Chapter 1 Solutions

1. The average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

With the distance in metre and the time in seconds, the velocity is

$$\begin{aligned}\bar{v} &= \frac{384\,400\,000m}{262\,140s} \\ &= 1466.4\,\tfrac{m}{s} \\ &= 5279\,\tfrac{km}{h}\end{aligned}$$

2. The average velocity of the *Deutschland* is found with

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

The average velocity is

$$23.15kn \times \frac{1.853\,\frac{km}{h}}{1kn} = 42.9\,\tfrac{km}{h} = 11.92\,\tfrac{m}{s}$$

and the crossing time is

$$\begin{aligned}\Delta t &= 5\,j \times \frac{24h}{1\,j} \times \frac{60\min}{1h} \times \frac{60s}{1\min} + 11h \times \frac{60\min}{1h} \times \frac{60s}{1\min} + 54\min \times \frac{60s}{1\min} \\ &= 474,840s\end{aligned}$$

The displacement is therefore

$$\begin{aligned}\bar{v} &= \frac{\Delta x}{\Delta t} \\ 11.92\,\tfrac{m}{2} &= \frac{\Delta x}{474,840s} \\ \Delta x &= 5,658,100m = 5658.1km\end{aligned}$$

With this distance, the crossing time of the Lusitania can be found. The average velocity of the Lusitania was

$$23.99kn \times \frac{1.853\frac{km}{h}}{1kn} = 44.45\frac{km}{h} = 12.35\frac{m}{s}$$

The crossing time is then

$$\begin{aligned} \overline{v} &= \frac{\Delta x}{\Delta t} \\ 12.35\tfrac{m}{s} &= \frac{5,658,100m}{\Delta t} \\ \Delta t &= 458,213s \end{aligned}$$

The time difference is then

$$t = 474,840s - 458,213s = 16,627s$$

This is 277 min and 7 seconds, so 4h 37 min 7 s. Let's round up to 4h 37 min.

3. This is solved with

$$x = x_0 + vt$$

If Richard initial position was x = 0, the equation becomes

$$\begin{aligned} x &= x_0 + vt \\ 350m &= 0m + 1.389\tfrac{m}{s} \cdot t \\ t &= 252s = 4\min 12s \end{aligned}$$

4. The collision will occur in

$$\begin{aligned} t &= \frac{L}{v_1 - v_2} \\ t &= \frac{2000m}{13.89\frac{m}{s} - -19.44\frac{m}{s}} \\ t &= 60s \end{aligned}$$

The French submarine position is then (if we set that its initial position was x = 0 m)

$$\begin{aligned} x &= x_0 + vt \\ &= 0m + 13.89\tfrac{m}{s} \cdot 60s = 833m \end{aligned}$$

The collision is 833 m to the right of the starting position of the French submarine.

5. Let's calculate first how long it takes Nicole to reach the car. Starting from $x = 0$ m, she arrives at $x = 100$ m with a velocity of 15 km/h. The time is therefore

$$\begin{aligned} x &= x_0 + vt \\ 100m &= 0m + 4.167\tfrac{m}{s} \cdot t \\ t &= 24s \end{aligned}$$

Then, we must find out how much it will take for the bears to catch up with Nicole.

$$\begin{aligned} t &= \frac{L}{v_1 - v_2} \\ t &= \frac{30m}{6.944\frac{m}{s} - 4.167\frac{m}{s}} \\ t &= 10.8s \end{aligned}$$

As bears the catch up before she reaches the car, small Nicole must kindly give back the fish to the grizzly bears.

6. Let's split this motion into two parts at a constant speed. If Dieudonné started at the position $x = 0$ m, then its position at the end of the first part is

$$\begin{aligned} x &= x_0 + vt \\ &= 0km + 110\tfrac{km}{h} \cdot 4h \\ &= 440km \end{aligned}$$

Using this position as the initial position of the second part, the position at the end of the second part is

$$\begin{aligned} x &= x_0 + vt \\ &= 440km + 130\tfrac{km}{h} \cdot 2h \\ &= 700km \end{aligned}$$

His displacement is therefore 700 km.

His average velocity is then

$$\begin{aligned}\overline{v} &= \frac{\Delta x}{\Delta t} \\ \overline{v} &= \frac{700km}{6h} \\ \Delta t &= 116.7\tfrac{km}{h}\end{aligned}$$

7. Let's split this motion into two parts at a constant speed. If Phil started at the position $x = 0$ m, then its position at the end of the first part is

$$\begin{aligned}x &= x_0 + vt \\ &= 0m + 30\tfrac{m}{s} \cdot 80s \\ &= 2400m\end{aligned}$$

The displacement during this part is then 2400 m.

Using this position as the initial position of the second part, the position at the end of the second part is

$$\begin{aligned}x &= x_0 + vt \\ &= 2400m + -20\tfrac{m}{s} \cdot 15s \\ &= 2100m\end{aligned}$$

The displacement during this part is therefore

$$\Delta x = x_f - x_i = 2100m - 2400m = -300m$$

The total displacement is

$$\Delta x = x_f - x_i = 2100m - 0m = 2100m$$

The total distance is

$$d = 2400m + 300m = 2700m$$

The average velocity is

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{2100m}{95s} = 22.1\tfrac{m}{s}$$

The average speed is

$$\text{Average speed} = \frac{\text{distance}}{\Delta t} = \frac{2700m}{95s} = 28.42\tfrac{m}{s}$$

8. Setting that the epicentre is at $x = 0$ at the observer is at $x = D$, the arrival time of the primary waves is

$$x = x_0 + vt$$
$$D = 0 + v_p t_p$$
$$t_p = \frac{D}{v_p}$$

The arrival time of the secondary waves is

$$x = x_0 + vt$$
$$D = 0 + v_s t_s$$
$$t_s = \frac{D}{v_s}$$

These times are not known, but it is known that the secondary waves arrive 40 seconds after the primary waves. Therefore

$$t_s - t_p = 40s$$

This means that

$$\frac{D}{v_s} - \frac{D}{v_p} = 40s$$
$$D\left(\frac{1}{v_s} - \frac{1}{v_p}\right) = 40s$$

Using the values of the speeds, the distance can be found

$$D\left(\frac{1}{5\frac{km}{s}} - \frac{1}{8\frac{km}{s}}\right) = 40s$$
$$D = 533km$$

9. a) At t = 0 s, the position is x = 0 m. At t = 9 s, the position is x = - 8 m. The displacement is therefore

$$\Delta x = x_f - x_i = -8m - 0m = -8m$$

b) Lets's calculate the displacement for each part where the speed is constant, so for each part where the slope is constant.

