*	0

The equations of force are then

$$\sum F_x = ma_x$$
$$\rightarrow 176.4N\cos\left(-30°\right) + \mu_k F_{N2} - T = m_2 a$$
$$\sum F_y = ma_y$$
$$\rightarrow 176.4N\sin\left(-30°\right) + F_{N2} = 0$$

The normal force can be found with the sum of the *y*-component of the forces.

$$176.4N\sin\left(-30°\right) + F_{N2} = 0$$
$$F_{N2} = -176.4N\sin\left(-30°\right)$$
$$F_{N2} = 88.2N$$

The sum of the *x*-component of the forces then becomes

$$176.4N\cos\left(30°\right) + \mu_k F_{N2} - T = m_2 a$$
$$176.4N\cos\left(30°\right) + 0.4\cdot 88.2N - T = m_2 a$$
$$188.047N - T = m_2 a$$

The two equation for the sum of the *x*-component of the forces are then

$$T + 94.08N = m_1 a$$
$$188.047N - T = m_2 a$$

This system can be solved by adding these equations.

$$\left(T + 94.08N\right) + \left(188.047N - T\right) = m_1 a + m_2 a$$
$$94.08N + 188.047N = \left(m_1 + m_2\right)a$$
$$282.127N = 42kg\cdot a$$
$$a = 6.7173\tfrac{m}{s^2}$$

The tension can then be found

$$T + 94.08N = m_1 a$$
$$T + 94.08N = 24kg \times 6.7173\tfrac{m}{s^2}$$
$$T = 67.135N$$

5. Since the friction coefficients are different for each block, it would be difficult to find the acceleration of the system by considering it as a single object. Instead, the forces on each block must be found.

The forces exerted on the 12 kg object are (m_1 = 12 kg) (using an x-axis directed to the right):

1) The weight, 117.6 N downwards.
2) A normal force F_{N1}, upwards.
3) A friction force F_{f1} towards the right.
4) The 30 N force at 37°.
5) The tension force (T_1) of the rope between the 12 kg block and the 8 kg block, directed towards the left.

The equations of force are then

$$\sum F_x = ma_x$$
$$\rightarrow 30N\cos(37°) + \mu_{k1}F_{N1} - T_1 = m_1 a$$
$$\sum F_y = ma_y$$
$$\rightarrow -117.6N + F_{N1} + 30N\sin(37°) = 0$$

The normal force can be found with the sum of the y-component of the forces.

$$-117.6N + F_{N1} + 30N\sin(37°) = 0$$
$$F_{N1} = 117.6N - 30N\sin(37°)$$
$$F_{N1} = 99.55N$$

The sum of the x-component of the forces then becomes

$$30N\cos(37°) + \mu_{k1}F_{N1} - T_1 = m_1 a$$
$$30N\cos(37°) + 0.5 \times 99.55N - T_1 = m_1 a$$
$$73.732N - T_1 = m_1 a$$

The forces exerted on the 8 kg object are (using an x-axis directed to the right):

1) The weight, 78.4 N downwards.
2) A normal force F_{N2}, upwards.
3) A friction force F_{f2} towards the right.
4) The tension force (T_1) of the rope between the 12 kg block and the 8 kg block, directed towards the right.
5) The tension force (T_2) of the rope between the 8 kg block and the 5 kg block, directed towards the left.

The equations of force are then

$$\begin{aligned}
&\sum F_x = ma_x \\
&\qquad \to \mu_{k2} F_{N2} + T_1 - T_2 = m_2 a \\
&\sum F_y = ma_y \\
&\qquad \to -78,4N + F_{N2} = 0
\end{aligned}$$

The sum of the y-component of the forces allows us to find that F_{N2} = 78.4 N. The sum of the x-component of the forces then becomes

$$\begin{aligned}
\mu_{k2} F_{N2} + T_1 - T_2 &= m_2 a \\
0.4 \times 78.4N + T_1 - T_2 &= m_2 a \\
31.36N + T_1 - T_2 &= m_2 a
\end{aligned}$$

The forces exerted on the 5 kg box are (m_3 = 5 kg) (using an x-axis directed to the right):

1) The weight, 49 N downwards.
2) A normal force F_{N3}, upwards.
3) A friction force F_{f3} towards the right.
4) The tension force (T_2) of the rope between the 8 kg block and the 5 kg block, directed towards the right.

The equations of force are then

$$\begin{aligned}
&\sum F_x = ma_x \\
&\qquad \to \mu_{k3} F_{N3} + T_2 = m_3 a \\
&\sum F_y = ma_y \\
&\qquad \to -49N + F_{N3} = 0
\end{aligned}$$

The sum of the y-component of the forces allows us to find that F_{N3} = 49 N. The sum of the x-component of the forces then becomes

$$\begin{aligned}\mu_{k3}F_{N3}+T_2&=m_3a\\0.3\times 49N+T_2&=m_3a\\14.7N+T_2&=m_3a\end{aligned}$$

The three equations for the sum of the x-component of the forces are then

$$\begin{aligned}73.732N-T_1&=m_1a\\31.36N+T_1-T_2&=m_2a\\14.7N+T_2&=m_3a\end{aligned}$$

This system can be solved by adding these three equations.

$$\begin{aligned}(73.732N-T_1)+(31.36N+T_1-T_2)+(14.7N+T_2)&=m_1a+m_2a+m_3a\\73.732N+31.36N+14.7N&=(m_1+m_2+m_3)a\\119.792N&=25kg\times a\\a&=4.792\tfrac{m}{s^2}\end{aligned}$$

Then, the tensions can be found. T_1 is found with the sum of the x-component of the forces for the 12 kg block.

$$\begin{aligned}73.732N-T_1&=m_1a\\T_1&=16.232N\end{aligned}$$

T_2 is found with the sum of the x-component of the forces for the 5 kg block.

$$\begin{aligned}14.7N+T_2&=m_3a\\14.7N+T_2&=5kg\times 4.792\tfrac{m}{s^2}\\T_2&=9.258N\end{aligned}$$

6. The forces exerted on the 100 kg block of ice are (using an x-axis directed to the right):

1) The weight, 980 N downwards.
2) A normal force F_N, upwards.
3) A friction force $F_f = \mu_k F_N$ towards the left.
4) The tension force T at 25°.

The equations of force are then

$$\sum F_x = ma_x$$
$$\to -\mu_k F_N + F\cos(25°) = 0$$
$$\sum F_y = ma_y$$
$$\to -980N + F_N + F\sin(25°) = 0$$

The normal force can be found with the sum of the y-component of the forces.

$$F_N = 980N - F\sin(25°)$$

This value is then substituted in the sum of the x-component of the forces.

$$-\mu_k F_N + F\cos(25°) = 0$$
$$-\mu_k (980N - F\sin(25°)) + F\cos(25°) = 0$$
$$-\mu_k \cdot 980N + \mu_k F\sin(25°) + F\cos(25°) = 0$$
$$F(\mu_k \sin(25°) + \cos(25°)) = \mu_c \cdot 980N$$
$$F(0.1 \cdot \sin(25°) + \cos(25°)) = 0.1 \cdot 980N$$
$$F = 103.31N$$

7. The forces exerted on the sled are (using an x-axis directed uphill):

1) The weight, 78.4 N downwards (at -100°).
2) A normal force F_N, towards the positive y-axis.
3) A friction force $F_f = \mu_k F_N$ directed downhill (towards the negative x-axis).
4) The tension force of the rope T, at 20°.

