Chapter 4 Solutions

1. The forces acting on William are:

1) The weight (705.6 N) directed downwards.
2) A normal force F_N directed upwards exerted by the floor of the lift.

The sum of the y-component of the forces is

$$\sum F_y = ma_y$$
$$-705.6N + F_N = ma_y$$

a) If the speed is constant, the acceleration is zero and we have

$$-705.6N + F_N = 0$$
$$F_N = 705.6N$$

b) If the speed increases, the acceleration is in the same direction as the velocity. It is then directed upwards. Then

$$-705.6N + F_N = ma_y$$
$$-705.6N + F_N = 72kg \times \left(2\tfrac{m}{s^2}\right)$$
$$F_N = 849.6N$$

c) If the speed decreases, the acceleration is in the opposite direction to the velocity. It is then directed downwards. Then

$$-705.6N + F_N = ma_y$$
$$-705.6N + F_N = 72kg \times \left(-3\tfrac{m}{s^2}\right)$$
$$F_N = 489.6N$$

2. The forces acting on the 5 kg box are:

1) The weight (49 N) directed downwards.
2) A normal force F_{N1} directed upwards exerted by the 10 kg block.

If the elevator slows down, then the acceleration is in the direction opposite to the velocity. It is then directed downwards.

The sum of the y-component of the forces is

$$\sum F_y = m_1 a_y$$
$$-49N + F_{N1} = 5kg \times \left(-1\tfrac{m}{s^2}\right)$$

The normal force is therefore

$$-49N + F_{N1} = 5kg \times \left(-1\tfrac{m}{s^2}\right)$$
$$F_{N1} = 44N$$

The forces acting on the 10 kg box are:

1) The weight (98 N) directed downwards.
2) A normal force F_{N1} directed downwards exerted by the 5 kg block, which has the same magnitude has the normal force exerted by the 10 kg block on the 5 kg block.
3) A normal force F_{N2} directed upwards exerted by the floor.

The sum of the y-component of the forces is

$$\sum F_y = m_2 a_y$$
$$-49N - F_{N1} + F_{N2} = m_2 a_y$$

As F_{N1} = 44 N and the acceleration is -1 m/s², the normal force is

$$-98N - 44N + F_{N2} = 10kg \times -1\tfrac{m}{s^2}$$
$$F_{N2} = 132N$$

3. The forces acting on the 300 kg part are:

1) The weight (2940 N) directed downwards.
2) The tension force T directed upwards.

The sum of the y-component of the forces is

$$\sum F_y = ma_y$$
$$T - 2940N = ma_y$$

a) If there is no acceleration, then

$$T - 2940N = ma_y$$
$$T - 2940N = 0$$
$$T = 2940N$$

b) If the acceleration is 3 m/s² upwards, then

$$T - 2940N = ma_y$$
$$T - 2940N = 300kg \times 3\tfrac{m}{s^2}$$
$$T = 3840N$$

c) If the acceleration is 2 m/s² downwards, then

$$T - 2940N = ma_y$$
$$T - 2940N = 300kg \times -2\tfrac{m}{s^2}$$
$$T = 2340N$$

4. The forces acting on the 10 kg box are (with a *y*-axis directed upwards).

1) The weight (98 N) directed downwards.
2) The tension force T_2 directed upwards.

The sum of the *y*-component of the forces is

$$\sum F_y = ma_y$$
$$-98N + T_2 = 10kg \cdot a$$

The forces acting on the 6 kg box are (with a *y*-axis directed upwards):

1) The weight (58.8 N) directed downwards.
2) The tension force T_1 directed downwards.
3) The tension force T_2 directed upwards.

The sum of the *y*-component of the forces is

$$\sum F_y = ma_y$$
$$-58.8N + T_1 - T_2 = 6kg \cdot a$$

a) If the acceleration is 2.4 m/s² downwards, the two equations become

$$-98N + T_2 = 10kg \cdot \left(-2.4 \tfrac{m}{s^2}\right)$$
$$-58.8N + T_1 - T_2 = 6kg \cdot \left(-2.4 \tfrac{m}{s^2}\right)$$

The first equation gives T_2 = 74 N. By substituting this value in the second equation, we get T_1 = 118.4 N.

b) The peak acceleration is found by setting each of the strings at its maximum tension and find the accelerations. The lower of the two maximum accelerations will be the limit.

If T_1 is set at its maximal tension of 200 N, we have

$$-98N + T_2 = 10kg \cdot a_{\max}$$
$$-58.8N + 200N - T_2 = 6kg \cdot a_{\max}$$

Summing these equations, we get

$$\left(-98N + T_2\right) + \left(-58.8N + 200N - T_2\right) = 10kg \cdot a_{\max} + 6kg \cdot a_{\max}$$
$$43.2N = 16kg \cdot a_{\max}$$
$$a_{\max} = 2.7 \tfrac{m}{s^2}$$

If T_2 is set at its maximal tension of 200 N, we have

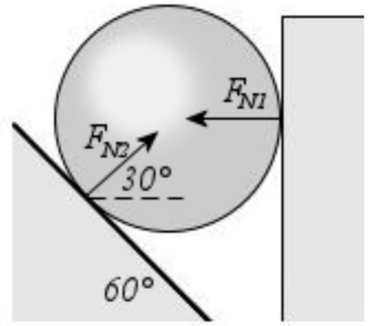
$$-98N + 200N = 10kg \cdot a_{\max}$$
$$-58.8N + T_1 - 200N = 6kg \cdot a_{\max}$$

With the first equation, we have

$$-98N + 200N = 10kg \cdot a_{\max}$$
$$a_{\max} = 10.2 \tfrac{m}{s^2}$$

Therefore, rope 1 sets the limit to the acceleration at 2.7 m/s².

5. The forces acting on the ball are:



1) The weight (3.92 N) directed downwards.
2) A normal force F_{N1} directed towards the left exerted by the wall.
3) A normal force F_{N2} directed at 30° exerted by the inclined.