At t = 0 s, the position is x = 0 m. At t = 3 s, the position is x = 8 m. The displacement is therefore

$$\Delta x = x_f - x_i = 8m - 0m = 8m$$

At t = 3 s, the position is x = 8 m. At t = 3 s, the position is x = 8 m. The displacement is zero.

At t = 5 s, the position is x = 8 m. At t = 9 s, the position is x = -8 m. The displacement is therefore

$$\Delta x = x_f - x_i = -8m - 8m = -16m$$

The distance is the sum of the absolute values of the displacements.

Distance = 8 m + 0 m + 16 m = 24 m

c) At t = 3 s, the position is x = 8 m. At t = 9 s, the position is x = -8 m. The displacement is therefore

$$\Delta x = x_f - x_i = -8m - 8m = -16m$$

The average velocity is therefore

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{-16m}{9s - 3s} = -2.67 \tfrac{m}{s}$$

d) The velocity at t = 1 s is equal to the slope of the graph at t = 1 s. As it is a straight line, any two points can be taken on this line to calculate the slope. We'll take the points (0 s, 0 m) and (3 s, 8 m). The velocity is therefore

$$v = slope = \frac{x_2 - x_1}{t_2 - t_1} = \frac{8m - 0m}{3s - 0s} = 2.67 \tfrac{m}{s}$$

e) The velocity at t = 8 s is equal to the slope of the graph at t = 8 s. As it is a straight line, any two points can be taken on this line to calculate the slope. We'll take the points (7 s, 0 m) and (9 s, - 8 m). The velocity is therefore

$$v = slope = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-8m - 0m}{9s - 7s} = -4\tfrac{m}{s}$$

10. a) To find the average velocity between t = 0 s and t = 2 s, we need to find the position at these two times. At t = 0 s, the position is

$$\begin{aligned} x &= 4\tfrac{m}{s^2} \cdot (0s)^2 - 5\tfrac{m}{s} \cdot (0s) + 10m \\ &= 10m \end{aligned}$$

At t = 2 s, the position is

$$\begin{aligned} x &= 4\tfrac{m}{s^2} \cdot (2s)^2 - 5\tfrac{m}{s} \cdot (2s) + 10m \\ &= 16m \end{aligned}$$

The average velocity is therefore

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{16m - 10m}{2s - 0s} = 3\tfrac{m}{s}$$

b) We find the velocity at t = 2 s by calculating the value of the derivative at t = 2 s. The derivative is

$$v = \frac{dx}{dt} = \frac{d\left(4\tfrac{m}{s^2}t^2 - 5\tfrac{m}{s}t + 10m\right)}{dt} = 8\tfrac{m}{s^2}t - 5\tfrac{m}{s}$$

At t = 2 s, the velocity is therefore

$$v = 8\tfrac{m}{s^2} \cdot (2s) - 5\tfrac{m}{s} = 11\tfrac{m}{s}$$

11. The acceleration is

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{27.78\tfrac{m}{s} - 0\tfrac{m}{s}}{6.1s} = 4.554\tfrac{m}{s^2}$$

12. The acceleration is

$$\overline{a}=\frac{\Delta v}{\Delta t}=\frac{0\frac{m}{s}-33.33\frac{m}{s}}{3.6s}=-9.26\frac{m}{s^2}$$

13. a) At $t = 0$ s, the velocity is $v = 0$ m/s. At $t = 4$ s, the velocity is $v = 8$ m/s. The average acceleration is therefore

$$\overline{a}=\frac{\Delta v}{\Delta t}=\frac{8\frac{m}{s}-0\frac{m}{s}}{4s-0s}=2\frac{m}{s^2}$$

b) At $t = 10$ s, the velocity is $v = 8$ m/s. At $t = 12$ s, the velocity is $v = 4$ m/s. The average acceleration is therefore

$$\overline{a}=\frac{\Delta v}{\Delta t}=\frac{4\frac{m}{s}-8\frac{m}{s}}{12s-10s}=-2\frac{m}{s^2}$$

c) At $t = 4$ s, the velocity is $v = 8$ m/s. At $t = 8$ s, the velocity is $v = 8$ m/s. The average acceleration is therefore

$$\overline{a}=\frac{\Delta v}{\Delta t}=\frac{8\frac{m}{s}-8\frac{m}{s}}{8s-4s}=0\frac{m}{s^2}$$

d) The acceleration at $t = 1$ s is equal to the slope of the graph at $t = 1$ s. As it is a straight line, any two points can be taken on this line to calculate the slope. We'll take the points (0 s, 0 m/s) and (2 s, 8 m/s). The acceleration is therefore

$$v=slope=\frac{v_2-v_1}{t_2-t_1}=\frac{8\frac{m}{s}-0\frac{m}{s}}{2s-0s}=4\frac{m}{s^2}$$

e) The acceleration at $t = 14$ s is equal to the slope of the graph at $t = 14$ s. As this is a straight line whose slope is zero, the acceleration is zero.

