Chapter 10 Solutions

1. The forces exerted on the box are:

1) The weight, 1176 N downwards.
2) A normal force, 1176 N upwards.
3) A friction force, 352.8 N towards the left.
4) The force exerted by Karim, 800 N towards the right.

a) The components of the impulse are

$$I_x = F_x \Delta t = 0N \cdot 10s = 0\tfrac{kgm}{s}$$

$$I_y = F_y \Delta t = -1176N \cdot 10s = -11,760\tfrac{kgm}{s}$$

b) The components of the impulse are

$$I_x = F_x \Delta t = 0N \cdot 10s = 0\tfrac{kgm}{s}$$

$$I_y = F_y \Delta t = 1176N \cdot 10s = 11,760\tfrac{kgm}{s}$$

c) The components of the impulse are

$$I_x = F_x \Delta t = -352.8N \cdot 10s = -3528\tfrac{kgm}{s}$$

$$I_y = F_y \Delta t = 0N \cdot 10s = 0\tfrac{kgm}{s}$$

d) The components of the impulse are

$$I_x = F_x \Delta t = 800N \cdot 10s = 8000\tfrac{kgm}{s}$$

$$I_y = F_y \Delta t = 0N \cdot 10s = 0\tfrac{kgm}{s}$$

2. The forces exerted on the crate are:

1) The weight, 294 N at -110°.
2) A normal force towards the positive *y*-axis.
3) A friction force, 70 N towards the negative *x*-axis.
4) The force exerted by Gilbert towards the positive *x*-axis.

To find the magnitude of the normal force and the force exerted by Gilbert, the sum of the *x* and *y* components of the forces must be made. These sums are

$$\sum F_x = ma_x$$
$$\rightarrow 294N \cdot \cos(-110°) + F - 70N = 30kg \cdot 1\tfrac{m}{s^2}$$
$$\sum F_y = ma_y$$
$$\rightarrow 294N \cdot \sin(-110°) + F_N = 0$$

Using the force of the *x*-component of the forces, we find F = 200.55 N.
Using the force of the *y*-component of the forces, we find F_N = 276.27 N.

a) The components of the impulse given by gravity are

$$I_x = F_x \Delta t = 294N\cos(-110°) \cdot 20s = -2011.1\tfrac{kgm}{s}$$

$$I_y = F_y \Delta t = 294N\sin(-110°) \cdot 20s = -5525.4\tfrac{kgm}{s}$$

b) The components of the impulse given by the friction force are

$$I_x = F_x \Delta t = -70N \cdot 20s = -1400\tfrac{kgm}{s}$$

$$I_y = F_y \Delta t = 0N \cdot 20s = 0\tfrac{kgm}{s}$$

c) The components of the impulse given by Gilbert are

$$I_x = F_x \Delta t = 200.55N \cdot 20s = 4011.1\tfrac{kgm}{s}$$

$$I_y = F_y \Delta t = 0N \cdot 10s = 0\tfrac{kgm}{s}$$

d) The components of the impulse given by the normal force are

$$I_x = F_x \Delta t = 0N \cdot 20s = 0 \tfrac{kgm}{s} \qquad I_y = F_y \Delta t = 276.27N \cdot 20s = 5525.4 \tfrac{kgm}{s}$$

e) The *x*-component of the net impulse is then

$$\begin{aligned} I_x &= -2011.1 \tfrac{kgm}{s} + -1400 \tfrac{kgm}{s} + 4011.1 \tfrac{kgm}{s} + 0 \tfrac{kgm}{s} \\ &= 600 \tfrac{kgm}{s} \end{aligned}$$

The *y*-component of the net impulse is then

$$\begin{aligned} I_y &= -5525.4 \tfrac{kgm}{s} + 0 \tfrac{kgm}{s} + 0 \tfrac{kgm}{s} + 5525.4 \tfrac{kgm}{s} \\ &= 0 \tfrac{kgm}{s} \end{aligned}$$

3. The components of the net impulse during the first 3 seconds are

$$I_x = F_x \Delta t = 2N \cdot 3s = 6 \tfrac{kgm}{s} \qquad I_y = F_y \Delta t = 1N \cdot 3s = 3 \tfrac{kgm}{s} \qquad I_z = F_z \Delta t = -4N \cdot 3s = -12 \tfrac{kgm}{s}$$

The components of the net impulse during the next 5 seconds are

$$I_x = F_x \Delta t = -4N \cdot 5s = -20 \tfrac{kgm}{s} \qquad I_y = F_y \Delta t = 5N \cdot 5s = 25 \tfrac{kgm}{s} \qquad I_z = F_z \Delta t = 2N \cdot 5s = 10 \tfrac{kgm}{s}$$

The components of the total net impulse are thus

$$I_{net\,x} = 6 \tfrac{kgm}{s} - 20 \tfrac{kgm}{s} = -14 \tfrac{kgm}{s} \qquad I_{net\,y} = 3 \tfrac{kgm}{s} + 25 \tfrac{kgm}{s} = 28 \tfrac{kgm}{s} \qquad I_{net\,z} = -12 \tfrac{kgm}{s} + 10 \tfrac{kgm}{s} = -2 \tfrac{kgm}{s}$$

4. To find the impulse, the area under the curve from $t = 0$ s and $t = 8$ s must be found. First, there is a small triangle under the time axis. The area of this triangle is

$$Area_1 = \frac{base \times height}{2} = \frac{2s \times 500N}{2} = 500 \tfrac{kgm}{s}$$

As this triangle is under the time axis, the area corresponds to a negative impulse. Therefore I_{1x} = -500 kgm/s.

Then, there is a large triangle above the time axis. The area of this triangle is

$$Area_2 = \frac{base \times height}{2} = \frac{6s \times 2000N}{2} = 6000\,\tfrac{kgm}{s}$$

As this triangle is above the time axis, the area corresponds to a positive impulse. Therefore I_{2x} = 6000 kgm/s.

The area on the small triangle to the left must not be counted as it is after t = 8 s.

The total impulse is thus

$$I_{net\,x} = -500\,\tfrac{kgm}{s} + 6000\,\tfrac{kgm}{s} = 5500\,\tfrac{kgm}{s}$$

5. The impulse is

$$\begin{aligned} I_x &= \int_t^{t'} F_x dt \\ &= \int_{0s}^{5s} 9\,\tfrac{N}{s^2} \cdot t^2 dt \\ &= \left[3\,\tfrac{N}{s^2} \cdot t^3 \right]_{0s}^{5s} \\ &= 3\,\tfrac{N}{s^2} \cdot (5s)^3 - 3\,\tfrac{N}{s^2} \cdot (0s)^3 \\ &= 375\,\tfrac{kgm}{s} \end{aligned}$$

6. The impulse given is

$$\begin{aligned} I_x &= F_x \Delta t \\ &= 250N \cdot 20s \\ &= 5000\,\tfrac{kgm}{s} \end{aligned}$$

We thus have

$$I_{net\,x} = \Delta p_x$$
$$5000\tfrac{kgm}{s} = mv'_x - mv_x$$
$$5000\tfrac{kgm}{s} = 2000kg \cdot v'_x - 2000kg \cdot \left(-5\tfrac{m}{s}\right)$$
$$v'_x = -2.5\tfrac{m}{s}$$

7. Using an *x*-axis directed towards the motion of the arrow, the average force is

$$\begin{aligned}\overline{F}_x &= \frac{\Delta p_x}{\Delta t} \\ &= \frac{mv'_x - mv_x}{\Delta t} \\ &= \frac{0,1kg \cdot 150\tfrac{m}{s} - 0,1kg \cdot 0\tfrac{m}{s}}{0,05s} \\ &= 300N\end{aligned}$$

8. Using an *x*-axis directed towards the motion of the bullet, the average force is

$$\begin{aligned}\overline{F}_x &= \frac{\Delta p_x}{\Delta t} \\ &= \frac{mv'_x - mv_x}{\Delta t} \\ &= \frac{0,02kg \cdot 0\tfrac{m}{s} - 0,02kg \cdot 900\tfrac{m}{s}}{0,004s} \\ &= -4500N\end{aligned}$$

The magnitude of the force is therefore 4500 N.