The table of force is

Forces	x	y
Weight	0	-3.92N
Normal Force 1	$-F_{N1}$	0
Normal Force 2	$F_{N2}\cos 30°$	$F_{N2}\sin 30°$

Since the components of the acceleration are zero, the equations of forces are

$$\sum F_x = -F_{N1} + F_{N2}\cos 30° = 0$$
$$\sum F_y = -3.92N + F_{N2}\sin 30° = 0$$

From the sum of the y-component of the forces, we have

$$-3.92N + F_{N2}\sin 30° = 0$$
$$F_{N2} = 7.84N$$

This answer is then substituted in the sum of the x-component of the forces to obtain

$$-F_{N1} + 7.84N\cos 30° = 0$$
$$F_{N1} = 6.79N$$

6. First, the tension will be found with the equation of the forces on the 4 kg box. The forces acting on the 4 kg box are:

1) The weight (39.2 N) directed downwards.
2) The tension force T directed upwards.

The sum of the y-component of the forces is

$$\sum F_y = ma_y$$
$$T - 39.2N = 0$$
$$T = 39.2N$$

Now, the forces acting on the 12 kg box are:

1) The weight (117.6 N) directed downwards.
2) The tension force T directed upwards.
3) A normal force F_N directed upwards exerted by the ground.

The sum of the y-component of the forces is

$$\sum F_y = ma_y$$
$$T - 117.6N + F_N = 0$$
$$39.2N - 117.6N + F_N = 0$$
$$F_N = 78.4N$$

7. First, the tension will be found with the equation of the forces on the 4 kg box. The forces acting on the 4 kg box are:

1) The weight (39.2 N) directed downwards.
2) The tension force T directed upwards.

The sum of the y-component of the forces is

$$\sum F_y = ma_y$$
$$T - 39.2N = 0$$
$$T = 39.2N$$

The forces acting on the 12 kg box are:

1) The weight (117.6 N) directed downwards.
2) The tension force T directed upwards.
3) A normal force F_{N1} directed upwards exerted by 20 kg box.

The sum of the y-component of the forces is

$$\sum F_y = ma_y$$
$$T - 117.6N + F_{N1} = 0$$
$$39.2N - 117.6N + F_{N1} = 0$$
$$F_{N1} = 78.4N$$

Finally, the forces acting on the 12 kg box are:

1) The weight (196 N) directed downwards.
2) A normal force F_{N1} directed downwards exerted by the 12 kg block, which has the same magnitude has the normal force exerted by the 20 kg block on the 12 kg block.
3) A normal force F_{N2} directed upwards exerted by the ground.

The sum of the *y*-component of the forces is

$$\sum F_y = ma_y$$
$$F_{N2} - 196N - F_{N1} = 0$$
$$F_{N2} - 196N - 78.4N = 0$$
$$F_{N2} = 274.4N$$

8. The forces acting on the snowball are:

1) The weight (392 N) directed downwards.
2) A normal force F_N directed upwards exerted by the ground.
3) The force exerted by Gontran (100 N) directed at -25°.
4) The force exerted by Philemon (75 N) directed at 30°.

The table of force is

Forces	*x*	*y*
Weight	0	-392N
Normal force	0	F_N
Gontran	100N cos(-25°)	100N sin(-25°)
Philemon	75N cos30°	75N sin30°

The acceleration can be found with the sum of the *x*-component of the forces.

$$\sum F_x = ma_x$$
$$100N\cos(-25°) + 75N\cos(30°) = 40kg \times a_x$$
$$a_x = 3.89\tfrac{m}{s^2}$$

The normal force can be found with the sum of the *y*-component of the forces.

$$\sum F_y = ma_y$$
$$-392N + F_N + 100N\sin(-25°) + 75N\sin(30°) = 40kg \times 0\tfrac{m}{s^2}$$
$$F_N = 396.76N$$

9. The forces acting on Irina are:

1) The weight (588 N) directed downwards.
2) A normal force F_N directed at 15° exerted by the cliff.
3) The tension force T directed at 121°.

The table of force is

Forces	***x***	***y***
Weight	0	-588N
Normal force	$F_N\cos(15°)$	$F_N\sin(15°)$
Tension	$T\cos 121°$	$T\sin 121°$

Since the components of the acceleration are zero, the equations of forces are

$$\sum F_x = ma_x$$
$$\rightarrow F_N\cos(15°) + T\cos(121°) = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -588N + F_N\sin(15°) + T\sin(121°) = 0$$

We have two equations and two unknowns. To solve, we will solve for the normal force in the first equation

$$F_N = \frac{-T\cos(121°)}{\cos(15°)}$$

And substitute this value in the second equation.

$$-588N + F_N \sin(15°) + T\sin(121°) = 0$$

$$-588N + \frac{-T\cos(121°)}{\cos(15°)}\sin(15°) + T\sin(121°) = 0$$

$$-588N + T\left(\frac{-\cos(121°)}{\cos(15°)}\sin(15°) + \sin(121°)\right) = 0$$

$$-588N + T(0.9952) = 0$$

$$T = 590.85N$$

With this tension force, the normal force can now be found.

$$\begin{aligned} F_N &= \frac{-T\cos(121°)}{\cos(15°)} \\ &= \frac{-590.85N\cos(121°)}{\cos(15°)} \\ &= 315.05N \end{aligned}$$

10. The forces acting on Indiana are:

1) The weight (637 N) directed downwards.
2) The tension force T directed at 5°.
3) The tension force T directed at 175°.

The table of force is

Forces	x	y
Weight	0	-637N
Tension 5°	$T\cos 5°$	$T\sin 5°$
Tension 175°	$T\cos 175°$	$T\sin 175°$

Since the components of the acceleration are zero, the equations of forces are

$$\sum F_x = ma_x$$

$$\rightarrow T\cos(5°) + T\cos(175°) = 0$$

$$\sum F_y = ma_y$$

$$\rightarrow -637N + T\sin(5°) + T\sin(175°) = 0$$

The first equation gives no information. The sum is always zero, regardless of the value of T, since cos (5°) = - cos (175°). However, the tension can be found with the second equation.

$$-637N + T\sin(5°) + T\sin(175°) = 0$$
$$-637N + T(\sin(5°) + \sin(175°)) = 0$$
$$T = 3654N$$

11. The forces acting on the box are:

1) The weight (392 N) directed downwards.
2) The tension force T_1 directed at 20° exerted by the string on the right.
3) The tension force T_2 directed at 120° exerted by the string on the left.