14. The answer is found with the concavity of the curve.

15. a) To find the average acceleration between $t = 0$ s and $t = 1$ s, we need to find the velocity at these two times. Find these velocities we need, to derive the formula for the position to obtain the formula of velocity versus time. The derivative is

$$v=\frac{dx}{dt}=\frac{d\left(3\frac{m}{s^3}t^3-8\frac{m}{s^2}t^2+2\frac{m}{s}t-6m\right)}{dt}=9\frac{m}{s^3}t^2-16\frac{m}{s^2}t+2\frac{m}{s}$$

At t = 0 s, the velocity is

$$v=9\frac{m}{s^3}\cdot(0s)^2-16\frac{m}{s^2}\cdot(0s)+2\frac{m}{s}=2\frac{m}{s}$$

At t = 1 s, the velocity is

$$v=9\frac{m}{s^3}\cdot(1s)^2-16\frac{m}{s^2}\cdot(1s)+2\frac{m}{s}=-5\frac{m}{s}$$

The average acceleration is then

$$\overline{a}=\frac{\Delta v}{\Delta t}=\frac{-5\frac{m}{s}-2\frac{m}{s}}{1s-0s}=-7\frac{m}{s^2}$$

b) To find the acceleration, the formula for the velocity must be derived. The derivative is

$$a=\frac{dv}{dt}=\frac{d\left(9\frac{m}{s^3}t^2-16\frac{m}{s^2}t+2\frac{m}{s}\right)}{dt}=18\frac{m}{s^3}t-16\frac{m}{s^2}$$

At t = 2 s, the acceleration is

$$a=18\frac{m}{s^3}\cdot(2s)-16\frac{m}{s^2}=20\frac{m}{s^2}$$

c) To find the jerk, the formula for the acceleration must be derived. The derivative is

$$j=\frac{da}{dt}=\frac{d\left(18\frac{m}{s^3}t-16\frac{m}{s^2}\right)}{dt}=18\frac{m}{s^3}$$

The jerk is always constant at 18 m/s³.

16. a) The distance travelled is

$$\begin{aligned} x &= x_0 + v_0 t + \frac{1}{2}at^2 \\ &= 0m + 0m + \frac{1}{2}5\tfrac{m}{s^2} \cdot (6s)^2 \\ &= 90m \end{aligned}$$

b) The velocity is

$$\begin{aligned} v &= v_0 + at \\ &= 0\tfrac{m}{s} + 5\tfrac{m}{s^2} \cdot 6s \\ &= 30\tfrac{m}{s} \end{aligned}$$

17. a) The time is given by this formula.

$$x = x_0 + \frac{1}{2}(v_0 + v)t$$

$$100m = 0m + \frac{1}{2}\left(25\tfrac{m}{s} + 15\tfrac{m}{s}\right) \cdot t$$

$$t = 5s$$

b) The acceleration is

$$v = v_0 + at$$

$$15\tfrac{m}{s} = 25\tfrac{m}{s} + a \cdot (5s)$$

$$a = -2\tfrac{m}{s^2}$$

c) The velocity at the third pole speed is given by

$$2a(x - x_0) = v^2 - v_0^2$$

$$2 \cdot \left(-2\tfrac{m}{s^2}\right) \cdot (200m - 0m) = v^2 - \left(25\tfrac{m}{s}\right)^2$$

$$v^2 = -175m^2$$

As there is no solution, the car does not reach the third post.

18. a) The acceleration is found with

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$
$$1600m = 1400m + 5\tfrac{m}{s} \cdot (25s) + \frac{1}{2} a \cdot (25s)^2$$
$$a = 0.24\tfrac{m}{s^2}$$

b) The velocity at the end of the race is

$$\begin{aligned} v &= v_0 + at \\ &= 5\tfrac{m}{s} + 0.24\tfrac{m}{s} \cdot (25s) \\ &= 11\tfrac{m}{s} \end{aligned}$$

(Okay, this is a little fast to finish a 1600 m race…)

19. The distance is

$$2a(x - x_0) = v^2 - v_0^2$$
$$2 \cdot \left(-0.2\tfrac{m}{s^2}\right) \cdot (x - 0m) = \left(9.65\tfrac{m}{s}\right)^2 - \left(10\tfrac{m}{s}\right)^2$$
$$x = 17.19m$$

20. The velocity is found with

$$x = x_0 + \frac{1}{2}(v_0 + v)t$$
$$30m = 0m + \frac{1}{2}\left(v_0 + 20\tfrac{m}{s}\right) \cdot 1,2s$$
$$v_0 = 30\tfrac{m}{s}$$

21. The acceleration is found with

$$2a(x - x_0) = v^2 - v_0^2$$
$$2 \cdot a(32m - 0m) = \left(24\tfrac{m}{s}\right)^2 - \left(30\tfrac{m}{s}\right)^2$$
$$a = -5.0625\tfrac{m}{s^2}$$

The braking distance with an initial velocity of 42 m/s is then

$$\begin{aligned} 2a\left(x-x_0\right) &= v^2 - v_0^2 \\ 2\cdot\left(-5.0625\tfrac{m}{s^2}\right)\left(x-0m\right) &= \left(0\tfrac{m}{s}\right)^2 - \left(42\tfrac{m}{s}\right)^2 \\ x &= 174.2m \end{aligned}$$

22. There are two parts to this movement: a first part where there's no acceleration, which lasts 0.5 s, and a second part where the car slows down.

The position at the end of the first part is

$$\begin{aligned} x &= x_0 + v_0 t \\ &= 0m + 30\tfrac{m}{s}\cdot 0,5s \\ &= 15m \end{aligned}$$

This position is the initial position of the second part. Therefore

$$\begin{aligned} 2a\left(x-x_0\right) &= v^2 - v_0^2 \\ 2\cdot\left(-4\tfrac{m}{s^2}\right)\left(x-15m\right) &= \left(0\tfrac{m}{s}\right)^2 - \left(30\tfrac{m}{s}\right)^2 \\ x &= 127.5m \end{aligned}$$

As the moose was 100 m from the car, she hits the moose.

23. Let's find out how long it takes for each racer to complete the race. There are two phases to the movement of each racer: an acceleration phase and deceleration phase.

Bob's acceleration phase.