9. The average force on the car is

$$\begin{aligned}\bar{F}_x &= \frac{\Delta p_x}{\Delta t} \\ &= \frac{mv'_x - mv_x}{\Delta t} \\ &= \frac{1150kg \cdot 2.6\tfrac{m}{s} - 1150kg \cdot \left(-15\tfrac{m}{s}\right)}{0.10s} \\ &= 202{,}400N\end{aligned}$$

As it is positive, the force exerted on the car by the wall is towards the right.

According to Newton's third law, the force exerted on the wall by the car is therefore 202,400 N towards the left.

10. Using an *x*-axis towards the right and a *y*-axis upwards, the *x*-component of the average force is

$$\begin{aligned}\bar{F}_x &= \frac{\Delta p_x}{\Delta t} \\ &= \frac{mv'_x - mv_x}{\Delta t} \\ &= \frac{0.05kg \cdot 8\tfrac{m}{s}\cos\left(25°\right) - 0.05kg \cdot 12\tfrac{m}{s}\cos\left(-45°\right)}{0.06s} \\ &= -1.029N\end{aligned}$$

The *y*-component of the average force is

$$\begin{aligned}\bar{F}_y &= \frac{\Delta p_y}{\Delta t} \\ &= \frac{mv'_y - mv_y}{\Delta t} \\ &= \frac{0.05kg \cdot 8\tfrac{m}{s}\sin\left(25°\right) - 0.05kg \cdot 12\tfrac{m}{s}\sin\left(-45°\right)}{0.06s} \\ &= 9.889N\end{aligned}$$

The magnitude of force is therefore

$$\begin{aligned}\bar{F} &= \sqrt{\bar{F}_x^2 + \bar{F}_y^2} \\ &= \sqrt{\left(-1.029N\right)^2 + \left(9.889N\right)^2} \\ &= 9.942N\end{aligned}$$

and its direction is

$$\theta = \arctan \frac{\overline{F}_y}{\overline{F}_x}$$
$$\theta = \arctan \frac{9.889N}{-1.029N}$$
$$\theta = 95.94°$$

11. The average force is

$$\overline{F}_x = \frac{I_x}{\Delta t}$$

The impulse is found by calculating the area under the curve. As it is a triangle, the area is

$$\begin{aligned} Area &= \frac{base \times height}{2} \\ &= \frac{30s \times 1000N}{2} \\ &= 15,000 \tfrac{kgm}{s} \end{aligned}$$

The average force is therefore

$$\begin{aligned} \overline{F}_x &= \frac{I_x}{\Delta t} \\ &= \frac{15,000 \tfrac{kgm}{s}}{30s} \\ &= 500N \end{aligned}$$

12. An x-axis in the direction of the movement of the ball is used here. In the absence of external forces, the momentum is conserved. Therefore

$$p_{tot\,x} = p'_{tot\,x}$$
$$65.8kg \cdot 0 \tfrac{m}{s} = 0.8kg \cdot 20 \tfrac{m}{s} + 65kg \cdot v'_{Ed}$$
$$v'_{Ed} = -0.246 \tfrac{m}{s}$$

Edward thus goes at 0.246 m/s in the direction opposite to the movement of the ball.

13. An x-axis in the direction of the movement of the ball is used here. In the absence of external forces, the momentum is conserved. Therefore

$$p_{tot\,x} = p'_{tot\,x}$$
$$60kg \cdot 0\tfrac{km}{h} + 15kg \cdot 20\tfrac{km}{h} = 75kg \cdot v'$$
$$v' = 4\tfrac{km}{h}$$

Marie-Sophie (with the ball in her hands) thus goes at 4 km/h towards the right.

14. *First step: Yuri launches the ball.*

An x-axis in the direction of the movement of the ball is used here. In the absence of external forces, the momentum is conserved. Therefore

$$p_{tot\,x} = p'_{tot\,x}$$
$$100kg \cdot 0\tfrac{m}{s} = 20kg \cdot 5\tfrac{m}{s} + 80kg \cdot v'_{Yuri}$$
$$v'_{Yuri} = -1.25\tfrac{m}{s}$$

Yuri thus goes at 1.25 m/s towards the left.

Second step: Valentina catches the ball.

In the absence of external force, the momentum is conserved. Therefore

$$p_{tot\,x} = p'_{tot\,x}$$
$$20kg \cdot 5\tfrac{m}{s} + 70kg \cdot 0\tfrac{m}{s} = 90kg \cdot v'$$
$$v' = 1.111\tfrac{m}{s}$$

Valentina (with the ball in the hands) thus goes at 1.111 m/s towards the right.

15. An x-axis towards the right is used here. In the absence of external forces, the momentum is conserved. Therefore

$$p_{tot\,x} = p'_{tot\,x}$$
$$25kg \cdot 0\tfrac{m}{s} + 70kg \cdot 0\tfrac{m}{s} = 25kg \cdot 10\tfrac{m}{s} + 70kg \cdot v'_{Helmut}$$
$$v'_{Helmut} = -3.571\tfrac{m}{s}$$

Helmut thus travels at 3.571 m/s towards the left.

16. An x-axis towards the right is used here. In the absence of external forces, the momentum is conserved. Therefore

$$\begin{aligned} p_{tot\,x} &= p'_{tot\,x} \\ 94kg \cdot 0\tfrac{m}{s} + 345kg \cdot 0\tfrac{m}{s} &= 94kg \cdot 2.7\tfrac{m}{s} + 345kg \cdot v'_{\log} \\ v'_{\log} &= -0.736\tfrac{m}{s} \end{aligned}$$

The log therefore goes at 0.736 m/s towards the left. The time required to reach the other end is therefore

$$\begin{aligned} t &= \frac{L}{v_1 - v_2} \\ &= \frac{5m}{2.7\frac{m}{s} - -0.736\frac{m}{s}} \\ &= 1.455s \end{aligned}$$

17. An x-axis in the direction of the movement of the cannonballs is used here. In the absence of external forces, the momentum is conserved. Therefore

$$\begin{aligned} p_{tot\,x} &= p'_{tot\,x} \\ 1,200,000kg \cdot 0\tfrac{m}{s} + 180kg \cdot 0\tfrac{m}{s} &= 1,200,000kg \cdot v'_{ship} + 180kg \cdot 425\tfrac{m}{s} \\ v'_{ship} &= -0.06375\tfrac{m}{s} \end{aligned}$$

The ship will therefore move at 0.06375 m/s in the direction opposite to the movement of the cannonballs.

18. An x-axis towards the right and a y-axis upwards is used here. In the absence of external forces, the momentum is conserved. We thus have, for the x-component,

$$\begin{aligned} p_{tot\,x} &= p'_{tot\,x} \\ 10kg \cdot v_{ball\,x} &= 2kg \cdot 80\tfrac{m}{s}\cos(80°) + 5kg \cdot 50\tfrac{m}{s}\cos(-60°) + 3kg \cdot 60\tfrac{m}{s}\cos(220°) \\ v_{ball\,x} &= 1.4896\tfrac{m}{s} \end{aligned}$$

For the x-component, we have

$$p_{tot\ y} = p'_{tot\ y}$$
$$10kg \cdot v_{ball\ y} = 2kg \cdot 80\tfrac{m}{s}\sin(80°) + 5kg \cdot 50\tfrac{m}{s}\sin(-60°) + 3kg \cdot 60\tfrac{m}{s}\sin(220°)$$
$$v_{ball\ y} = -17.4639\tfrac{m}{s}$$

The speed is therefore

$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{\left(1.4896\tfrac{m}{s}\right)^2 + \left(-17.4639\tfrac{m}{s}\right)^2} \\ &= 17.53\tfrac{m}{s} \end{aligned}$$

And the direction of the velocity is

$$\begin{aligned} \theta &= \arctan\frac{v_y}{v_x} \\ &= \arctan\frac{-17.4639\tfrac{m}{s}}{1.4896\tfrac{m}{s}} \\ &= -85{,}1° \end{aligned}$$

19. An x-axis in the direction of the movement of the alpha particle is used here. In the absence of external forces, the momentum is conserved. Therefore

$$p_{tot\ x} = p'_{tot\ x}$$
$$6.64\times10^{-27}kg \cdot 0\tfrac{m}{s} + 388.6\times10^{-27}kg \cdot 0\tfrac{m}{s} = 6.64\times10^{-27}kg \cdot v'_\alpha + 388.6\times10^{-27}kg \cdot v'_{nucleus}$$
$$0\tfrac{kgm}{s} = 6.64 \cdot v'_\alpha + 388.6 \cdot v'_{nucleus}$$

To solve, you must use the fact that the sum of the kinetic energies is 1.3 x 10^{-13} J. Therefore

$$E'_{k\ \alpha} + E'_{k\ nucleus} = 1{,}3\times10^{-13}J$$
$$\tfrac{1}{2}6.64\times10^{-27}kg \cdot \left(v'_\alpha\right)^2 + \tfrac{1}{2}388.6\times10^{-27}kg \cdot \left(v'_{nucleus}\right)^2 = 1.3\times10^{-13}J$$
$$6.64 \cdot \left(v'_\alpha\right)^2 + 388.6 \cdot \left(v'_{nucleus}\right)^2 = 2.6\times10^{14}\tfrac{m^2}{s^2}$$

We now have two equations and two unknowns.