The table of force is

Forces	*x*	*y*
Weight	78.4 N cos (-100°)	78.4 N sin (-100°)
Normal force	0	F_N
Friction force	$-\mu_k F_N$	0
Tension force	T cos20°	T sin20°

Since the acceleration is zero, the equations of force are

$$\sum F_x = ma_x$$
$$\rightarrow 78.4N\cos(-100°) - \mu_k F_N + T\cos(20°) = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow 78.4N\sin(-100°) + F_N + T\sin(20°) = 0$$

We have two equations and two unknowns. To resolve, we will solve for the normal force in the sum of the *y*-component of the forces.

$$F_N = -78.4N\sin(-100°) - T\sin(20°)$$
$$F_N = 78.4N\sin(100°) - T\sin(20°)$$

This value is then substituted in the sum of the *x*-component of the forces.

$$78.4N\cos(100°) - \mu_k F_N + T\cos(20°) = 0$$
$$78.4N\cos(100°) - \mu_k\left(78.4N\sin(100°) - T\sin(20°)\right) + T\cos(20°) = 0$$
$$78.4N\cos(100°) - \mu_k 78.4N\sin(100°) + \mu_k T\sin(20°) + T\cos(20°) = 0$$
$$78.4N\cos(100°) - \mu_k 78.4N\sin(100°) = -\mu_k T\sin(20°) - T\cos(20°)$$
$$78.4N\cos(100°) - \mu_k 78.4N\sin(100°) = T\left(-\mu_k\sin(20°) - \cos(20°)\right)$$
$$T = \frac{78.4N\cos(100°) - \mu_k 78.4N\sin(100°)}{-\mu_k\sin(20°) - \cos(20°)}$$
$$T = \frac{-13.614N - 9.265N}{-0.0410 - 0.9397}$$
$$T = 23.33N$$

8. The forces acting on the 50 kg box are (an *x*-axis towards the right is used) :

1) The weight (490 N) directed downwards.
2) A normal force directed towards the positive *y*-axis.
3) A friction force $F_f = \mu_c F_N$ directed towards the left.
4) The 300 N force exerted by the rope at the angle θ.

Thus, the table of force is

Forces	*x*	*y*
Weight	0	$-mg$
Normal force	0	F_N
Friction force	$-\mu_c F_N$	0
Tension force	$T\cos\theta$	$T\sin\theta$

The equations of forces are

$$\begin{aligned}&\sum F_x = ma_x\\&\qquad\rightarrow -\mu_c F_N + T\cos\theta = ma\\&\sum F_y = ma_y\\&\qquad\rightarrow -mg + F_N + T\sin\theta = 0\end{aligned}$$

With the second equation, the normal force is found.

$$F_N = mg - T\sin\theta$$

This value can then be substituted in the equation for the *x*-components of the forces.

$$\begin{aligned}-\mu_c\left(mg - T\sin\theta\right) + T\cos\theta &= ma\\a &= \frac{-\mu_c\left(mg - T\sin\theta\right) + T\cos\theta}{m}\\a &= -\mu_c g + \frac{T}{m}\mu_c\sin\theta + \frac{T}{m}\cos\theta\end{aligned}$$

At the maximum acceleration, we have *da/dq* = 0. Thus,

$$\frac{da}{d\theta} = 0 + \frac{T}{m}\mu_c\cos\theta - \frac{T}{m}\sin\theta = 0$$

This leads to

$$\begin{aligned}\frac{T}{m}\mu_c\cos\theta - \frac{T}{m}\sin\theta &= 0\\\mu_c\cos\theta - \sin\theta &= 0\\\mu_c\cos\theta &= \sin\theta\\\mu_c &= \tan\theta\end{aligned}$$

Therefore, the angle required to have the largest acceleration is

$$\begin{aligned}\theta &= \arctan 0.7\\&= 35^\circ\end{aligned}$$

9. Let's consider first the situation where the box remains at rest to find the force of friction required for the object to remain at rest. The forces exerted on the crate are:

1) The weight, 980 N downwards.
2) A normal force F_N, upwards.
3) A 600 N tension force T, towards the right.
4) A friction force F_f towards the left.

Since the acceleration is zero, the equations of force are

$$\sum F_x = ma_x$$
$$\rightarrow 600N - F_f = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -980N + F_N = 0$$

The sum of the x-component of the forces allows us to find the friction force.

$$600N - F_f = 0$$
$$F_f = 600N$$

A friction force of 600 N is required for the object to remain at rest.

The maximum value of the force of friction is

$$\begin{aligned} F_{f\max} &= \mu_s F_N \\ &= 0.6 \times 980N \\ &= 588N \end{aligned}$$

With a maximum friction force of 588 N, we cannot have the 600 N of friction required to keep the object at rest. The crate will therefore move.

10. The forces exerted on the box are (using an x-axis directed to the right):

1) The weight, mg downwards.
2) A normal force F_N, upwards.
3) A horizontal friction force F_f.

The equations of force are then

$$\sum F_x = ma_x$$
$$\to F_f = ma$$
$$\sum F_y = ma_y$$
$$\to -mg + F_N = 0$$

The first equation tells us that the force of friction and the acceleration have the same sign. With a positive acceleration, the frictional force must be positive, therefore towards the right.

Also, the equation tells us that the frictional force to the right is the force which accelerated the box to the right so that it can stay with the truck. Furthermore, we have

$$ma = F_f \le \mu_s F_N$$

This becomes

$$ma \le \mu_s F_N$$
$$ma \le \mu_s mg$$
$$a \le \mu_s g$$
$$a \le 0.65 \times 9.8 \tfrac{N}{kg}$$
$$a \le 6.37 \tfrac{m}{s^2}$$

The maximum acceleration of the box is therefore of 6.37 m/s². If the truck has a greater acceleration than this, the box slips.

11. Let's consider first the situation where the block remains at rest in order to find the force of friction required for the object to remain at rest. The forces exerted on the block are (using an *x*-axis directed downhill):

1) The weight, *mg* downwards, therefore at -65°.
2) A normal force F_N perpendicular to the slope, so towards the positive *y*-axis.
3) A friction force F_f directed uphill.

Since the acceleration is zero, the equations of force are

$$\sum F_x = ma_x$$
$$\rightarrow mg\cos(-55°) - F_f = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow mg\sin(-55°) + F_N = 0$$

The sum of the *x*-component of the forces allows us to find the friction force.

$$mg\cos(-55°) - F_f = 0$$
$$F_f = mg\cos(-55°)$$
$$F_f = mg\cos(55°)$$

The maximum value of the force of friction is

$$F_{f\max} = \mu_s F_N$$
$$= 0,8 \times mg\sin(55°)$$

(The normal force was found with the sum of the *y*-component of the forces.)