Bob's position at the end of this phase is

$$\begin{aligned} x_B &= x_{B0} + v_{B0}t + \frac{1}{2}a_B t^2 \\ &= 0m + 0m + \frac{1}{2}\left(5\tfrac{m}{s^2}\right)\left(1.8s\right)^2 \\ &= 8.1m \end{aligned}$$

Bob's velocity at the end of this phase is

$$\begin{aligned} v_B &= v_{B0} + a_B t^2 \\ &= 0\tfrac{m}{s} + \left(5\tfrac{m}{s^2}\right)\left(1.8s\right) \\ &= 9\tfrac{m}{s} \end{aligned}$$

Bob's deceleration phase.

These values for the position and the velocity are taken as the initial values of the second phase of the movement. Therefore, the time to arrive at x = 100 m is

$$\begin{aligned} x_B &= x_{B0} + v_{B0}t + \frac{1}{2}a_B t^2 \\ 100m &= 8.1m + 9\tfrac{m}{s}\cdot t + \frac{1}{2}\left(-0.1\tfrac{m}{s^2}\right)t^2 \\ 0m &= -91.9m + 9\tfrac{m}{s}\cdot t - 0.05\tfrac{m}{s^2}t^2 \end{aligned}$$

The solutions to this quadratic equation are

$$t = \frac{-9 \pm \sqrt{9^2 - 4\cdot -0.05 \cdot -91.9}}{2\cdot -0.05} \begin{array}{l} \nearrow t = 10.867s \\ \\ \searrow t = 169.13s \end{array}$$

Only the first solution is good. The second is the time that would take Bob to return to the finish line if his negative acceleration continued forever. In this case, Bob would slow down continuously until it stops then go backwards to return to x = 100 m after 169 s.

Bob therefore completed the second part of his race (the deceleration phase) in 10.867 s. The duration of his race is therefore 1.8 s + 10.867 s = 12.667 s.

Gilles's acceleration phase.

Gilles's position at the end of this phase is

$$\begin{aligned} x_G &= x_{G0} + v_{G0}t + \frac{1}{2}a_G t^2 \\ &= 0m + 0m + \frac{1}{2}\left(6\tfrac{m}{s^2}\right)\left(1.7s\right)^2 \\ &= 8.67m \end{aligned}$$

Gilles's velocity at the end of this phase is

$$\begin{aligned} v_G &= v_{G0} + a_G t \\ &= 0\tfrac{m}{s} + \left(6\tfrac{m}{s^2}\right)(1.7s) \\ &= 10.2\tfrac{m}{s} \end{aligned}$$

<u>*Gilles's deceleration phase.*</u>

These values for the position and the velocity are taken as the initial values of the second phase of the movement. Therefore, the time to arrive at x = 100 m is

$$x_G = x_{B0} + v_{B0}t + \frac{1}{2}a_B t^2$$

$$100m = 8.67m + 10.2\tfrac{m}{s}\cdot t + \frac{1}{2}\left(-0.24\tfrac{m}{s^2}\right)t^2$$

$$0m = -91.33m + 10.2\tfrac{m}{s}\cdot t - 0.12\tfrac{m}{s^2}t^2$$

The solutions to this quadratic equation are

$$t = \frac{-10.2 \pm \sqrt{10.2^2 - 4\cdot -0.12 \cdot -91.33}}{2\cdot -0.12} \quad \begin{matrix} \nearrow t = 10.171s \\ \\ \searrow t = 74.83s \end{matrix}$$

Once again, only the first solution is good.

Gilles therefore completed the second part of his race (the deceleration phase) in 10.171 s. The duration of his race is therefore 1.7 s + 10.171 s = 11.871 s.

Bob completed the race 12.667 s whereas Gilles completed the race 11.871 s. Gilles therefore wins by 0.796 s.

24. There are two phases to this movement.

<u>*First phase*</u>: only the first rocket is accelerating

a_1 = 5 m/s² a_2 = 0 m/s² duration = 2 s.

At the end of this phase, the position and the velocity of the first rocket are

$$\begin{aligned} x_1 &= x_{10} + v_{10}t + \frac{1}{2}a_1 t^2 \\ &= 0m + 0m + \frac{1}{2}\left(5\tfrac{m}{s^2}\right)\left(2s\right)^2 \\ &= 10m \end{aligned}$$

$$\begin{aligned} v_1 &= v_{10} + a_1 t \\ &= 0\tfrac{m}{s} + \left(5\tfrac{m}{s^2}\right)\left(2s\right) \\ &= 10\tfrac{m}{s} \end{aligned}$$

As for the 2 rocket, it remains at $x = 0$ m and its speed remains zero during this phase.

<u>Second phase</u>: both rockets are accelerating

$a_1 = 5$ m/s² $\quad a_2 = 6$ m/s²

The positions of the rockets during this phase are

$$\begin{aligned} x_1 &= x_{10} + v_{10}t + \frac{1}{2}a_1 t^2 \\ &= 10m + 10\tfrac{m}{s}\cdot t + \frac{1}{2}\left(5\tfrac{m}{s^2}\right)t^2 \\ &= 10m + 10\tfrac{m}{s}\cdot t + 2.5\tfrac{m}{s^2}t^2 \end{aligned}$$

$$\begin{aligned} x_2 &= x_{20} + v_{20}t + \frac{1}{2}a_2 t^2 \\ &= 0m + 0m + \frac{1}{2}\left(6\tfrac{m}{s^2}\right)t^2 \\ &= 3\tfrac{m}{s^2}t^2 \end{aligned}$$

When rocket 2 catches up with rocket 1, they are in the same place ($x_1 = x_2$). Therefore

$$\begin{aligned} x_1 &= x_2 \\ 10m + 10\tfrac{m}{s}\cdot t + 2.5\tfrac{m}{s^2}t^2 &= 3\tfrac{m}{s^2}t^2 \\ 10m + 10\tfrac{m}{s}\cdot t - 0.5\tfrac{m}{s^2}t^2 &= 0 \end{aligned}$$

The solutions to the quadratic equation are

$$t = \frac{-10 \pm \sqrt{10^2 - 4\cdot -0.5\cdot 10}}{2\cdot -0.5} \quad \begin{matrix} \nearrow t = -0.954s \\ \\ \searrow t = 20.954s \end{matrix}$$

Rocket catches up with rocket 1 20.954 s after the departure of rocket 2 or 22.954 s after the departure of rocket 1.