We first solve for the speed of the alpha particle in the momentum equation

$$0\tfrac{kgm}{s} = 6.64 \cdot v'_\alpha + 388.6 \cdot v'_{nucleus}$$
$$6.64 \cdot v'_\alpha = -388.6 \cdot v'_{nucleus}$$
$$v'_\alpha = -58.52 \cdot v'_{nucleus}$$

and substitute this value in the kinetic energy equation.

$$6.64 \cdot (v'_\alpha)^2 + 388.6 \cdot (v'_{nucleus})^2 = 2.6\times 10^{14} \tfrac{m^2}{s^2}$$
$$6.64 \cdot (-58.52 \cdot v'_{nucleus})^2 + 388.6 \cdot (v'_{nucleus})^2 = 2.6\times 10^{14} \tfrac{m^2}{s^2}$$
$$22{,}742 \cdot (v'_{nucleus})^2 + 388.6 \cdot (v'_{gros})^2 = 2.6\times 10^{14} \tfrac{m^2}{s^2}$$
$$23{,}131 \cdot (v'_{nucleus})^2 = 2.6\times 10^{14} \tfrac{m^2}{s^2}$$
$$v'_{nucleus} = \pm 1.06\times 10^5 \tfrac{m}{s}$$

According to the figure, it is quite clear that the negative answer is the right answer.

$$v'_{nucleus} = -1.06\times 10^5 \tfrac{m}{s}$$

The other speed is therefore

$$v'_\alpha = -58.52 \cdot v'_{nucleus}$$
$$= 6.20\times 10^6 \tfrac{m}{s}$$

20. An x-axis towards the right is used here. In the absence of external forces, the momentum is conserved. Therefore

$$p_{tot\,x} = p'_{tot\,x}$$
$$0\tfrac{kgm}{s} = 0.03kg \cdot v'_{ball\,x} + 150kg \cdot v'_{carriage}$$
$$0\tfrac{kgm}{s} = 0.03kg \cdot 900\tfrac{m}{s}\cos 30° + 150kg \cdot v'_{carriage}$$
$$v'_{carriage} = -0.1559\tfrac{m}{s}$$

The trolley therefore moves at 0.1559 m/s towards the left.

21. An x-axis towards the right is used here. In a collision, the momentum is conserved. Therefore

$$p_{tot\,x} = p'_{tot\,x}$$
$$10,000kg \cdot 20\tfrac{m}{s} + 20,000kg \cdot 2\tfrac{m}{s} = 30,000kg \cdot v'$$
$$v' = 8\tfrac{m}{s}$$

22. An x-axis towards the right is used here. In a collision, the momentum is conserved. Therefore

$$p_{tot\,x} = p'_{tot\,x}$$
$$80kg \cdot 5\tfrac{m}{s} + 110kg \cdot \left(-4\tfrac{m}{s}\right) = 190kg \cdot v'$$
$$v' = -0.2105\tfrac{m}{s}$$

23. An x-axis towards the right is used here. In a collision, the momentum is conserved. Therefore

$$p_{tot\,x} = p'_{tot\,x}$$
$$5kg \cdot 10\tfrac{m}{s} + m_2 \cdot \left(-2\tfrac{m}{s}\right) = \left(5kg + m_2\right) \cdot \left(-1\tfrac{m}{s}\right)$$

If we solve for m_2, we have

$$5kg \cdot 10\tfrac{m}{s} + m_2 \cdot \left(-2\tfrac{m}{s}\right) = \left(5kg + m_2\right) \cdot \left(-1\tfrac{m}{s}\right)$$
$$50\tfrac{kgm}{s} - m_2 \cdot 2\tfrac{m}{s} = -5\tfrac{kgm}{s} - m_2 \cdot 1\tfrac{m}{s}$$
$$55\tfrac{kgm}{s} = m_2 \cdot 1\tfrac{m}{s}$$
$$m_2 = 55kg$$

24. *<u>First Collision</u>*

An x-axis towards the right is used here. In a collision, the momentum is conserved. Therefore

$$
\begin{aligned}
p_{tot\,x} &= p'_{tot\,x} \\
2kg \cdot 12 \tfrac{m}{s} + 1kg \cdot 0 \tfrac{m}{s} &= 3kg \cdot v_2 \\
v_2 &= 8 \tfrac{m}{s}
\end{aligned}
$$

Second Collision

We have

$$
\begin{aligned}
p_{tot\,x} &= p'_{tot\,x} \\
3kg \cdot 8 \tfrac{m}{s} + 3kg \cdot 0 \tfrac{m}{s} &= 6kg \cdot v_3 \\
v_3 &= 4 \tfrac{m}{s}
\end{aligned}
$$

25. *Collision*

An x-axis towards the right is used here. In a collision, the momentum is conserved. Therefore

$$
\begin{aligned}
p_{tot\,x} &= p'_{tot\,x} \\
0.02kg \cdot 800 \tfrac{m}{s} + 2kg \cdot 0 \tfrac{m}{s} &= 2.02kg \cdot v' \\
v' &= 7.921 \tfrac{m}{s}
\end{aligned}
$$

Rise of the Pendulum

We then have the rise of the pendulum. The maximum height reached by the block can be found with the conservation of mechanical energy. As the system is composed of a single object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

Initially (pendulum at its lowest point, immediately after the collision), the mechanical energy is

$$
\begin{aligned}
E_{mec} &= \frac{1}{2}mv^2 + mgy \\
&= \frac{1}{2} 2.02kg \left(7.921 \tfrac{m}{s}\right)^2 + 0J \\
&= 63.37J
\end{aligned}
$$

We have chosen to put the $y = 0$ at the lowest point of the pendulum.

At the maximum angle, the energy is

$$\begin{aligned} E_{mec} &= \frac{1}{2}mv^2 + mgy \\ &= 0J + 2.02kg \cdot 9.8\tfrac{N}{kg} \cdot y \\ &= 19.796N \cdot y \end{aligned}$$

According to the principle of conservation of mechanical energy, we have

$$\begin{aligned} E &= E' \\ 63.37J &= 19.796N \cdot y \\ y &= 3.201m \end{aligned}$$

The angle is thus

$$\begin{aligned} \cos\theta &= \frac{L-y}{L} \\ \cos\theta &= \frac{8m - 3.201m}{8m} \\ \theta &= 53.1^\circ \end{aligned}$$

26. An x-axis towards the right is used here. In a collision, the momentum is conserved. Therefore

$$\begin{aligned} p_{tot\,x} &= p'_{tot\,x} \\ 10kg \cdot 8\tfrac{m}{s} \cdot \cos(-70^\circ) + 200kg \cdot 4\tfrac{m}{s} &= 210kg \cdot v' \\ v' &= 3.94\tfrac{m}{s} \end{aligned}$$

The trolley therefore moves at 3.94 m/s towards the right.