It's hard to tell if F_f required for the object to remain at rest is greater than F_{fmax}. To find out, we will calculate the ratio of these forces.

$$\frac{F_{f\max}}{F_f} = \frac{\mu_s mg\sin(55°)}{mg\cos(55°)}$$
$$\frac{F_{f\max}}{F_f} = \frac{\mu_s \sin(55°)}{\cos(55°)}$$
$$\frac{F_{f\max}}{F_f} = \mu_s \tan(55°)$$
$$\frac{F_{f\max}}{F_f} = 0.8 \times \tan(55°)$$
$$\frac{F_{f\max}}{F_f} = 1.1425$$

This means that the maximum frictional force is 1.1425 times larger than the force required to keep the object at rest. As it is larger than the friction required to keep the object at rest, the object remains at rest.

12. The forces exerted on the 5 kg box (box A) are

1) The weight, 49 N downwards.
2) A normal force F_{N1}, upwards, exerted by box B.
3) A tension force T_1 towards the left.
4) A friction force F_{f1} towards the right, exerted by box B.

The equations of force are then

$$\sum F_x = ma_x$$
$$\to -T + F_{f1} = 0$$
$$\sum F_y = ma_y$$
$$\to -49N + F_{N1} = 0$$

The normal force can be found with the sum of the y-component of the forces. It is F_{N1} = 49N.

Now let's look at the forces on the 10 kg box (box B). Let's assume that this box is at rest. This will allow us to know what is required for this box to be at rest. The forces exerted on box B are:

1) The weight 98 N, downwards.
2) A normal force F_{N1} downwards, exerted by box A.
3) A normal force F_{N2}, upwards exerted by the ground.
4) A friction force F_{f1} towards the left, exerted by box A.
5) A friction force F_{f2} towards the left, exerted by the ground.
6) The force F, towards the right.

Since the acceleration is zero, the equations of force are

$$\sum F_x = ma_x$$
$$\to -F_{f1} - F_{f2} + F = 0$$
$$\sum F_y = ma_y$$
$$\to -98N - F_{N1} + F_{N2} = 0$$

Since F_{N1} = 49 N, this last equation gives us F_{N2} = 147 N.

With the sum of the x-component of the forces on box B, we find

$$F = F_{f1} + F_{f2}$$

Since

$$F_{f1} \leq \mu_{s1} F_{N1} \qquad\qquad F_{f2} \leq \mu_{s2} F_{N2}$$

we have

$$F = F_{f1} + F_{f2} \leq \mu_{s1} F_{N1} + \mu_{s1} F_{N1}$$
$$F \leq \mu_{s1} F_{N1} + \mu_{s1} F_{N1}$$
$$F \leq 0.8 \times 49N + 0.6 \times 147N$$
$$F \leq 127.4N$$

This is what is required for the box B to remain at rest. If we want this box to move, we must pull with a force larger than 127.4 N.

13. The forces exerted on Donald are:

1) The weight, 490 N downwards.
2) A normal force F_{N1} towards the left, exerted by the wall on the right.
3) A normal force F_{N2} towards the right, exerted by the wall on the left.
4) A friction force F_{f1} upwards, exerted by the wall on the right.
5) A friction force F_{f2} upwards, exerted by the wall on the left.

Since the acceleration of Donald is zero, the equations of force are

$$\sum F_x = ma_x$$
$$\rightarrow -F_{N1} + F_{N2} = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -490N + F_{f1} + F_{f2} = 0$$

With the sum of the y-component of the forces, we find

$$-490N + F_{f1} + F_{f2} = 0$$
$$F_{f1} + F_{f2} = 490N$$

Since

$$F_{f1} \leq \mu_{s1} F_{N1} \qquad\qquad F_{f2} \leq \mu_{s2} F_{N2}$$

we have

$$490N = F_{f1} + F_{f2} \leq \mu_s F_{N1} + \mu_s F_{N2}$$
$$490N \leq \mu_s F_{N1} + \mu_s F_{N2}$$
$$490N \leq \mu_s F_{N1} + \mu_s F_{N1}$$

(since the two normal forces are equal according to the sum of the *x*-component of the forces).

$$490N \leq \mu_s \left(F_{N1} + F_{N1}\right)$$
$$490N \leq 2\mu_s F_{N1}$$
$$F_{N1} \geq \frac{490N}{2\mu_s}$$
$$F_{N1} \geq \frac{490N}{2\times 1.4}$$
$$F_{N1} \geq 175N$$

Donald must, therefore, push on the walls so that the normal forces are at least 175 N.

14. The forces exerted on the box are (using an *x*-axis directed to the right):

1) The weight, 980 N downwards.
2) A normal force F_N, upwards.
3) A friction force F_f towards the left.
4) The force exerted by Boris (at -30°).

The table of force is

Forces	*x*	*y*
Weight	0	-980 N
Normal force	0	F_N
Friction force	$-F_f$	0
Boris	F cos (-30°)	F sin (-30°)

Let's assume that the box is motionless. This will allow us to find out what is required for this box to be at rest. If we want the box to move, we'll only have to do the opposite of what is required for this box to remain at rest.

Since the acceleration is zero, the equations of force are

$$\sum F_x = ma_x$$
$$\rightarrow -F_f + F\cos(30°) = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -980N + F_N + F\sin(-30°) = 0$$

With the sum of the *x*-component of the forces, we have

$$-F_f + F\cos(30°) = 0$$
$$F\cos(30°) = F_f$$

Since we must have

$$F_f \leq \mu_s F_N$$

then

$$F\cos(30°) = F_f \leq \mu_s F_N$$
$$F\cos(30°) \leq \mu_s F_N$$

The normal force can be found with the sum of the *y*-component of the forces.

$$-980N + F_N + F\sin(-30°) = 0$$
$$F_N = 980N - F\sin(-30°)$$
$$F_N = 980N + F\sin(30°)$$

Therefore

$$F\cos(30°) \leq \mu_s F_N$$
$$F\cos(30°) \leq \mu_s\left(980N + F\sin(30°)\right)$$
$$F\cos(30°) \leq \mu_s 980N + \mu_s F\sin(30°)$$
$$F\cos(30°) - \mu_s F\sin(30°) \leq \mu_s 980N$$
$$F\left(\cos(30°) - \mu_s\sin(30°)\right) \leq \mu_s 980N$$
$$F\left(\cos(30°) - 0,5\times\sin(30°)\right) \leq 0,5\times 980N$$
$$F(0.616) \leq 490N$$
$$F \leq 795.4N$$

The object remains at rest if the applied force is less than 795.4 N. Therefore, if the force exerted is larger than 795.4 N, the object will move.

15. The forces exerted on the block are:

1) The weight, 9.8 N downwards.
2) A normal force F_N, towards the left.
3) A friction force F_f, upwards or downwards.
4) The 30 N force F, at 30°.

The direction of the frictional force is not known because we do not know if the vertical component of the force F is larger or smaller than the weight of the block. If this component is smaller than the weight, there must be a friction directed upwards to help support the block. If this component is larger than the weight, there must be a friction force downwards to prevent the block from sliding upwards. We will, therefore, presume that friction force F_f is directed upwards and then found its magnitude. If we get a positive answer, the force is in the assumed direction (upwards). If we get a negative answer, the force is in the direction opposite to the direction assumed (downwards).