The position of the rockets is then

$$\begin{aligned} x_2 &= 3\tfrac{m}{s^2} t^2 \\ &= 3\tfrac{m}{s^2}(20.954s)^2 \\ &= 1317m \end{aligned}$$

25. As the equation of motion is

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

we must have

$$\begin{aligned} 5m &= x_0 + 0m + 0m \\ 5m &= x_0 + (1s)v_0 + (0.5s^2)a \\ 9m &= x_0 + (2s)v_0 + (2s^2)a \end{aligned}$$

The first equation gives us directly x_0 = 5 m. The other two equations then become

$$\begin{aligned} 5m &= 5m + (1s)v_0 + (0.5s^2)a \\ 0 &= v_0 + (0.5s)a \end{aligned} \qquad \begin{aligned} 9m &= 5m + (2s)v_0 + (2s^2)a \\ 4m &= (2s)v_0 + (2s^2)a \end{aligned}$$

We will solve for v_0 in the equation to the left and the substitute it in the equation to the right

$$\begin{aligned} 4m &= (2s)v_0 + (2s^2)a \\ 4m &= (2s)(-(0.5s)a) + (2s^2)a \\ 4m &= (-1s^2)a + (2s^2)a \\ 4m &= (1s^2)a \\ a &= 4\tfrac{m}{s^2} \end{aligned}$$

Therefore

$$0 = v_0 + (0.5s)a$$
$$v_0 = -(0.5s)a$$
$$v_0 = -(0.5s)\cdot 4\tfrac{m}{s^2}$$
$$v_0 = -2\tfrac{m}{s}$$

Therefore, the equation of motion is

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$x = 5m - 2\tfrac{m}{s}\cdot t + 2\tfrac{m}{s^2}\cdot t^2$$

At t = 5 s, the position is

$$x = 5m - 2\tfrac{m}{s}\cdot(5s) + 2\tfrac{m}{s^2}\cdot(5s)^2$$
$$= 45m$$

26. a) The velocity is given by (downwards positive axis, y_0 = 0 at the start.)

$$2a(y - y_0) = v^2 - v_0^2$$
$$2\cdot(9.8\tfrac{m}{s^2})(12m - 0m) = v^2 - (0\tfrac{m}{s})^2$$
$$v = 15.34\tfrac{m}{s}$$

b) The free-fall time is (downwards positive axis, y_0 = 0 at the start.)

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$
$$12m = 0m + 0m + \frac{1}{2}(9.8\tfrac{m}{s^2})t^2$$
$$t = 1.565s$$

27. (In this problem, an upwards positive axis is used, y_0 = 0 at the start.)

a) At the maximum height, the velocity is zero. Therefore

$$2a(y - y_0) = v^2 - v_0^2$$
$$2\cdot(-9.8\tfrac{m}{s^2})(y - 0m) = (0\tfrac{m}{s})^2 - (28\tfrac{m}{s})^2$$
$$y = 40m$$

b) The time is found with the following formula.

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$
$$25m = 0m + 28\tfrac{m}{s} \cdot t + \frac{1}{2}\left(-9.8\tfrac{m}{s^2}\right)t^2$$

The solutions to this quadratic equation are

t = 1.108 s and t = 4.607s

Both solutions are good.

c) The time is found with the following formula.

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$
$$-25m = 0m + 28\tfrac{m}{s} \cdot t + \frac{1}{2}\left(-9.8\tfrac{m}{s^2}\right)t^2$$

The solutions to this quadratic equation are

t = -0.785s s and t = 6.499s

Only the second solution is good.

d) This is solved with the following formula

$$2a\left(y - y_0\right) = v^2 - v_0^2$$
$$2 \cdot \left(-9.8\tfrac{m}{s^2}\right)\left(20m - 0m\right) = v^2 - \left(28\tfrac{m}{s}\right)^2$$
$$v = \pm 19.8\tfrac{m}{s}$$

Both solutions are good.

e) There are two possibilities here since the velocity can be positive or negative.

If the velocity is positive, we have

$$v = v_0 + at$$
$$10\tfrac{m}{s} = 28\tfrac{m}{s} + \left(-9.8\tfrac{m}{s^2}\right)t$$
$$t = 1.837s$$

If the velocity is negative, we have

$$\begin{gathered} v = v_0 + at \\ -10\tfrac{m}{s} = 28\tfrac{m}{s} + \left(-9.8\tfrac{m}{s^2}\right)t \\ t = 3.876s \end{gathered}$$

f) There are two possibilities here since the velocity can be positive or negative.

If the velocity is positive, we have

$$\begin{gathered} 2a\left(y - y_0\right) = v^2 - v_0^2 \\ 2\cdot\left(-9.8\tfrac{m}{s^2}\right)\left(y - 0m\right) = \left(12\tfrac{m}{s}\right)^2 - \left(28\tfrac{m}{s}\right)^2 \\ y = 32.65m \end{gathered}$$

If the velocity is negative, we have

$$\begin{gathered} 2a\left(y - y_0\right) = v^2 - v_0^2 \\ 2\cdot\left(-9.8\tfrac{m}{s^2}\right)\left(y - 0m\right) = \left(-12\tfrac{m}{s}\right)^2 - \left(28\tfrac{m}{s}\right)^2 \\ y = 32.65m \end{gathered}$$

This is the same height.

g) The velocity is found with

$$\begin{gathered} 2a\left(y - y_0\right) = v^2 - v_0^2 \\ 2\cdot\left(-9.8\tfrac{m}{s^2}\right)\left(-80m - 0m\right) = v^2 - \left(28\tfrac{m}{s}\right)^2 \\ v = -48.50\tfrac{m}{s} \end{gathered}$$

Only the negative answer is kept, because the stone is moving downwards.

h) The flight time is

$$\begin{gathered} v = v_0 + at \\ -48.50\tfrac{m}{s} = 28\tfrac{m}{s} + \left(-9.8\tfrac{m}{s^2}\right)t \\ t = 7.806s \end{gathered}$$

28. (In this problem, a downwards positive axis is used, $y_0 = 0$ at the start.)