27. An x-axis towards the right is used here. In a collision, the momentum is conserved. Therefore

$$p_{tot\,x} = p'_{tot\,x}$$
$$0.2kg \cdot 10\tfrac{m}{s} + 0.55kg \cdot -4\tfrac{m}{s} = 0.2kg \cdot v'_1 + 0.55kg \cdot v'_2$$
$$-0.2\tfrac{kgm}{s} = 0.2kg \cdot v'_1 + 0.55kg \cdot v'_2$$
$$-1\tfrac{m}{s} = v'_1 + 2.75 \cdot v'_2$$

In an elastic collision, kinetic energy is also conserved.

$$E_{k\,tot} = E'_{k\,tot}$$
$$\tfrac{1}{2}0.2kg \cdot \left(10\tfrac{m}{s}\right)^2 + \tfrac{1}{2}0.55kg \cdot \left(-4\tfrac{m}{s}\right)^2 = \tfrac{1}{2}0.2kg \cdot v_1'^2 + \tfrac{1}{2}0.55kg \cdot v_2'^2$$
$$14.4J = \tfrac{1}{2}0.2kg \cdot v_1'^2 + \tfrac{1}{2}0.55kg \cdot v_2'^2$$
$$144\tfrac{m^2}{s^2} = v_1'^2 + 2.75 \cdot v_2'^2$$

We can solve for v_1 in the first equation

$$v'_1 = -1\tfrac{m}{s} - 2.75 \cdot v'_2$$

And substitute in the second equation to get

$$144\tfrac{m^2}{s^2} = \left(-1\tfrac{m}{s} - 2.75 \cdot v'_2\right)^2 + 2.75 \cdot v_2'^2$$
$$144\tfrac{m^2}{s^2} = 1\tfrac{m^2}{s^2} + 5.5\tfrac{m}{s} \cdot v'_2 + 7.5625 \cdot v_2'^2 + 2.75 \cdot v_2'^2$$
$$0 = -143\tfrac{m^2}{s^2} + 5.5\tfrac{m}{s} \cdot v'_2 + 10.3125 \cdot v_2'^2$$
$$v'_2 = -4\tfrac{m}{s} \quad \text{and} \quad v'_2 = 3.467\tfrac{m}{s}$$

The first solution is obviously the speed before the collision. The other solution is the speed after the collision. Thus

$$v'_2 = 3.467\tfrac{m}{s}$$

The speed of the other ball is

$$v'_1 = -1\tfrac{m}{s} - 2.75 \cdot v'_2$$
$$= -10.53\tfrac{m}{s}$$

28. a) An x-axis towards the right is used here. In a collision, the momentum is conserved. Therefore

$$p_{tot\ x} = p'_{tot\ x}$$
$$500kg \cdot 20\tfrac{m}{s} + 250kg \cdot 10\tfrac{m}{s} = 500kg \cdot v'_1 + 250kg \cdot 12\tfrac{m}{s}$$
$$v'_1 = 19\tfrac{m}{s}$$

b) The kinetic energy before the collision is

$$\begin{aligned} E_k &= \tfrac{1}{2}500kg \cdot \left(20\tfrac{m}{s}\right)^2 + \tfrac{1}{2}250kg \cdot \left(10\tfrac{m}{s}\right)^2 \\ &= 112,500J \end{aligned}$$

The kinetic energy after the collision is

$$\begin{aligned} E'_k &= \tfrac{1}{2}500kg \cdot \left(19\tfrac{m}{s}\right)^2 + \tfrac{1}{2}250kg \cdot \left(12\tfrac{m}{s}\right)^2 \\ &= 108,250J \end{aligned}$$

The change of kinetic energy is thus

$$\begin{aligned} \Delta E_k &= 108,250J - 112,500J \\ &= -4250J \end{aligned}$$

Therefore, the fraction lost is

$$\frac{-4250J}{112,500J} = -0.0378$$

Therefore, 3.78% of the kinetic energy is lost.

c) The change of momentum is

$$\begin{aligned} \Delta p_x &= p'_x - p_x \\ &= 500kg \cdot 19\tfrac{m}{s} - 500kg \cdot 20\tfrac{m}{s} \\ &= -500\tfrac{kgm}{s} \end{aligned}$$

d) As momentum of the system is conserved and the 500 kg asteroid lost 500 kgm/s, then the momentum of the 250 kg asteroid must increase by 500 kgm/s. You can easily check this with

$$\begin{aligned}\Delta p_x &= p'_x - p_x \\ &= 250kg \cdot 12\tfrac{m}{s} - 250kg \cdot 10\tfrac{m}{s} \\ &= 500\tfrac{kgm}{s}\end{aligned}$$

29. An x-axis towards the right is used here. In a collision, the momentum is conserved. Therefore

$$\begin{aligned}p_{tot\,x} &= p'_{tot\,x} \\ 0.004kg \cdot 750\tfrac{m}{s} + 1.15kg \cdot 0\tfrac{m}{s} &= 0.004kg \cdot 320\tfrac{m}{s} + 1.15kg \cdot v'_{bloc\,x} \\ v'_{bloc\,x} &= 1.496\tfrac{m}{s}\end{aligned}$$

30. *First Part: The Descent of the Block*

The speed of the block at the end of its descent will be found with the conservation of mechanical energy. As the system is composed of a single object, the mechanical energy of the system is

$$E_{mec} = \frac{1}{2}mv^2 + mgy$$

Initially (configuration shown in the figure), the mechanical energy is

$$\begin{aligned}E &= \frac{1}{2}mv^2 + mgy \\ &= 0J + 2kg \cdot 9.8\tfrac{N}{kg} \cdot 4m \\ &= 78.4J\end{aligned}$$

We have chosen to set the origin $y = 0$ at the level of the horizontal surface.

After the descent, the mechanical energy is

$$\begin{aligned}E' &= \frac{1}{2}mv'^2 + mgy' \\ &= \frac{1}{2}2kg \cdot v'^2 + 0J \\ &= 1kg \cdot v'^2\end{aligned}$$

According to the conservation of mechanical energy, we have

$$E = E'$$
$$78.4J = 1kg \cdot v'^2$$
$$v' = 8.854 \tfrac{m}{s}$$

Second Part: The Collision

In a collision, momentum is conserved. Thus

$$p_{tot\,x} = p'_{tot\,x}$$
$$2kg \cdot 8.854 \tfrac{m}{s} + 3kg \cdot 0 \tfrac{m}{s} = 5kg \cdot v'$$
$$v' = 3.542 \tfrac{m}{s}$$

This is the speed of the blocks after the collision.

Third Part: The Sliding

According to the sum of the x-components of the force, the acceleration is

$$\sum F_x = ma_x$$
$$-F_f = ma_x$$
$$-\mu_k F_N = ma_x$$
$$-\mu_k mg = ma_x$$
$$a_x = -\mu_k g$$
$$a_x = -2.45 \tfrac{m}{s^2}$$

With an initial velocity of 3.542 m/s, the stopping distance is

$$2a_x (x - x_0) = v_x^2 - v_{0x}^2$$
$$2\left(-2.45 \tfrac{m}{s^2}\right)(x - 0m) = \left(0 \tfrac{m}{s}\right)^2 - \left(3.542 \tfrac{m}{s}\right)^2$$
$$x = 2.56m$$

31. In a collision, momentum is conserved. Therefore,

$$m_1 v_1 = m_1 v_1' + m_2 v_2'$$

In an elastic collision the kinetic energy is conserved. Therefore,

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

Solving for v_1' in the first equation

$$v_1' = v_1 - \tfrac{m_2}{m_1} v_2'$$

and using this results in the second equation yields

$$\begin{aligned}
\frac{1}{2} m_1 v_1^2 &= \frac{1}{2} m_1 \left(v_1 - \tfrac{m_2}{m_1} v_2' \right)^2 + \frac{1}{2} m_2 v_2'^2 \\
m_1 v_1^2 &= m_1 \left(v_1 - \tfrac{m_2}{m_1} v_2' \right)^2 + m_2 v_2'^2 \\
m_1 v_1^2 &= m_1 v_1^2 - 2 m_2 v_1 v_2' + \tfrac{m_2^2}{m_1} v_2'^2 + m_2 v_2'^2 \\
0 &= -2 m_2 v_1 v_2' + \tfrac{m_2^2}{m_1} v_2'^2 + m_2 v_2'^2 \\
2 m_2 v_1 v_2' &= \tfrac{m_2^2}{m_1} v_2'^2 + m_2 v_2'^2 \\
2 m_2 v_1 &= \tfrac{m_2^2}{m_1} v_2' + m_2 v_2' \\
2 m_2 m_1 v_1 &= m_2^2 v_2' + m_1 m_2 v_2' \\
2 m_1 v_1 &= m_2 v_2' + m_1 v_2' \\
2 m_1 v_1 &= \left(m_2 + m_1 \right) v_2' \\
v_2' &= \frac{2 m_1 v_1}{m_1 + m_2}
\end{aligned}$$