The table of force is

Forces	***x***	***y***
Weight	0	-392N
Tension 1	$T_1\cos 20°$	$T_1\sin 20°$
Tension 2	$T_2\cos 120°$	$T_2\sin 120°$

Since the components of the acceleration are zero, the equations of forces are

$$\sum F_x = ma_x$$
$$\rightarrow T_1\cos(20°) + T_2\cos(120°) = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -392N + T_1\sin(20°) + T_2\sin(120°) = 0$$

We have two equations and two unknowns. To obtain a solution, we will solve for T_2 in the first equation

$$T_2 = -\frac{T_1\cos(20°)}{\cos(120°)}$$

And substitute the value in the second equation.

$$-392N + T_1 \sin(20°) + T_2 \sin(120°) = 0$$

$$-392N + T_1 \sin(20°) + -\frac{T_1 \cos(20°)}{\cos(120°)} \sin(120°) = 0$$

$$-392N + T_1 \left(\sin(20°) - \frac{\cos(20°)}{\cos(120°)} \sin(120°) \right) = 0$$

$$-392N + T_1 (1.9696) = 0$$

$$T_1 = 199.02N$$

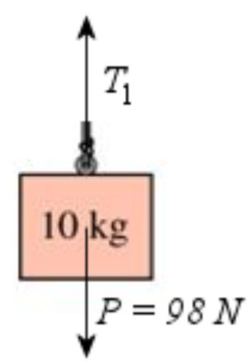
From this tension, the other tension can be found.

$$\begin{aligned} T_2 &= -\frac{T_1 \cos(20°)}{\cos(120°)} \\ &= -\frac{199.02N \times \cos(20°)}{\cos(120°)} \\ &= 374.04N \end{aligned}$$

12. Let's start by finding the tension of the string that supports the box.

The forces acting on the 10 kg box are:

1) The weight (98 N) directed downwards.
2) The tension force T_1 directed upwards.



The sum of the y-component of the forces is

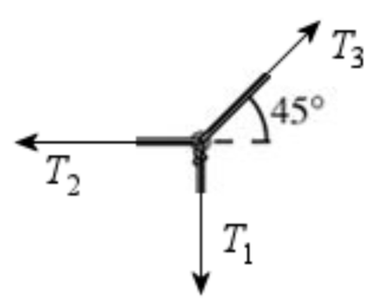
$$\sum F_y = ma_y$$

$$T_1 - 98N = 0$$

$$T_1 = 98N$$

Now let's look at the forces on the node that connects the three strings. The forces are:

1) The tension force T_1 = 98 N directed downwards.
2) The tension force T_2 directed towards the left exerted by the string on the left.
3) The tension force T_3 directed towards the left at 45° exerted by the string on the right.



The table of force is

Forces	x	y
Tension 1	0	-98N
Tension 2	$-T_2$	0
Tension 3	$T_3\cos 45°$	$T_3\sin 45°$

Since the components of the acceleration are zero, the equations of forces are

$$\sum F_x = ma_x$$
$$\to -T_2 + T_3\cos(45°) = 0$$
$$\sum F_y = ma_y$$
$$\to -98N + T_3\sin(45°) = 0$$

With the sum of the y-component of the forces, we find

$$-98N + T_3\sin(45°) = 0$$
$$T_3 = 138.59N$$

This value is then used in the sum of the x-component of the forces to find T_2.

$$-T_2 + T_3\cos(45°) = 0$$
$$-T_2 + 138.59N\cos(45°) = 0$$
$$T_2 = 98N$$

13. The forces acting on the 10 kg block are:

1) The weight (98 N) directed downwards.
2) The tension force T directed at 60° exerted by the string.
3) The force F directed towards the negative x-axis.

The table of force is

Forces	x	y
Weight	0	-98N
Tension	$T\cos 60°$	$T\sin 60°$
Force *F*	$-F$	0

Since the components of the acceleration are zero, the equations of forces are

$$\sum F_x = ma_x$$
$$\rightarrow T\cos(60°) - F = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -98N + T\sin(60°) = 0$$

The tension can be found with the sum of the *y*-component of the forces.

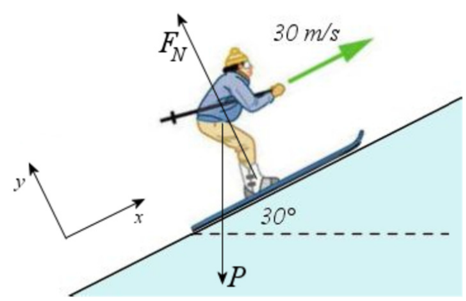
$$-98N + T\sin(60°) = 0$$
$$T = 113.16N$$

This value is then used in the sum of the *x*-component of the forces to find *F*.

$$T\cos(60°) - F = 0$$
$$113.16N\cos(60°) - F = 0$$
$$F = 56.58N$$

14. On the slope, the forces acting on Yannick are:

1) The weight *mg* directed downwards.
2) A normal force F_N perpendicular to the slope surface.



With the axes shown in the figure, the table of force is

Forces	x	y
Weight	*mg* cos (-120°)	*mg* sin (-120°)
Normal force	0	F_N

The equations of forces are then

$$\sum F_x = ma_x$$
$$\rightarrow mg\cos(-120°) = ma_x$$
$$\sum F_y = ma_y$$
$$\rightarrow mg\sin(-120°) + F_N = 0$$

The acceleration can be found with the sum of the *x*-component of the forces.

$$mg\cos(-120°) = ma_x$$
$$g\cos(-120°) = a_x$$
$$a_x = -4.9\tfrac{m}{s^2}$$

From there, the distance travelled can be found with

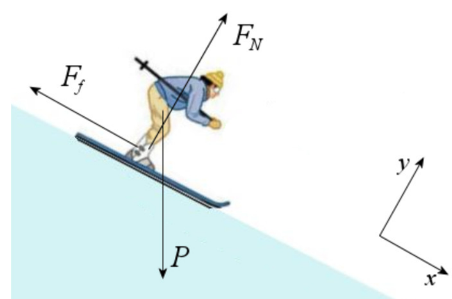
$$2a_x(x - x_0) = v^2 - v_0^2$$
$$2\cdot\left(-4.9\tfrac{m}{s^2}\right)(x - 0m) = \left(0\tfrac{m}{s}\right)^2 - \left(30\tfrac{m}{s}\right)^2$$
$$x = 91.84m$$

And the stopping time with

$$v_x = v_{0x} + a_x t$$
$$0\tfrac{m}{s} = 30\tfrac{m}{s} + \left(-4.9\tfrac{m}{s^2}\right)\cdot t$$
$$t = 6.122s$$

15. On the slope, the forces acting on Wolfgang are:

1) The weight mg directed downwards.
2) A normal force F_N perpendicular to the slope surface.
3) A friction force F_f opposed to the velocity.