If the block remains at rest, the acceleration is zero and the equations of the forces are

$$\sum F_x = ma_x$$
$$\to -F_N + 30N\cos(30°) = 0$$
$$\sum F_y = ma_y$$
$$\to -9.8N + F_f + 30N\sin(30°) = 0$$

a) The friction force can be found with the sum of the y-component of the forces.

$$-9.8N + F_f + 30N\sin(30°) = 0$$
$$F_f = 9.8N - 30N\sin(30°)$$
$$F_f = -5.2N$$

Since the answer is negative, the frictional force is 5.2 N downwards.

b) The minimal friction coefficient is found with

$$F_f \le \mu_s F_N$$

The normal force can be found with the sum of the y-component of the forces.

$$-F_N + 30N\cos(30°) = 0$$
$$F_N = 30N\cos(30°)$$
$$F_N = 25.98N$$

Therefore

$$5.2N \leq \mu_s 25.98N$$
$$\mu_s \geq 0.2001$$

The coefficient must, therefore, be larger than 0.2001.

16. To know the force of friction, we need to know if we have to deal with kinetic friction or static friction. We will first calculate if the stone remains at rest. The forces exerted on the stone are:

1) The weight, 980 N downwards.
2) A normal force F_N, upwards.
3) The tension force T towards the right.
4) A friction force F_f towards the left.

If the acceleration is zero, the equations of force are

$$\sum F_x = ma_x$$
$$\to T - F_f = 0$$
$$\sum F_y = ma_y$$
$$\to -980N + F_N = 0$$

The friction force is, therefore, equal to the tension of the rope. This tension is found by considering the forces on Thierry. The forces exerted on Thierry are:

1) The weight, 686 N downwards.
2) The tension force T upwards.

The sum of the y-component of the forces is then

$$-686N + T = 0$$

The tension is therefore 686 N. This means that the force of friction on the stone must also be 686 N so that the stone remains at rest. Can we have a frictional force as large as that? The maximum value of the friction force is

$$F_{f\max} = \mu_s F_N$$

The normal force can be found with the sum of the *y*-component of the forces acting on the stone. It is 980 N. The maximum value of the friction force is therefore

$$\begin{aligned} F_{f\max} &= 0.5 \times 980N \\ &= 490N \end{aligned}$$

As 686 N are required to keep the stone at rest, the stone will slip.

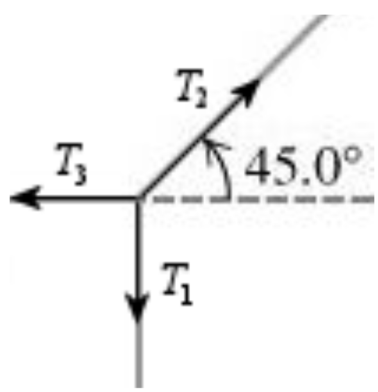
If the stone is slipping, we have kinetic friction, whose magnitude is

$$\begin{aligned} F_f &= \mu_k F_N \\ &= 0.45N \times 980N \\ &= 441N \end{aligned}$$

The force of friction is therefore 441 N towards the left.

17. Let's assume that the system is at rest. This will allow us to find the necessary condition so that this system is at rest.

Let's start by finding the forces acting on the node where the three strings meet.



Obviously, the tension T_1 is equal to the weight of the 20 kg block, thus T_1 = 196 N.

The table of force is

Forces	x	y
Tension 1	0	-196 N
Tension 2	$T_2 \cos 45°$	$T_2 \sin 45°$
Tension 3	$-T_3$	0

The equations of force are then

$$\sum F_x = ma_x$$
$$\rightarrow T_2 \cos(45°) - T_3 = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -196N + T_2 \sin(45°) = 0$$

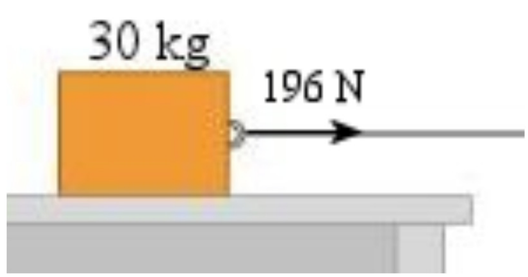
The tension T_2 can be found with the sum of the y-component of the forces.

$$-196N + T_2 \sin(45°) = 0$$
$$T_2 = 277.2N$$

Then, the tension T_3 can be found with the sum of the x-component of the forces.

$$T_2 \cos(45°) - T_3 = 0$$
$$277.2N \cos(45°) - T_3 = 0$$
$$T_3 = 196N$$

We now have the following situation.



The forces exerted on the block are:

1) The weight, 294 N downwards.
2) A normal force F_N, upwards.
3) The tension force, 196 N, towards the positive x-axis.
4) A friction force F_f towards the negative x-axis.

The table of force is

Forces	*x*	*y*
Weight	0	-294 N
Normal force	0	F_N
Tension force	196 N	0
Friction force	$-F_f$	0

The equations of force are then

$$\begin{aligned}&\sum F_x = ma_x\\&\qquad \to 196N - F_f = 0\\&\sum F_y = ma_y\\&\qquad \to -294N + F_N = 0\end{aligned}$$

The friction force can be found with the sum of the *x*-component of the forces.

$$196N - F_f = 0$$
$$F_f = 196N$$

We must then have

$$F_f \leq \mu_s F_N$$
$$294N \leq \mu_s \times 196N$$

(The normal comes from the sum of the *y*-component of the forces acting on the 30 kg block.)

We then have

$$196N \leq \mu_s \times 294N$$
$$\tfrac{2}{3} \leq \mu_s$$

18. The force is

$$\begin{aligned}F_d &= \frac{1}{2} C_x A \rho v^2\\&= \frac{1}{2} \times 0.47 \times \pi \left(0.11m\right)^2 \times 1.3 \tfrac{kg}{m^3} \times \left(20 \tfrac{m}{s}\right)^2\\&= 4.645N\end{aligned}$$

19. The force is

$$\begin{aligned} F_d &= \frac{1}{2}(C_x A)\rho v^2 \\ &= \frac{1}{2}\times 0.682m^2 \times 1.3\tfrac{kg}{m^3}\times\left(33.33\tfrac{m}{s}\right)^2 \\ &= 492.6N \end{aligned}$$

20. The terminal velocity is

$$\begin{aligned} v_t &= \sqrt{\frac{2mg}{C_x A\rho}} \\ &= \sqrt{\frac{2\times 0.44kg\times 9.8\frac{N}{kg}}{0.47\times\pi(0.11m)^2\times 1.3\frac{kg}{m^3}}} \\ &= 19.27\tfrac{m}{s} \end{aligned}$$