We have

$$2a\left(y-y_0\right)=v^2-v_0^2$$
$$2\cdot\left(9.8\tfrac{m}{s^2}\right)\left(10m-0m\right)=\left(24\tfrac{m}{s}\right)^2-v_0^2$$
$$v=\pm 19.49\tfrac{m}{s}$$

Both answers are good. Tryphon can have thrown the balloon upwards or downwards.

29. (In this problem, an upwards positive axis is used, $y_0 = 0$ at the start.)

The velocity is found with

$$2a\left(y-y_0\right)=v^2-v_0^2$$
$$2\cdot\left(-9.8\tfrac{m}{s^2}\right)\left(80m-0m\right)=\left(0\tfrac{m}{s}\right)^2-v_0^2$$
$$v=39.6\tfrac{m}{s}$$

30. (In this problem, an upwards positive axis is used, $y_0 = 0$ at the start.)

We have

$$y=y_0+v_0t+\frac{1}{2}at^2$$
$$0m=0m+v_0\cdot t+\frac{1}{2}\left(-9.8\tfrac{m}{s^2}\right)t^2$$
$$v_0\cdot t=\frac{1}{2}\left(9.8\tfrac{m}{s^2}\right)t^2$$
$$v_0=\frac{1}{2}\left(9.8\tfrac{m}{s^2}\right)t$$
$$v_0=\frac{1}{2}\left(9.8\tfrac{m}{s^2}\right)\cdot\left(12s\right)$$
$$v_0=58.8\tfrac{m}{s}$$

31. (In this problem, an upwards positive axis is used, $y_0 = 0$ at the start.)

This problem is solved with

$$2a(y-y_0)=v^2-v_0^2$$
$$2\cdot\left(-9.8\tfrac{m}{s^2}\right)(5m-0m)=(0.3v_0)^2-v_0^2$$
$$2\cdot\left(-9.8\tfrac{m}{s^2}\right)(5m-0m)=(0.3)^2v_0^2-v_0^2$$
$$2\cdot\left(-9.8\tfrac{m}{s^2}\right)(5m-0m)=\left((0.3)^2-1\right)v_0^2$$
$$v_0=10.38\tfrac{m}{s}$$

32. (In this problem, an upwards positive axis is used, $y_0 = 0$ at the start.)

There are two phases to this movement: a 4 m/s² upwards acceleration for 20 seconds, and then a 9.8 m/s² downwards acceleration when the motors stop.

First phase: a = 4 m/s²

At the end of this phase, we have

$$\begin{aligned}x&=x_0+v_0t+\frac{1}{2}at^2\\&=0m+0m+\frac{1}{2}\left(4\tfrac{m}{s^2}\right)(20s)^2\\&=800m\end{aligned}$$

$$\begin{aligned}v&=v_0+at\\&0\tfrac{m}{s}+\left(4\tfrac{m}{s^2}\right)(20s)\\&=80\tfrac{m}{s}\end{aligned}$$

Second phase: a = -9.8 m/s²

The final values of the phase 1 are the initial values for this second phase.

At the highest point, the speed is zero. The maximum height is then found with

$$2a(y-y_0)=v^2-v_0^2$$
$$2\cdot\left(-9.8\tfrac{m}{s^2}\right)(y-800m)=\left(0\tfrac{m}{s}\right)^2-\left(80\tfrac{m}{s}\right)^2$$
$$y=1127m$$

The time elapsed during this phase so that the rocket is back to the ground is

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$0m = 800m + 80\tfrac{m}{s} \cdot t + \frac{1}{2}\left(-9.8\tfrac{m}{s^2}\right)t^2$$

The solutions to this quadratic equation are

t = -6.999 s and t = 23.326 s

Only the positive response is good.

The total flight time is the sum of the times of the two phases of the movement. Therefore, the total time is 20 s + 23.326 s = 43.326 s.

33. (In this problem, a downwards positive axis is used, y_0 = 0 at the start.)

The position and the velocity of Kim's ball after 1 s are

$$\begin{aligned} y &= y_0 + v_0 t + \frac{1}{2} a t^2 \\ &= 0m + 0m + \frac{1}{2}\left(9.8\tfrac{m}{s^2}\right)(1s)^2 \\ &= 4.9m \end{aligned}$$

$$\begin{aligned} v &= v_0 + at \\ &= 0\tfrac{m}{s} + \left(9.8\tfrac{m}{s^2}\right)(1s) \\ &= 9.8\tfrac{m}{s} \end{aligned}$$

Then began a second phase when Leo throws his ball. The position of the two balls as a function of time is then

$$\begin{aligned} x_K &= x_{K0} + v_{K0} t + \frac{1}{2} a_K t^2 \\ &= 4{,}9m + 9.8\tfrac{m}{s} \cdot t + \frac{1}{2}\left(9.8\tfrac{m}{s^2}\right)t^2 \\ &= 4.9m + 9.8\tfrac{m}{s} \cdot t + 4.9\tfrac{m}{s^2} t^2 \end{aligned}$$

$$\begin{aligned} x_L &= x_{L0} + v_{L0} t + \frac{1}{2} a_L t^2 \\ &= 0m + 12\tfrac{m}{s} t + \frac{1}{2}\left(9.8\tfrac{m}{s^2}\right)t^2 \\ &= 12\tfrac{m}{s} t + 4.9\tfrac{m}{s^2} t^2 \end{aligned}$$

When Leon's ball catches up with Kim's ball, the two positions are equal. We therefore have

$$
\begin{aligned}
x_K &= x_L \\
4{,}9m + 9.8\tfrac{m}{s}\cdot t + 4.9\tfrac{m}{s^2}t^2 &= 12\tfrac{m}{s}t + 4.9\tfrac{m}{s^2}t^2 \\
4.9m + 9.8\tfrac{m}{s}\cdot t &= 12\tfrac{m}{s}t \\
4.9m &= 2.2\tfrac{m}{s}t \\
t &= 2.227s
\end{aligned}
$$

Leon's ball catches up Kim's ball 2.227 s following the departure of Leon's ball, therefore 3.227 s after the departure of Kim's ball.