With the axes shown in the figure, the table of force is

Forces	x	y
Weight	mg cos(-60°)	mg sin(-60°)
Normal force	0	F_N
Friction force	$-F_f$	0

The equations of forces are then

$$\sum F_x = ma_x$$
$$\rightarrow mg\cos(-60°) - F_f = ma_x$$
$$\sum F_y = ma_y$$
$$\rightarrow mg\sin(-60°) + F_N = 0$$

In order to resolve, we will need the acceleration, which can be found with

$$2a_x\left(x - x_0\right) = v^2 - v_0^2$$
$$2a_x\left(50m - 0m\right) = \left(20\tfrac{m}{s}\right)^2 - \left(10\tfrac{m}{s}\right)^2$$
$$a_x = 3\tfrac{m}{s^2}$$

The force of friction can then be found with the sum of the x-component of the forces.

$$mg\cos\left(-60°\right) - F_f = ma_x$$
$$70kg \cdot 9.8\tfrac{m}{s^2} \cdot \cos\left(-60°\right) - F_f = 70kg \cdot 3\tfrac{m}{s^2}$$
$$F_f = 133N$$

16. The forces acting on the 30 kg block are (with a horizontal axis (x) and a vertical axis (y)):

1) The weight (294 N) directed downwards.
2) The tension force T directed towards the right.
3) A normal force F_N directed at 115° exerted by the inclined.

The table of force is

Forces	x	y
Weight	0	-294 N
Tension	T	0
Normal force	F_N cos115°	F_N sin115°

Since the components of the acceleration are zero, the equations of forces are

$$\sum F_x = ma_x$$
$$\rightarrow T + F_N\cos\left(115°\right) = 0$$
$$\sum F_y = ma_y$$
$$\rightarrow -294N + F_N\sin\left(115°\right) = 0$$

The normal force can be found with the sum of the y-component of the forces.

$$-294N + F_N\sin\left(115°\right) = 0$$
$$F_N = 324.39N$$

This value is then substituted into the sum of the *x*-component of the forces to find *T*.

$$T + F_N \cos(115°) = 0$$
$$T + 324.39N \cos(115°) = 0$$
$$T = 137.09N$$

17. The forces acting on the 180 kg crate are (with axes tilted so that the *x*-axis is directed uphill.):

1) The weight (784 N) directed downwards, therefore at -130°.
2) A normal force F_N directed towards the positive *y*-axis exerted by the ground.
3) The 800 N force *F* directed at -40°.

The table of force is

Forces	*x*	*y*
Weight	784N cos (-130°)	784N sin (-130°)
Normal force	0	F_N
Force *F*	800N cos (-40°)	800N sin (-40°)

The equations of forces are then

$$\sum F_x = ma_x$$
$$\rightarrow 784N \cos(-130°) + 800N \cos(-40°) = ma_x$$
$$\sum F_y = ma_y$$
$$\rightarrow 784N \sin(-130°) + F_N + 800N \sin(-40°) = 0$$

The acceleration can be found with the sum of the *x*-component of the forces.

$$784N \cos(-130°) + 800N \cos(-40°) = ma_x$$
$$-503.95N + 612.84N = 80kg \cdot a_x$$
$$a_x = 1.361 \tfrac{m}{s^2}$$

Then F_N can be found with the sum of the *y*-component of the forces.

$$784N \sin(-130°) + F_N + 800N \sin(-40°) = 0$$
$$F_N = 1114.81N$$

18. Let's find the acceleration by considering both boxes as a single object whose mass is 5 kg. The forces acting on this object are:

1) The weight (49 N) directed downwards.
2) A normal force F_N directed upwards, exerted by the ground.
3) The 50 N force F directed towards the positive x-axis.
4) An 18 N friction force F_f directed towards the negative x-axis.

The table of force is

Forces	*x*	*y*
Weight	0	-49N
Normal force	0	F_N
Force *F*	50N	0
Friction force	-18N	0

The acceleration can be found with the sum of the x-component of the forces.

$$\sum F_x = ma_x$$
$$50N - 18N = 5kg \cdot a_x$$
$$a_x = 6.4 \tfrac{m}{s^2}$$

To find the normal force between the boxes, the forces on one of the two boxes must be looked at. Let's take the 2 kg box. The forces on this box are:

1) The weight (19.6 N) directed downwards.
2) A normal force F_{N1} directed upwards exerted by the ground.
3) A normal force F_{N2} directed towards the positive x-axis exerted by the 3 kg box.
4) An 8 N friction force directed towards the negative x-axis.

The table of force is

Forces	*x*	*y*
Weight	0	-19,6 N
Normal force 1	0	F_{N1}
Normal force 2	F_{N2}	0
Friction force	-8 N	0

F_{N2} can then be found with the sum of the x-component of the forces.

$$\sum F_x = ma_x$$
$$F_{N2} - 8N = 2kg \cdot 6.4 \tfrac{m}{s^2}$$
$$F_{N2} = 20.8N$$

19. The forces acting on the 12 kg block are (with an *x*-axis directed downwards):

1) The weight (117.6 N) directed downwards.
2) The tension force *T* directed upwards exerted by the rope.

The sum of the *x*-component of the forces is

$$\sum F_x = m_1 a_x$$
$$117.6N - T = 12kg \cdot a_x$$

The forces acting on the 10 kg block are (with an *x*-axis directed upwards):

1) The weight (98 N) directed downwards.
2) The tension force *T* directed upwards exerted by the rope.

The sum of the *x*-component of the forces is

$$\sum F_x = m_1 a_x$$
$$-98N + T = 10kg \cdot a_x$$

The two equations are

$$117.6N - T = 12kg \cdot a_x$$
$$-98N + T = 10kg \cdot a_x$$

This system can be solved by adding these two equations.

$$(117.6N - T) + (-98N + T) = 12kg \cdot a_x + 10kg \cdot a_x$$
$$117.6N - 98N = 22kg \cdot a_x$$
$$a_x = 0.891 \tfrac{m}{s^2}$$

Therefore, the tension is

$$117.6N - T = 12kg \cdot a_x$$
$$117.6N - T = 12kg \cdot 0.891\tfrac{m}{s^2}$$
$$T = 106.91N$$

20. The forces acting on the 24 kg block are (with an x-axis directed to the right):

1) The weight (235.2 N) directed downwards.
2) A normal force F_{N1} directed upwards exerted by the ground.
3) The tension force T directed towards the right.
4) The 300 N force F directed at 160°.