21. The terminal velocity is

$$v_t = \sqrt{\frac{2mg}{C_x A\rho}}$$

If the volume is 0.01 m³, then the edge of the cube has a length of

$$\begin{aligned} L^3 &= 0.01m^3 \\ L &= 0.21544m \end{aligned}$$

The mass of the cube is then

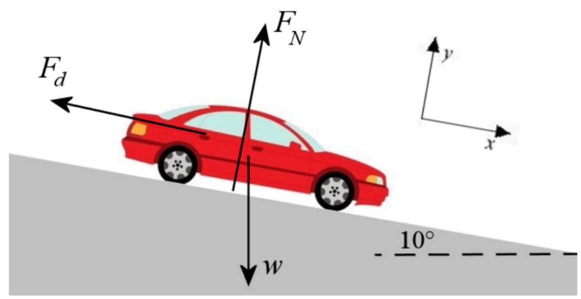
$$\begin{aligned} m &= \rho\times(volume) \\ &= 7320\tfrac{kg}{m^3}\times 0.01m^3 \\ &= 73.2kg \end{aligned}$$

Since the value of C_x is 1.05, the terminal velocity is

$$
\begin{aligned}
v_t &= \sqrt{\frac{2mg}{C_x A\rho}} \\
&= \sqrt{\frac{2\times 73.2kg \times 9.8\frac{N}{kg}}{1.05\times(0.21544m)^2\times 1.3\frac{kg}{m^3}}} \\
&= 150.5\tfrac{m}{s}
\end{aligned}
$$

22. The forces acting on the Honda Civic are (using an axis directed downhill):

1) The weight, mg downwards (-80°).
2) A normal force F_N, towards the positive y-axis.
3) A drag force F_d directed downhill (towards the negative x-axis).



The table of force is

Forces	x	y
Weight	mg cos (-80°)	mg sin (-80°)
Normal Force	0	F_N
Drag Force	$-F_d$	0

At the terminal velocity, the acceleration is zero. The equations of the forces are, therefore,

$$
\begin{aligned}
&\sum F_x = ma_x \\
&\qquad \to mg\cos(-80°) - F_d = 0 \\
&\sum F_y = ma_y \\
&\qquad \to mg\sin(-80°) + F_N = 0
\end{aligned}
$$

The first of these equations gives

$$
\begin{aligned}
mg\cos(-80°) - F_d &= 0 \\
mg\cos(80°) - \frac{1}{2}C_x A\rho v_t^2 &= 0 \\
v_t &= \sqrt{\frac{2mg\cos(80°)}{C_x A\rho}}
\end{aligned}
$$

For a 2001 Honda Civic, C_xA = 0.682 m². With an 1142 kg mass, the terminal velocity is

$$\begin{aligned} v_t &= \sqrt{\frac{2\cdot 1142kg\cdot 9.8\frac{N}{kg}\cdot\cos(80°)}{0.682m^2\cdot 1.3\frac{kg}{m^3}}} \\ &= 66.21\tfrac{m}{s} \\ &= 238.4\tfrac{km}{h} \end{aligned}$$

23. With a terminal velocity of 30 m/s, we find that

$$\begin{aligned} v_t &= \sqrt{\frac{2mg}{C_xA\rho}} \\ 30\tfrac{m}{s} &= \sqrt{\frac{2\times 100kg\times 9.8\frac{N}{kg}}{CA_x\times 1.3\frac{kg}{m^3}}} \\ CA_x &= 1.675m^2 \end{aligned}$$

The frictional force at half the terminal velocity is therefore

$$\begin{aligned} F_d &= \frac{1}{2}(C_xA)\rho v^2 \\ &= \frac{1}{2}\times 1.675m^2\times 1.3\tfrac{kg}{m^3}\times\left(15\tfrac{m}{s}\right)^2 \\ &= 245N \end{aligned}$$

24. The forces exerted on the 2 kg carriage are (using an *x*-axis directed upwards):

1) The weight, 19.6 N downwards.
2) A normal force F_N, upwards.
3) The spring force F_{sp}, towards the right.

The sum of the *x*-component of the forces is then

$$\begin{aligned} \sum F_x &= ma_x \\ &\to F_{sp} = m\cdot a_x \end{aligned}$$

The spring force is then

$$F_{sp} = 2kg \times 4\tfrac{m}{s^2}$$
$$F_{sp} = 8N$$

Therefore, the stretching of the spring is

$$F_{sp} = kx$$
$$8N = 200\tfrac{N}{m} \cdot x$$
$$x = 0.04m = 4cm$$

25. a) Let's assume that the box is at rest. This will allow us to know the condition that must be met for the box to be at rest. The forces exerted on the 100 kg box are (using an x-axis directed to the right):

1) The weight, 980 N downwards.
2) A normal force F_N, upwards.
3) The spring force F_{sp} towards the right.
4) A friction force F_f towards the left.

The equations of force are then

$$\sum F_x = ma_x$$
$$\rightarrow F_{sp} - F_f = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -980N + F_N = 0$$

We easily found that $F_{sp} = F_f$. Therefore

$$F_{sp} = F_f \le \mu_s F_N$$
$$F_{sp} \le \mu_s F_N$$
$$F_{sp} \le 0.5 \times 980N$$
$$F_{sp} \le 490N$$

(The normal comes from the sum of the y-component of the forces.)

The equation then becomes

$$F_{sp} \leq 490N$$
$$kx \leq 490N$$
$$x \leq \frac{490N}{k}$$
$$x \leq \frac{490N}{1000\frac{N}{m}}$$
$$x \leq 0.49m$$

Thus, the box remains in place as long as the stretching of the spring is less than 49 cm. Therefore, if the stretching of the spring is larger than 49 cm, then the box moves.

b) The forces exerted on the 100 kg box are (using an *x*-axis directed to the right):

1) The weight, 980 N downwards.
2) A normal force F_N, upwards.
3) The spring force F_{sp} towards the right.
4) A friction force $F_f = \mu_k F_N$ towards the left.

A constant velocity, the acceleration is zero. The equations of force are then

$$\sum F_x = ma_x$$
$$\rightarrow F_{sp} - \mu_k F_N = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -980N + F_N = 0$$

We therefore have

$$F_{sp} = \mu_k F_N$$
$$F_{sp} = 0.4 \times 980N$$
$$F_{sp} = 392N$$

(The normal comes from the sum of the *y*-component of the forces.)

This gives us

$$F_{sp} = 392N$$
$$kx = 392N$$
$$x = \frac{392N}{k}$$
$$x = \frac{392N}{1000\frac{N}{m}}$$
$$x = 0.392m$$

The spring is so stretched 39.2 cm.

26. The forces exerted on the 5 kg box are (using an *x*-axis directed downhill):

1) The weight, 49 N downwards
2) A normal force F_N, upwards
3) The spring force $F_{sp} = kx$ towards the left
4) A friction force $F_f = \mu_k F_N$ towards the left

The equations of force are then

$$\sum F_x = ma_x$$
$$\rightarrow -F_{sp} - \mu_k F_N = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -49N + F_N = 0$$

The sum of the *x*-component of the forces gives us

$$-kx - \mu_k F_N = ma$$
$$-5000\tfrac{N}{m} \times 0.1m - 0.6 \times 49N = 5kg \times a$$
$$a = -105.88\tfrac{m}{s^2}$$

(The normal comes from the sum of the *y*-component of the forces.)