Then, the position of Leon's ball is

$$
\begin{aligned}
x_L &= 12\tfrac{m}{s}t + 4.9\tfrac{m}{s^2}t^2 \\
&= 12\tfrac{m}{s}\cdot(2.227s) + 4.9\tfrac{m}{s^2}\cdot(2.227s)^2 \\
&= 51.03m
\end{aligned}
$$

The ball therefore catches up with Kim's ball before hitting the ground (which is at $y = 400$ m).

The ball is then at a height of 400 m – 51.03m = 348.97 m.

34. (In this problem, an upwards positive axis is used, $y_0 = 0$ at the ground.)

a) The position as function of time of the two balls is

$$
\begin{aligned}
x_J &= x_{J0} + v_{J0}t + \frac{1}{2}a_J t^2 \\
&= 51m + -5\tfrac{m}{s}\cdot t + \frac{1}{2}\left(-9.8\tfrac{m}{s^2}\right)t^2 \\
&= 51m - 5\tfrac{m}{s}\cdot t - 4.9\tfrac{m}{s^2}t^2
\end{aligned}
\qquad
\begin{aligned}
x_F &= x_{F0} + v_{F0}t + \frac{1}{2}a_F t^2 \\
&= 1m + 15\tfrac{m}{s}t + \frac{1}{2}\left(-9.8\tfrac{m}{s^2}\right)t^2 \\
&= 1m + 15\tfrac{m}{s}t - 4.9\tfrac{m}{s^2}t^2
\end{aligned}
$$

When Johnny's ball hit Frederique's ball, the two positions are equal. We then have

$$
\begin{aligned}
x_J &= x_F \\
51m - 5\tfrac{m}{s}\cdot t - 4.9\tfrac{m}{s^2}t^2 &= 1m + 15\tfrac{m}{s}t - 4.9\tfrac{m}{s^2}t^2 \\
51m - 5\tfrac{m}{s}\cdot t &= 1m + 15\tfrac{m}{s}t \\
50m &= 20\tfrac{m}{s}t \\
t &= 2.5s
\end{aligned}
$$

b) Then, the position of the balls is

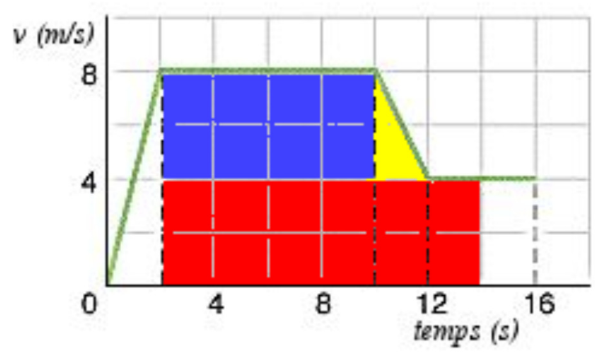
$$\begin{aligned} x_J &= 51m - 5\tfrac{m}{s}\cdot t - 4,9\tfrac{m}{s^2}t^2 \\ &= 51m - 5\tfrac{m}{s}\cdot(2.5s) - 4.9\tfrac{m}{s^2}\cdot(2.5s)^2 \\ &= 7.875m \end{aligned}$$

c) The velocities of the balls are

$$\begin{aligned} v_J &= v_{J0} + a_J t \\ &= -5\tfrac{m}{s} + \left(-9.8\tfrac{m}{s^2}\right)\cdot(2.5s) \\ &= -29.5\tfrac{m}{s} \end{aligned} \qquad \begin{aligned} v_F &= v_{F0} + a_F t \\ &= 15\tfrac{m}{s} + \left(-9.8\tfrac{m}{s^2}\right)\cdot(2.5s) \\ &= -9.5\tfrac{m}{s} \end{aligned}$$

It can be noted that the two balls are moving downwards. Frederique's ball had time to reach its maximum height and begin to descend before being hit by Johnny's ball.

35. The displacement is found with the area under the curve. This area is



The area of the red rectangle is 4 m/s x 12 s = 48 m.

The area of the blue rectangle is 4 m/s x 8 s = 32 m.

The area of the yellow triangle is ½ (4m/s x 2 s) = 4 m.

As the total area is 84m, the displacement is 84 m.

36. To obtain the position from the formula of the velocity, we must integrate. We then have

$$\begin{aligned} x &= \int v\,dt \\ &= \int 2\tfrac{m}{s} \cdot e^{\frac{-t}{2s}} dt \\ &= 2\tfrac{m}{s} \cdot e^{\frac{-t}{2s}} \cdot (-2s) + cst \\ &= -4m \cdot e^{\frac{-t}{2s}} + cst \end{aligned}$$

Since the position at $t = 0$ is $x_0 = 5$ m. We have

$$\begin{aligned} 5m &= -4m \cdot e^{\frac{-0s}{2s}} + cst \\ 5m &= -4m + cst \\ cst &= 9m \end{aligned}$$

The position is therefore

$$x = -4m \cdot e^{\frac{-t}{2s}} + 9m$$

At $t = 4$s, the position is then

$$\begin{aligned} x &= -4m \cdot e^{\frac{-2s}{2s}} + 9m \\ &= -4m \cdot e^{-1} + 9m \\ &= 7.528m \end{aligned}$$

37. a) The velocity is found by integrating the acceleration.