The table of force is

Forces	x	y
Weight	0	-235,2N
Normal force 1	0	F_{N1}
Tension force	T	0
Force *F*	300N cos (160°)	300 N sin (160°)

The equations of forces are then

$$\sum F_x = m_1 a_x$$
$$\rightarrow T + 300N\cos(160°) = m_1 a_x$$
$$\sum F_y = m_1 a_y$$
$$\rightarrow -235,2N + F_{N1} + 300N\sin(160°) = 0$$

The forces acting on the 18 kg box are (with an x-axis directed downhill):

1) The weight (176.4 N) directed downwards, therefore at -30°.
2) A normal force F_{N2} perpendicular to the slope surface.
3) The tension force T directed towards the negative x-axis.

The table of force is

Forces	x	y
Weight	176.4 N cos (-30°)	176.4 N sin (-30°)
Normal force 2	0	F_{N2}
Tension force	$-T$	0

The equations of forces are then

$$\sum F_x = m_2 a_x$$
$$\to 176.4N\cos(-30°) - T = m_2 a_x$$
$$\sum F_y = m_2 a_y$$
$$\to 176.4N\sin(-30°) + F_{N2} = 0$$

a and b) The two equations for the sum of the *x*-component of the forces are

$$T + 300N\cos(160°) = m_1 a_x$$
$$176.4N\cos(-30°) - T = m_2 a_x$$

The solution to this system of equations can be found by adding these equations

$$\left(T + 300N\cos(160°)\right) + \left(176.4N\cos(-30°) - T\right) = m_1 a_x + m_2 a_x$$
$$300N\cos(160°) + 176.4N\cos(-30°) = (m_1 + m_2) a_x$$
$$-281.91N + 152.77N = 42kg \cdot a_x$$
$$a_x = -3.075 \tfrac{m}{s^2}$$

The tension is therefore

$$T + 300N\cos(160°) = m_1 a_x$$
$$T + 300N\cos(160°) = 24kg \cdot \left(-3.075 \tfrac{m}{s^2}\right)$$
$$T = 208.11N$$

c) The normal force in the 24 kg block is found with the sum of the *y*-component of the forces on this block.

$$-235.2N + F_{N1} + 300N\sin(160°) = 0$$
$$F_{N1} = 132.59N$$

The normal force in the 18 kg block is found with the sum of the *y*-component of the forces on this block.

$$176.4N\sin(-30°) + F_{N2} = 0$$
$$F_{N2} = 88.2N$$

21. The forces acting on the 2 kg block are (with an *x*-axis directed upwards):

1) The weight (19.6 N) directed downwards.

2) The tension force T directed upwards.

The sum of the x-component of the forces is

$$\sum F_x = m_1 a_x$$
$$-19.6N + T = 2kg \cdot a_x$$

The forces acting on the box of mass m are (with an x-axis directed downhill):

1) The weight mg directed downwards, therefore at -70°.
2) A normal force F_N perpendicular to the slope surface.
3) The tension force T directed towards the negative x-axis.

The table of force is

Forces	***x***	***y***
Weight	mg cos (-70°)	mg sin (-70°)
Normal force	0	F_N
Tension force	$-T$	0

The sum of the x-component of the forces is

$$\sum F_x = m_2 a_x$$
$$mg\cos(-70°) - T = ma_x$$

The two equations are then

$$-19.6N + T = 2kg \cdot a_x$$
$$mg\cos(-70°) - T = ma_x$$

a) Since the acceleration is -2 m/s², we have

$$-19.6N + T = -4N$$
$$mg\cos(-70°) - T = m \cdot \left(-2\tfrac{m}{s^2}\right)$$

The first equation allows us to find T = 15.6 N. If this value is substituted in the second equation, we have

$$mg\cos(-70°)-15.6N = m\cdot\left(-2\tfrac{m}{s^2}\right)$$
$$mg\cos(-70°)+m\cdot\left(2\tfrac{m}{s^2}\right)=15.6N$$
$$m\left(g\cos(-70°)+2\tfrac{m}{s^2}\right)=15.6N$$
$$m\left(5.352\tfrac{m}{s^2}\right)=15.6N$$
$$m=2.915kg$$

b) If the tension is 25 N, we have

$$-19.6N+25N=2kg\cdot a_x$$
$$mg\cos(-70°)-25N=ma_x$$

The first equation allows us to find a_x = 2.7 m/s². If this value is substituted in the second equation, we have

$$mg\cos(-70°)-25N=m\cdot\left(2.7\tfrac{m}{s^2}\right)$$
$$mg\cos(-70°)-m\cdot\left(2.7\tfrac{m}{s^2}\right)=25N$$
$$m\left(g\cos(-70°)-2.7\tfrac{m}{s^2}\right)=25N$$
$$m\left(0.6518\tfrac{m}{s^2}\right)=25N$$
$$m=38.36kg$$

22. The forces acting on the 20 kg block are (with an x-axis directed upwards):

1) The weight (196 N) directed downwards.
2) The tension force T_1 directed upwards.

The sum of the x-component of the forces is

$$\sum F_x = m_1 a_x$$
$$-196N+T_1=m_1a_x$$

The forces acting on the 80 kg block are (with an x-axis directed to the left):

1) The weight (784 N) directed downwards.
2) A normal force F_N directed upwards.
3) The tension force T_1 directed towards the negative x-axis.
4) The tension force T_2 directed towards the positive x-axis.

The table of force is

Forces	x	y
Weight	0	-784 N
Normal force	0	F_N
Tension force 1	$-T_1$	0
Tension force 2	T_2	0

The sum of the x-component of the forces is

$$\sum F_x = m_2 a_x$$
$$-T_1 + T_2 = m_2 a_x$$

(The sum of the y-component of the forces is useless here.)

The forces acting on the 30 kg block are (with an x-axis directed downwards):

1) The weight (294 N) directed downwards.
2) The tension force T_2 directed upwards.