27. The forces exerted on the 5 kg block are:

1) The weight, 49 N downwards.
2) The tension force T upwards.
3) The spring force F_{sp} upwards or downwards.

As the direction of the force exerted by the spring is not known (because we do not know if the spring is stretched or compressed), we assume that the force is positive, i.e. directed upwards. When we have the solution, we'll know that this direction is correct if the force is positive. If it negative, the force is in the direction opposite to the direction assumed, i.e. downwards.

The sum of the y-component of the forces

$$\sum F_y = ma_y$$
$$-49N + T + F_{sp} = 0$$

Since the rope also supports a 2 kg block in equilibrium, the tension of the rope is 19.6 N. We then have

$$-49N + 19.6N + F_{sp} = 0$$
$$F_{sp} = 29.4N$$

This positive answer tells us that the force exerted by the spring is directed upwards. This also means that the spring is compressed. The compression is

$$F_{sp} = 29.4N$$
$$kx = 29.4N$$
$$x = \frac{29.4N}{k}$$
$$x = \frac{29.4N}{250\frac{N}{m}}$$
$$x = 0.1176m$$

The spring is therefore compressed 11.76 cm.

28. The forces exerted on the 100 g object are:

1) The weight, 0.98 N downwards.
2) The spring force F_{sp} upwards.

The sum of the y-component of the forces is then

$$\sum F_y = ma_x$$
$$-0.98N + F_{sp} = 0$$

The spring, therefore, exerts a 0.98 N force. We thus have

$$\begin{aligned} kx &= 0.98N \\ k \times 0.04m &= 0.98N \\ k &= 24.5 \tfrac{N}{m} \end{aligned}$$

With the 400 g mass, we now have the following forces:

1) The weight, 3.92 N downwards.
2) The spring force F_{sp} upwards.

The sum of the y-component of the forces is then

$$\begin{aligned} \sum F_y &= ma_x \\ -3.92N + F_{sp} &= 0 \end{aligned}$$

The spring, therefore, exerts a 3.92 N force. We thus have

$$\begin{aligned} kx &= 3.92N \\ 24.5 \tfrac{N}{m} \cdot x &= 3.92N \\ x &= 0.16m \end{aligned}$$

The spring is thus stretched 16 cm.

29. The force exerted on one side of the box is

$$\begin{aligned} F_P &= PA \\ &= 300{,}000Pa \times (0.2m)^2 \\ &= 12{,}000N \end{aligned}$$

30. The forces exerted on the cover are

1) A pressure force $F_p = PA$ downwards
2) The spring force $F_{sp} = kx$ upwards

The sum of the y-component of the forces is then

$$\begin{aligned} \sum F_y &= ma_y \\ -PA + kx &= 0 \end{aligned}$$

We then have

$$-PA + kx = 0$$
$$-102,000Pa \times \pi (0.1m)^2 + 10,000\tfrac{N}{m} \cdot x = 0$$
$$x = 0.3204m$$

The spring is therefore compressed 32.04 cm.

31. The forces exerted on the block of cedar are

1) The weight, 1.96 N downwards.
2) The buoyant force F_B upwards.
3) The tension force T downwards.

The sum of the y-component of the forces is then

$$\sum F_y = ma_y$$
$$-1{,}96N + F_B - T = 0$$

To find the buoyant force, the volume of the block must be found. This volume is found with the density.

$$m = \rho \times (volume)$$
$$0.2kg = 490\tfrac{kg}{m^3} \times (volume)$$
$$volume = 0.0004082m^3$$

The buoyant force is therefore

$$\begin{aligned} F_B &= \rho g V_f \\ &= 1000\tfrac{kg}{m^3} \times 9.8\tfrac{N}{kg} \times 0.0004081m^3 \\ &= 4N \end{aligned}$$

The sum of the y-component of the forces then becomes

$$-1.96N + 4N - T = 0$$
$$T = 2.04N$$

32. In air, the forces exerted on the piece of aluminum are

1) The weight, *mg* downwards.
2) The spring force $F_{sp} = kx$ upwards.

The sum of the *y*-component of the forces is then

$$\sum F_y = ma_y$$
$$-mg + kx = 0$$

The mass of the aluminum piece can then be found

$$-mg + kx = 0$$
$$-m \times 9.8 \tfrac{N}{kg} + 200 \tfrac{N}{m} \times 0.1m = 0$$
$$m = 2.0408kg$$

In water, the forces exerted on the piece of aluminum are

1) The weight, 20 N downwards.
2) The spring force $F_{sp} = kx$ upwards.
3) The buoyant force F_B upwards.

The sum of the *y*-component of the forces is then

$$\sum F_y = ma_y$$
$$-20N + kx + F_B = 0$$

To find the buoyant force, the volume of the piece must be found. This volume is found with the density.

$$m = \rho \times (volume)$$
$$2.0408kg = 2700 \tfrac{kg}{m^3} \times (volume)$$
$$volume = 0.0007559m^3$$

The buoyant force is then

$$\begin{aligned} F_B &= \rho g V_f \\ &= 1000 \tfrac{kg}{m^3} \times 9.8 \tfrac{N}{kg} \times 0.0007559m^3 \\ &= 7.4074N \end{aligned}$$

kx can then be found with the sum of the *y*-component of the forces.

$$-20N + kx + 7.4074N = 0$$
$$kx = 12.5926N$$

Therefore, the stretching of the spring is

$$kx = 12.5926N$$
$$200\tfrac{N}{m} \cdot x = 12.5926N$$
$$x = 0.062963m$$

The stretching of the spring is therefore 6.2963 cm.

33. The forces acting on the object are :

1) The weight mg directed downwards.
2) The buoyant force of the denser fluid (fluid 1) directed upwards.
3) The buoyant force of the less dens fluid (fluid 2) directed upwards.

The equation for the y-component of the force is

$$\sum F_y = ma_y$$
$$-mg + \rho_1 g V_{f1} + \rho_2 g V_{f2} = 0$$

The mass of the object is

$$m = \rho_{object} \cdot Volume$$
$$= \rho_{object} \cdot Ah$$

where A is the area of the base of the object.

The immersed volumes

$$V_{f1} = A \cdot \tfrac{3}{4} h$$
$$V_{f2} = A \cdot \tfrac{1}{4} h$$

Using these formulas, the equation of forces then becomes

$$-mg + \rho_1 g V_{f1} + \rho_2 g V_{f2} = 0$$
$$-\rho_{object} \cdot Ahg + \rho_1 g A \tfrac{3}{4} h + \rho_2 g A \tfrac{1}{4} h = 0$$
$$-\rho_{object} + \rho_1 \tfrac{3}{4} + \rho_2 \tfrac{1}{4} = 0$$
$$\rho_{object} = \tfrac{3}{4}\rho_1 + \tfrac{3}{4}\rho_2$$
$$\rho_{object} = \tfrac{3}{4}1000 \tfrac{kg}{m^3} + \tfrac{1}{4}800 \tfrac{kg}{m^3}$$
$$\rho_{object} = 950 \tfrac{kg}{m^3}$$

34. Before that Sylvain gets in the boat, the forces on the boat are :

1) The weight *mg* directed downwards.
2) The buoyant force of the fluid directed upwards.

