Chapter 3 Solutions

1. The acceleration is found with

$$\sum F_x = ma_x$$
$$120N = 80kg \times a_x$$
$$a_x = 1.5\tfrac{m}{s^2}$$

2. The acceleration of the car is

$$2a(x - x_0) = v^2 - v_0^2$$
$$2a(80m - 0m) = \left(0\tfrac{m}{s}\right)^2 - \left(27.78\tfrac{m}{s}\right)^2$$
$$a = -4.823\tfrac{m}{s^2}$$

The force is therefore

$$\sum F_x = ma_x$$
$$F = 1200kg \times -4.823\tfrac{m}{s^2}$$
$$F = -5787N$$

The answer is negative because it is in the opposite direction to the velocity, which we had set as positive. The magnitude of the force is therefore 5787 N.

3. Without the trailer, the acceleration is

$$v = v_0 + at$$
$$8\tfrac{m}{s} = 0\tfrac{m}{s} + a \times (1s)$$
$$a = 8\tfrac{m}{s^2}$$

Therefore, the force that accelerates the truck is

$$\sum F_x = ma_x$$
$$F = 1800kg \times 8\tfrac{m}{s^2}$$
$$F = 14,400N$$

With the trailer, the total mass is now 134,800 kg. The acceleration is therefore

$$\sum F_x = ma$$
$$14,400N = 134,800kg \times a$$
$$a = 0.1068 \tfrac{m}{s^2}$$

The time required to reach 10 km/h is therefore

$$v = v_0 + at$$
$$2.778 \tfrac{m}{s} = 0 \tfrac{m}{s} + 0.1068 \tfrac{m}{s^2} \times t$$
$$t = 26s$$

4. The acceleration of the plane is

$$\sum F_x = ma_x$$
$$48,900N \times 2 = 23,500kg \times a$$
$$a = 4.16 \tfrac{m}{s^2}$$

The length of the runway is therefore

$$2a_x \left(x - x_0 \right) = v_x^2 - v_{x0}^2$$
$$2 \times 4.16 \tfrac{m}{s^2} \left(x - 0m \right) = \left(80 \tfrac{m}{s} \right)^2 - \left(0 \tfrac{m}{s} \right)^2$$
$$x = 768.9m$$

5. We will resolve into *x* and *y* components, using an *x*-axis to the right and a *y*-axis upwards.

The components of the 25 N force at 0° are

$$F_{1x} = 25N \times \cos\left(0°\right) = 25N$$
$$F_{1y} = 25N \times \sin\left(0°\right) = 0N$$

The components of the 30 N force at 45° are

$$F_{2x} = 30N \times \cos\left(45°\right) = 15\sqrt{2}N$$
$$F_{2y} = 30N \times \sin\left(45°\right) = 15\sqrt{2}N$$

The components of the 20 N force at 90° are

$$F_{3x} = 20N \times \cos(90°) = 0N$$
$$F_{3y} = 20N \times \sin(90°) = 20N$$

The components of the 25 N force at 225° are

$$F_{4x} = 20N \times \cos(225°) = -10\sqrt{2}N$$
$$F_{4y} = 20N \times \sin(225°) = -10\sqrt{2}N$$

The components of the 50 N force at 270° are

$$F_{5x} = 50N \times \cos(270°) = 0N$$
$$F_{5y} = 50N \times \sin(270°) = -50N$$

The *x*-component of the total force is then

$$\begin{aligned}\sum F_x &= F_{1x} + F_{2x} + F_{3x} + F_{4x} + F_{5x} \\ &= 25N + 15\sqrt{2}N + 0N + -10\sqrt{2}N + 0N \\ &= 32.07N\end{aligned}$$

The *y*-component of the total force is then

$$\begin{aligned}\sum F_y &= F_{1y} + F_{2y} + F_{3y} + F_{4y} + F_{5y} \\ &= 0N + 15\sqrt{2}N + 20N + -10\sqrt{2}N + -50N \\ &= -22.93N\end{aligned}$$

The *x*-component of the acceleration is then

$$\begin{aligned}\sum F_x &= ma_x \\ 32.07N &= 5kg \times a_x \\ a_x &= 6.414 \tfrac{m}{s^2}\end{aligned}$$

The *y*-component of the acceleration is then

$$\begin{aligned}\sum F_y &= ma_y \\ -22.93N &= 5kg \times a_y \\ a_y &= -4.586 \tfrac{m}{s^2}\end{aligned}$$

The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = 7.885 \tfrac{m}{s^2}$$

and its direction is

$$\theta = \arctan \frac{a_y}{a_x} = -35.56°$$

6. Let's find first the sum of the forces on the sled and Aaron. We will resolve into *x* and *y* components using an *x*-axis to the right and a *y*-axis upwards.