The speed of the other ball is thus

$$\begin{aligned}
v_1' &= v_1 - \tfrac{m_2}{m_1} v_2' \\
&= v_1 - \frac{m_2}{m_1} \frac{2 m_1 v_1}{m_1 + m_2} \\
&= v_1 - \frac{2 m_2 v_1}{m_1 + m_2} \\
&= \frac{m_1 + m_2}{m_1 + m_2} v_1 - \frac{2 m_2 v_1}{m_1 + m_2} \\
&= \frac{m_1 + m_2 - 2 m_2}{m_1 + m_2} v_1 \\
&= \frac{m_1 - m_2}{m_1 + m_2} v_1
\end{aligned}$$

32. The variation of kinetic energy is

$$\begin{aligned}\Delta E_k &= E_k' - E_k \\ &= \frac{1}{2}m_{tot}v'^2 - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)\end{aligned}$$

The second ball has no speed before the collision. Thus,

$$\Delta E_k = \frac{1}{2}m_{tot}v'^2 - \frac{1}{2}m_1v_1^2$$

The speed of the object after the collision is found with momentum conservation.

$$\begin{aligned}m_1v_1 &= (m_1 + m_2)v' \\ v' &= \frac{m_1v_1}{m_1 + m_2}\end{aligned}$$

Therefore, the kinetic energy variation is

$$\begin{aligned}\Delta E_k &= \frac{1}{2}m_{tot}v'^2 - \frac{1}{2}m_1v_1^2 \\ &= \frac{1}{2}(m_1 + m_2)\left(\frac{m_1v_1}{m_1 + m_2}\right)^2 - \frac{1}{2}m_1v_1^2 \\ &= \frac{1}{2}\frac{m_1^2v_1^2}{m_1 + m_2} - \frac{1}{2}m_1v_1^2 \\ &= \frac{1}{2}\left(\frac{m_1^2}{m_1 + m_2} - m_1\right)v_1^2 \\ &= \frac{1}{2}\left(\frac{m_1^2}{m_1 + m_2} - \frac{m_1(m_1 + m_2)}{m_1 + m_2}\right)v_1^2 \\ &= \frac{1}{2}\left(\frac{m_1^2}{m_1 + m_2} - \frac{m_1^2 + m_1m_2}{m_1 + m_2}\right)v_1^2 \\ &= \frac{1}{2}\left(\frac{m_1^2 - m_1^2 - m_1m_2}{m_1 + m_2}\right)v_1^2 \\ &= -\frac{1}{2}\frac{m_1m_2}{m_1 + m_2}v_1^2\end{aligned}$$

33. An x-axis towards the right and a y-axis upwards is used here. In the absence of external forces, the momentum is conserved. We thus have, for the x-component

$$\begin{aligned} p_{tot\,x} &= p'_{tot\,x} \\ 3kg \cdot 0\tfrac{m}{s} + 2kg \cdot -3\tfrac{m}{s} &= 5kg \cdot v'_x \\ v'_x &= -1.2\tfrac{m}{s} \end{aligned}$$

For the y-component, we have

$$\begin{aligned} p_{tot\,y} &= p'_{tot\,y} \\ 3kg \cdot 2\tfrac{m}{s} + 2kg \cdot 0\tfrac{m}{s} &= 5kg \cdot v'_y \\ v'_y &= 1.2\tfrac{m}{s} \end{aligned}$$

The speed is therefore

$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{\left(-1.2\tfrac{m}{s}\right)^2 + \left(1.2\tfrac{m}{s}\right)^2} \\ &= 1.697\tfrac{m}{s} \end{aligned}$$

And its direction is

$$\begin{aligned} \theta &= \arctan\frac{v_y}{v_x} \\ &= \arctan\frac{1.2\frac{m}{s}}{-1.2\frac{m}{s}} \\ &= 135° \end{aligned}$$

The angle in the figure is therefore 45°.

34. An x-axis towards the right and a y-axis upwards is used here. In the absence of external forces, the momentum is conserved. We thus have, for the x-component

$$\begin{aligned} p_{tot\,x} &= p'_{tot\,x} \\ 1500kg \cdot v_A \cos(45°) + 2000kg \cdot -v_B &= 3500kg \cdot 12\tfrac{m}{s}\cos(60°) \\ 1060.7kg \cdot v_A + 2000kg \cdot -v_B &= 21,000\tfrac{kgm}{s} \end{aligned}$$

For the y-component, we have

$$p_{tot\,x} = p'_{tot\,x}$$
$$1500kg \cdot v_A \sin(45°) + 2000kg \cdot 0\tfrac{m}{s} = 3500kg \cdot 12\tfrac{m}{s}\sin(60°)$$
$$1060.7kg \cdot v_A = 36,373\tfrac{kgm}{s}$$

This equation allows us to find the speed of car A.

$$1060.7kg \cdot v_A = 36,373\tfrac{kgm}{s}$$
$$v_A = 34.29\tfrac{m}{s}$$

With this information, the speed of car B can be found with the equation for the *x*-component.

$$1060.7kg \cdot v_A + 2000kg \cdot -v_B = 21,000\tfrac{kgm}{s}$$
$$1060.7kg \cdot 34.29\tfrac{m}{s} + 2000kg \cdot -v_B = 21,000\tfrac{kgm}{s}$$
$$36,373\tfrac{kgm}{s} + 2000kg \cdot -v_B = 21,000\tfrac{kgm}{s}$$
$$2000kg \cdot -v_B = -15,373\tfrac{kgm}{s}$$
$$v_B = 7.69\tfrac{m}{s}$$

35. An *x*-axis towards the right and a *y*-axis upwards is used here. In the absence of external forces, the momentum is conserved. We thus have, for the *x*-component

$$p_{tot\,x} = p'_{tot\,x}$$
$$1kg \cdot 5\tfrac{m}{s} + 3kg \cdot 0\tfrac{m}{s} = 1kg \cdot v'_{1x} + 3kg \cdot v'_2 \cos(-50°)$$
$$5\tfrac{kgm}{s} = 1kg \cdot v'_{1x} + 3kg \cdot v'_2 \cos(-50°)$$
$$5\tfrac{m}{s} = v'_{1x} + 3v'_2 \cos(-50°)$$

For the *y*-component, we have

$$p_{tot\,y} = p'_{tot\,y}$$
$$1kg \cdot 0\tfrac{m}{s} + 3kg \cdot 0\tfrac{m}{s} = 1kg \cdot v'_{1y} + 3kg \cdot v'_2 \sin(-50°)$$
$$0\tfrac{kgm}{s} = 1kg \cdot v'_{1y} + 3kg \cdot v'_2 \sin(-50°)$$
$$0\tfrac{m}{s} = v'_{1y} + 3v'_2 \sin(-50°)$$

As this is an elastic collision, the kinetic energy is also conserved. Therefore

$$E_{k\,tot} = E'_{k\,tot}$$
$$\tfrac{1}{2}1kg \cdot \left(5\tfrac{m}{s}\right)^2 + \tfrac{1}{2}3kg \cdot \left(0\tfrac{m}{s}\right)^2 = \tfrac{1}{2}1kg \cdot v_1'^2 + \tfrac{1}{2}3kg \cdot v_2'^2$$
$$12.5J = \tfrac{1}{2}1kg \cdot v_1'^2 + \tfrac{1}{2}3kg \cdot v_2'^2$$
$$25\tfrac{m^2}{s^2} = v_1'^2 + 3v_2'^2$$

Since $v_1'^2 = v_{1x}'^2 + v_{1y}'^2$, this equation becomes

$$25\tfrac{m^2}{s^2} = v_1'^2 + 3v_2'^2$$
$$25\tfrac{m^2}{s^2} = v_{1x}'^2 + v_{1y}'^2 + 3v_2'^2$$