The equation for the *y*-component of the force is

$$\sum F_y = ma_y$$
$$-mg + \rho g V_f = 0$$

Which leads to

$$mg = \rho g V_f$$

After Sylvain gets in the boat, we have the following forces acting on the boat:

1) The weight *mg* directed downwards.
2) The buoyant force of the fluid directed upwards which has increased since the immersed volume has increased by ΔV.
3) The normal force made by Sylvain, equal to its weight $m_s g$, directed downwards

The equation for the *y*-component of the force is

$$\sum F_y = ma_y$$
$$-mg + \rho g \left(V_f + \Delta V\right) - m_s g = 0$$

But since

$$mg = \rho g V_f$$

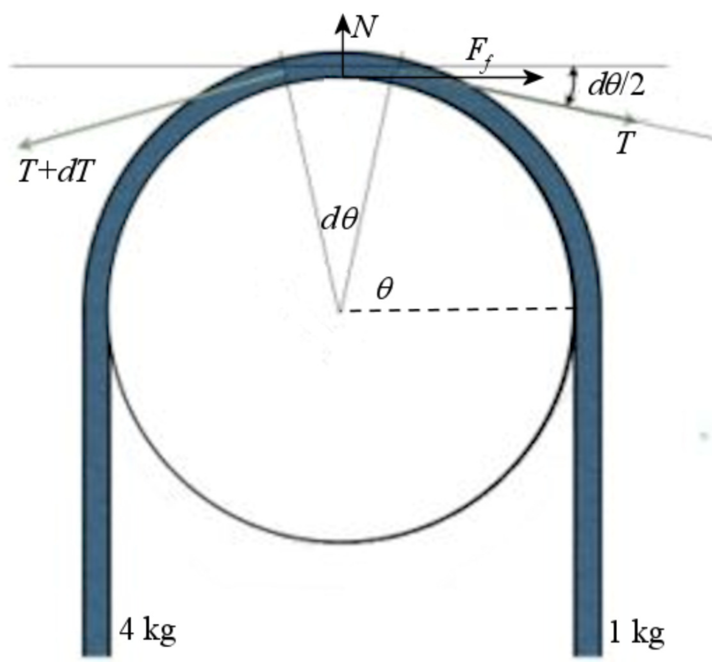
The equation becomes

$$-mg+\rho g\left(V_f+\Delta V\right)-m_s g=0$$
$$-\rho g V_f+\rho g\left(V_f+\Delta V\right)-m_s g=0$$
$$\rho g\Delta V-m_s g=0$$
$$m_s=\rho\Delta V$$

The increase in volume being 4 m² x 0.05 m = 0.2 m³, the mass is

$$\begin{aligned} m_s &= \rho\Delta V \\ &= 1000\tfrac{kg}{m^3}\cdot 0,2m^3 \\ &= 200kg \end{aligned}$$

35. We know that tension is 39.2 N on one side of the rope and 9.8 N the other side of the rope. To see how the friction changes the tension of the rope, we will consider a small piece of rope.



wiki.imga.org.il/index.php?title=%3FHow_do_friction_devices_work

The sum of the forces acting on the small piece of rope is (the mass of the rope is neglected)

$$\sum F_x = T\cos\left(\tfrac{d\theta}{2}\right)-\left(T+dT\right)\cos\left(\tfrac{d\theta}{2}\right)+F_f=0$$
$$\sum F_y = -T\sin\left(\tfrac{d\theta}{2}\right)-\left(T+dT\right)\sin\left(\tfrac{d\theta}{2}\right)+N=0$$

Since the angle is small, we can use cos x = 1 and sin x = x. then, the equation become

$$\sum F_x = T - (T + dT) + F_f = 0$$
$$\sum F_y = -T\frac{d\theta}{2} - (T + dT)\frac{d\theta}{2} + N = 0$$

The second equation gives

$$-T\frac{d\theta}{2} - (T + dT)\frac{d\theta}{2} + N = 0$$
$$N = T\frac{d\theta}{2} + (T + dT)\frac{d\theta}{2}$$
$$N = T\frac{d\theta}{2} + T\frac{d\theta}{2} + dT\frac{d\theta}{2}$$

The third term is very small and can be neglected to obtain

$$\begin{aligned} N &= T\frac{d\theta}{2} + T\frac{d\theta}{2} \\ &= Td\theta \end{aligned}$$

Since the friction force is at its maximum, the sum of the x-component of the forces becomes

$$\sum F_x = T - (T + dT) + \mu N = 0$$
$$dT = \mu N$$

With the value of the normal force, this equation is

$$dT = \mu T d\theta$$

The solution of this equation is

$$\frac{dT}{T} = \mu d\theta$$
$$\int \frac{dT}{T} = \int \mu d\theta$$
$$\ln T = \mu\theta + cst$$

at $\theta = 0$, the tension is 9.8 N. Therefore,

$$\ln 9.8N = 0 + cst$$
$$cst = \ln 9.8N$$

This, the tension is

$$\ln T = \mu\theta + \ln 9.8N$$
$$\ln T - \ln 9.8N = \mu\theta$$
$$\ln \tfrac{T}{9.8\text{N}} = \mu\theta$$
$$\frac{T}{9.8N} = e^{\mu\theta}$$
$$T = 9.8Ne^{\mu\theta}$$

At 180°, the tension is 39,2 N. Thus,

$$39.2N = 9.8Ne^{\mu\pi}$$

(We must work in radians because we used the approximation of small angles, which is true only for angles in radians.)

It only remains to solve for μ.

$$4 = e^{\mu\pi}$$
$$\ln 4 = \mu\pi$$
$$\mu = \frac{\ln 4}{\pi}$$
$$\mu = 0.4413$$

36. The only force acting on the ball is the drag. With an *x*-axis in the direction of the velocity, the sum of the *x*-component of the forces is

$$\sum F_x = ma$$
$$-F_d = ma$$
$$-\frac{1}{2}C\rho Av^2 = ma$$

To find the speed of the ball depending on the time, we use

$$a = \frac{dv}{dt}$$

Therefore

$$-\frac{1}{2}C\rho A v^2 = m\frac{dv}{dt}$$

This equation can then be solved to obtain

$$\begin{aligned}
\frac{dv}{dt} &= -\frac{C\rho A v^2}{2m} \\
\frac{dv}{v^2} &= -\frac{C\rho A}{2m}dt \\
\int\frac{dv}{v^2} &= -\int\frac{C\rho A}{2m}dt \\
-\frac{1}{v} &= -\frac{C\rho A t}{2m} + Cst
\end{aligned}$$

Since the speed is v_0 at $t = 0$, the value of the constant can be found.

$$\begin{aligned}
-\frac{1}{v_0} &= 0 + Cst \\
Cst &= -\frac{1}{v_0}
\end{aligned}$$

The formula of velocity as a function of time is thus

$$-\frac{1}{v} = -\frac{C\rho A t}{2m} - \frac{1}{v_0}$$

Solving for v, it becomes

$$\begin{aligned}
\frac{1}{v} &= \frac{C\rho A t}{2m} + \frac{1}{v_0} \\
\frac{1}{v} &= \frac{C\rho A t v_0}{2m v_0} + \frac{2m}{2m v_0} \\
\frac{1}{v} &= \frac{C\rho A t v_0 + 2m}{2m v_0} \\
v &= \frac{2m v_0}{C\rho A t v_0 + 2m}
\end{aligned}$$