The sum of the x-component of the forces is

$$\sum F_x = m_3 a_x$$
$$294N - T_2 = m_3 a_x$$

The three equations are then

$$-196N + T_1 = m_1 a_x$$
$$-T_1 + T_2 = m_2 a_x$$
$$294N - T_2 = m_3 a_x$$

a) This system can be resolved by adding these three equations.

$$(-196N + T_1) + (-T_1 + T_2) + (294N - T_2) = m_1 a_x + m_2 a_x + m_3 a_x$$
$$-196N + 294N = (m_1 + m_2 + m_3) a_x$$
$$98N = 130kg \cdot a_x$$
$$a_x = 0.7538 \tfrac{m}{s^2}$$

b) With the acceleration, the tensions can now be found. For T_1, we have

$$-196N + T_1 = m_1 a_x$$
$$-196N + T_1 = 20kg \cdot 0.7538 \tfrac{m}{s^2}$$
$$T_1 = 211.1N$$

For T_2, we have

$$294N - T_2 = m_3 a_x$$
$$294N - T_2 = 30kg \cdot 0.7538 \tfrac{m}{s^2}$$
$$T_2 = 271.4N$$

23. The forces acting on the 20 kg block are (with an x-axis directed downhill):

1) The weight (196 N) directed downwards, therefore at -60°.
2) A normal force F_{N1} perpendicular to the slope surface.
3) The tension force T directed towards the negative x-axis.

The table of force is

Forces	***x***	***y***
Weight	196 N cos (-60°)	196 N sin (-60°)
Normal force	0	F_{N1}
Tension	-T	0

The sum of the x-component of the forces is

$$\sum F_x = m_1 a_x$$
$$196N \cos(-60°) - T = m_1 a_x$$

(The sum of the y-component of the forces is useless here.)

The forces acting on the 12 kg block are (with an x-axis directed uphill):

1) The weight (117.6 N) directed downwards, therefore at -150°.
2) A normal force F_{N2} perpendicular to the slope surface.
3) The tension force T directed towards the positive x-axis.

The table of force is

Forces	***x***	***y***
Weight	117.6 N cos (-150°)	117.6 N sin (-150°)
Normal force	0	F_{N2}
Tension	T	0

The sum of the *x*-component of the forces is

$$\sum F_x = m_2 a_x$$
$$117{,}6N\cos(-150°) + T = m_2 a_x$$

(The sum of the *y*-component of the forces is useless here.)

The two equations are then

$$196N\cos(-60°) - T = m_1 a_x$$
$$117.6N\cos(-150°) + T = m_2 a_x$$

This system can be solved by adding the equations.

$$\left(196N\cos(-60°) - T\right) + \left(117.6N\cos(-150°) + T\right) = m_1 a_x + m_2 a_x$$
$$196N\cos(-60°) + 117.6N\cos(-150°) - T = (m_1 + m_2)\, a_x$$
$$98N + -101.84N = 32kg \cdot a_x$$
$$a_x = -0.12 \tfrac{m}{s^2}$$

Therefore, the tension is

$$196N\cos(-60°) - T = m_1 a_x$$
$$196N\cos(-60°) - T = 20kg \cdot \left(-0.12 \tfrac{m}{s^2}\right)$$
$$T = 100.4N$$

24. a) The acceleration is found by considering the carrier as a single 820 kg object. The forces exerted on the carrier are (with an *x*-axis directed to the right):

1) The weight (8036 N) directed downwards.
2) A normal force F_N directed upwards.
3) An 800 N force exerted by the tractor.

The sum of the *x*-component of the forces gives

$$\sum F_x = m_1 a_x$$
$$800N = 820kg \cdot a_x$$
$$a_x = 0.9756 \tfrac{m}{s^2}$$

b) T_1 is found by considering the force on the last carriage. The forces exerted on this carriage are (with an x-axis directed to the right)

1) The weight (2352 N) directed downwards.
2) A normal force F_N directed upwards.
3) The tension force T_1 directed towards the right.

The sum of the x-component of the forces gives

$$\sum F_x = m_1 a_x$$
$$T_1 = 240kg \cdot 0.9756 \tfrac{m}{s^2}$$
$$T_1 = 234.1N$$

T_2 is found by considering the last two carriages as a single 400 kg object. The forces exerted on this object are (with an x-axis directed to the right):

1) The weight (3920 N) directed downwards.
2) A normal force F_N directed upwards.
3) The tension force T_2 directed towards the right.

The sum of the x-component of the forces gives

$$\sum F_x = m_1 a_x$$
$$T_2 = 400kg \cdot 0.9756 \tfrac{m}{s^2}$$
$$T_2 = 390.2N$$

T_3 is found by considering the last three carriages as a single 620 kg object. The forces exerted on this object are (with an x-axis directed to the right):

1) The weight (6076 N) directed downwards.
2) A normal force F_N directed upwards surface.
3) The tension force T_3 directed towards the right.

The sum of the x-component of the forces gives

$$\sum F_x = m_1 a_x$$
$$T_3 = 620kg \cdot 0.9756 \tfrac{m}{s^2}$$
$$T_3 = 604.9N$$

25. The forces acting on block A are (with an x-axis directed to the left):

1) The weight $m_a g$ directed downwards.
2) A normal force F_N directed upwards
3) The tension force T directed towards the negative x-axis
4) The force F directed towards the positive x-axis

The table of force is

Forces	x	y
Weight	0	$-m_a g$
Normal force	0	F_N
Tension	$-T$	0
Force *F*	F	0

The sum of the x-component of the forces is

$$\sum F_x = m_a a_x$$
$$-T + F = m_a a_x$$

(The sum of the y-component of the forces is useless here.)

The forces acting on block B are (with an x-axis directed upwards):

1) The weight $m_b g$ directed downwards.
2) The tension force T directed upwards.

The sum of the x-component of the forces is

$$\sum F_x = m_b a_x$$
$$-m_b g + T = m_b a_x$$

The two equations are then

$$-T + F = m_a a_x$$
$$-m_b g + T = m_b a_x$$

If these equations are added, we get

$$(-T + F) + (-m_b g + T) = m_a a_x + m_b a_x$$
$$F - m_b g = (m_a + m_b) a_x$$

We also know that the acceleration is -1 m/s² when the force is 100 N and that the acceleration is 2 m/s² when the force is 200 N. We, therefore, have the two following equations.

$$100N - m_b g = (m_a + m_b)\cdot\left(-1\tfrac{m}{s^2}\right)$$
$$200N - m_b g = (m_a + m_b)\cdot\left(2\tfrac{m}{s^2}\right)$$

If we solve for m_a in the first equation.