If we solve for v'_{1x} in the *x*-component equation, we have

$$5\tfrac{m}{s} = v'_{1x} + 3v'_2 \cos(-50°)$$
$$v'_{1x} = 5\tfrac{m}{s} - 3v'_2 \cos(-50°)$$

If we solve for v'_{1y} in the *y*-component equation, we have

$$0\tfrac{m}{s} = v'_{1y} + 3v'_2 \sin(-50°)$$
$$v'_{1y} = -3v'_2 \sin(-50°)$$

We then substitute these values in the conservation of kinetic energy equation.

$$25\tfrac{m^2}{s^2} = v_{1x}'^2 + v_{1y}'^2 + 3v_2'^2$$
$$25\tfrac{m^2}{s^2} = \left(5\tfrac{m}{s} - 3v'_2 \cos(-50°)\right)^2 + \left(-3v'_2 \sin(-50°)\right)^2 + 3v_2'^2$$

It remains to solve for v'_2.

$$25\tfrac{m^2}{s^2} = \left(5\tfrac{m}{s} - 3v'_2 \cos(-50°)\right)^2 + \left(-3v'_2 \sin(-50°)\right)^2 + 3v_2'^2$$
$$25\tfrac{m^2}{s^2} = 25\tfrac{m^2}{s^2} - 30\tfrac{m}{s} \cdot v'_2 \cos(-50°) + 9v_2'^2 \cos^2(-50°) + 9v_2'^2 \sin^2(-50°) + 3v_2'^2$$
$$0 = -30\tfrac{m}{s} \cdot v'_2 \cos(-50°) + 9v_2'^2 + 3v_2'^2$$
$$0 = -30\tfrac{m}{s} \cdot v'_2 \cos(-50°) + 12v_2'^2$$
$$0 = -30\tfrac{m}{s} \cdot \cos(-50°) + 12v'_2$$
$$v'_2 = 1.607\tfrac{m}{s}$$

With this answer $v_{1x}'^2$ can be found

$$\begin{aligned} v'_{1x} &= 5\tfrac{m}{s} - 3v'_2\cos(-50°) \\ &= 1.901\tfrac{m}{s} \end{aligned}$$

and v'^2_{1y} can be found

$$\begin{aligned} v'_{1y} &= -3v'_2\sin(-50°) \\ &= 3.693\tfrac{m}{s} \end{aligned}$$

The speed is therefore

$$\begin{aligned} v'_1 &= \sqrt{v'^2_{1x} + v'^2_{1y}} \\ &= \sqrt{\left(1.901\tfrac{m}{s}\right)^2 + \left(3.693\tfrac{m}{s}\right)^2} \\ &= 4.154\tfrac{m}{s} \end{aligned}$$

And its direction is

$$\begin{aligned} \theta &= \arctan\frac{v_y}{v_x} \\ &= \arctan\frac{3.693\tfrac{m}{s}}{1.901\tfrac{m}{s}} \\ &= 62.8° \end{aligned}$$

The two answers are therefore

$$\begin{aligned} v'_1 &= 4.154\tfrac{m}{s} \text{ at } 62.8° \\ v'_2 &= 1.607\tfrac{m}{s} \end{aligned}$$

36. a) An x-axis towards the right and a y-axis upwards is used here. In the absence of external forces, the momentum is conserved. We thus have, for the x-component

$$\begin{aligned} p_{tot\,x} &= p'_{tot\,x} \\ 4kg\cdot 8\tfrac{m}{s} + 6kg\cdot 0\tfrac{m}{s} &= 4kg\cdot 6\tfrac{m}{s}\cos(30°) + 6kg\cdot v'_{2x} \\ 32\tfrac{kgm}{s} &= 20.785\tfrac{kgm}{s} + 6kg\cdot v'_{2x} \\ v'_{2x} &= 1.869\tfrac{m}{s} \end{aligned}$$

For the y-component, we have

$$p_{tot\ y} = p'_{tot\ y}$$
$$4kg \cdot 0\tfrac{m}{s} + 6kg \cdot 0\tfrac{m}{s} = 4kg \cdot 6\tfrac{m}{s}\sin(30°) + 6kg \cdot v'_{2y}$$
$$0\tfrac{kgm}{s} = 12\tfrac{kgm}{s} + 6kg \cdot v'_{2y}$$
$$v'_{2y} = -2\tfrac{m}{s}$$

The speed is therefore

$$\begin{aligned} v'_2 &= \sqrt{v'^2_{2x} + v'^2_{2y}} \\ &= \sqrt{\left(1,869\tfrac{m}{s}\right)^2 + \left(2\tfrac{m}{s}\right)^2} \\ &= 2,738\tfrac{m}{s} \end{aligned}$$

And its direction is

$$\begin{aligned} \theta &= \arctan\frac{v_y}{v_x} \\ &= \arctan\frac{-2\tfrac{m}{s}}{1,869\tfrac{m}{s}} \\ &= -46,9° \end{aligned}$$

b) The kinetic energy before the collision is

$$E_{k\ tot} = \tfrac{1}{2}4kg \cdot \left(8\tfrac{m}{s}\right)^2 + \tfrac{1}{2}6kg \cdot \left(0\tfrac{m}{s}\right)^2 = 128J$$

The kinetic energy after the collision is

$$E'_{k\ tot} = \tfrac{1}{2}4kg \cdot \left(6\tfrac{m}{s}\right)^2 + \tfrac{1}{2}6kg \cdot \left(2.738\tfrac{m}{s}\right)^2 = 94.48J$$

The energy loss is therefore

$$\Delta E_k = 94,48J - 128J = -33,52J$$

c) No, because there is a loss of kinetic energy in the collision.

37. The impulse given by the force of 50 N is

$$\begin{aligned} I_x &= F_x \Delta t \\ &= 50N \cdot 3s \\ &= 150 \tfrac{kgm}{s} \end{aligned}$$

Therefore

$$\begin{aligned} p'_x &= p_x + I_x \\ 30kg \cdot v' &= \left(10kg \cdot 0 \tfrac{m}{s} + 20kg \cdot 0 \tfrac{m}{s}\right) + 150 \tfrac{kgm}{s} \\ 30kg \cdot v' &= 150 \tfrac{kgm}{s} \\ v' &= 5 \tfrac{m}{s} \end{aligned}$$

38. There is a momentum given by gravity during 0.5 s after the collision. This impulse is

$$\begin{aligned} I_y &= F_y \Delta t \\ &= \left(-1.02kg \cdot 9.8 \tfrac{N}{kg}\right) \cdot 0.5s \\ &= -4.998 \tfrac{kgm}{s} \end{aligned}$$

Therefore

$$\begin{aligned} p'_y &= p_y + I_y \\ 1.02kg \cdot v' &= \left(0.02kg \cdot 500 \tfrac{m}{s} + 1kg \cdot 0 \tfrac{m}{s}\right) + -4.998 \tfrac{kgm}{s} \\ 1.02kg \cdot v' &= 5.002 \tfrac{kgm}{s} \\ v' &= 4.904 \tfrac{m}{s} \end{aligned}$$

39. a) The thrust force is

$$\begin{aligned} F_{thrust} &= V_{\exp} R \\ &= 3200 \tfrac{m}{s} \cdot 3400 \tfrac{kg}{s} \\ &= 1.088 \times 10^7 N \end{aligned}$$

b) The speed will be

$$\begin{aligned} v' &= v + v_{exp} \ln \frac{m}{m - RT} \\ &= 0\tfrac{m}{s} + 3200\tfrac{m}{s} \ln \frac{710{,}000kg}{710{,}000kg - 3400\tfrac{kg}{s} \cdot 60s} \\ &= 0\tfrac{m}{s} + 3200\tfrac{m}{s} \ln \frac{710{,}000kg}{506{,}000kg} \\ &= 1084\tfrac{m}{s} \end{aligned}$$

c) The speed will be

$$\begin{aligned} v' &= v + v_{exp} \ln \frac{m}{m - RT} - gt \\ &= 0\tfrac{m}{s} + 3200\tfrac{m}{s} \ln \frac{710{,}000kg}{710{,}000kg - 3400\tfrac{kg}{s} \cdot 30s} - 9.8\tfrac{m}{s^2} \cdot 30s \\ &= 0\tfrac{m}{s} + 3200\tfrac{m}{s} \ln \frac{710{,}000kg}{608{,}000kg} - 294\tfrac{m}{s} \\ &= 202.3\tfrac{m}{s} \end{aligned}$$