The speed cannot yet be calculated because we don't know how long it will take for the ball to reach its target. To find out, the formula of the position as a function of time is needed. This position is found with

$$x = \int v\,dt$$

Thus

$$\begin{aligned}
x &= \int \frac{2mv_0}{C\rho Atv_0 + 2m}dt \\
&= \frac{2m}{C\rho A}\int \frac{1}{\left(t + \frac{2m}{C\rho Atv_0}\right)}dt \\
&= \frac{2m}{C\rho A}\ln\left(t + \frac{2m}{C\rho Atv_0}\right) + Cst
\end{aligned}$$

Since the position is 0 at $t = 0$, the value of the constant can be found.

$$0 = \frac{2m}{C\rho A}\ln\left(0 + \frac{2m}{C\rho Atv_0}\right) + Cst$$

$$Cst = -\frac{2m}{C\rho A}\ln\left(\frac{2m}{C\rho Atv_0}\right)$$

Therefore

$$\begin{aligned}
x &= \frac{2m}{C\rho A}\ln\left(t + \frac{2m}{C\rho Atv_0}\right) - \frac{2m}{C\rho A}\ln\left(\frac{2m}{C\rho Atv_0}\right) \\
&= \frac{2m}{C\rho A}\left(\ln\left(t + \frac{2m}{C\rho Atv_0}\right) - \ln\left(\frac{2m}{C\rho Atv_0}\right)\right) \\
&= \frac{2m}{C\rho A}\ln\left(\frac{t + \frac{2m}{C\rho Atv_0}}{\frac{2m}{C\rho Atv_0}}\right) \\
&= \frac{2m}{C\rho A}\ln\left(\frac{C\rho Atv_0t + 2m}{2m}\right) \\
&= \frac{2m}{C\rho A}\ln\left(\frac{C\rho Av_0}{2m}t + 1\right)
\end{aligned}$$

The time needed to reach the target can now be found. For the baseball, we have

$$\begin{aligned}\frac{2m}{C\rho A} &= \frac{2\cdot 0.145kg}{0.47\cdot 1.3\frac{kg}{m^3}\cdot \pi\left(0.037m\right)^2}\\ &= 110.36m\end{aligned}$$

Therefore, the position is

$$\begin{aligned}x &= 110.36m\cdot \ln\left(\frac{30\frac{m}{s}}{110.36m\cdot}\cdot t+1\right)\\ x &= 110.36m\cdot \ln\left(0.27184\tfrac{1}{s}\cdot t+1\right)\end{aligned}$$

The time to reach x = 100 m is thus

$$\begin{aligned}100m &= 110.36m\cdot \ln\left(0.27184\tfrac{1}{s}\cdot t+1\right)\\ 0.90614 &= \ln\left(0.27184\tfrac{1}{s}\cdot t+1\right)\\ e^{0.90614} &= 0.27184\tfrac{1}{s}\cdot t+1\\ 2.4748 &= 0.27184\tfrac{1}{s}\cdot t+1\\ 1.4748 &= 0.27184\tfrac{1}{s}\cdot t\\ t &= 5.425s\end{aligned}$$

The speed is, therefore,

$$\begin{aligned}v &= \frac{2mv_0}{C\rho Atv_0+2m}\\ &= \frac{v_0}{\frac{C\rho Av_0}{2m}t+1}\\ &= \frac{30\frac{m}{s}}{\frac{30\frac{m}{s}}{110.36m}5.425s+1}\\ &= 12.12\tfrac{m}{s}\end{aligned}$$

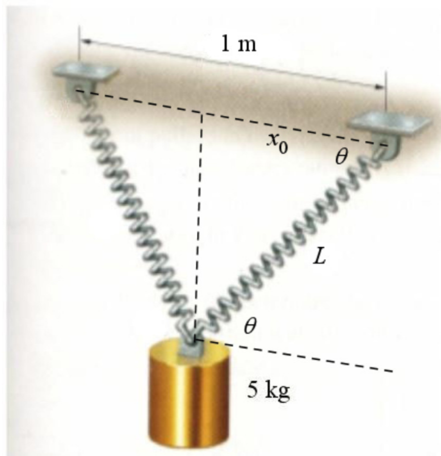
37. The sums of the forces on the mass are

$$\begin{aligned}\sum F_x &= -F_{sp}\cos\theta+F_{sp}\cos\theta=0\\ \sum F_y &= F_{sp}\sin\theta+F_{sp}\sin\theta-mg=0\end{aligned}$$

The second equation gives

$$F_{sp} = \frac{mg}{2\sin\theta}$$

The force made by the spring and the angle both depend on the stretching of the spring.



The length of the spring is L. Thus, the force exerted by the spring is

$$F_{sp} = k\left(L - x_0\right)$$

where x_0 is the length of the unstretched spring (50 cm).

The sine of the angle is

$$\sin\theta = \frac{y}{L} = \frac{\sqrt{L^2 - x_0^2}}{L}$$

Therefore,

$$F_{sp} = \frac{mg}{2\sin\theta}$$

$$k\left(L - x_0\right) = \frac{mgL}{2\sqrt{L^2 - x_0^2}}$$

It only remains to solve this equation for L.

$$\left(L-x_0\right)\sqrt{L^2-x_0^2}=\frac{mgL}{2k}$$

$$\left(L-x_0\right)^2\left(L^2-x_0^2\right)=\left(\frac{mg}{2k}\right)^2 L^2$$

$$\left(L-x_0\right)^2\left(L-x_0\right)\left(L+x_0\right)=\left(\frac{mg}{2k}\right)^2 L^2$$

$$\left(L-x_0\right)^3\left(L+x_0\right)=\left(\frac{mg}{2k}\right)^2 L^2$$

$$\left(L^3-3x_0L^2+3x_0^2L-x_0^3\right)\left(L+x_0\right)=\left(\frac{mg}{2k}\right)^2 L^2$$

$$L^4-3x_0L^3+3x_0^2L^2-x_0^3L+x_0L^3-3x_0^2L^2+3x_0^3L-x_0^4=\left(\frac{mg}{2k}\right)^2 L^2$$

$$L^4-2x_0L^3+2x_0^3L-x_0^4=\left(\frac{mg}{2k}\right)^2 L^2$$

$$L^4-2x_0L^3-\left(\frac{mg}{2k}\right)^2 L^2+2x_0^3L-x_0^4=0$$

This is a 4$^{\text{th}}$-degree equation. With the values, it is

$$L^4-(1m)L^3-\left(\frac{1}{16}m^2\right)L^2+\left(\frac{1}{4}m^3\right)L-\left(\frac{1}{16}m^4\right)=0$$

$$16L^4-(16m)L^3-\left(1m^2\right)L^2+\left(4m^3\right)L-\left(1m^4\right)=0$$

The only positive solution of this equation is

$$L=0.81623\text{ m}$$