$$100N - m_b g = (m_a + m_b)\cdot\left(-1\tfrac{m}{s^2}\right)$$
$$100N - m_b \cdot 9.8\tfrac{m}{s^2} = -m_a \cdot 1\tfrac{m}{s^2} + -m_b \cdot 1\tfrac{m}{s^2}$$
$$100N - m_b \cdot 8.8\tfrac{m}{s^2} = -m_a \cdot 1\tfrac{m}{s^2}$$
$$m_a = 8.8 \cdot m_b - 100kg$$

And substitute in the second equation, we have

$$200N - m_b g = (m_a + m_b)\cdot\left(2\tfrac{m}{s^2}\right)$$
$$200N - m_b g = (8.8m_b - 100kg + m_b)\cdot\left(2\tfrac{m}{s^2}\right)$$
$$200N - m_b g = (9.8m_b - 100kg)\cdot\left(2\tfrac{m}{s^2}\right)$$
$$200N - m_b \cdot 9.8\tfrac{m}{s^2} = m_b \cdot 19.6\tfrac{m}{s^2} - 200N$$
$$400N = m_b \cdot 29.4\tfrac{m}{s^2}$$
$$m_b = 13.61kg$$

Then, m_a is

$$m_a = 8.8 \cdot m_b - 100kg$$
$$m_a = 19.73kg$$

26. The forces acting on the bottom pulley are:

1) A tension force directed downwards, equal to the weight of the 100 kg mass. This is therefore a 980 N force, directed downwards.
2) 3 times the tension force T directed upwards exerted by the string passing through the pulleys.

Since the acceleration is zero, the sum of the y-components of the force are

$$\sum F_y = ma_y$$
$$-980N + 3T = 0$$
$$T = 326.7N$$

27. The forces acting on the pulley attached to the 5 kg block are:

1) The weight (49 N) directed downwards.
2) Twice the tension force T directed upwards exerted by the string.

The sum of the y-component of the forces is

$$\sum F_y = ma_y$$
$$-49N + 2T = ma_y$$

a) If the acceleration is zero, we have

$$-49N + 2T = 0$$
$$T = 24.5N$$

b) If the tension is 20 N, the acceleration is

$$-49N + 2T = ma_y$$
$$-49N + 2 \cdot 20N = 5kg \cdot a_y$$
$$a_y = -1.8 \tfrac{m}{s^2}$$

28. The forces acting on the bottom pulley are:

1) A tension force directed downwards, equal to the weight of the 25 kg mass. This is therefore a 245 N force, directed downwards.
2) Twice the tension force T directed upwards exerted by the string passing through the pulleys.

Since the acceleration is zero, the sum of the y-components of the force are

$$\sum F_y = ma_y$$
$$-245N + 2T = 0$$
$$T = 122.5N$$

The sum of the forces on the bucket is then

$$\begin{aligned}\sum F_y &= ma_y \\ T - mg &= 0 \\ 122.5N - mg &= 0 \\ m &= 12.5kg\end{aligned}$$

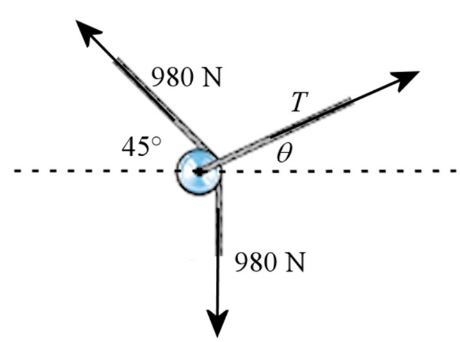
29. The forces acting on Romeo are:

1) The weight (392 N) directed downwards.
2) Twice the tension force T = 40 N directed upwards exerted by the string.

The sum of the y-component of the forces is

$$\begin{aligned}\sum F_y &= ma_y \\ -392N + 2T &= ma_y \\ -392N + 500N &= 40kg \cdot a_y \\ a_y &= 2.7 \tfrac{m}{s^2}\end{aligned}$$

30. The forces acting on the pulley are:



1) The tension of the rope that supports the 100 kg mass, which is 980 N
2) The tension of the rope on the left, which is also 980 N since it's the same rope.
3) The tension of the rope on the right (T).

With no acceleration, the equations of the forces are:

$$\sum F_x = ma_x$$
$$\to 980N\cos(135°) + T\cos\theta = 0$$
$$\sum F_y = ma_y$$
$$\to -980N + 980N\sin(135°) + T\sin\theta = 0$$

This leads to

$$\begin{aligned} T\cos\theta &= -980N\cos(135°) \\ &= 692.96N \end{aligned}$$

and

$$\begin{aligned} T\sin\theta &= 980N - 980N\sin(135°) \\ &= 287.03N \end{aligned}$$

T is found with

$$\begin{aligned} (T\cos\theta)^2 + (T\sin\theta)^2 &= (692.96N)^2 + (287.03N)^2 \\ T^2\cos^2\theta + T^2\sin^2\theta &= 562\,589N^2 \\ T^2\left(\cos^2\theta + \sin^2\theta\right) &= 562\,589N^2 \\ T^2 &= 562\,589N^2 \\ T &= 750N \end{aligned}$$

The angle is found with

$$\begin{aligned} \frac{T\sin\theta}{T\cos\theta} &= \frac{287.03N}{692.96N} \\ \frac{\sin\theta}{\cos\theta} &= 0.4142 \\ \tan\theta &= 0.4142 \\ \theta &= 22.5° \end{aligned}$$

31. The forces acting on pulley C are:

(F is the tension of the rope passing through pulleys C and A, while T is the tension of the rope passing through the pulley B.)

1) 2 times the tension F directed downwards.
2) The tension force T directed upwards.

Since the acceleration is zero, the sum of the y-components of the force are

$$\begin{aligned} \sum F_y &= ma_y \\ -2F + T &= 0 \\ T &= 2F \end{aligned}$$

The forces acting on pulley A are:

1) A tension force directed downwards, equal to the weight of the 10 kg mass. This is therefore a 98 N force directed downwards.
2) 2 times the tension F directed upwards.
3) The tension force T directed upwards.

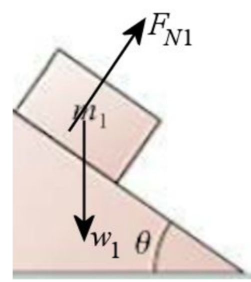
Since the acceleration is zero, the sum of the y-components of the force are

$$\sum F_y = ma_y$$
$$-98N + 2F + T = 0$$

Since $T = 2F$, we have

$$-98N + 2F + 2F = 0$$
$$-98N + 4F = 0$$
$$F = 24.5N$$

32. The forces exerted on the block are (with an x-axis directed towards the right):



1) The weight m_1g directed downwards.
2) A normal force F_{N1} directed at 90° - θ.