The components of the 57 N force to the right are

$$\begin{aligned} F_{1x} &= 57N \times \cos(0°) = 57N \\ F_{1y} &= 57N \times \sin(0°) = 0N \end{aligned}$$

The components of the force exerted by the mom are

$$\begin{aligned} F_{2x} &= 55N \times \cos(145°) = -45.05N \\ F_{2y} &= 55N \times \sin(145°) = 31.55N \end{aligned}$$

The components of the force exerted by the dad are

$$\begin{aligned} F_{3x} &= 55N \times \cos(215°) = -45.05N \\ F_{3y} &= 55N \times \sin(215°) = -31.55N \end{aligned}$$

The *x*-component of the total force is

$$\begin{aligned} \sum F_x &= F_{1x} + F_{2x} + F_{3x} \\ &= 57N + -45.05N + -45.05N \\ &= -33.11N \end{aligned}$$

The *y*-component of the total force is

$$\begin{aligned}\sum F_y &= F_{1y} + F_{2y} + F_{3y} \\ &= 0N + 31.55N + -31.55N \\ &= 0N\end{aligned}$$

To find the mass, the acceleration must be found. If the sled moves 6 m in 2 seconds, its acceleration is

$$\begin{aligned}x &= x_0 + v_0 t + \tfrac{1}{2} a_x t^2 \\ -6m &= 0m + 0\tfrac{m}{s} \times (2s) + \tfrac{1}{2} a_x \times (2s)^2 \\ a_x &= -3\tfrac{m}{s^2}\end{aligned}$$

The mass is therefore

$$\begin{aligned}\sum F_x &= ma_x \\ -33.11N &= m \times \left(-3\tfrac{m}{s^2}\right) \\ m &= 11.04kg\end{aligned}$$

Aaron's mass is therefore

$$\begin{aligned}m &= m_{tot} - m_{sled} \\ &= 11.04kg - 2kg \\ &= 9.04kg\end{aligned}$$

7. Let's find first the sum of the forces on the box. We will resolve into *x* and *y* components using an *x*-axis to the right and a *y*-axis upwards.

The components of the 400 N force are

$$\begin{aligned}F_{1x} &= 400N \times \cos(30°) = 346.4N \\ F_{1y} &= 400N \times \sin(30°) = 200N\end{aligned}$$

The components of the 600 N force are

$$\begin{aligned}F_{2x} &= 600N \times \cos(140°) = -459.6N \\ F_{2y} &= 600N \times \sin(140°) = 385.7N\end{aligned}$$

The *x*-components of the total force are

$$\sum F_x = F_{1x} + F_{2x} + F_{3x}$$
$$0N = 346.4N + -459.6N + F_{3x}$$
$$F_{3x} = 113.2N$$

The y-components of the total force are

$$\sum F_y = F_{1y} + F_{2y} + F_{3y}$$
$$850N = 200N + 385.7N + F_{3y}$$
$$F_{3y} = 264.3N$$

The magnitude of the force is thus

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = 287.5N$$

and its direction is

$$\theta = \arctan\frac{F_{3y}}{F_{3x}} = 66.8°$$

8. a) The acceleration of the spaceship is

$$a = \frac{F}{m} = \frac{-200N}{2500kg} = -0.08\,\tfrac{m}{s^2}$$

The velocity after 0.5 s is therefore

$$v = at = -0.08\,\tfrac{m}{s^2} \times 0.5s = -0.04\,\tfrac{m}{s}$$

b) The acceleration of the astronaut is

$$a = \frac{F}{m} = \frac{200N}{100kg} = 2\,\tfrac{m}{s^2}$$

The velocity after 0.5 s is therefore

$$v = at = 2\,\tfrac{m}{s^2} \times 0.5s = 1\,\tfrac{m}{s}$$

9. The acceleration of the object is

$$a = \frac{F}{m} = \frac{-8\frac{N}{m}x}{2kg} = -4\tfrac{1}{s^2}x$$

Since $a = dv/dt$, this equation becomes

$$\frac{dv}{dt} = -4\tfrac{1}{s^2}x$$

And now, the magic trick to solve.

$$\frac{dv}{dx}\frac{dx}{dt} = -4\tfrac{1}{s^2}x$$
$$\frac{dv}{dx}v = -4\tfrac{1}{s^2}x$$
$$vdv = -4\tfrac{1}{s^2}xdx$$

Integrating each side, the equation becomes

$$\int vdv = \int -4\tfrac{1}{s^2}xdx$$
$$\frac{v^2}{2} = -\frac{4\frac{1}{s^2}}{2}x^2 + Cst$$

Knowing that the speed is 10 m/s at $x = 0$ the constant can be found.

$$\frac{\left(10\frac{m}{s}\right)^2}{2} = 0 + Cst$$
$$Cst = 50\tfrac{m^2}{s^2}$$

Therefore, the speed is given by

$$\frac{v^2}{2} = -\frac{4\frac{1}{s^2}}{2}x^2 + 50\tfrac{m^2}{s^2}$$
$$v^2 = -4\tfrac{1}{s^2}x^2 + 100\tfrac{m^2}{s^2}$$

The position when the speed is zero can now be found.

$$\begin{aligned} 0 &= -4\tfrac{1}{s^2}x^2 + 100\tfrac{m^2}{s^2} \\ 4\tfrac{1}{s^2}x^2 &= 100\tfrac{m^2}{s^2} \\ x &= 5m \end{aligned}$$

10. The acceleration of the object is

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \frac{d(2\tfrac{1}{sm}x^2)}{dt} \\ &= \frac{d(2\tfrac{1}{sm}x^2)}{dx}\frac{dx}{dt} \\ &= 4\tfrac{1}{sm}x\frac{dx}{dt} \end{aligned}$$

Since dx/dt = v, the equation becomes

$$a = 4\tfrac{1}{sm}xv$$

Since

$$v = 2\tfrac{1}{sm}x^2$$

The acceleration becomes

$$\begin{aligned} a &= 4\tfrac{1}{sm}x \cdot 2\tfrac{1}{sm}x^2 \\ &= 8\tfrac{1}{s^2m^2}x^3 \end{aligned}$$

Therefore, the force is

$$\begin{aligned} F &= ma \\ &= 2kg \cdot 8\tfrac{1}{s^2m^2}x^3 \\ &= 16\tfrac{kg}{s^2m^2}x^3 \end{aligned}$$