40. We have

$$\begin{aligned} v' &= v + v_{exp} \ln \frac{m}{m - RT} \\ 18{,}000\tfrac{m}{s} &= 15{,}000\tfrac{m}{s} + v_{exp} \ln \frac{100{,}000kg}{100{,}000kg - 750\tfrac{kg}{s} \cdot 60s} \\ 3000\tfrac{m}{s} &= v_{exp} \ln \frac{100{,}000kg}{55{,}000kg} \\ v_{exp} &= 5018\tfrac{m}{s} \end{aligned}$$

41. When the spring is compressed at the maximum, the two masses have the same speed. Thus, according to the law of conservation of momentum, we have

$$\begin{aligned} p_{tot\,x} &= p'_{tot\,x} \\ 1kg \cdot 30\tfrac{m}{s} &= 1kg \cdot v' + 2kg \cdot v' \\ v' &= 10\tfrac{m}{s} \end{aligned}$$

In addition, be mechanical energy must be conserved. With two masses and a spring, the mechanical energy is

$$E_{mec} = \frac{1}{2}m_1 v_1^2 + m_1 g y_1 + \frac{1}{2}m_2 v_2^2 + m_2 g y_2 + \frac{1}{2}kx^2$$

Before the collision, the mechanical energy is (using a $y = 0$ on the ground)

$$\begin{aligned} E_{mec} &= \frac{1}{2}m_1 v_1^2 + m_1 g y_1 + \frac{1}{2}m_2 v_2^2 + m_2 g y_2 + \frac{1}{2}kx^2 \\ &= \frac{1}{2}1kg \cdot \left(30\tfrac{m}{s}\right)^2 + 0 + 0 + 0 + 0 \\ &= 450J \end{aligned}$$

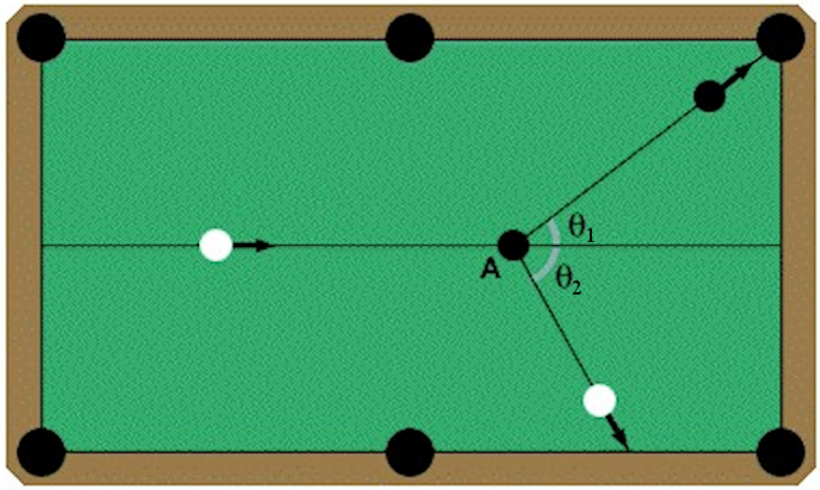
When the spring is compressed at its maximum, the mechanical energy is

$$\begin{aligned} E'_{mec} &= \frac{1}{2}m_1 v_1'^2 + m_1 g y_1' + \frac{1}{2}m_2 v_2'^2 + m_2 g y_2' + \frac{1}{2}kx'^2 \\ &= \frac{1}{2}1kg \cdot \left(10\tfrac{m}{s}\right)^2 + 0 + \frac{1}{2}2kg \cdot \left(10\tfrac{m}{s}\right)^2 + 0 + + \frac{1}{2}kx'^2 \\ &= 50J + 100J + \frac{1}{2}kx'^2 \\ &= 150J + \frac{1}{2}kx'^2 \end{aligned}$$

Then, the conservation of mechanical energy gives

$$\begin{aligned} E_{mec} &= E'_{mec} \\ 450J &= 150J + \frac{1}{2}kx'^2 \\ 300J &= \frac{1}{2} \cdot 9600\tfrac{N}{m} x'^2 \\ x' &= 0.25m \end{aligned}$$

42. The collision is as follow.



The equation of conservation of the *x*-component of the momentum is

$$\begin{aligned} p_x &= p'_x \\ mv_{1x} + \cancel{mv_{2x}} &= mv'_{1x} + mv'_{2x} \\ v_{1x} &= v'_{1x} + v'_{2x} \\ v_1 &= v'_1 \cos\theta_1 + v'_2 \cos\theta_2 \end{aligned}$$

The equation of conservation of the *y*-component of the momentum is

$$\begin{aligned} p_y &= p'_y \\ \cancel{mv_{1y}} + \cancel{mv_{2y}} &= mv'_{1y} + mv'_{2y} \\ 0 &= v'_{1y} + v'_{2y} \\ 0 &= v'_1 \sin\theta_1 - v'_2 \sin\theta_2 \end{aligned}$$

The equation for the conservation of kinetic energy is

$$\begin{aligned} E_k &= E'_k \\ \frac{1}{2}mv_1^2 + \cancel{\frac{1}{2}mv_2^2} &= \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2 \\ v_1^2 &= v_1'^2 + v_2'^2 \end{aligned}$$

First, θ_2 is eliminated with $\cos^2\theta_2 + \sin^2\theta_2 = 1$. The cosine comes from the conservation of the *x*-component of the momentum

$$v'_2 \cos\theta_2 = v_1 - v'_1 \cos\theta_1$$

The sine comes from the conservation of the *y*-component of the momentum

$$v_2' \sin\theta_2 = v_1' \sin\theta_1$$

Then

$$\begin{aligned}
\left(v_2' \cos\theta_2\right)^2 + \left(v_2' \sin\theta_2\right)^2 &= \left(v_1 - v_1' \cos\theta_1\right)^2 + \left(v_1' \sin\theta_1\right)^2 \\
v_2'^2 \cos^2\theta_2 + v_2'^2 \sin^2\theta_2 &= v_1^2 - 2v_1 v_1' \cos\theta_1 + v_1'^2 \cos^2\theta_1 + v_1'^2 \sin^2\theta_1 \\
v_2'^2 \left(\cos^2\theta_2 + \sin^2\theta_2\right) &= v_1^2 - 2v_1 v_1' \cos\theta_1 + v_1'^2 \left(\cos^2\theta_1 + \sin^2\theta_1\right) \\
v_2'^2 &= v_1^2 - 2v_1 v_1' \cos\theta_1 + v_1'^2
\end{aligned}$$

Using the conservation of energy formula $v_1^2 = v_1'^2 + v_2'^2$, the equation becomes

$$\begin{aligned}
v_2'^2 &= v_1^2 - 2v_1 v_1' \cos\theta_1 + v_1'^2 \\
v_1^2 - v_1'^2 &= v_1^2 - 2v_1 v_1' \cos\theta_1 + v_1'^2 \\
-v_1'^2 &= -2v_1 v_1' \cos\theta_1 + v_1'^2 \\
-2v_1'^2 &= -2v_1 v_1' \cos\theta_1 \\
v_1' &= v_1 \cos\theta_1
\end{aligned}$$

Then v_2' is easily found with $v_1^2 = v_1'^2 + v_2'^2$.

$$\begin{aligned}
v_2'^2 &= v_1^2 - v_1'^2 \\
v_2'^2 &= v_1^2 - v_1^2 \cos^2\theta_1 \\
v_2'^2 &= v_1^2 \left(1 - \cos^2\theta_1\right) \\
v_2'^2 &= v_1^2 \sin^2\theta_1 \\
v_2' &= v_1 \sin\theta_1
\end{aligned}$$

Finally, θ_2 is found with $0 = v_1' \sin\theta_1 - v_2' \sin\theta_2$.