The table of force is

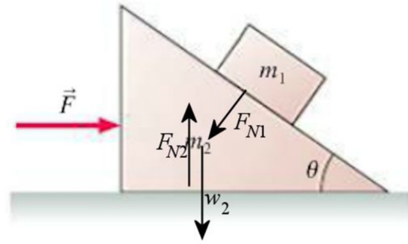
Forces	x	y
Weight	0	$-m_1g$
Normal Force	F_{N1} cos (90°-θ)	F_{N1} sin (90°-θ)

The equations of force are, therefore,

$$\sum F_x = ma_x$$
$$\rightarrow F_{N1}\cos(90° - \theta) = m_1a$$
$$\sum F_y = ma_y$$
$$\rightarrow -m_1g + F_{N1}\sin(90° - \theta) = 0$$

The y-component of the acceleration vanishes since the block does not slide. The x-component of the acceleration is not zero as the triangle and the small block accelerate when the force F is exerted on the triangle.

The forces exerted on the triangle are (with an x-axis directed towards the right):



1) The weight m_2g directed downwards.
2) A normal force F_{N1} directed at $-(90°+\theta)$.
3) A normal force F_{N2} directed upwards exerted by the ground.
4) The applied force F directed towards the right.

The table of force is

Forces	*x*	*y*
Weight	0	$-m_2g$
Normal force 1	$F_{N1}\cos -(90°+\theta)$	$F_{N1}\sin -(90°+\theta)$
Normal force 2	0	F_{N2}
***F* Force**	F	0

The equations of force are, therefore,

$$\sum F_x = ma_x$$
$$\to F_{N1}\cos-(90°+\theta)+F = m_2a$$
$$\sum F_y = ma_y$$
$$\to -m_2g + F_{N1}\sin-(90°+\theta)+F_{N2} = 0$$

Using trigonometric identities, the equations of the small block become

$$F_{N1}\sin\theta = m_1a$$
$$-m_1g + F_{N1}\cos\theta = 0$$

Using trigonometric identities, the equation of the *x*-component of the triangle becomes (the equation for the *y*-component is useless)

$$-F_{N1}\sin\theta + F = m_2a$$

F_{N1} is found with $-m_1g + F_{N1}\cos\theta = 0$. It is

$$F_{N1} = \frac{m_1g}{\cos\theta}$$

Substituting in the other two equations, we obtain

$$F_{N1}\sin\theta = m_1 a$$
$$\frac{m_1 g}{\cos\theta}\sin\theta = m_1 a$$
$$m_1 g\tan\theta = m_1 a$$
$$g\tan\theta = a$$

and

$$-F_{N1}\sin\theta + F = m_2 a$$
$$-\frac{m_1 g}{\cos\theta}\sin\theta + F = m_2 a$$
$$-m_1 g\tan\theta + F = m_2 a$$

Our two equations are now

$$g\tan\theta = a$$
$$-m_1 g\tan\theta + F = m_2 a$$

Using a from the first equation into the second, F is found

$$-m_1 g\tan\theta + F = m_2 g\tan\theta$$
$$F = m_2 g\tan\theta + m_1 g\tan\theta$$
$$F = (m_1 + m_2)\, g\tan\theta$$

33. We're going to treat the rope as two objects: the part that hangs and the part on the table. We'll assume that these two parts are connected by a small massless string of zero length.

For the hanging part, the forces are:

1) The weight $m_1 g$ directed downwards.
2) The tension of the string that connects the two pieces directed upwards

The equation of the y-component of the forces is (with an axis directed downwards)

$$\sum F_y = ma_y$$
$$\to m_1 g - T = m_1 a$$

For the part of the rope on the table, the forces are:

1) The weight m_2g directed downwards.
2) A normal force F_N directed upwards
3) The tension of the string that connects the two pieces directed towards the right.

The equation of the x-component of the forces is (with an axis directed to the right)

$$\sum F_x = ma_x$$
$$\rightarrow T = m_2 a$$

Adding these two equations, the result is

$$m_1 g - T + T = m_1 a + m_2 a$$
$$m_1 g = (m_1 + m_2) a$$
$$m_1 g = Ma$$

where M is the total mass of the rope.

The problem is that m_1 (the mass of the hanging part) change constantly as the rope slides. Let's suppose that the length of the hanging part of the rope is x. Then, the mass of the hanging part is equal to the total mass multiplied by the proportion of the rope that is hanging.

$$m_1 = M\frac{x}{L}$$

where L is the total length of the rope. Thus, the acceleration is

$$m_1 g = Ma$$
$$x\frac{M}{L}g = Ma$$
$$a = x\frac{g}{L}$$

We now need to find speed knowing that x = 20 cm initially and x = 60 cm at the end. The speed can be found with

$$a = \frac{dv}{dt}$$

Then

$$\frac{dv}{dt} = x\frac{g}{L}$$

The equation is solved in the following manner

$$\begin{aligned}
\frac{dv}{dt} &= x\frac{g}{L} \\
\frac{dv}{dx}\frac{dx}{dt} &= x\frac{g}{L} \\
\frac{dv}{dx}v &= x\frac{g}{L} \\
vdv &= x\frac{g}{L}dx
\end{aligned}$$

If both sides are integrated, the result is

$$\begin{aligned}
\int vdv &= \int x\frac{g}{L}dx \\
\frac{v^2}{2} &= x^2\frac{g}{2L} + Cst
\end{aligned}$$

Since $v = 0$ when $x = 0.2$ m, the value of the constant can be found

$$\begin{aligned}
0 &= (0.2m)^2\frac{g}{2L} + Cst \\
Cst &= -(0.2m)^2\frac{g}{2L}
\end{aligned}$$

Therefore, the formula for the speed is

$$\begin{aligned}
\frac{v^2}{2} &= x^2\frac{g}{2L} - (0.2m)^2\frac{g}{2L} \\
v &= \sqrt{\frac{g}{L}\left(x^2 - (0.2m)^2\right)}
\end{aligned}$$

Thus, the speed when $x = 0.6$ m is

$$\begin{aligned}
v &= \sqrt{\frac{9.8\frac{N}{m}}{0.6m}\left((0.6m)^2 - (0.2m)^2\right)} \\
&= 2.286\tfrac{m}{s}
\end{aligned}$$