$$\begin{aligned} v &= \int a\,dt \\ &= \int \left(36\tfrac{m}{s^4} t^2 + 10\tfrac{m}{s^2}\right) dt \\ &= 12\tfrac{m}{s^4} t^3 + 10\tfrac{m}{s^2} t + cst \end{aligned}$$

As the initial velocity is 4 m/s, we have

$$\begin{aligned} v &= 12\tfrac{m}{s^4} t^3 + 10\tfrac{m}{s^2} t + cst \\ 4\tfrac{m}{s} &= 12\tfrac{m}{s^4} \cdot (0s)^3 + 10\tfrac{m}{s^2}(0s) + cst \\ cst &= 4\tfrac{m}{s} \end{aligned}$$

The velocity is therefore

$$v = 12\tfrac{m}{s^4}t^3 + 10\tfrac{m}{s^2}t + 4\tfrac{m}{s}$$

The velocity at t = 1 s is therefore

$$\begin{aligned} v &= 12\tfrac{m}{s^4}\cdot(1s)^3 + 10\tfrac{m}{s^2}\cdot(1s) + 4\tfrac{m}{s} \\ &= 26\tfrac{m}{s} \end{aligned}$$

b) The position is found by integrating the velocity.

$$\begin{aligned} x &= \int v dt \\ &= \int \left(12\tfrac{m}{s^4}t^3 + 10\tfrac{m}{s^2}t + 4\tfrac{m}{s}\right)dt \\ &= 3\tfrac{m}{s^4}t^4 + 5\tfrac{m}{s^2}t^2 + 4\tfrac{m}{s}t + cst \end{aligned}$$

If the origin is chosen so that the object is at x = 0 m at the start, the equation becomes

$$\begin{aligned} 0m &= 3\tfrac{m}{s^4}\cdot(0s)^4 + 5\tfrac{m}{s^2}\cdot(0s)^2 + 4\tfrac{m}{s}\cdot(0s) + cst \\ cst &= 0m \end{aligned}$$

The position is therefore given by the formula

$$x = 3\tfrac{m}{s^4}t^4 + 5\tfrac{m}{s^2}t^2 + 4\tfrac{m}{s}t$$

At t = 4 s, the position is then

$$\begin{aligned} x &= 3\tfrac{m}{s^4}\cdot(4s)^4 + 5\tfrac{m}{s^2}\cdot(4s)^2 + 4\tfrac{m}{s}\cdot(4s) \\ &= 864m \end{aligned}$$

As we set that the position at t = 0 s is 0 m, the displacement is 864 m.

38. a) Using x_0 = 0, le position at the end of the first phase is

$$x = \frac{1}{2}at_1^2$$

At the end of the constant speed phase, the position is

$$400m = x_0 + 148.33\tfrac{m}{s}t_2$$

As the position at the end of the first phase is the initial position of the second phase, the position at the end of the second phase is

$$\begin{aligned} 400m &= x_0 + 148.33\tfrac{m}{s} t_2 \\ &= \frac{1}{2}at_1^2 + 148.33\tfrac{m}{s} t_2 \end{aligned}$$

Since the total time was 4.503 s, we know that

$$t_2 = 4.503s - t_1$$

and

$$\begin{aligned} 400m &= \frac{1}{2}at_1^2 + 148.33\tfrac{m}{s} t_2 \\ &= \frac{1}{2}at_1^2 + 148.33\tfrac{m}{s}\left(4.503s - t_1\right) \end{aligned}$$

We also know that the speed is at_1 at the end of the acceleration period. Therefore,

$$\begin{aligned} v &= at_1 \\ 148.33\tfrac{m}{s} &= at_1 \\ a &= \frac{148.33\tfrac{m}{s}}{t_1} \end{aligned}$$

With these two equations, we find that

$$\begin{aligned} 400m &= \frac{1}{2}at_1^2 + 148.33\tfrac{m}{s}\left(4.503s - t_1\right) \\ 400m &= \frac{1}{2}\frac{148.33\tfrac{m}{s}}{t_1}t_1^2 + 148.33\tfrac{m}{s}\left(4.503s - t_1\right) \\ 400m &= 74.165\tfrac{m}{s}\cdot t_1 + 667.945m - 148.33\tfrac{m}{s} t_1 \\ -267.945m &= 74.165\tfrac{m}{s}\cdot t_1 - 148.33\tfrac{m}{s} t_1 \\ -267.945m &= -74.165\tfrac{m}{s}\cdot t_1 \\ t_1 &= 3.613s \end{aligned}$$

b) The acceleration is

$$\begin{aligned} a &= \frac{148.33\frac{m}{s}}{t_1} \\ &= \frac{148.33\frac{m}{s}}{3,613s} \\ &= 41.06\frac{m}{s^2} \end{aligned}$$

39. Sophie's position is

$$x_1 = vt$$

The position of the train is

$$x_2 = \frac{1}{2}3\frac{m}{s^2}t^2 + 10m$$

If Sophie catches the train, then

$$\begin{aligned} x_1 &= x_2 \\ vt &= \frac{1}{2}3\frac{m}{s^2}t^2 + 10m \end{aligned}$$

The solution of this equation is

$$\begin{aligned} vt &= \frac{1}{2}3\frac{m}{s^2}t^2 + 10m \\ 3\frac{m}{s^2}t^2 - 2vt + 20m &= 0 \\ t &= \frac{2v \pm \sqrt{4v^2 - 4\cdot 3\frac{m}{s^2}\cdot 20m}}{6\frac{m}{s^2}} \end{aligned}$$

If Sophie catches the train, then there must be a solution. To have a solution, the term inside the root must be positive. This means that

$$\begin{aligned} 4v^2 - 4\cdot 3\frac{m}{s^2}\cdot 20m &> 0 \\ v^2 - 60\frac{m^2}{s^2} &\geq 0 \\ v^2 &\geq 60\frac{m^2}{s^2} \\ v &\geq 7.746\frac{m}{s} \end{aligned}$$

The minimum speed is, therefore, 7.746 m/s.