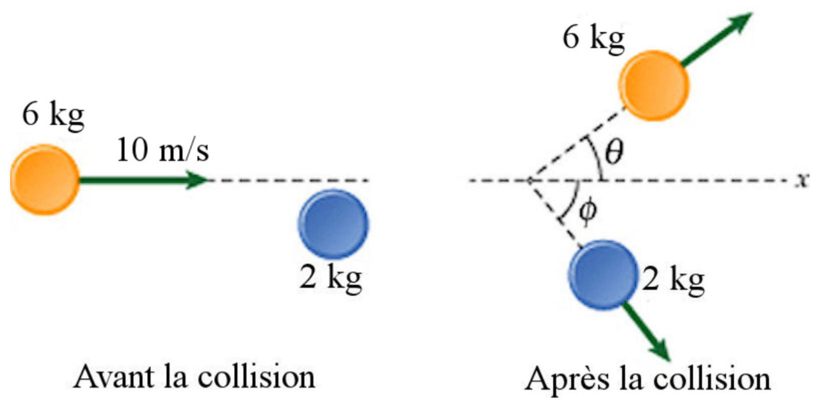
$$\begin{aligned}
\sin\theta_2 &= \frac{v_1' \sin\theta_1}{v_2'} \\
\sin\theta_2 &= \frac{\left(v_1 \cos\theta_1\right) \sin\theta_1}{v_1 \sin\theta_1} \\
\sin\theta_2 &= \cos\theta_1
\end{aligned}$$

Since $\cos\theta = \sin(90° - \theta)$, this equation becomes

$$\sin\theta_2 = \cos\theta_1$$
$$\sin\theta_2 = \sin(90° - \theta_1)$$
$$\theta_2 = 90° - \theta_1$$
$$\theta_1 + \theta_2 = 90°$$

Since the sum of the angles is 90°, the trajectories are perpendicular.

43. The collision is as follow



The equation of conservation of the *x*-component of the momentum is

$$p_x = p'_x$$
$$m_1 v_{1x} + \cancel{m_2 v_{2x}} = m_1 v'_{1x} + m_2 v'_{2x}$$
$$m_1 v_{1x} = m_1 v'_{1x} + m_2 v'_{2x}$$
$$m_1 v_{1x} = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$$
$$60 \tfrac{kgm}{s} = 6kg \cdot v'_1 \cos\theta_1 + 2kg \cdot v'_2 \cos\theta_2$$
$$30 \tfrac{m}{s} = 3 \cdot v'_1 \cos\theta_1 + v'_2 \cos\theta_2$$

The equation of conservation of the *y*-component of the momentum is

$$p_y = p'_y$$
$$\cancel{m_1 v_{1y}} + \cancel{m_2 v_{2y}} = m_1 v'_{1y} + m_2 v'_{2y}$$
$$0 = m_1 v'_{1y} + m_2 v'_{2y}$$
$$0 = m_1 v'_1 \sin\theta_1 - m_2 v'_2 \sin\theta_2$$
$$0 = 6kg \cdot v'_1 \sin\theta_1 - 2kg \cdot v'_2 \sin\theta_2$$
$$0 = 3 \cdot v'_1 \sin\theta_1 - v'_2 \sin\theta_2$$

The equation for the conservation of kinetic energy is

$$E_k = E'_k$$
$$\frac{1}{2} m_1 v_1^2 + \cancel{\frac{1}{2} m_2 v_2^2} = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_1 v_2'^2$$
$$m_1 v_1^2 = m_1 v_1'^2 + m_2 v_2'^2$$
$$600 \tfrac{kgm^2}{s^2} = 6kg \cdot v_1'^2 + 2kg \cdot v_2'^2$$
$$300 \tfrac{m^2}{s^2} = 3 \cdot v_1'^2 + v_2'^2$$

First, θ_2 is eliminated with $\cos^2\theta_2 + \sin^2\theta_2 = 1$. The cosine comes from the conservation of the *x*-component of the momentum

$$v'_2 \cos\theta_2 = 30 \tfrac{m}{s} - 3 \cdot v'_1 \cos\theta_1$$

The sine comes from the conservation of the *y*-component of the momentum

$$v'_2 \sin\theta_2 = 3 \cdot v'_1 \sin\theta_1$$

Thus

$$\left(v'_2 \cos\theta_2\right)^2 + \left(v'_2 \sin\theta_2\right)^2 = \left(30 \tfrac{m}{s} - 3v'_1 \cos\theta_1\right)^2 + \left(3v'_1 \sin\theta_1\right)^2$$
$$v_2'^2 \cos^2\theta_2 + v_2'^2 \sin^2\theta_2 = 900 \tfrac{m^2}{s^2} - 180 \tfrac{m}{s} v'_1 \cos\theta_1 + 9v_1'^2 \cos^2\theta_1 + 9v_1'^2 \sin^2\theta_1$$
$$v_2'^2 \left(\cos^2\theta_2 + \sin^2\theta_2\right) = 900 \tfrac{m^2}{s^2} - 180 \tfrac{m}{s} v'_1 \cos\theta_1 + 9v_1'^2 \left(\cos^2\theta_1 + \sin^2\theta_1\right)$$
$$v_2'^2 = 900 \tfrac{m^2}{s^2} - 180 \tfrac{m}{s} v'_1 \cos\theta_1 + 9v_1'^2$$

Using the conservation of energy formula $300 \tfrac{m^2}{s^2} = 3 \cdot v_1'^2 + v_2'^2$, the equation becomes

$$v_2'^2 = 900\tfrac{m^2}{s^2} - 180\tfrac{m}{s} v_1' \cos\theta_1 + 9v_1'^2$$
$$300\tfrac{m^2}{s^2} - 3\cdot v_1'^2 = 900\tfrac{m^2}{s^2} - 180\tfrac{m}{s} v_1' \cos\theta_1 + 9v_1'^2$$
$$-3\cdot v_1'^2 = 600\tfrac{m^2}{s^2} - 180\tfrac{m}{s} v_1' \cos\theta_1 + 9v_1'^2$$
$$-v_1'^2 = 200\tfrac{m^2}{s^2} - 60\tfrac{m}{s} v_1' \cos\theta_1 + 3v_1'^2$$
$$0 = 200\tfrac{m^2}{s^2} - 60\tfrac{m}{s} v_1' \cos\theta_1 + 4v_1'^2$$

Solving for the cosine, the result is

$$0 = 200\tfrac{m^2}{s^2} - 60\tfrac{m}{s} v_1' \cos\theta_1 + 4v_1'^2$$
$$60\tfrac{m}{s} v_1' \cos\theta_1 = 200\tfrac{m^2}{s^2} + 4v_1'^2$$
$$\cos\theta_1 = \frac{10\tfrac{m}{s}}{3}\frac{1}{v_1'} + \frac{1}{15\tfrac{m}{s}} v_1'$$

The value of v'_1 can vary depending on how the collision occurs. For a specific value of v'_1 there's a maximum angle. Since the derivative of a function is zero at an extremum, the maximum value of the angle is found with

$$\frac{d\left(\cos\theta_1\right)}{dv_1'} = 0$$

Thus, at the maximum angle

$$\frac{d\left(\cos\theta_1\right)}{dv_1'} = 0$$
$$\frac{-10\tfrac{m}{s}}{3}\frac{1}{v_1'^2} + \frac{1}{15\tfrac{m}{s}} = 0$$
$$\frac{1}{15\tfrac{m}{s}} = \frac{10\tfrac{m}{s}}{3}\frac{1}{v_1'^2}$$
$$v_1'^2 = 50\tfrac{m^2}{s^2}$$

Therefore, the angle is

$$\cos\theta_1 = \frac{10\frac{m}{s}}{3}\frac{1}{v_1'} + \frac{1}{15\frac{m}{s}}v_1'$$

$$\cos\theta_1 = \frac{10\frac{m}{s}}{3}\sqrt{\frac{1}{50\frac{m^2}{s^2}}} + \frac{1}{15\frac{m}{s}}\sqrt{50\frac{m^2}{s^2}}$$

$$\cos\theta_1 = 0,94281$$

$$\theta_1 = 19,47°$$